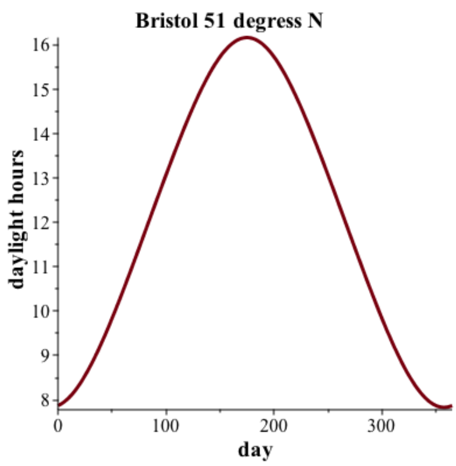
The length of days 1

(by Alan Champneys)

Have you ever thought about what a graph of the number of hours of daylight throughout the year might look like?

Clearly it depends on lattitude. Here is a graph for Bristol which has an angle of lattitude $\phi = 51°$ (north).



Looks like a sine wave, doesn't it? But is it exactly a sine wave? and how would we prove it? Think about what exactly causes the length of the days to change. Discuss in your group.

For the rest of this exercise we are going to try to look test a formula for the daylength anywhere on the Earth.

The length of days 2

(by Alan Champneys)

It turns out that there is a good approximate equation for calculating the length of a day

Sunrise equation

The **sunrise equation** defines an hour angle ω measured in terms of something called the sun's declination angle δ (the apparent angle of the sun to the vertical at noon) and the latitude ϕ.

$$\cos(\omega) = -\tan(\phi)\tan(\delta)$$

where $-180° < \omega < 0$ corresponds to sunrise, and $0 < 180°$ corresponds to sunset.

The sun's declination angle δ can be defined as 23.45° times a sine wave with amplitude 1 and period of 365 days. That is the angle varies sinusoidally between +23.45○ (midsummer) and -23.45° (midwinter) over the course of a year. In the northern hemisphere midsummer is 21st June which, ignoring leap years, is

$$\text{21st June } \equiv d_0 = \qquad\qquad \text{days into the year}$$

But we need to convert days into degrees. Can you therefore find an expression for can the declination angle in terms days d since New Year?

The length of days 3

(by Alan Champneys)

You should get something like

$$\delta \approx 23.45 \cos\left(\frac{360^\circ}{365}[d-172]\right) \text{ or } 23.45 \sin\left(\frac{360^\circ}{365}[d+10]\right)$$

Next, given the sunrise equation

$$\cos(\omega) = -\tan(\phi)\tan(\delta)$$

we need to turn the hour angle ω into a time. The angle is defined as zero at 12 noon and 180° at 12 midnight. Let $\omega > 0$ be the hour angle of sunset. Given that the Earth spins 360° in 24 hours. We can define the sunset and sunrise times in terms of ω as

$$\text{sunrise} = \qquad\qquad\qquad, \quad \text{sunset} =$$

Hence we can now use the sunrise equation to find an experession for the length of a day in terms of the latitude angle ϕ and day number d

$$\text{daylength} = \text{sunset} - \text{sunrise} =$$

Try feeding the numbers for Bristol at different days of the year into this formula to see if you get the right answer.

The length of days 4

(by Alan Champneys)

What happens to the angle ω defined by the sunrise equation, at midsummer when $\phi = 90° - 23.45 = 66.55°$?

What hour does sunrise occur for that location on that day?

Note that $\phi = 66.55°$ precisely defines the arctic circle. What happens to solutions of the sunrise equation at midsummer when $\phi > 66.55°$?

Can you explain this in terms of what you understand happens in summertime inside the arctic circle?

The length of days 5

(by Alan Champneys)

Can you derive the sunrise equation?

$$\cos(\omega) = -\tan(\phi)\tan(\delta)$$

The following diagram may help.