Available PhD projects
supervised by Márton Balázs

I have been mostly working with stochastic interacting systems. This is an area of probability theory and stochastic processes where several stochastic components, each rather simple on its own, interact to form a complicated mathematical model with fascinating behaviour. Both areas offered below fall in this general category.

Various phenomena in last passage percolation

Place i.i.d. random weights on the vertices of the planar lattice $\mathbb{Z}^2$, and consider two points in that lattice. The first passage percolation (FPP) problem asks about the paths on $\mathbb{Z}^2$ between these two points that collect the least total weight along the way. Questions include existence, uniqueness, and geometric properties (e.g., fluctuations from the straight line between the two points) of such paths, as well as asymptotics (Law of Large Numbers, fluctuations) of the total weight collected by them. While this model is very natural, even some of the most basic questions seem very hard.

The last passage percolation problem asks the same questions, but for the paths collecting the largest total weight along the way. To make sense of this, one restricts the relative position of the two points to North-East, as well as the possible steps of the paths to either North or East on the lattice. Suddenly more structure is available and many more questions have been answered than in FPP. There are still several things to explore, this is what this topic will be about. We'll use probabilistic arguments, admiration for those will be useful. For taster, a few papers I wrote with coauthors in the topics are [1, 2, 3].

Fluctuations in interacting particle systems

In this field particles are placed on the sites of the integer line $\mathbb{Z}$, and a stochastic dynamics is run on the resulting configurations. The main feature, which makes these models both interesting and difficult at the same time, is that the particles influence each other, hence the word "interacting" in the title. Under rather general assumptions the models exhibit non-conventional scaling properties: rather than square-root scaling and Normal distributional limits, one often finds one-third power of scaling and other limit distributions. This has been proved for a handful of models and not yet proved for many others in the area. This project will concentrate on using probabilistic arguments to prove sharper and/or more general results than available on such exotic scalings. Some of my papers with coauthors in the topics are [4, 5, 6, 7]. (I know, these appeared a while ago but I have some new ideas to start on!)

References

[1] M. Balázs, E. Cator, and T. Seppäläinen. Cube root fluctuations for the corner growth model associated to the exclusion process. *Electr. J. Prob.*, 11:no. 42, 1094–1132 (electronic), 2006.

[2] M. Balázs, O. Busani, and T. Seppäläinen. Non-existence of bi-infinite geodesics in the exponential corner growth model. *Forum of Math., Sigma*, 8:e46, 2020.

[3] M. Balázs, O. Busani, and T. Seppäläinen. Local stationarity of exponential last passage percolation. *Pr. Th. Rel. Fields*, 180:113–162, 2021.

[4] M. Balázs and J. Komjáthy. Order of current variance and diffusivity in the rate one totally asymmetric zero range process. *J. Stat. Phys.*, 133(1):59–78, 2008.

[5] M. Balázs and T. Seppäläinen. Order of current variance and diffusivity in the asymmetric simple exclusion process. *Ann. Math.*, 171(2):1237–1265, 2010.

[6] M. Balázs, J. Komjáthy, and T. Seppäläinen. Fluctuation bounds in the exponential bricklayers process. *J. Stat. Phys.*, 147(1):35–62, 2012.

[7] M. Balázs, J. Komjáthy, and T. Seppäläinen. Microscopic concavity and fluctuation bounds in a class of deposition processes. *Ann. Inst. H. Poincaré Pr. Stat.*, 48(1):151–187, 2012.