Resale Price Maintenance and Horizontal Cartel

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Abstract
An often expressed idea to motivate the per se illegality of RPM is that it can limit both inter- and intra-brand competition. This paper analyses this argument in a context where manufacturers and retailers have interlocking relationships. It is shown that even as part of purely bilateral vertical contracts, RPM indeed limits the exercise of both inter- and intra-brand competition and can even generate industry-wide monopoly pricing. The final impact on prices depends on the substitutability between retailers and between manufacturers, and on the extent of potential competition at the retail level.

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1 Introduction

The attitude of competition authorities and courts towards vertical restraints varies significantly from one country to another or from one period to another.[1] Still, it emerges a consensus against resale price maintenance (RPM), a restraint according to which the manufacturer sets the final price that retailers charge to consumers. While competition authorities are sometimes tolerant towards some variants of RPM such as price ceilings and recommended or advertised prices, they usually treat price floors and strict RPM as per se illegal. For example, the European Commission recently adopted a more open attitude towards nonprice restrictions but it maintained RPM on a black list – with only one other restraint. In France, price floors are per se illegal and, in Lypobar vs. La Croissanterie (1989), the Paris Court of Appeal ruled that RPM was an abuse of franchisees' economic dependence.

In contrast with the consensus of the jurisprudence against RPM, the economic analysis of vertical restraints is more ambiguous: it is not straightforward that RPM has a more negative impact on welfare than other vertical restraints that limit as well intrabrand competition; instead, both price (e.g., RPM) and non-price restraints (e.g., exclusive territories) have positive and negative effects on welfare, depending on the context in which they are used.[2] Moreover, a comparison of the welfare effects of exclusive territories, RPM and exclusive dealing shows that the balance is not clearly in favor of nonprice restrictions.[3]

Minimum prices might of course be sponsored by retailers to maintain a downstream cartel: that is, retailers may try to enforce an illegal horizontal agreement through vertical arrangements that restrict prices.[4] Less clear is the case of purely vertical contracts, where manufacturers and retailers bilaterally negotiate their own wholesale and retail prices.

A few papers have stressed that RPM can help a manufacturer to better exert its

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[1]For an overview of the legal frameworks regarding vertical restraints, see OECD (1994) or the European Commission's *Green Paper on Vertical Restraints* (1996). Comanor-Rey (1996) also compares the evolution of the attitudes of the U.S. competition authorities and within the European Community.

[2]In particular, the arguments that courts have put forward to justify territorial restraints could often be used as well in favor of RPM.

[3]See Caballero-Sanz and Rey (1996).

[4]For example, in response to increased pressure from their national cartel office, Swiss bookstores entered into an exclusive agreement with a unique importer to maintain high prices for German books.

market power. Hart and Tirole (1990) show for example that a manufacturer is tempted to free-ride on its retailers when vertical contracts are privately negotiated and not publicly observed; as a result, downstream competition percolates to the upstream level and prevents the manufacturer from fully exerting its market power.[5] In this context, an industry wide price-floor would prevent the risk of opportunistic behavior and help the manufacturer to exert its market power. O'Brien and Shaffer (1992) further show that bilaterally negotiated price ceilings, too, can help prevent opportunism.[6]

Those papers thus stress that RPM can help restore pre-existing market power. Dobson and Waterson (1997) study instead a bilateral duopoly with interlocking relationships. Assuming that manufacturers use (inefficient) linear wholesale prices, they show that the welfare effects of RPM depend on the relative degree of upstream and downstream differentiation as well as on retailers' and manufacturers' bargaining powers; RPM can be socially preferable when retailers are in a weak bargaining position, because the double-marginalization problems generated by the restriction to linear wholesale prices is more severe in such circumstances.

However, one argument often mentioned against RPM, and not yet much formally analyzed, is that RPM can eliminate or reduce interbrand competition.[7] A first step in that direction is provided by Jullien and Rey (2001), who stress that, by making retail prices less responsive to local shocks on retail cost or demand, RPM yields more uniform prices that facilitate tacit collusion – by making deviations easier to detect. In contrast, we will focus here on a static bilateral duopoly with interlocking relationships, as in Dobson and Waterson, but will allow for efficient (two-part) wholesale tariffs, in order to eliminate

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[5]The idea is that, when secretly contracting with one retailer, the manufacturer has an incentive to free-ride on the others and ends-up selling more than the monopoly quantity. This insight is reminiscent of the Coasian pricing problem for durable goods – or of a franchisor's incentive to sell too many franchises – and has further been explored by McAfee and Schwartz (1994) and O'Brien and Schaffer (1992). Rey and Tirole (2003) provides an overview of this literature.

