Coordinating Climate Action Under Uncertainty*

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Abstract: This paper studies the strategic interaction between two agents/countries deciding whether to take climate action. Depending on the unobserved state of the environment, modelled as the critical mass of agents needed to take action, successful climate action provides a public good creating free riding incentives. If one agent's action is sufficient to restore the environment actions exhibit strategic substitutes. If the state is critical though, both agents need to coordinate and actions exhibit strategic complements. We extend the global games to environments where agents' actions change between strategic substitutes and complements discontinuously in the underlying parameter. We show that risk-dominant actions can be strictly dominant at signals around the parameter value of such discontinuity, even if they are nowhere strictly dominant in the underlying parameter, and iteratively strictly dominant in the whole range of signals at which they are risk-dominant. We provide conditions on agents' utilities that warrant this outcome. This result applied to climate action implies that for a range of policies that climate action is not strictly dominant in the complete information game, it is so in the incomplete information environment. Taking into account the strategic interactions and incomplete information can offer new tools for policymakers to coordinate climate action. (*JEL* Codes: C72, D83)

Keywords: global games, public good, free riding, strategic complements/substitutes.

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1 Introduction

Despite continuous discussions, increased public awareness and an increase in the measures goverments take, climate change remains one of the most urgent issues of modern societies. Until 2100, we expect an increase of 4°C in global average temperature compared to pre-industrial levels, a scenario of catastrophic estimated consequences. In the absence of a global institution able to monitor and enforce commitments, countries seem reluctant or unable to implement the proposed policies and the switch to renewable energy sources is slow. With the adoption of carbon markets nowhere near the desired level [1] and countries being unlikely to reach the Paris Agreement goal of limiting global warming to well below 2°C, preferably to 1.5°C increase, compared to pre-industrial levels[2], scientists urge for higher coordinated effort.

Economic theory has largely modeled the phenomenon in a prisoner's dilemma strategic context (see bellow for details). There is a growing climate literature[3], press coverage[4] and institutional studies[5] that highlight the coordination nature of the issue. As the planet gets warmer, the payoffs of the cooperative solution seem to increase, due to the enormous costs that a climate disaster implies, while the payoffs from unilateral deviation decrease since even topical mitigation starts having an effect. Lastly the payoffs of what would be the equilibrium of the prisoners dilemma decrease. If everyone free rides forever, a climate catastrophe is sure to occur. This argument implies that as the environment affects the payoffs, there exists a natural threshold such that the agents will want to coordinate their effort. In this case, a climate disaster might occur not due to free riding incentives but rather due to a coordination failure.

Importantly, this type of analysis highlights how the state of the climate can affect the strategic interactions between agents. On the other hand, the climate is a highly non-linear dynamic system with sensitive dependence on initial conditions and exhibits chaotic behavior. Since we can determine initial conditions only with finite precision, the long-term prediction of exact future climate states is not possible.

[1]DICE model finds that the price should be $\$36/ton$ while the temperature-limiting level is at \$100–\$250 per ton. Today we are less than 10% of that price. (Nordhaus (2019))

[2]The latest UNEP (UN Environment Programme) "Emissions Gap Report" found out that all efforts would prevent only 7.5% of greenhouse gas emissions by 2030. To reach the 1.5°C target, however, it would have to be 55%.The models included the updated nationally determined contributions (NDCs) to climate protection in the context of the 2021's UN Climate Change Conference in Glasgow. Even with these new targets, we would have a warming of 2.7°C within this century.

[3]See for example Carattini et al. (2019), DeCanio and Fremstad (2013), Milinski et al. (2008)

[4]Is Climate Change a Prisoner's Dilemma or a Stag Hunt?, The Atlantic (2012).

[5]UN Says Unprecedented Coordination Needed to Tackle Climate and Disaster Risks, UN climate change (2018) and coverage in all COPs since.

Instead we have increasingly accurate probabilistic forecasts. With the state of nature being uncertain, agents in turn face uncertainty on the strategic interaction they face. This paper offers a framework to study these issues in an incomplete environment where incentives may change discontinuously depending on the underlying state of nature. We explore equilibrium in these situations and describe policies that can facilitate coordination in this environment. Importantly, we find that in this environment, information frictions matter. Taking into account the incomplete information nature of the environment, we can offer a larger range of policies that can facilitate coordination than what previous literature suggests.

There is a large economic literature that has explored the incentives that lead economic agents to overuse common pool resources and the similar problem of public goods. At the heart of the analysis is that unilateral deviation from the cooperative action is beneficial to agents, resulting to a prisoner's dilemma strategic interaction. With this in mind economics proposed solutions to ameliorate the issue via taxation (Pigou (1928)), assigning property rights/introducing markets (Coase (1960)) or creating communities able to monitor and punish deviation from the cooperative behaviour (Ostrom (1990)). These works have made distinct contributions to the literature of governance. The common role of the proposed policies, from a game theoretic perspective, is that by internalising the externalities climate change causes, one can change the game's payoffs in a way that cooperation will be the strictly dominant strategy of the game. The proposed solution is very appealing since it describes a way to implement the cooperative solution even in the worst case, when agents have the least incentive to cooperate. On the other hand, the proposed change in utility seems to imply the adoption of measures that would be too costly for other sectors in the economy resulting in slow adoption by countries.

As a result the climate situation has severely worsened and the strategic situation may not still resemble a prisoner's dilemma. As the planet gets warmer we have reached a stage that partial or full coordination of countries is recquired to avoid a climate catastrophe. In this case, the game resembles a pure free riding and ultimately a coordination game depending on the participation needed. Even though this analysis seems to describe the current situation, this is one of the first papers that studies the strategic interactions and potential policies in this environment.

To make ideas more concrete consider a game between the US and China deciding whether they should switch from a fossil fuel based production to a renewable energy production. Since climate action remains costly, even if agents enjoy great benefits from mitigation, they would rather free ride others' action if their participation is not pivotal. On the other hand, as the state of nature worsens, there exists a natural threshold such that an agent's action will have an effect only if others coordinate in acting as well. If for example the prediction for the increase in temperature was 2.5^{o}C (lower than the actual one) then China or the US [6] unilaterally could achieve the goal of 2^{o}C by switching technologies. But both countries however, would have an incentive to free ride the others effort. Under the current prediction though, the Paris agreement goal can be reached only if both countries adopt the policy, opening the door to the possibility of self-fulfilling coordination failures.

This paper studies what policies, interpreted as changes in agents' utilities, can guarantee coordination in this situation. The result crucially depends on information. In a complete information environment, in order to guarantee coordination, a policy maker needs to implement measures that would change utilities up to the point that agents will find in strictly dominant to cooperate. A policy comparable with previous recommendations. In the incomplete information version of the game though, there exists a policy such that coordination can be achieved with a smaller change in utility. This implies that by taking into account these different strategic interactions and the incomplete information, one could propose a less harsh policy, potentially easier to be adopted by countries, that still resolves the coordination issue. This policy might be especially relevant for the most reluctant, to adopt climate policies, countries.

Coordination issues are prominent in many economic and social phenomena. Examples include bank runs, currency attacks, protest participation, and regime changes. These situations have been depicted as coordination games with multiple equilibria, some of which exhibit coordination failure. However, for economic theory to retain predictive power in the face of multiple equilibria, it needs to provide a compelling reason to select a particular equilibrium as the unambiguously right one.

Carlsson and van Damme (1993) were the first to show that the multiplicity of equilibria is not robust to the introduction of a small uncertainty in the payoffs, correlated among players via noisy private signals of some underlying fundamental. Instead the risk dominant action is the one uniquely selected after iterated deletion of strictly dominated strategies. The result depends on agents' actions being strategic complements, payoffs being continuous to the fundamental and on the existence of extreme regions of the fundamental for which agents have a strictly dominant action. With the predictive power thus reinstated, global games have been fruitfully applied to various contexts, such as financial markets and social situations.

There are many situations though in which actions are not always strategic complements, potentially changing discontinuously between complements and substitutes. Coordination to mitigate the effects of climate change exhibits such characteristics. Consider two countries deciding whether to take the costly action of reducing carbon emissions in favor of a cleaner environment. If the environmental damage is moderate

[6] Accounting for 33% and 12% of total carbon emissions (World bank (2019)).

and one country adopting the policy would suffice for restoration, adopting the policy exhibits strategic substitutes. Either country would rather have its opponent act and enjoy the benefits of the clean environment without incurring any cost. If the existing damage is severe, on the other hand, both countries need to coordinate their effort in order to tackle the issue, implying that actions exhibit strategic complements. If there exists some uncertainty about the environmental damage, the actions of adopting a carbon reducing policy are not always strategic complements.

