1. For the Pauli operators,[1] compute in the standard basis:[2]
   (a) The Dirac form of each operator. [3]
   (b) The matrix of each operator.

2. Let $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ be a normalised single qubit state.
   (a) What are the possible outcomes, if the two qubit state $|\Psi_1\rangle = |\psi\rangle \otimes |\psi\rangle$ is measured in the computational basis, and what are their probabilities?
   (b) What are the possible outcomes, if the two qubit state $|\Psi_2\rangle = CNOT|\psi\rangle \otimes |\psi\rangle$ is measured in the computational basis, and what are their probabilities?

3. Let A be any operator that satisfies $A^2 = I$, where I is the identity operator, and let θ be a real parameter. Use the power series for the exponential to show that $e^{iA\theta} = I\cos\theta + iA\sin\theta$.

4. (a) Let U and V be unitary operators. Show that UV and U^2V^3 are both unitary.
   (b) If H and G are Hermitian, is HG always Hermitian?

5. Give a table of values for the following functions $f : B_3 \to B_1$:[4] [5]

$$f_1(x_1x_2x_3) = x_1 \oplus x_2, \qquad f_2(x_1x_2x_3) = x_1x_2, \qquad f_3(x_1x_2x_3) = x_1x_2 \oplus x_2x_3 \oplus x_3x_1.$$

   Which of these functions are balanced?

6. Let $|\psi_1\rangle = |0\rangle$, $|\psi_2\rangle = |1\rangle$ and $|\psi_3\rangle = \frac{|0\rangle - |1\rangle}{\sqrt{2}}$ be single qubit states. A source emits each state with probability $\frac{1}{3}$.
   (a) Find the density operator ρ associated with this source, expressed both in Dirac form and as a matrix in the standard basis.
   (b) If Y is measured on the source, what are the possible outcomes, and what are their corresponding probabilities?

[1] The Pauli operators X, Y and Z on $\mathbb{C}^2$ are defined by

$$\begin{aligned} X|0\rangle &= |1\rangle; & X|1\rangle &= |0\rangle \\ Y|0\rangle &= i|1\rangle; & Y|1\rangle &= -i|0\rangle \\ Z|0\rangle &= |0\rangle; & Z|1\rangle &= -|1\rangle. \end{aligned}$$

[2] The standard basis for single qubit states means $\{|0\rangle, |1\rangle\}$
[3] Dirac form means $\sum_{j,k} a_{jk}|j\rangle\langle k|$
[4] B_3 denotes the set of all 3-bit strings, and $B_1 = \{0, 1\}$.
[5] $\oplus$ means addition modulo 2