Priority and Particle Physics: structure, dependence, and moderation in all things

Kerry McKenzie

ph07km@leeds.ac.uk

My talk

1. Why structuralism needs dependence
2. Fine's theory of essential dependence
3. The priority of structure 1: quantum objects and entanglement relations
4. The priority of structure 2: elementary particles and group structure

- In each case, while the dependence of objects on structures can be established without difficulty, the question of reciprocated dependence is left hanging.
- This has nothing to do with any unclarity in our understanding of dependence, but rests with our failure to fully articulate what it is that we mean by 'structure' in the physics context.

The Motivations for Structuralism

- Theory change
- Quantum mechanics, especially entanglement
- 'Century of Symmetry' in particle physics: prediction of particles via assumptions about the structure of fundamental equations

The Structuralist Intuition

- A recommended *reconceptualization* of fundamental physical objects in structural terms;
- A claim about the *ontological priority* of structure over objects.

  Ladyman has characterized a structuralist thesis as "any ontological or metaphysical thesis that inflates the ontological priority of structure and relations" (Stanford)

  $\Rightarrow$ Structures are *not secondary* to objects. (The 'core claim'.)

Two structuralist positions:

- 'Radical' structuralism: structures have one-way priority over objects (French, Ladyman)
- 'Moderate' structuralism: structures and objects are 'ontologically on a par' (Esfeld, Lam, Eddington)

Analyzing Priority

In terms of *supervenience*:

> "OSR is the view that the world has an objective modal structure that is ontologically fundamental, in the sense of *not supervening* on the intrinsic properties of a set of individuals." (Ladyman and Ross [2007], p130.)

In terms of *dependence*:

> "I shall take it that a core feature of OSR is the claim that putative objects are *dependent* in some manner upon the relevant relations (and hence these putative objects can be reconceptualized as mere nodes in the relevant structure)."(French [2010], p104)

- We should be clear that these are different, non-coextensive relations.

Supervenience Vs Dependence

- I will make no *a priori* stipulations about the logical form of either relation.

Supervenience:

- clear and well-understood...
- but not explanatory.

Dependence:

- has deep connections with explanation...
- but has not been viewed as sufficiently clear.

Supervenience Vs Dependence

- Structuralism is not just a thesis about the priority of structure over objects. It is also an invocation for us to *reconceptualize* objects in structural terms.
- Ideally then, attributions of priority "should be appropirately tied to the nature of the dependent item" (Fine [1995] p272). But this is the starting point for the essentialst analysis of dependence (cf. Fine's).

  $\Rightarrow$ It is *dependence* that structuralism should use to cash out its priority claims.

Introducing Fine's Analysis: Essential Dependence

For Fine, purely modal analyses of ontological dependence are doomed to failure: instead,

> "The necessity of the conditional x exists only if y does should be appropriately tied to the nature of the dependent item x." (Fine [1995], p272)

By 'nature', Fine means something close to what has traditionally meant *essence*.
However,

> "essentialism has not typically been viewed all that favourably in the context of modern physics" (French [2010], p106).

- Is talk of essence *inappropriate* in fundamental physics?
- Should we speak only of *identity*?

Introducing Fine's Analysis: Essential Dependence

> "The conception of essence Fine has in mind is a traditional conception according to which what is essential to an object *pertains to what the object is*, or *defines the object* (at least in part)" (Correia [2008], p1018).

The properties which we may take to feature in a fundamental particle's essence are

- Its fundamental, determinate, state-independent properties
- (Some of the) properties involved in conferring distinctness from other members of its kind.

Fine's Analysis: Essential Dependence

$\Box_x$='it is true in virtue of the identity of x that'

$$\Box_x \phi(x) = \text{`}\phi \text{ is an essential property of } x\text{'}$$

> "I accept that if an object essentially has a certain property then it is necessary that it has that property (or has the property if it exists); but I reject the converse" (Fine [1994], p4)

$$\Box_x \phi(x) \rightarrow \Box(Ex \rightarrow \phi(x)) \tag{1}$$

According to Fine, the conditionals on the RHS are "not necessary simpliciter" but "are true in virtue of the identity of the objects in question" (ibid. p7); hence we may strengthen (1) to

$$\Box_x \phi(x) \rightarrow \Box_x(Ex \rightarrow \phi(x)) \tag{2}$$

Call (2) the 'basic schema'.