Similar strategic interactions emerge in other collective action situations such as contribution to a public good or protest participations. A defining feature of these situations is that the successful outcome of the coordinated actions is a public good that benefits everyone regardless of whether or not they contributed. This precludes the coordinating actions from being invariably strategic complements due to free-riding incentives, which is a crucial departure from the global games literature hitherto.

From a theoretical standpoint, we depart from the Carlsson and van Damme (1993) framework by assuming that there exists a critical fundamental value where actions discontinuously change from being substitutes to complements. The key observation is that, in the incomplete information version of the game, the risk dominant action can be strictly dominant around the discontinuity, even though it is nowhere strictly dominant in the complete information game. This allows for an iterative process similar to the one in global games to select that action for the entire range of the fundamentals in which it is risk dominant. We examine the conditions on the utilities around the critical value that warrant reverberation of the iterative process throughout the risk-dominant region.

We consider a stylised regime change model, vastly studied in the global games literature, modified to include free riding incentives. Two agents/countries simultaneously decide whether to take the costly action of adopting carbon emission reducing policies or not. The policy will be successful if, given the current environmental damage, enough countries adopt the policy benefiting both agents identically; the policy fails otherwise, with no benefit to either. The cost of adopting the climate policy is interpreted as the result of a global rule. If the rule is harsh, say the carbon price is high due to the way that property rights were assigned, then the cost of adopting the green policy is low and vice versa. We assume that harsher rules and smaller costs are less likely to be adopted and thus we are interested in the largest cost that can accommodate coordination. Importantly, taking climate action and adopting the proposed rule is the same thing in our model.

The state of the environment, which dictates how many agents are needed to adopt the policy, is modelled as the unobservable (underlying) fundamental which is a continuous random variable. However, each agent observes a private signal on the state of the environment with a small random noise, from which they make inferences, before deciding whether to adopt a policy. The signal also allows the agent to make inference about the other agent's signal and their inference on the fundamental. If their signals are below 1 (above 1 but below 2, resp.) their policy adoptions are likely to be strategic substitutes (complements, resp.); if they are well above 2, non-adoption is dominant as there is no chance to succeed.[7]

We start by characterizing a class of equilibria in which action to protect the environment will be taken whenever a success is possible (asymptotically as the noise in the signal vanishes) provided that acting is the risk-dominant action whenever actions are strategic complements. In this equilibrium, referred to as an interval-threshold equilibrium, one agent participates when their signal is below a threshold near 2 and the other when their signal is approximately in the interval $[1, 2]$. Essentially, in this class of equilibria, the participation is implemented whenever the policy can be successful, in particular, avoiding coordination failure when actions feature strategic complements. But for large noise these are not the only equiliria that exist.

The main finding is that the coordination failure is always prevented in any equilibrium, because the risk-dominant action uniquely survives iterated eliminations of strictly dominated strategies so long as the cost of adopting the policy is below a bound which we identify, as noise vanishes. The key observation is that at the borderline signal of 1, where agents' participations are equally likely to be strategic complements and substitutes, an agent is pivotal in succeeding with one half probability whether the other agent participates or not. Consequently, his minimal expected benefit from participating is bounded away from zero however the other agent mixes between participating and not across her possible signals. If participation cost is below this minimal benefit, therefore, participation is strictly dominant for him upon observing a signal in a small neighborhood of 1, even though no action is strictly dominant in the complete information version of the game. This allows for an iterative procedure, similar to the one developed in the global games, to select the risk dominant action for signals in an interval converging to [1,2] as the noise vanishes. This insight about equilibrium selection in discontinuous global games can be generalised for any 2×2 environment as shown by Park and Smyrniotis (2022).

Crucially the costs that will warrant this result, would still imply a coordination game in the complete information environment, and no action would be strictly dominant. This implies that the range of costs we have in mind are strictly larger than the ones that would warrant coordination in the complete information game. By

[7] That a strictly dominant action exists only when the fundamental is large enough, is a further departure from the global games framework where it exists for low enough fundamentals as well.

taking into account the incomplete information nature of the environment, one can propose a policy that is less demanding in terms of changes in utility. To the extend that changes in utility are associated with harsher environmental measures, the proposed policy will be easier to adopt. By taking into account the incompleteness of information, one could propose an easier to adopt measure that yet would guarantee coordination.

The rest of the paper is organised as follows. Section 2 discusses the relevant literature. Section 3 describes the model and the equilibrium concept. Section 4 characterizes interval-threshold equilibria and discusses equilibria when noise is large. Section 5 establishes the iterative dominance of risk-dominant actions in the strategic complements region and discusses equilibria with vanishingly small noise. Section 6 discusses and section 7 concludes.

2 Related Literature

This paper is related to a very large literature that studies the "tragedy of the commons" (Hardin (1968)) and whether economic agents provided with the right incentives can avoid such situations. At the heart of the problem lie the externalities that are present in collective action problems (Samuelson (1954), Bator (1958)). Pigou (1928) was the first to observe that by providing appropriate tax schemes, a designer could reduce these externalities while Coase (1960) showed how by assigning property rights and introducing markets the issue can be ameliorated. The strategic interactions these papers study can be boiled down on an one shot $n-$person prisoner's dilemma strategic situation. Ostrom (1990) extended the analysis by considering dynamic repeated environments and by showing how the introduction of communities able to enforce credible threats could achieve the cooperative solution. This is a very incomplete list of very influential works in the area. Compared to this literature this paper studies a problem of "global commons" where actions do not always exhibit strategic substitutes but also can potentially be complements depending on the underlying state of the environment. In this modified environment, we argue that taking into account the incomplete information nature of the issue can changes the proposed policy.

The theoretical underpinnings of the model are closely related with the global games framework, firstly studied by Carlsson and van Damme (1993). They showed that equilibrium selection in coordination games is possible if we embed complete information games in incomplete information environments. The risk dominant action (Harsanyi et al. (1988)) is the uniquely selected equilibrium. Their framework was later expanded by Morris and Shin (2001) and Morris and Shin (2002) who highlighted the effects of public information in such environments. The framework has been utilized to study coordination issues in many theoretical and applied papers studying a variety of phenomena. Examples include Angeletos et al. (2006) who demonstrated how policy interventions can act as an endogenous signal and reintroduce multiplicity, Angeletos et al. (2007) who studied coordination games in a dynamic environment and many others. This line of literature assumes that actions feature strategic complements and that utilities are continuous to the fundamentals. In our environment actions change discontinuously from strategic complements to substitutes. Moreover, main result does not require the existence of two dominance regions, an assumption commonly made in this framework.

This paper is related to the line of literature that attempts to relax the complements assumption in global games. Karp et al. (2007) were the first to consider a global game with the addition of congestion effects. Their result was later challenged by Hoffmann and Sabarwal (2015) who argued that their existence result was incomplete. Bunsupha and Ahuja (2018) completed their result fully characterizing an equilibrium for this game with infinitely switching strategies. They showed that this equilibrium is unique under any strategy in which the aggregate action is monotone to the state of the fundamentals. Harrison and Jara-Moroni (2021) expand the global games framework to games that feature only strategic substitutes with overlapping dominance regions. Unlike us their payoffs are continuous to the fundamental and they do not consider pure free riding alongside with coordination incentives.

Equilibrium existence issues in games that feature both strategic substitutes and complements are discussed in Hoffmann and Sabarwal (2019a). While uniqueness in such environments is considered in Hoffmann and Sabarwal (2019b). Their result is different from ours since they assume that agents' utility is continuous to the state. Moreover, their uniqueness result relies on a contagion argument starting from a dominance region. If an action is strictly dominant for some realizations of the fundamentals, and if that dominance region is strong enough (they use a p-dominance condition to measure the influence of the dominant region to nearby values of the state), then for realizations of the fundamental close to this dominant region agents will take the same action, allowing an iterative argument to select an equilibrium. Our result is different to this line of literature since it relies on the discontinuity between strategic substitutes and complements in order to establish an iterative process.

Moreover, this paper is related to the literature that employs the global games framework to study collective action problems(Tullock (1971), Olson (1965)). Shadmehr (2018) study a collective action game where the strength of the regime is commonly known while there exists uncertainty on the participation cost of the agents. The decision to act depends on that cost and they characterise a symmetric equilibrium with a cutoff strategy. Their equilibrium is unique if the uncertainty is not too small. Actions can feature either strategic substitutes or complements depending on the commonly known strength of the regime thus both cannot exist in the same model as in our environment. Morris and Shadmehr (2020) study a problem where the uncertainty is about the strength of the regime like us. The benefit that agents receive from a successful collective action however depends on the individual's effort, a continuous variable. Thus actions do not necessarily feature free riding. Their focus is the incentives that a leader needs to provide to heterogeneous agents to induce coordination. Other examples that study different aspects of collective action within this framework include Edmond (2013) who studies information manipulation in regime change movements; Shadmehr and Bernhardt (2011) who study the effects that uncertainty about the alternative regimes can have in the participation decision and others.

