Comparative advantage 1

(by Filippo Simini)

The manager of a furniture company that manufactures tables hires you to increase productivity.

Four table legs and one table top are needed to produce one table.
On average, after a day worth of effort one worker produces $l = 10$ legs or $t = 5$ table tops.

Suppose that n out of the N workers of the company are assigned to the production of legs, and the remaining $(N - n)$ to the production of tops.

How many full tables will be produced in one day, on average, as a function of n ?

Solution

Parameters:

l = number of legs produced in one day by one worker, on average.
t = number of tops produced in one day by one worker, on average.
N = total number of workers.

Variables:

n = number of workers assigned to legs.

Relationships:

$N - n$ = number of workers assigned to tops.
ln = total number of legs produced in one day, on average.
$t(N - n)$ = total number of T produced in one day, on average.
$P = \min\{nl/4, (N - n)t\}$ = total number of tables produced in one day, on average.

The last relationship is obtained considering that to produce one full table one top and four legs are needed.
Hence, the number of tables cannot exceed the number of tops and the number of groups of four legs.

Comparative advantage 2

(by Filippo Simini)

To maximise the production, workers should be assigned to the production of legs or tops in order to manufacture 4 legs in the time needed to complete 1 table top.

How many workers should produce legs and how many should produce tops, in order to maximise the production of tables ?

Solution

We have to find the value of n that maximises the total number of tables produced per day.

We use the result of part 1 for the total number of tables produced per day as a function of n:

$$P = \min\{nl/4, (N-n)t\}$$

Intuitively, we should maximise the production by producing legs and tops in the right proportions, i.e. 4 legs per 1 top.
Then the question becomes:
What is the n that solves

$$nl/4 = (N-n)t \quad ?$$

Solving for n yields:

$$n = N4t/(l+4t)$$

If $l = 10$ and $t = 5$, then $n/N = 2/3$ of the workers should produce legs, and $1/3$ tops.

The average number of tables produced in one day is equal to $N5/3$.

Comparative advantage 3

(by Filippo Simini)

We found that the average number of tables produced in one day is equal to $P = N5/3$. Is it possible to further increase the production of tables?

Previously we assumed that the productivity of each worker is exactly equal to the average productivity. This may not be true in general, as it is common to find more and less productive workers.

Suppose that you find out that in the company there are some workers that are faster than the average at producing table tops, and some other workers that are faster at producing legs.
Considering this information, you are able to form two groups:
Group 1 has 2/3 of the workers and they produce 12 legs and 4 tops per day, on average; Group 2 has 1/3 of the workers and they produce 6 legs and 7 tops per day, on average.

Average number of units produced by one worker per day			
	Group 1	Group 2	All
fraction of workers	2/3	1/3	1
legs	12	6	10
tops	4	7	5

How can you use this new information to increase the productivity ?

Solution

Group 1 is more efficient at producing Legs than the overall pool of workers.
Group 2 is more efficient at producing Tops than the overall pool of workers.
Hence, assign Group 1 to the production of legs only, and Group 2 to the production of tops only !

Let's compare the production of tables between the previous solution (Model 1) and this new solution considering the two groups (Model 2).

Model 1: $P_1 = 5/3N$

Model 2: $P_2 = \min\{(N2/3)12/4, (N1/3)7\} = \min\{6/3N, 7/3N\} = 6/3N$.

Since $P_2 = 6/3 > 5/3 = P_1$, productivity of Model 2 is higher than Model 1.

This situation is an example of what economists call *absolute advantage*:

If two countries produce two products of the same quality, but the first country is faster at producing one product and the second country is faster at producing the other product, then they should specialise and trade: each country will produce only the product they make faster and they will trade part of it for the other product.

Comparative advantage 4

(by Filippo Simini)

Suppose instead that you were not able to split workers into two groups such that each group is more efficient than the other at producing one item, tops or legs.
This can happen when there are some workers that are more efficient than others at both tasks, while all other workers are less efficient than the average at both tasks.

Dividing the workers into two groups of equal size according to their efficiency, you are able to form the following two groups:
Group 1 is formed by the most efficient workers that produce 14 legs and 6 tops per day, on average; Group 2 comprises the least efficient workers that produce 6 legs and 4 tops per day, on average.

Average number of units produced by one worker per day			
	Group 1	Group 2	All
fraction of workers	1/2	1/2	1
legs	14	6	10
tops	6	4	5

How should the work be divided among the two groups in order to have balanced production, such that one table top and four legs are produced at the same rate ?