[6]O'Brien and Shaffer use a concept of "contract equilibrium" which concentrates on pairwise deviations; therefore, they do not consider multilateral deviations which can indeed be profitable, thereby generating existence problems for standard Nash equilibria in contracts – see Rey and Vergé (2003).

[7]For example, in *Continental T.V.* vs. *GTE Sylvania* the US Supreme Court mentioned that a clear distinction had to be made between price and nonprice restraints, since price restrictions seemed to limit interbrand competition, thus facilitating cartellization –see 433 U.S. (1977) at 55.

double marginalization problems and focus instead on the impact of RPM on interbrand and intrabrand competition.[8] Our analysis suggests that RPM can prevent any effective competition –at the interbrand level as well as at the intrabrand level– and yield instead the monopoly outcome.

This paper is organized as follows. Section 2 presents our framework, where two rival manufacturers distribute their goods through two competing (but possibly differentiated) retailers; this framework allows for interlocking relationships ("double common agency"): each manufacturer can use both retailers, and conversely each retailer can carry both brands. Section 3 provides a preliminary analysis of such double common agency situations: while retail prices are lower than the monopoly price in the absence of RPM, with RPM there exist many equilibria, including one in which retail prices and manufacturers' profits are at the monopoly level; in addition, introducing (arbitrarily small) retail efforts singles out the equilibrium with monopoly prices and profits. We then endogenize the market structure. Section 4 first studies situations with potential downstream competition, which is captured by assuming that manufacturers can bypass established retailers and find alternative retailers to distribute their products. Both manufacturers' products are then always present at each retail location and, when RPM is allowed, there always exists an equilibrium with double common agency and monopoly prices and profits. Section 5 turns to the case of retail bottlenecks, where manufacturers cannot bypass established retailers. Manufacturers must then leave a rent to retailers to induce them to sell their products; relatedly, they may try to eliminate competitors by signing up the retailers into exclusive relationships. As a result, there may exist no equilibrium where both manufacturers are present in both retail outlets, even though there is demand for each brand at each store. In addition, while there may as well still exist a continuum of equilibria, equilibria with higher retail prices now involve larger rents for the retailers and lower profits for the manufacturer – implying that manufacturers favor equilibria with rather "competitive" prices. Section 6 concludes.

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[8]Another difference concerns the equilibrium concept. To reflect different bargaining power, Dobson and Waterson assume that wholesale prices are determined by simultaneous pairwise bargaining; this supposes that a manufacturer has two independent divisions, each of them negotiating with one retailer and not taking into account the impact of its own negotiation on the other division.

2   The Basic Framework

Two manufacturers, A and B, each produce their own brand of a good and market them through two differentiated retailers, 1 and 2. [Retailers could for example differ in the services they provide to consumers, the location of their stores, etc.] If each retailer carries both products, consumers can thus find two competing brands at two competing stores, and can thus choose among four imperfectly substitute "products", each manufacturer producing two of them ($\{A1, A2\}$ and $\{B1, B2\}$, respectively) and each retailer distributing two of them as well ($\{A1, B1\}$ and $\{A2, B2\}$, respectively).

In order to avoid that one firm - manufacturer or retailer - plays a particular role, we suppose that demand functions are symmetric (this implies that the differentiation between the brands and between the stores is horizontal rather than vertical): for any price vector $p = (p_{A1}, p_{B1}, p_{A2}, p_{B2})$, any $i \neq h \in \{A, B\}$ and any $j \neq k \in \{1, 2\}$,

$$D_{ij}(\mathbf{p}) \equiv D(p_{ij}, p_{hj}, p_{ik}, p_{hk}),$$

where the demand function $D(.)$ is continuously differentiable. In what follows, we will drop the arguments in D_{ij} when there is no risk of confusion, and will systematically use indexes i, h for the two manufacturers and j, k for the two retailers. The products being (imperfect) substitutes, we will suppose that the demand for one product decreases with the price of that product and increases with the other prices:[9,10]

$$\partial_1 D < 0 \text{ and } \partial_l D > 0 \text{ for } l = 2, 3, 4.$$

Furthermore, we will suppose that direct effects dominate, so that demand decreases if all prices increase:

$$\sum_{i=1}^{4} \partial_i D < 0. \tag{1}$$

We will also assume that both production and distribution marginal costs are symmetric and constant, and denote them respectively by c and γ.[11]

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[9]We denote by $\partial_x f$ the partial derivative of f with respect to its ith argument.