Lastly, this paper is subject to Weinstein and Yildiz (2007) critique who demonstrated that the particular departure from the complete information that is assumed in the global games framework is with loss of generality. In their paper, they show that the modelling choice of information can be modified in such a way that any action is uniquely rationalizable. By considering more general perturbations, they were able to recreate the global games result for any action. In a later paper Morris et al. (2016) showed that the particular departure of global games coincides with the epistemic foundation that has players being agnostic about their rank beliefs. That is players do not know whether their type is higher compared to their opponents'. Although we restrict ourselves to a less general class of games by considering the perturbation developed by Carlsson and van Damme, this form of incomplete information is believed to be suitable for the phenomena that this paper considers.

3 Model

Two risk neutral agents denoted by $i \in I = \{1, 2\}$ simultaneously make a binary choice $a_i \in \{0, 1\}$. We refer to $a_i = 1$ as the agent i's choice to "adopt the carbon reducing policy", or simply "act" for short, and $a_i = 0$ as his choice to "not adopt/not act," respectively. The two agents' choices succeed in restoring the environment if the number of agents who act exceeds θ. The random variable $\theta \in (0, 3)$ is realized at the beginning of the game. We interpret θ as the state of the environment which dictates how many agents need to act to restore it. Each agent receives a benefit of $b > 0$ if the environment is restored. Each agent i incurs a cost c_i if they act.

We interpret $c_i = P_{renew} - P_{carbon}$ as the difference between the cost of fossil fuel and renewable energy. Since carbon is more efficient, $P_{renew} > P_{carbon}$, implying that changing to a green technology is costly, $c_i > 0$. On the other hand, if policies are in place this relationship could reverse. We interpret a global policy as a change in P_{carbon} since this is what policies have predominantly focused so far[8]. A large increase in the prices of carbon, even though it implies more incentives for an agent/country to adopt the green technology, may be rejected for other reasons not modelled here. For example there may be political reasons (large body of voters adversely affected) or lobbying from oil producers that would make this policy very hard for governments to implement. A small c_i would imply a larger P_{carbon} and thus a strict policy that we assume to be harder to implement due to exogenous reasons. Moreover, if $c_i \leq 0$ then agents would have a weakly dominant action to act. In this case, switching technologies would be beneficial for the economy. This is the benchmark policy that previous research suggests. Instead we focus on policies that are bounded away from this, $c_i > 0$ and are easier to implement. We ask whether such policies or equivalently costs exist such that coordination is guaranteed. Lastly, notice that participation in a carbon market and adopting the green technology is the same thing in this formulation.

Thus, agent i's utility is

$$u_i(a_i, a_{-i}, \theta) = \begin{cases} b - a_i c_i & \text{if } \; a_1 + a_2 \geq \theta \\ -a_i c_i & \text{otherwise} \end{cases}$$

and assume $b \geq c_i$. It is trivial that agent i would never act if $b < c_i$.

We describe the state of nature as "moderate" if $\theta \leq 1$, "critical" if $1 < \theta \leq 2$, and "irreversible" if $\theta > 2$. If the state is moderate, the two agents' choices to act are strategic substitutes as just one acting is enough to restore the environment, generating free-riding incentives for the agents. If the state is critical, choices to act are strategic complements since both agents need to act to succeed. If the state of nature is irreversible, clearly both agents have a strictly dominant choice to not act because regardless of agents' actions the environment cannot be restored. The description above is common knowledge, as is the information structure on θ explained below.

In the complete information benchmark where the value of θ is common knowledge, multiple equilibria arise due to standard coordination issues (when $\theta \leq 2$). When the state is moderate and agents' actions are strategic substitutes, there are two pure-strategy equilibria depending on who acts and a mixed-strategy equilibrium in which both agents randomize between acting and not. When the state is critical and agents' actions are strategic complements, there exist an equilibrium in which neither agent acts (coordination failure) as well as one in which both act. When $\theta > 2$, there

[8]The analysis would be the same for changes in P_{renew} or both.

is a unique dominant-strategy equilibrium where neither agent acts. Importantly regardless how low the cost of adopting the policy is, all of these equilibria exist. Only policies that would push $c_i > 0$ could guarantee coordination. This implies that in a complete information environment, changing $c_i < 0$ has no effect on agents' strategic interactions and do not facilitate coordination.

We study an incomplete information environment where each agent privately observes a noisy signal of the underlying fundamental θ drawn from a uniform distribution over the interval $(0, 3)$.[9] Specifically, each agent i observes a signal $x_i = \theta + \epsilon_i$ where ϵ_i is an unbiased noise independently and identically distributed according to a cdf F supported on $[-\sigma, \sigma]$, with an associated density function f which is symmetric around and single-peaked at 0. Being interested in the impact of departure from complete information, we assume that the noise is relatively small, in particular, $\sigma \in (0, 1/6)$. With a slight abuse of notation, we denote the cdf of the random variable $\theta + \epsilon_i$ by $F(\cdot|\theta)$ and the density function by $f(\cdot|\theta)$, both with $[\theta - \sigma, \theta + \sigma]$ as their support.

Then, the posterior distribution (cdf) of θ conditional on any signal $x_i \in \mathbb{R}$ is $F(\cdot|x_i)$ is because

$$\frac{\int_{x_i-\sigma}^{\theta} f(x_i|\theta')d\theta'}{\int_{x_i-\sigma}^{x_i+\sigma} f(x_i|\theta')d\theta'} = \frac{\int_{x_i-\sigma}^{\theta} f(x_i-\theta')d\theta'}{\int_{x_i-\sigma}^{x_i+\sigma} f(x_i-\theta')d\theta'} = 1 - F(x_i - \theta) = F(\theta - x_i) = F(\theta|x_i),$$

where the third equality is due to symmetry distribution of noise around 0. That is, upon observing a signal x_i, agent i's posterior belief on θ is also F, centered at $\theta = x_i$ with a support $[x_i - \sigma, x_i + \sigma]$; thus, the posterior distribution $F(\cdot|x_i)$ shifts to the right as x_i increases by the same amount: $F(\theta|x_i) = F(\theta'|x_i')$ if $\theta' - \theta = x_i' - x_i$.

Finally, we assume that the cost of adopting the policy/acting satisfies

$$c_1 + c_2 < 1 \quad \text{and} \quad c_1 \leq c_2.$$

The first inequality ensures that acting is risk dominant for the range of the fundamentals for which agents' choices to act are strategic complements.[10] The second inequality is withought loss. When it strictly holds, it implies that agent 1 has a risk dominant action to act whenever $\theta \leq 1$ and it is not risk dominant for agent 2 to act is this range of signals.

[9] The distribution of θ is inessential for qualitative results so long as it has continuous and strictly positive density on an interval containing $[0, 2]$, but uniform distribution facilitates exposition greatly. Moreover, for the limit results, as noise vanishes any prior would approximate a uniform distribution.

[10] The case where $c_1 + c_2 > 1$ would imply that "not act" would be the risk dominant action. This would trivialise the problem and no agent would act in the strategic complements region in the incomplete information version of the game.

A strategy of agent i is a measurable function $s_i : \mathbb{R} \to [0,1]$ that specifies, contingently on every possible signal $x_i \in \mathbb{R}$, a probability with which agent i chooses to act. Agent i's expected utility from taking $a_i \in \{0,1\}$ upon observing a signal x_i, conditional on the other agent's strategy s_{-i}, is

$$U_i(a_i, s_{-i}, x_i) := \int\int \big[s_{-i}(x_{-i})u_i(a_i,1,\theta)+(1-s_{-i}(x_{-i}))u_i(a_i,0,\theta)\big]\,dF(x_{-i}|\theta)\,dF(\theta|x_i).$$

Let $U_i(\alpha, s_{-i}, x_i) = \alpha\, U_i(1, s_{-i}, x_i) + (1-\alpha)U_i(0, s_{-i}, x_i)$ for $\alpha \in (0,1)$.

Definition 1 A strategy profile (s_1^*, s_2^*) is a *Bayesian Nash equilibrium* (BNE) if

$$U_i(s_i^*(x_i), s_{-i}^*, x_i) \;\geq\; U_i(a_i, s_{-i}^*, x_i) \qquad \forall a_i \in \{0,1\}, \quad \forall x_i \in \mathbb{R}, \quad i = 1,2.$$

4 Interval-threshold equilibrium

We start the analysis with characterising existence of equilibria in the incomplete information environment. Ideally, the two agents would like to coordinate on both acting when $\theta \in (1,2)$ and only one of them acting when $\theta < 1$, but this is infeasible because they observe only noisy signals of θ. Since the noise is small, however, such coordination may be approximated if one agent acts on all signals roughly below 2, and the other agent acts on all signals roughly in the interval $[1,2]$.