Fine's Analysis: Ontological Dependence

We know that, for Fine, "ontological dependence should be tied to the nature of the dependent entity".
This we can express with

$$\Box_x(Ex \rightarrow Ey) \tag{3}$$

Generalization of the basic schema to two objects:

$$\Box_{x,y}\psi(x,y) \rightarrow \Box_{x,y}(Ex\&Ey \rightarrow \psi(x,y)) \tag{4}$$

and the analogous statement of the ontological dependence of x and y on some z:

$$\Box_{x,y}(Ex\&Ey \rightarrow Ez) \tag{5}$$

Fine's Analysis: Consequential Essence

"A property belongs to the *constitutive* essence of an object if it is not had in virtue of being a logical consequence of some more basic essential properties; and a property might be said to belong to the *consequential* essence of an object if it is a logical consequence of properties that belong to the constitutive essence... Thus a property of containing Socrates as a member will presumably be part of the constitutive essence of singleton Socrates, whereas the property of containing some member or other will presumably only be part of its consequential essence." (Fine [1995], p276).

- A further test: "The proposal is... that x depends upon y just in case y cannot be 'generalized out' of the consequentialist essence of x, or, in other words, just in case some proposition $P(y)$ belongs to the essence [of x] without its generalization belonging to the essence." (ibid., p278).

The Priority of Structure 1: Entangled Quantum Objects

- Principle of the Indiscernibility of Identicals:

  If $x = y$, then

  for all monadic properties P, if Px then Py;

  and for all two-place relations R, then for all z, if Rxz then Ryz, and if Rzx then Rzy;

  and so on for n-ary relations and appropriate permutations.

- Principle of the Identity of Indiscernibles ('PII'):

  If, for all monadic properties P, Px iff Py; and

  for all two-place relations R, and for all z, Rxz iff Ryz, and Rzx iff Rzy; ...

  then $x = y$.

The Priority of Structure 1: Entangled Quantum Objects

Both of these principles may be regarded as having an image in modern logic.

Principle of the Indiscernibility of Identicals - gives the essentials of the Hilbert-Bernays analysis of identity in first-order predicate logic.

Principle of the Identity of Indiscernibles - may be argued to follow from the Hilbert-Bernays analysis, modulo some observations about the *completeness* of the predicate calculus. (See Saunders [2003] for details.)

The Priority of Structure 1: Entangled Quantum Objects

Take two particles of the same kind - two electrons in a helium atom.

- The particles will be in an *entangled state*.
- Since they're of the same kind, they are alike in all their (perfectly natural) monadic properties.
- Since they're entangled, all the relations they stand in are symmetric.
- Whatever we can say about the one we can say about the other (cf. Max Black's two spheres): how, then, do we individuate them? In what sense may it really be said that there are two?

The Priority of Structure 1: Entangled Quantum Objects

- Solution: though our objects satisfy only symmetric relations, so long as they satify at least one that is also *irreflexive*, they will be secured as distinct.
- We have, as part of the Hilbert-Bernays analysis / Principle of Indiscernibility of Identicals that

  If $x = y$, then for all two-place relations R, and for all z, if Rxz then Ryz, and if Rzx then Rzy;

  but if R is irreflexive, this is *false* under the assignment of x or y to z.

- In the two-spheres case, the relation of *being 3m apart from* is irreflexive;
- In the case of entangled electrons, we have relations like

$$\frac{1}{\sqrt{2}}(\psi_x(\uparrow)\psi_y(\downarrow) - \psi_x(\downarrow)\psi_y(\uparrow)) \tag{6}$$

Securing the Priority of Structure 1

- QM guarantees the presence of an irreflexive relation between entangled objects.
- From the Hilbert-Bernays analysis, we know that

$$E(R : R^{irref}(x, y)) \rightarrow x \neq y$$

Securing the Priority of Structure 1

A theorem relating essence and identity (Fine [1995b]):

$$x \neq y \rightarrow \Box_{x,y} x \neq y \tag{7}$$

(Contrast with:

$$x = y \rightarrow \Box_x x = y \tag{8}$$

> "Whereas a true identity $x = y$ depends upon the nature of the one object x, a true non-identity depends upon the nature of both objects." (Fine, [1995b], p256).)

- The relation of *being distinct from* holds of x and y essentially.
- Whatever can be deduced from this relation will belong to the *consequential essence* of x and y, *provided* it can't be universalized.

Securing the Priority of Structure 1

Given that they are intrinsically indistinguishable objects that satisfy only symmetric relations, the PII allows us to deduce that

$$x \neq y \rightarrow E(R : R^{irref}(x, y)) \tag{9}$$

But

$$x \neq y \nrightarrow All(R : R^{irref}(x, y)) \tag{10}$$

Therefore we have

$$\Box_{x,y} E(R : R^{irref}(x, y)). \tag{11}$$

Securing the Priority of Structure 1

Return to the 'basic schema' extended to two objects:

$$\Box_{x,y}\psi(x,y) \rightarrow \Box_{x,y}(Ex\&Ey \rightarrow \psi(x,y)) \tag{4}$$

Substituting we get

$$\Box_{x,y}E(R : R^{irref}(x,y) \rightarrow \Box_{x,y}(Ex\&Ey \rightarrow E(R : R^{irref}(x,y)) \tag{12}$$

$$\therefore \Box_{x,y}((Ex\&Ey) \rightarrow E(R : R^{irref}(x,y)) \tag{13}$$

- The dependence of objects on relations, and hence on structures, is in this way established.