[10]This assumption seems reasonable but is not always maintained. For example, Dobson and Waterson (1997) consider a linear model where (considering inverse demand functions) the price of one product decreases when the quantity of *any* product increases; in that case, the demand for one brand in one store necessarily decreases when the price of the competing brand increases in the competing store ($\partial_4 D < 0$).

[11]We assume constant returns to scale only for expositional simplicity. The following analysis would

The industry-wide monopoly profit is equal to

$$\Pi^M\left(\mathbf{p}\right) \equiv \sum_{\substack{i=A,B \\ j=1,2}} \left(p_{ij} - c - \gamma\right) D_{ij}\left(\mathbf{p}\right).$$

Throughout the paper, we assume that this monopoly profit is concave in p and maximal for symmetric prices, $p^M = (p^M, p^M, p^M, p^M)$.

3 Preliminary Analysis: Intrinsic Double Common Agency

We assume in this section that manufacturers have all the bargaining power and that the market structure is necessarily that of a double common agency, by supposing that the market "breaks down" whenever a retailer refuses to carry a brand. This second assumption is admittedly ad-hoc and is only introduced to present the main intuition in a simple way; it is relaxed in the following sections.[12] Together, the two assumptions capture in an easy way two essential features of (potential) competition for each retail outlet: (i) retailers cannot get any rent; and (ii) manufacturers cannot exclude rival products from either outlet location; as a result, as in this section, double common agency prevails and furthermore manufacturers obtain all the profits. The simplifying assumptions adopted here allow us to focus on the determination of prices in this context. As we will see, the analysis of price determination remains relevant when relaxing the assumptions, although the distribution of rents and the existence of double common agency equilibrium situations then become relevant issues when retailers have market power.

We thus consider in this section the following simplified game G:

1. Upstream competition:

    (a) Each manufacturer $i = A, B$ proposes a contract to each retailer $j = 1, 2$.

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remain unchanged when fixed costs are for example taken into consideration; more generally, it should become clear to the reader that the thrust of the argument does not rely on a specific formulation of upstream and downstream costs.

[12]This preliminary analysis is similar in spirit to the "intrinsic common agency" game that Bernheim and Whinston (1985) use to present their main insight.

Contract offers are simultaneous and publicly observable,[13] and consist of a wholesale two-part tariff (w_{ij}, F_{ij}) and, if allowed, of a retail price (p_{ij}).[14]

(b) Retailers simultaneously accept or reject the offers; each retailer can accept both contracts, accept only one, or refuse both, and these acceptance decisions are public.

2. Downstream competition: if all offers have been accepted, retailers simultaneously set their retail prices (as imposed by the manufacturer under RPM), demands are satisfied and payments made according to the contracts. Otherwise, no product is sold and all firms earn zero profit.

The simplifying "market break-down" assumption ensures that manufacturers offer contracts that are acceptable by both retailers, and that retailers never obtain more than their reservation utility, which we normalize to zero.

3.1   Two-Part Tariffs

Let us first suppose that contracts can only consist of two-part tariffs. In the last stage, if all offers have been accepted each retailer $j = 1, 2$ sets its prices p_{Aj} and p_{Bj} so as to maximize its profit, given by:

$$\sum_{i=A,B} (p_{ij} - w_{ij} - \gamma) D_{ij} - F_{ij}.$$

We will assume that there exists a unique retail price equilibrium for any vector of wholesale prices $w = (w_{A1}, w_{B1}, w_{A2}, w_{B2})$, and denote by:

$$\mathbf{p}^r(\mathbf{w}) = (p_{A1}^r(\mathbf{w}), p_{B1}^r(\mathbf{w}), p_{A2}^r(\mathbf{w}), p_{B2}^r(\mathbf{w}))$$

the equilibrium retail prices, and by $D_{ij}^r(\mathbf{w}) = D_{ij}(\mathbf{p}^r(\mathbf{w}))$ the resulting demand for each product.