We characterize the conditions under which such a strategy profile indeed constitutes a BNE, specifically where one agent i acts below a threshold x_i^* and the other agent $-i$ acts in an interval $[\underline{x}_{-i}, x_{-i}^*]$ where $\max\{x_1^*, x_2^*\} \in (2-\sigma, 2+\sigma)$. We refer to such equilibrium as an *interval-threshold equilibrium.*

Intuitively, upon observing their respective upper threshold signals, the agent with the higher threshold, say i with $x_i^* > x_{-i}^*$, infers that the regime is more likely to be invincible (i.e., $\theta > 2$ is more likely) and also that the other agent is less likely to act, than the other agent $-i$ does upon observing x_{-i}^*. Hence, the agent with the higher upper threshold takes more risk by acting on his upper threshold signal and therefore, his cost of acting should be lower. We start with this observation stated below (and proved in Appendix).

Lemma 1 *In every interval-threshold equilibrium, $x_2^* \leq x_1^*$ where the inequality is strict if and only if $c_1 < c_2$.*

An agent brings a benefit of $b = 1$ to himself by acting when his acting is pivotal in restoring the environment, namely, when either (i) the state is critical (i.e., $1 < \theta < 2$)

and the other agent acts or (ii) the state is moderate (i.e., $\theta < 1$) and the other agent does not. The probability of an agent's action being pivotal is:

$$Pv(x_i) = \text{Prob}(\text{agent } -i \text{ acts}, \theta \in (1,2) \,|\, x_i) + \text{Prob}(\text{agent } -i \text{ not act}, \theta < 1 \,|\, x_i) \ .$$

Hence, conditional on his signal x_i, it is optimal for an agent i to act if the probability that his action is pivotal exceeds his cost of acting $Pv(x_i) > c_i$, not act if $Pv(x_i) < c_i$ and he is indifferent between acting and not if they coincide:

Since the LHS (left-hand side) of (4) is continuous in x_i, (4) holds at each boundary signals x_i^*, x_{-i}^* and $\underline{x}_{-i}$. We first determine the boundary signal levels from this indifference condition, then verify optimality at other signals.

We start with the configuration that agent 1 acts below a threshold x_1^*, called a "threshold-player," and player 2 acts on signals in an interval $[\underline{x}_2, x_2^*]$, called an "interval-player."' Subsequently, we examine the other configuration which is analyzed analogously subject to suitable modifications due to $c_1 \leq c_2$.

Agent 1 acts on all signals below $x_1^* \in (2-\sigma, 2+\sigma)$ in the considered configuration. Observing a signal $x_2 < x_1^* - 2\sigma$, therefore, agent 2 infers that agent 1 will act for sure and thus, that he is pivotal if and only if the state is critical. Since the state must be critical if $x_2 > 1+\sigma$, he should act at signals $x_2 \in (1+\sigma, x_1^* - 2\sigma)$, implying that $\underline{x}_2 < 1+\sigma < x_1^* - 2\sigma < x_2^*$.

Moreover, upon observing $\underline{x}_2$, agent 2 is pivotal with the posterior probability that the state is critical, $1 - F(1|\underline{x}_2)$. Hence, the indifference condition for agent 2 at the lower boundary signal $\underline{x}_2$ simplifies to the first term of (4) being equal to c_2:

$$1 - F(1|\underline{x}_2) \;=\; c_2 \quad \Longrightarrow \quad \underline{x}_2 \in \begin{cases} (1-\sigma, 1] & if \quad c_2 \leq 0.5 \\ (1, 1+\sigma) & if \quad c_2 > 0.5. \end{cases} \tag{1}$$

This equation determines the value of $\underline{x}_2$ uniquely and independently of x_1^* and x_2^*.

To determine the upper threshold levels, note that upon observing their respective upper boundary signal x_i^*, both agents deduce that the state is never moderate (i.e., $\theta > 1$) because $1+\sigma < x_2^*$ as verified above. Hence, either agent is pivotal if and only if the state is critical ($\theta < 2$) and the other agent acts, simplifying the indifference condition at x_i^* to the first term of (4) being equal to c_i:

$$\int_{x_1^*-\sigma}^{2} F(x_2^*|\theta)\, dF(\theta|x_1^*) \;=\; c_1 \quad \text{and} \quad \int_{x_2^*-\sigma}^{2} F(x_1^*|\theta)\, dF(\theta|x_2^*) \;=\; c_2. \tag{2}$$

Here, the integrand $F(x_i^*|\theta)$ is the probability that agent i would act conditional on θ, from the perspective of agent $-i$ upon observing x_{-i}^*. This is clear for agent $i = 1$, the threshold-player, because he is supposed to act at all signals below x_1^*; and so is

$F(x_2^*|\theta)$ because, upon observing x_1^*, agent 1 infers that x_2 is at most 2σ away from $x_1^* > 2-\sigma$, hence $x_2 > 2-3\sigma = 1+3\sigma > \underline{x}_2$. Thus, the upper boundary signals x_1^* and x_2^* are determined as the solution to the two equations in (2), independently of $\underline{x}_2$.

As we show in Appendix, there is a unique solution to (2) and $1+3\sigma < x_2^* < x_1^* \in (2-\sigma, 2+\sigma)$. It is clear that $x_1^*, x_2^* < 2+\sigma$ because if $x_i^* \geq 2+\sigma$ then the state must be irreversible (i.e., $\theta > 2$) and there is no chance to restore the environment. If $x_1^* \leq 2-\sigma$ so that $x_2^* < 2-\sigma$ as well, on the other hand, upon observing their respective upper boundary signal x_i^*, either agent i would infer that the state must be critical and thus that he is pivotal when the other agent observes a signal below x_{-i}^*. The probabilities for the two agents to be pivotal upon observing x_i^* as such are complementary, implying that the LHS of the two equations in (2) add up to 1, but this would contradict the assumption that $c_1 + c_2 < 1$.

We have so far determined the boundary signal levels by (1) and (2) in an equilibrium where agents 1 and 2 adopt a threshold strategy and an interval strategy, respectively. We now verify optimality of these strategies at other signals.

Conditional on agent 1's strategy of acting on all signals below x_1^*, it is straightforward to see that it is optimal for agent 2 to act precisely at signals $x_2 \in [\underline{x}_2, x_2^*]$ because the expected gain from acting is lower at $x_2 < \underline{x}_2$ than at $\underline{x}_2$ since the state is less likely to be critical (while agent 1 will act for sure because $x_1 \leq \underline{x}_2 + 2\sigma < x_1^*$); and it increases as x_2 increases from $\underline{x}_2$ because the state is more likely to be critical, until x_2 gets high enough so that the state starts to become more likely to be irreversible and/or the other agent starts to be less likely to act; at that point the expected gain starts to decline, down to c_2 at $x_2 = x_2^*$ by (2) and lower afterward.

Next, we check optimality of agent 1 acting at every $x_1 < x_1^*$. Conditional on agent 2 acting if and only if $x_2 \in [\underline{x}_2, x_2^*]$, agent 1's expected gain from acting on observing a signal x_1, i.e., the LHS of (4), is

$$\int_{-\infty}^{1} F(\underline{x}_2|\theta)dF(\theta|x_1) \;+\; \int_{1}^{2} [F(x_2^*|\theta) - F(\underline{x}_2|\theta)]dF(\theta|x_1). \tag{3}$$

It is verified (in Appendix) that (3) decreases in $x_1 \leq 1-\sigma$ (when the second integral vanishes), but for $x_1 \geq 1+\sigma$ (when the first integral vanishes) it initially increases then declines (when the posterior probability of agent 2 acting declines), down to c_2 at $x_1 = x_1^*$ and further afterwards. Therefore, it suffices to show that (3) exceeds c_1 at every $x_1 \in [1-\sigma, 1+\sigma]$. Note that $F(x_2^*|\theta) = 1$ in (3) for $x_1 \leq 1+\sigma$ because $x_2^* > 1+3\sigma$ as noted above.

First, consider the case that $\underline{x}_2 \in (1-\sigma, 1]$, that is, $c_2 \leq 0.5$ by (1). Recall that agent 2's expected gain from acting on observing $x_2 = \underline{x}_2$, which equals c_2 by definition of $\underline{x}_2$, is the probability that $\theta \in (1,2)$, i.e., $1 - F(1|\underline{x}_2)$. Thus, upon observing the same signal $x_1 = \underline{x}_2$, if agent 1 is pivotal with a probability at least 0.5 conditional on θ being in a subset with a posterior probability at least $2(1 - F(1|\underline{x}_2))$, then agent 1's expected gain from acting is at least $1 - F(1|\underline{x}_2) = c_2 \geq c_1$. We identify, in Appendix, a subset of θ that works as such (the top end of feasible θ's upon observing $x_1 = \underline{x}_2$), and also show that the argument extends to other signals $x_1 \in (1-\sigma, 1+\sigma)$. In addition, a symmetric logic applies to the case that $\underline{x}_2 \in (1, 1+\sigma)$.

