

Modelling Exercises - Summer 2019 Edition

What are these resources? These exercises are based on the experience of the Engineering Mathematics Department at the University of Bristol. We have taught over 40 generations of students the art and practice of *ab initio* real-world problem solving, using mathematics — that is, mathematical modelling — at degree level. You can find out more about our degree programmes at the web address below.

Our core experience is that the best way to teach students to solve open-ended problems is by enabling them to investigate; allow them to think freely without fear of getting the "wrong" answer. We find it best if students are allowed to do this in pairs or small groups.

Together with the MEI and their involvement in the running the AMSP Programme to support A-level Mathematics in the UK, we have developed these set of mathematical modelling projects that are intended to be used in the school classroom.

The problems assume no more mathematics than already taught at GCSE. In particular no calculus is assumed, nor any knowledge of statistics or mechanics. Any additional formulae are included as part of the problem description. So it is also possible to use these exercises as stretching material for more able GCSE students.

Usage notes It is envisaged that the exercises will be tackled in groups of 2-5 students. Many of the exercises have multiple sheets that should be used consecutively. Copy the front sides of the sheets and then give the first sheet in a set to a group of students. We envisage that on average, the investigation of each sheet should take about 15 minutes; some much longer, especially the more open-ended ones.

Hand out the subsequent sheets only when the group has finished working on the previous one. (Sheets may contain solutions to previous ones and several sheets being handed out at the same time can overwhelm some students).

Many of the questions don't have clear cut answers, and sometimes alternative paths to solutions are possible.

The back side of each sheet shows notes on the solution. These notes are targeted at teachers and should not normally be provided to students.

The role of the teacher is to circulate around the classroom, to be ready to provide the next sheet in a sequence when appropriate. The ethos is not to encourage the students to get the "right" answers, but to encourage them to think, and to enable all group members to contribute.

New for 2019 Based on feedback so far, we have fixed some bugs and typos. We've also indicated a level of difficulty and which topics in the A-level syllabus each problem relates to; See the contents page. New problems this year are also highlighted with the symbol **NEW 2019**

It should be said that each problem is intended to teach the art of mathematical modelling, rather than to be associated with any particular part of the syllabus. Also, many of the 'hard' problems can easily be discussed and tackled using elementary approaches.

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We are constantly working on improving the exercises and definitely welcome feedback. Please contact Alan.champneys@bristol.ac.uk for any suggestions for improvements, or even new problems.

The complete set of exercises can be downloaded from



bristol.ac.uk/engmaths-modelling

Contents

E=Easy, M=Medium, H=Hard

Fermi Estimates

- Moving mount Fuji. E (*estimation, arithmetic*)
- The top notch burger joint. E (*estimation, arithmetic*)
- A Fermi challenge. E/M (*estimation, arithmetic, statistics & data*)

Geometry

- The Mathematics Removal Company. NEW 2019 E/M (*trig & Pythagoras*)
- Rolling a fifty pence piece. NEW 2019 E/M (*Pythagoras, circle theorems*)
- The length of days. NEW 2019 E/H (last part) (*trigonometry, curve sketching*)
- Greek geometry. M (*Pythagoras, similar triangles*)
- Rugby kicking. M/H (*trigonometry, curve sketching, circle theorems*)

Sequences and Series

- Payday loans. E/M (*iteration, indices, compound interest*)
- A secret of bees. M (*iteration, Fibonacci sequence, convergence*)
- The one-sided arch. NEW 2019 M/H (*geometry, iteration, harmonic series*)

Mechanics (no prior mechanics knowledge assumed)

- The weightless girl. M (*unit conversion, simple quadratic equation*)
- Interstellar flight. M (*SUVAT, estimation*)
- Ball bouncing. H (*SUVAT, simultaneous equations*)

Optimisation

- Sports betting. E/M (*algebra, line sketching, inequalities*)
- Simpson's paradox. M (*basic statistics, inequalities*)
- The railway line. M/H (*geometry, algebra, graph sketching*)
- Comparative advantage. M/H (*algebra, linear equations*)

Open-ended investigations

- Watering a sports field. NEW 2019 M (*estimation, circle theorems, tessellation*)
- Car parking. NEW 2019 H (*estimation, tessellation, graph theory*)

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¹apart, of course, for this sentence, and its footnote