Solution

Let's call $x \in [0,1]$ the fraction of workers of group 1 that are assigned to work on legs, and $(1-x)$ the fraction of workers of group 1 that work on tops.
Let's call $y \in [0,1]$ the fraction of workers of group 2 that are assigned to work on legs, and $(1-y)$ the fraction of workers of group 2 that work on tops.

Let's call $\hat{l}_i$ (t_i) the average number of legs (tops) produced by one worker of group i in one day. To simplify the notation of the following equations, let's call $l_i = \hat{l}_i/4$ the number of groups of 4 legs produced by a worker of group i in one day, on average. In our case, we have $l_1 = 3.5$, $l_2 = 1.5$, $t_1 = 6$, $t_2 = 4$.

The total daily production of 4 legs is $N/2[l_1x + l_2y]$, while the total production of tops per day is $N/2[t_1(1-x) + t_2(1-y)]$.
The overall production of tables is limited by the production of the least efficient component:

$$P = N/2 \min\{l_1x + l_2y, t_1(1-x) + t_2(1-y)\}$$

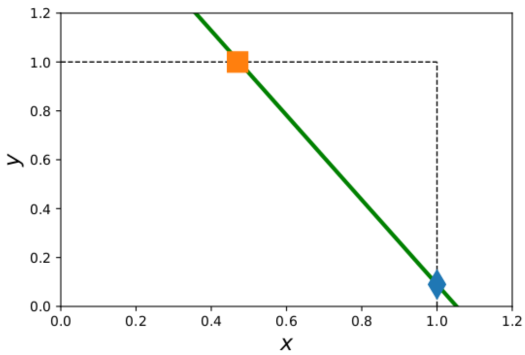
The maximum production can be achieved when 4 legs and 1 top are produced at the same rate, i.e. the production is balanced:

$$l_1x + l_2y = t_1(1-x) + t_2(1-y)$$

Rearranging:

$$y_{eq}(x) = \frac{t_1+t_2}{l_2+t_2} - x\frac{l_1+t_1}{l_2+t_2} \simeq 1.82 - x \cdot 1.73$$

The production is balanced for the (x, y) pairs on the green line:



The square and diamond correspond to the cases where group 2 only produces legs and group 1 only produces tops, respectively: $x = 1; y = 0.09$ and $y = 1; x = 0.47$.

Comparative advantage 5

(by Filippo Simini)

We found that the production is balanced (i.e. tops and 4-legs are produced at the same rate) if x workers of group 1 are assigned to produce legs, and $y_{eq}(x)$ workers of group 2 are assigned to produce legs, where

$$y_{eq}(x) = \frac{t_1 + t_2}{l_2 + t_2} - x\frac{l_1 + t_1}{l_2 + t_2}$$

In this case the productivity per worker is

$$P_3(x) = 1/2\,[l_1 x + l_2 y_{eq}(x)]$$

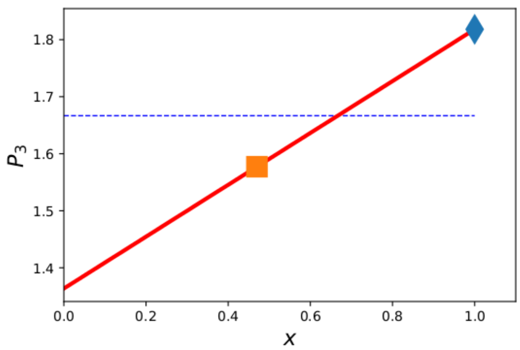
Is it still possible to divide the production among the two groups in order to increase the productivity with respect to the solution found in part 2 ?

Solution

When the production is balanced (along the green line obtained in the solution to part 4), the productivity per worker can be computed as

$$P_3(x) = 1/2[l_1 x + l_2 y_{eq}(x)]$$

and it is shown in the figure below:



The maximum productivity is achieved when $x = 1$, and all workers of group 1 produce legs. In this case the productivity is $P_3(x = 1) = 1.82 > 1.67 = P_1$, which is higher than Model 1 (dashed blue line).

This situation is related to what economists call *comparative advantage*:

Consider two countries that produce two products of the same quality. The first country is more efficient than the other at producing both products, however the second country is relatively more efficient than the first country at producing one of the products. Then they should specialise and trade: each country should produce the product they make more efficiently and they will trade part of it for the other product.