We now consider the alternative configuration in which agent 1 acts in an interval $[\underline{x}_1, x_1^*]$ and agent 2 below a threshold x_2^*. Analogously to the previous configuration, the upper boundary levels x_1^* and x_2^* are determined by (2) and $\underline{x}_1$ is determined by the condition $c_1 = 1 - F(1|\underline{x}_1)$. In the current configuration, $\underline{x}_1 \in (1-\sigma, 1)$ because $c_1 < 0.5$ by an analogous reasoning behind (1), and the previous analysis for the case $\underline{x}_2 \in (1-\sigma, 1)$ applies with the roles of agents 1 and 2 swapped. Specifically, conditional on agent 1 acting if and only if $x_1 \in [\underline{x}_1, x_1^*]$, agent 2's expected gain from acting at signal x_2 is

$$\int_{-\infty}^{2} F(\underline{x}_1|\theta) dF(\theta|x_2) \; + \; \int_{1}^{2} [F(x_1^*|\theta) - F(\underline{x}_1|\theta)] dF(\theta|x_2) \tag{4}$$

and the minimum value of (4) across all $x_2 < x_2^*$ exceeds c_1.

Note that (4) is a function of c_1 because $F(x_1^*|\theta) = 1$ for $x_2 \in (1-\sigma, 1+\sigma)$ and $\underline{x}_1$ is determined by $c_1 = 1 - F(1|\underline{x}_1)$; hence the minimum value of (4) across all $x_2 < x_2^*$ is also a function of c_1, which we denote by $\bar{c}_2(c_1)$. Therefore, the current configuration constitutes a BNE if and only if $c_2 \leq \bar{c}_2(c_1)$. Note that in the limit case as $c_1 \to 0$ so that $\underline{x}_1 \to 1-\sigma$, the value of (4) at $x_2 = 1-\sigma$ converges to 0.5. This implies that if $c_2 > 0.5$ then the current configuration fails to be a BNE for sufficiently small c_1.

Summarizing the discussion so far, we characterize interval-threshold equilibria as below.

Proposition 1 (a) *There exists an interval-threshold equilibrium in which agent 1 adopts the threshold strategy and agent 2 the interval strategy. This equilibrium is unique and achieves the efficiency of complete information asymptotically as* $\sigma \to 0$.
(b) *It is an equilibrium for agent 2 to adopt the threshold strategy and agent 1 the interval strategy if and only if* $c_2 \in [c_1, \bar{c}_2(c_1)] \neq \emptyset$ *where* $\bar{c}_2(c_1)$ *is the minimum value of (4) across all* $x_2 < x_2^*$ *and converges to* 0.5 *from above as* $c_1 \to 0$.

Recall that the upper boundary levels x_1^* and x_2^*, determined by the equation system (2), are the same regardless of which agent adopts the interval strategy. Therefore, both agents $i = 1, 2$ act at all signals in their respective range $[\underline{x}_i, x_i^*]$

in any interval-threshold equilibrium, thus largely coordinate when both need to act to restore the environment (since $[\underline{x}_i, x_i^*] \approx [1,2]$). In the next section, an iterated dominance argument shows that such coordination in the complementary region must prevail in every equilibrium if c_2 is not too large, as noise vanishes.

5 Iterative Dominance

Carlsson and van Damme (1993) establish the seminal result in 2-player, 2-action global games where the players' utilities change continuously in an underlying parameter θ and each player observes a noisy signal of θ: if an action, which is risk-dominant in some open range I of underlying parameter values, is strictly dominant at some $\theta \in I$ for at least one player, then it is iteratively dominant at all signals in I for both players in the global game as the noise vanishes.

Their result does not apply to the model analyzed in the previous section (in particular, to the complementary region) because no action is strictly dominant at any parameter values $\theta < 2$. Nevertheless, we show that acting ($a_i = 1$) is strictly dominant at signals near $x_i = 1$ in the global game, and through an iterative process its dominance extends to all signals in the complementary region as σ tends to 0. The key property behind this result is that acting, which is risk-dominant in the complete information game when θ is above the critical value of 1 (where the utilities are discontinuous), is also sufficiently attractive even if θ is slightly below 1 and the other agent switches to not acting ($a_{-i} = 0$). This may hedge the risk-dominant action sufficiently for it to be the dominant action at signals near the critical value, initiating the iterative expansion process.

Continuing with the model analyzed in the previous section, recall that an agent i is pivotal when either $\theta \in (1,2)$ and the other agent $-i$ acts or $\theta < 1$ and agent $-i$ does not. Given a strategy $s_{-i} : \mathbb{R} \to [0,1]$ of agent $-i$, therefore, the probability that agent i is pivotal upon observing a signal $x_i \in (1-\sigma, 1+\sigma)$ is

$$\begin{aligned} P(x_i|s_{-i}) &:= \int_{x_i-\sigma}^{1}\int_{\theta-\sigma}^{\theta+\sigma}[1 - s_{-i}(x_{-i})]dF(x_{-i}|\theta)dF(\theta|x_i) + \int_{1}^{x_i+\sigma}\int_{\theta-\sigma}^{\theta+\sigma} s_{-i}(x_{-i})dF(x_{-i}|\theta)dF(\theta|x_i) \\ &= F(1|x_i) + \int_{-\infty}^{\infty} s_{-i}(x_{-i})\Lambda(x_{-i}|x_i)\, dx_{-i} \\ \text{where} \quad \Lambda(x_{-i}|x_i) &:= \int_{1}^{x_i+\sigma} f(x_{-i}|\theta)f(\theta|x_i)d\theta - \int_{x_i-\sigma}^{1} f(x_{-i}|\theta)f(\theta|x_i)d\theta. \end{aligned} \tag{5}$$

If $P(x_i|s_{-i}) > c_i$ for every s_{-i}, then it is the dominant strategy for agent i to act at the signal x_i. To examine when this is the case, we observe that $P(x_i|s_{-i})$ is minimized when $s_{-i}(x_{-i}) = 0$ if $\Lambda(x_{-i}|x_i) \geq 0$ and when $s_{-i}(x_{-i}) = 1$ if $\Lambda(x_{-i}|x_i) < 0$.

Since $f(x_{-i}|\theta) = f(\theta|x_{-i})$ due to symmetry, $\Lambda(x_{-i}|x_i)$ is positive (negative, resp) if $\theta > 1$ is more (less, resp) likely than $\theta < 1$ conditional on observing two signals x_i and x_{-i}. Hence, $\Lambda(x_{-i}|x_i) = 0$ when x_{-i} and x_i are equidistant from 1 in opposite directions, i.e., $x_{-i} = 2 - x_i$, because then θ is equally likely to be above or below 1. Consequently,

$$\Lambda(x_{-i}|x_i) \begin{cases} < 0 & \text{if } \; x_{-i} < 2 - x_i \\ > 0 & \text{if } \; x_{-i} > 2 - x_i. \end{cases} \tag{6}$$

Thus, $P(x_i|s_{-i})$ is minimized when $s_{-i}(x_{-i}) = 0$ for $x_{-i} \geq 2 - x_i$ and $s_{-i}(x_{-i}) = 1$ for $x_{-i} < 2 - x_i$, which we denote by $\breve{s}_{-i}$. Let $\underline{P}(x_i) := P(x_i|\breve{s}_{-i})$ denote the minimum value of $P(x_i|s_{-i})$ across all s_{-i}.

If $x_i = 1$, in particular, $\breve{s}_{-i}$ assigns 0 for $x_{-i} \geq 1$ and 1 for $x_{-i} < 1$. Therefore, $\underline{P}(1)$ is the probability, conditional on $x_i = 1$, that θ is below 1 but x_{-i} is above 1, or the other way around. The two events are equally likely and the probability of the latter is $\int_1^{1+\sigma} F(1|\theta) f(\theta|1) d\theta$. Hence,

$$\underline{P}(1) = 2\int_1^{1+\sigma} F(1|\theta) f(\theta|1) d\theta = 2\int_1^{1+\sigma} F(1-\theta) f(1-\theta) d\theta \;=\; \frac{1}{4}$$

where the last equality follows because $\int_{-\infty}^{a} F(x) f(x) dx = F(a)^2/2$ for any cdf F.[11]

If $x_i = 1 - \sigma$ so that $\breve{s}_{-i}$ assigns 1 for all $x_{-i} < 1 + \sigma$, agent i is never pivotal because $\theta < 1$ for sure and the other agent were to always act, i.e., $\underline{P}(1 - \sigma) = 0$. Analogously, $\underline{P}(1+\sigma) = 0$ because if $x_i = 1+\sigma$ then $\theta > 1$ and the other agent never acts according to $\breve{s}_{-i}(x_{-i})$.