Radical or Moderate Structuralism?

Whether there is *reciprocated* dependence of structures on objects depends on what we take the *nature* of relations to be.

- If we interpret relations extensionally, then given the identity criteria for relations, we will obtain a reciprocated dependence.
- The radical structuralist must adopt an intensional construal:

  "We eschew an extensional understanding of relations... According to Zahar, the continuity in science is in the intension, not the extension, of its concepts..." (Ladyman and Ross [2007], p128
- But if we conceive of them intensionally, we don't *know* what their identity conditions are.
  $\Rightarrow$ An impasse as things stand.

The Priority of Structure 2: Group Structure and Elementary Particles

- *Equations* are intuitively structural: they describe relations between (determinate values of) properties and relations.
- Physicists are often interested in the *group structure* associated with their equations.
- Group structure encodes the *symmetries* of the equation: that is, the operations one can perform on the equation that leave the equation's form intact.

The Priority of Structure 2: Group Structure and Elementary Particles

- Example: the special theory of relativity. Suppose T is the operation such that $x_2 = Tx_1$. Then the statement that the laws of physics are the same for all observers becomes

$$\mathcal{L}(x_1) = \mathcal{L}(Tx_1) = \mathcal{L}(x_2)$$

  where '$\mathcal{L}$' denotes the form of the law.
- The set of these transformations $\{T_i\}$ form a *group* - the Poincare group.

Group Structure and Elementary Particles

- We can use the group structure associated with a law to construct the sorts of particles that will behave in accordance with that law.
- The key insight: the states that a particle can be in must transform into one another in exactly the same way that we transform between observers.
- That means that we should conceive of a relativistic particle as a set of possible states - a *vector space* - where the structure of that vector space is defined in terms of the transformations T_i.
- Such spaces are called *representations*.

Particles as Representations

An enormously important consequence of viewing particles as representations is that we can use the underlying group theory to deduce that while many properties of the particle change, some never do. We either get:

- particles with mass>0 and spin $= 0, 1/2, 1...$, or
- particles with mass$=0$ and spin $= 0, 1, 2....$

Each *representation* may be said to possess a determinate value for each property.

The known particles all fit into this scheme.

Since these properties can never change, it seems natural to call them *essential.*

Particles as Representations

- There has been a *re-conceptualization* of fundamental entities in physics in light of this.

  "Ever since the fundamental paper of Wigner on the [irreducible] representations of the Poincare group, it has been a (perhaps implicit) definition in physics that an elementary particle 'is' an [irreducible] representation of the group, G, of symmetries of nature" (Ne'eman and Sternberg, 1991)

- If elementary entities are those "whose existence and features have no further explanation," *are* there any such things?

The Priority of Structure 2: Group Structure and Elementary Particles

This reconceptualization of particles as representations has given rise to novel priority claims.

- "The group structure is primary and the group representations constructed from this structure have a mere derivative status." (Lyre [2004])

Likewise, the representations have been said to have

- "a lower ontological status" relative to the group structure (Kantorovich [2009]).

- We can sharpen up these priority claims, just as we did before.

Securing the Priority of Structure 2

- If what it is to be an elementary particle is *defined* in terms of it being a representation of the Poincare group, then that forms part of its essence.
- One can deduce the existence of the Poincare group from this essential property.
- But there is no corresponding *universalized* claim that one can deduce.
- It is therefore part of the consequential essence of elementary particles that the Poincare group exists.
- The ontological dependence of particles on this group can then be deduced via the basic schema as before.

Radical or Moderate Group-Structuralism?

Whether there is any parallel dependence of groups on representations depends on how we interpret group structure as a *physical*, not merely mathematical, concept.

- Isn't it at least *necessary* that we can produce a group's representations for it to qualify as realized in nature?
- Shouldn't the idea that structures are '*causal*' be framed in terms of its representations?

Conclusions

- There are good reasons for structuralists to adopt dependence-based accounts of priority.
- Fine's account secured the core structuralist claim in two prominent structuarlist contexts without any trouble, but we found ourselves hamstrung when we tried to either establish or deny the dependence of structures on objects.
- This has nothing to do with any shortcomings in our understanding of dependence, only with our failure to articulate fully what it is that we mean by 'dynamical structure' in physics.

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