As such, the function $\underline{P}(x_i)$ is defined continuously on the interval $[1 - \sigma, 1 + \sigma]$ and assumes strictly positive values in the interior and 0 at the boundaries. For each $c \in (0, \underline{P}(1))$, therefore, a largest interval $(\underline{x}^{(1)}(c), \widehat{x}^{(1)}(c))$ exists on which $\underline{P}(x_i) > c$.[12] Consequently,

[A] it is strictly dominant for an agent i to act at every signal $x_i \in (\underline{x}^{(1)}(c_i), \widehat{x}^{(1)}(c_i))$ if $c_i < \underline{P}(1)$.

Clearly, $1-\sigma < \underline{x}^{(1)}(c_1) < \underline{x}^{(1)}(c_2) < 1 < \widehat{x}^{(1)}(c_2) < \widehat{x}^{(1)}(c_1) < 1+\sigma$ if $c_1 < c_2 < \underline{P}(1)$.

From this initial range of signals on which acting is dominant, we expand the dominant range of signals iteratively in the usual manner. Given [A], an agent i with a signal $x_i \in [1 - \sigma, 1 + \sigma]$ is pivotal with a probability at least

$$\underline{P}_i^{(1)}(x_i) := \min_{s_{-i}} P(x_i|s_{-i}) \;\; \text{subject to} \;\; s_{-i}(x_{-i}) = 1 \quad \forall x_{-i} \in (\underline{x}^{(1)}(c_{-i}), \widehat{x}^{(1)}(c_{-i})). \tag{7}$$

[11] Letting $F(x) = t$ so that $f(x)dx = dt$, $\int_{-\infty}^{a} F(x)f(x)dx = \int_{-\infty}^{F(a)} t dt = F(a)^2/2$.

[12] Note that $1 - \underline{x}^{(1)}(c) = \widehat{x}^{(1)}(c) - 1$ by symmetry.

If $2 - x_i < \widehat{x}^{(1)}(c_{-i})$, the constraint in (7) requires s_{-i} to assign 1 to an interval of signals x_{-i} to which $\breve{s}_{-i}$ assigns 0, increasing the value of $\min_{s_{-i}} P(x_i|s_{-i})$. Therefore, $\underline{P}_i^{(1)}(x_i) > \underline{P}(x_i)$ for all $x_i \in [1, 1+\sigma]$, in particular, and consequently, the range of signals on which acting is (iteratively) strictly dominant for agent i expands to an interval $(\underline{x}^{(2)}(c_i), \widehat{x}^{(2)}(c_i))$ that contains $(\underline{x}^{(1)}(c_i), \widehat{x}^{(1)}(c_i))$ and $\widehat{x}^{(1)}(c_i) < \widehat{x}^{(2)}(c_i)$.

Repeating the process iteratively, one generates an increasing sequence of upper boundaries of dominant ranges $\{\widehat{x}^{(n)}(c_i)\}_n$ for each agent i. Suppose $\widehat{x}^{(n)}(c_i) \geq 1 + \sigma$ for both $i = 1, 2$ for some n, so that both agents are certain that $\theta > 1$ upon observing the boundary signal $\widehat{x}^{(n)}(c_i)$. Then, the probability of agent i being pivotal on observing $x_i \geq \widehat{x}^{(n)}(c_i)$ is minimized when agent $-i$ acts only in the then-dominant range of signals (which expands every round). Therefore, from then on, each agent's upper boundary of dominant range increases by at least the same amount as the other agent's boundary increased in the previous round (i.e., $\widehat{x}^{(n+1)}(c_i) - \widehat{x}^{(n)}(c_i) \geq \widehat{x}^{(n)}(c_{-i}) - \widehat{x}^{(n-1)}(c_{-i})$) until it reaches $2 - \sigma$, when the expansion slows down and settles at x_i^* for both players, i.e., the upper boundary signals of the interval-threshold equilibrium in the previous section. We show in Appendix that this is indeed the case if $c_1, c_2 < \underline{P}(1) = 1/4$.

Next, we determine $\underline{x}^{(\infty)}(c_i)$, the lower end of the signal range for which acting is iteratively dominant for agent i. Given that it is iteratively dominant for both agents to act at every $x_i \in (\underline{x}^{(1)}(c_i), x_i^*)$ as shown above, upon observing a signal $x_i \in (1-\sigma, \underline{x}^{(1)}(c_i))$, the probability that agent i is pivotal is minimized when $s_{-i}(x_{-i}) \equiv 1$ by (5). Thus, the minimized value is $1 - F(1|x_i)$ which increases in x_i from 0 at $x_i = 1-\sigma$ and exceeds c_2 at all $x_i \in (\underline{x}^{(1)}(c_i), 1)$ as shown in [A] above. Consequently, $\underline{x}^{(\infty)}(c_i)$ is the signal $x_i \in (1-\sigma, \underline{x}^{(1)}(c_i))$ that solves $1 - F(1|x_i) = c_i$ for $i = 1, 2$. Note that this is $\underline{x}_i$ defined in the previous section, namely, the lowest signal at which the interval-player acts in the interval-threshold equilibrium, which we now denote as $\underline{x}(c_i)$ to be explicit about its dependence on c_i (but not on i).

Proposition 2 *It is iteratively strictly dominant for agent i to act at every signal in the interval $[\underline{x}(c_i), x_i^*) \supset [1, 2-\sigma]$ if $c_1 \leq c_2 < \underline{P}(1) = 1/4$.*

We stated the result for $c_1, c_2 < 1/4$, but this is not necessary. Note that the lower c_1 is, the larger is the initial signal range where acting is dominant for agent 1, $(\underline{x}^{(1)}(c_1), \widehat{x}^{(1)}(c_1))$. This in turn means that a larger dominant signal range for agent 2 in the next stage, $(\underline{x}^{(2)}(c_2), \widehat{x}^{(2)}(c_2))$, and so on. As a result, the conclusion of Proposition 2 holds for higher c_2 (that goes above 1/4) if c_1 is lower. This negative relationship is made precise in the next section.

Finally, it is straightforward to show that agent i never acts at any signal $x_i > x_i^*$ in every equilibrium, leading to the following characterization of equilibrium in conjunction with Proposition 2.

Corollary 1 *If $c_2 < 1/4$, in every equilibrium both agents act for sure at all $x_i \in (\underline{x}_i, x_i^*) \supset [1, 2-\sigma]$ and never acts at any signal $x_i > x_i^*$.*

We already established that it constitutes an equilibrium that agent i acts if and only if $x_i \in (\underline{x}_i, x_i^*)$ and agent $-i$ acts if and only if $x_{-i} < x_{-i}^*$. This implies that the range of signals with dominance-solvable equilibrium actions cannot be expanded beyond $(\underline{x}_i, x_i^*)$.

Remark 1: Although we assumed that the noise ϵ has a bounded support (which simplified exposition), the arguments and results in Sections 4 and 5 extend straightforwardly when ϵ has an unbounded support. Moreover, the interval-threshold equilibrium in Section 4 extends to more than two agents in the obvious manner. For the uniqueness result to be extended to more than two agents, however, we need an unbounded support of ϵ: when there are $I > 2$ agents, at the borderline signal $s = I - 1$, each agent's participation is pivotal with a probability bounded away from 0 no matter how the other agents behave, because the critical mass can be any number between 1 and I, each with a positive probability. Hence, participation is dominant if participation cost is sufficiently small at signals around $I-1$, from which the dominant range of signals expands upward by an iterative process toward I.

Remark 2: In the two agents version of the model, full efficiency is achieved when only agent 1 acts whenever $\theta \in (0,1)$ and both agents act whenever $\theta \in (1,2)$, generating the social welfare

$$\widehat{W} \;=\; Prob(0 < \theta < 1)(2 - c_1) + Prob(1 < \theta < 2)(2 - c_1 - c_2).$$

Note from (2) and (1) that $x_i^* \to 2 + \sigma$ and $\underline{x}_i \to 1 - \sigma$ for both $i = 1, 2$ as $c_2 \to 0$, so that the social welfare in the limit is

$$\widehat{W} - c_2 \times Prob(1 - 2\sigma < \theta < 1) \times \int_{1-2\sigma}^{1} \big[1 - F(1-\sigma|\theta)\big] d\theta$$

which converges to $\widehat{W}$ as $c_2 \to 0$.

Section 5 demonstrated how low agents' costs facilitate uniqueness in the complements region. An important observation is that there is a friction between uniqueness in the strategic complements and substitutes region. By allowing equilibrium selection in the strategic complements region, we make it harder for equilibrium selection in the strategic substitutes region to be achieved. This is a known friction in the literature as demonstrated at Guesnerie (2004). They argue that conditions that facilitate equilibrium selection in a game of strategic complements will have the opposite effect in a game with strategic substitutes. In our game this is incorporated in the players' costs. For high enough costs we can achieve uniqueness in the strategic substitutes region but we will have multiplicity in the complements region. On the contrary, low costs result in a unique equilibrium in the complements region but multiplicity in the substitutes region. This is highlighted in the next proposition.

Proposition 3 *It is iteratively strictly dominant for agent* 1 *to act and agent* 2 *not to act at every signal* $x_i < 1$ *if* $c_2 > 3/4$.

Notice that equilibrium selection in the strategic substitutes region implies multiplicity whenever the fundamentals exhibit strategic complements and vice versa.

6 Discussion

The analysis so far has abstracted from many interesting strategic interactions that climate change presents. By choosing to focus on an one shot game, we have abstracted from issues of commitment, communication and negotiations all crucial parts of the analysis. One could consider a richer two stage game in which agents could negotiate or commit to some action which they have to take at the end of the second stage. Notice that any such interaction could only accommodate coordination. This work focuses on the minimum conditions required for agents to coordinate instead. Moreover, one could consider a more complete model that allows both the benefits and the costs to change with the state of the environment. Since all the results presented here are in level utilities, this implies that they would hold for a range of such functions.

A crucial assumption that we made is that policies have no informational value. In a more complete environment, one could model the policy proposed as a very strong public signal that both agents observe. The global games literature has shown that such signals can have great effects in equilibrium selection. Whether this remains the case in this discontinuous environment remains an open question.

Another important abstraction of the paper is focusing on a two agent, binary action model. A more comprehensive analysis of the topic should include multiple agents of different "size" who could choose to which extend they are willing to act. Depending on the impact they can have on the issue, these agents' actions can feature strategic substitutes, complements or different combinations of the two depending on whether they are compared with an agent of larger or smaller size. Nevertheless, this paper indicates that taking into account the different strategic interactions between countries and the incomplete information of the environment can have strong implications about the policies that can guarantee coordination. And proposes a framework of discontinuous incomplete information games to study the issue. One could imagine a more comprehensive model that takes into account all the various such strategic interactions/sizes of the countries and derive the limit of this finding. The analysis above in combination with the generalisation of the above result in Park and Smyrniotis (2022) hint that this analysis could be a fruitful area for future research.

7 Conclusion

This paper studies the strategic interactions between large players that decide whether to take climate action or not, after observing noisy signals about the state of the environment. If the state is moderate only one agent needs to adopt climate policies in order to restore the environment, while both agents' participation is needed if the state is critical. Actions can thus exhibit either strategic substitutes or complements with the possibility of a self-fulfilling coordination failure. The key implication of the model is that there exist utility levels such that a coordination failure will always be prevented, in the incomplete information environment of the game. The same utility levels would not guarantee coordination in the complete information game. This implies that a proposed policy, targeted towards coordination should take into account the incomplete information nature of climate change. By doing so one could propose policies that require less change in agents utility and thus it will be easier to be adopted by countries and be implemented, but achieve coordination nevertheless.

From a theoretical standpoint, we study a 2-player, 2-action coordination game in which agents' actions can feature either strategic complements or substitutes, the key characteristic of a public good provision problem. The departure from the previous literature stems from actions changing between substitutes and complements discontinuously to the underlying fundamental. We observe that around the critical level of the fundamental value, where such discontinuity occurs, agents can have a strictly dominant action in the incomplete information game even though no action is strictly dominant in the complete information version. That is because the risk dominant action from one side of the discontinuity, depending on agents' utilities, can be sufficiently attractive to the agents, in the contingency that their opponent takes the opposite action, on the other side of the discontinuity. This allows for an iterative process similar to the one developed in Carlsson, van Damme (1993) to select that action as the unique prediction, as noise vanishes, for all fundamental values for which it remains risk dominant. We derive conditions on the utilities of the agents that allow for such iterative process to take hold.

Appendix

Proof of Lemma 1. Consider the agent i whose upper threshold is higher, i.e., $x_i^* \geq x_{-i}^*$. Upon observing $x_i^* > 2 - \sigma$, this agent infers $\theta > 1$ and thus that he is pivotal if $\theta < 2$ and the other agent observes a signal to act, the likelihood of which is equal to c_i by (4). The probability that the other agent $-i$ observes a signal to act, however, is less than 0.5 because x_{-i} is equally likely to be above and below x_i^* and $x_{-i}^* < x_i^*$, from which we deduce that $c_i < 0.5$. Since this probability is positive, we also deduce that $x_{-i}^* > x_i^* - 2\sigma > 2 - 3\sigma > 1 + \sigma$. Then, upon observing x_{-i}^*, agent $-i$ also infers $\theta > 1$ and thus that he is pivotal if $\theta < 2$ and the other agent observes a signal to act, the likelihood of which is equal to c_{-i}.

Since $x_i^* > x_{-i}^* > 1 + \sigma$, the posterior probability that $\theta \in (1, 2)$ is higher at the signal x_{-i}^* than at x_i^*, and the probability of the other agent observing a signal to act conditional on agent i observing x_i^* is no higher than 0.5. Thus, if the agent i observes a signal to act with a probability exceeding 0.5 conditional agent $-i$ observing x_{-i}^*, then $c_i < c_{-i}$ ensues, i.e., $i = 1$. This is clearly the case if agent i is the threshold-player. If agent i is the interval-player, then $\underline{x}_i < 1 + \sigma$ because he should act upon observing a signal $x_i = 1 + \sigma$ given that $\theta > 1$ for sure and agent $-i$ acts with prob at least 0.5, as well as $c_i < 0.5$. Thus, $[\underline{x}_i, x_i^*]$ is an interval of length exceeding 2σ and contains x_{-i}^*, hence the agent i observes a signal to act with a probability exceeding 0.5 conditional agent $-i$ observing x_{-i}^*. ∎

Proof of Proposition 1. We provide the deferred proofs.

(1) To show there is a unique solution to (2) and $2 - 3\sigma < x_2^* < x_1^* \in (2 - \sigma, 2 + \sigma)$.

We have shown in the main text that $x_2^* \leq x_1^* \in (2 - \sigma, 2 + \sigma)$. For agent 1 to be indifferent between acting and not at x_1^*, he should be pivotal with a positive probability, which implies that $x_2^* > x_1^* - 2\sigma > 2 - 3\sigma$.

Next, suppose there are two solutions to (2, denoted by (x_1^*, x_2^*) and (x_1', x_2') where $x_1' = x_1^* - r < x_1^*$ wlog. Then, (2) dictates that

$$\int_{x_1^*-\sigma}^{2} F(x_2^* - \theta + \sigma) f(\theta - x_1^* + \sigma) d\theta \ = \ c_1 \ = \ \int_{x_1^*-r-\sigma}^{2} F(x_2' - \theta + \sigma) f(\theta - x_1^* + r + \sigma) d\theta.$$

Note that the RHS evaluated at $x_2' = x_2^* - r$, is $\int_{x_1^*-\sigma}^{2+r} F(x_2^* - \tilde{\theta} + \sigma) f(\tilde{\theta} - x_1^* + \sigma) d\tilde{\theta} > c_1$ by change of variable $\tilde{\theta} = \theta + r$. This implies that $x_2' < x_2^* - r$. On the other hand,

$$\int_{x_2^*-\sigma}^{2} F(x_1^* - \theta + \sigma) f(\theta - x_2^* + \sigma) d\theta \ = \ c_2 \ = \ \int_{x_2'-\sigma}^{2} F(x_1^* - r - \theta + \sigma) f(\theta - x_2' + \sigma) d\theta$$

by (2), but the RHS evaluated at $x_2' = x_2^* - r$, is $\int_{x_2^*-\sigma}^{2+r} F(x_1^* - \tilde{\theta} + \sigma) f(\tilde{\theta} - x_2^* + \sigma) d\tilde{\theta} > c_2$. This implies that $x_2' > x_2^* - r$ (because the RHS of the previous displayed equation decreases in x_2' due to symmetry and single-peakedness of f), contradicting the earlier assertion $x_2' < x_2^* - r$. Note that this argument presumes $x_2^* + \sigma > 2$. If $x_2^* + \sigma < 2$ then since $\theta < 2$ is evident to agent 2 upon observing x_2^* or x_2', $x_1^* - x_2^* = x_1' - x_2'$ must hold, again contradicting $x_2' < x_2^* - r$.

(2) To show (3) decreases in $x_1 \leq 1 + \sigma$; for $x_1 \geq 1 + \sigma$, it initially increases then declines.

The derivative of (3) wrt x_1 is

$$-\int_{-\infty}^{1} F(\underline{x}_2|\theta) f'(\theta|x_1) d\theta \;-\; \int_{1}^{2} [F(x_2^*|\theta) - F(\underline{x}_2|\theta)] f'(\theta|x_1) d\theta. \tag{8}$$

Note that f is symmetric around and single-peaked at $\theta = x_1$, that is, $f'(\theta|x_1) = -f'(2x_1 - \theta|x_1) > 0$ for $\theta \in (x_1 - \sigma, x_1]$, which is used repeatedly in the reasoning below. For $x_1 \leq 1 - \sigma$, only the first term is relevant (the second term vanishes) which is negative because $F(\underline{x}_2|\theta)$ decreases in $\theta \in [x_1 - \sigma, x_1 + \sigma]$. For $x_1 \geq 1 + \sigma$, only the second term is relevant (the first term vanishes). $F(x_2^*|\theta) = 1$ for $\theta \leq x_2^* - \sigma$, decreases for $\theta \in (x_2^* - \sigma, x_2^* + \sigma)$ and is 0 for $\theta \geq x_2^* + \sigma$. $F(\underline{x}_2|\theta) = 1$ for $\theta \leq \underline{x}_2 - \sigma$, decreases for $\theta \in (\underline{x}_2 - \sigma, \underline{x}_2 + \sigma)$ and is 0 for $\theta \geq \underline{x}_2 + \sigma$. Since $x_2^* - \underline{x}_2 > 2\sigma$, $F(x_2^*|\theta) - F(\underline{x}_2|\theta)$ increases for $\theta \in (1, \underline{x}_2)$ if nonempty, then stay constant at 1 until $\theta = x_2^* - \sigma$ (hence, for an interval of θ of length at least 2σ), from which point it declines down to 0 at $\theta = x_2^* + \sigma$. Due to symmetric and single-peaked f, therefore, as x_1 increases from $1 + \sigma$ the second term of (8) is positive, then 0 for a while before turning to negative. This means that for $x_1 \geq 1 + \sigma$, (3) initially increases then declines down to c_2 at $x_1 = x_1^*$ and further afterwards.

(3) To show that (3) exceeds c_1 at every $x_1 \in [1 - \sigma, 1 + \sigma]$.

Focus on the highest possible θ's with a posterior probability $2(1 - F(1|\underline{x}_2))$ upon observing $x_1 = \underline{x}_2$, that is, the interval $[\widehat{\theta}, \underline{x}_2 + \sigma]$ where $1 - F(\widehat{\theta}|\underline{x}_2) = 2(1 - F(1|\underline{x}_2))$. Agent 1's action is pivotal with a probability greater than 0.5 conditional on $\theta \in [\widehat{\theta}, \underline{x}_2 + \sigma]$, because then θ is equally likely to be above and below 1 (by construction) and

$$F(\underline{x}_2|\theta) > F(\underline{x}_2|\theta') \iff F(\underline{x}_2|\theta) + 1 - F(\underline{x}_2|\theta') > 1 \quad \text{if} \quad \theta < 1 < \theta', \tag{9}$$

that is, the average probability that agent 1's action is pivotal between any two $\theta, \theta' \in [\widehat{\theta}, \underline{x}_2 + \sigma]$, one below 1 and the other above 1, exceeds 0.5. This implies that (3) exceeds $1 - F(1|\underline{x}_2) = c_2$ at $x_1 = \underline{x}_2$.

The same conclusion obtains when agent 1 observes $x_1 > \underline{x}_2$ as well, because then θ is more likely to be above than below 1 subject to θ being in the top interval of possible θ's of measure $2(1 - F(1|\underline{x}_2))$ and, in addition to (9), we have $1 - F(\underline{x}_2|\theta) > 0.5$

for all $\theta > 1$. The same also holds at $x_1 < \underline{x}_2$, because then it is straightforward to verify that agent 1's action is pivotal with a probability exceeding 0.5 both conditional on $\theta < 1$ and conditional on $\theta \in (1,2)$.

Therefore, if $\underline{x}_2 \in (1-\sigma, 1)$, i.e., $c_2 \leq 0.5$, then the minimum value of (3) across all $x_1 < x_1^*$ exceeds c_2, hence exceeds c_1 as well, establishing it to be an equilibrium for agent 1 to adopt the threshold strategy below x_1^* and agent 2 the interval strategy on $[\underline{x}_2, x_2^*]$.

Next, consider the case that $\underline{x}_2 \in (1, 1+\sigma)$ so that $c_2 > 0.5$ by (1). Note the symmetry between this and the previous case: agent 1's action is pivotal if both θ and x_{-i} are one the same side (below or above) of 1 and $\underline{x}_2$, respectively, except that $\underline{x}_2$ is on the opposites of 1 in the two cases. From this symmetry it follows that the value of (3) at $x_1 \in (1-\sigma, 1+\sigma)$ in one case coincides with the value of (3) in the other case when x_1 is equidistant from 1 in the other direction and consequently, that the minimum value of (3) among all $x_1 < x_1^*$ is also the same in the two cases. Since this minimum value has been shown to exceed c_1 when $c_2 < 0.5$ above, so it must when $c_2 > 0.5$ as well, establishing it to be an equilibrium for agent 1 to adopt the threshold strategy below x_1^* and agent 2 the interval strategy on $[\underline{x}_2, x_2^*]$. ∎

Proof of Proposition 2. Recall the iterative process that generates an increasing sequence of upper boundaries of dominant ranges $\{\widehat{x}^{(n)}(c_i)\}_n$ for each agent i. It remains to verify that $\lim_{n\to\infty} \widehat{x}^{(n)}(c_i) = x_i^*$ for $i = 1,2$ if $c_1, c_2 < \underline{P}(1) = 1/4$.

Note that this will indeed be the case if the upper boundary $\widehat{x}^{(1)}(c_i)$ is already above $1+\sigma$ after the first round, i.e., $c_i \leq \min_{x_i \in [1-\sigma, 1+\sigma]} \underline{P}_i^{(1)}(x_i)$ for $i = 1,2$. For $c_i > 0$ small enough, this is the case because $\widehat{x}^{(1)}(c_i) \to 1+\sigma$ as $c_i \to 0$ and thus, $\underline{P}_i^{(1)}(x_i)$ is bounded away from 0 on $[1-\sigma, 1+\sigma]$.

From construction of the sequence of dominant intervals $\{(\underline{x}^{(n)}(c_i), \widehat{x}^{(n)}(c_i))\}_n$, it is clear that $(\underline{x}^{(n)}(c_i), \widehat{x}^{(n)}(c_i)) \subset (\underline{x}^{(n)}(c_i'), \widehat{x}^{(n)}(c_i'))$ for each n and $i = 1,2$, if $c_i \geq c_i'$ for $i = 1,2$. Therefore, if $\widehat{x}^{(\infty)}(c_i') < 1+\sigma$ for some i and some (c_1', c_2'), then $\widehat{x}^{(\infty)}(c_1) = \widehat{x}^{(\infty)}(c_2) < 1+\sigma$ for $c_1 = c_2 = \min\{c_1', c_2'\}$. Moreover, since $\widehat{x}^{(n)}(c_i)$ is continuous in c_i when $c_1 = c_2$, there is some $c > 0$ such that $\widehat{x}^{(\infty)}(c_1) = \widehat{x}^{(\infty)}(c_2) = 1+\sigma$ for $(c_1, c_2) = (c, c)$. This means that for $(c_1, c_2) = (c, c)$, we have c being equal to

$$\begin{aligned}
\underline{P}_i^{(\infty)}(1+\sigma) &= \min_{s_{-i}} P(1+\sigma|s_{-i}) \text{ subject to } s_{-i}(x_{-i}) = 1 \quad \forall x_{-i} \in (\underline{x}^{(\infty)}(c_{-i}), \widehat{x}^{(\infty)}(c_{-i})) \\
&\geq P(1+\sigma|s_{-i}) \text{ where } s_{-i}(x_{-i}) = 1 \quad \Leftrightarrow \quad x_{-i} \in (-\infty, 1-\sigma] \cup [1, 1+\sigma] \\
&> 1/4
\end{aligned}$$

where the weak equality is due to (5) and the strict inequality ensues because the regime is strong for sure on $x_i = 1+\sigma$, given which $Prob(x_{-i} \in [1, 1+\sigma]) > 1/4$, contradicting $c < \underline{P}(1) = 1/4$. ∎

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