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[13]The observability assumption is made for simplicity, to avoid technicalities such as a the definition of reasonable conjectures in the event of unexpected offers, and equilibrium existence problems –see Rey and Vergé (2003).

[14]A manufacturer can choose not to offer a contract, by "proposing" prohibitely high wholesale prices or franchise fees.

In the first stage each manufacturer i chooses wholesale prices w_{i1} and w_{i2}, and franchise fees F_{i1} and F_{i2} so as to maximize its profit subject to retailers' participation constraints. Since retailers can only accept both offers or earn zero profit, manufacturer i seeks to solve:

$$\max_{w_{i1}, w_{i2}, F_{i1}, F_{i2}} (w_{i1} - c)D_{i1}^r(\mathbf{w}) + F_{i1} + (w_{i2} - c)D_{i2}^r(\mathbf{w}) + F_{i2},$$

$$s.t. \quad (p_{i1}^r(\mathbf{w}) - w_{i1} - \gamma)D_{i1}^r(\mathbf{w}) - F_{i1} + (p_{j1}^r(\mathbf{w}) - w_{j1} - \gamma)D_{j1}^r(\mathbf{w}) - F_{j1} \geq 0$$

$$(p_{i2}^r(\mathbf{w}) - w_{i2} - \gamma)D_{i2}^r(\mathbf{w}) - F_{i2} + (p_{j2}^r(\mathbf{w}) - w_{j2} - \gamma)D_{j2}^r(\mathbf{w}) - F_{j2} \geq 0$$

Since the participation constraints are clearly binding, this program is equivalent to:

$$\max_{w_{A1}, w_{A2}} \Pi_i^r(\mathbf{w}) \equiv \sum_{,j=1,2} \left(p_{ij}^r(\mathbf{w}) - c - \gamma \right) D_{ij}^r(\mathbf{w}) + (p_{hj}^r(\mathbf{w}) - w_{hj} - \gamma)D_{hj}^r(\mathbf{w}).$$

In other words, through the franchise fees each manufacturer i internalizes the impact of its pricing decisions on (i) the entire margins $(p_{ij} - c - \gamma)$ on its own product (for $i = 1, 2$) and (ii) the retail margins $(p_{hj} - w_{hj} - \gamma)$ on the rival's product; it therefore ignores the rival's upstream margins $(w_{hj} - c)$. As a result, (symmetric) equilibrium prices are somewhat competitive (i.e., below the monopoly level) whenever the retail response to wholesale prices satisfies weak regularity conditions.

Assumption 1

i) For symmetric wholesale prices $w_{i1} = w_{i2} = w_i$ (with $i = A, B$), equilibrium retail prices are symmetric, $p_{i1}^r = p_{i2}^r \equiv \widetilde{p}(w_i, w_h)$ for $i \neq h = A, B$, thus leading to symmetric quantities $D_{i1}^r = D_{i2}^r \equiv \widetilde{D}(w_i, w_h)$; moreover:

ii) an increase in all wholesale prices increases retail prices: $\partial_1 \widetilde{p} + \partial_2 \widetilde{p} > 0$;

iii) an increase in the wholesale prices of one manufacturer decreases the demand for that manufacturer and increases the demand for its rival: $\partial_1 \widetilde{D} < 0 < \partial_2 \widetilde{D}$.

These conditions are for example satisfied when retail prices are strategic complements and direct effects dominate indirect ones.[15] In particular, they are satisfied in the linear demand case analyzed in section 5.

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[15]For example, $\partial_1 \widetilde{p} \geq \partial_2 \widetilde{p} \geq 0$ implies $\partial_1 \widetilde{D} < 0$ and $\partial_1 \widetilde{p} > \left(-\lambda_R / \hat{\lambda}_R \right) \partial_2 \widetilde{p} \geq 0$, where λ_R (respectively, $\hat{\lambda}_R$) denotes the impact on demand for the "product" ij of a uniform increase in retailer j's (respectively, retailer k's) prices, implies $\partial_2 \widetilde{D} > 0$.

Proposition 1 *Without RPM, under Assumption 1 any symmetric equilibrium of the form $w_{ij} = w^e$ and $p_{ij} = p^e$ is such that retailers earn zero profit and*

$$c < w^e < p^e < p^M.$$

Proof. See Appendix A. ∎

If there were a monopoly at either level, (public) two-part tariffs would instead lead to retail prices equal to monopoly prices. If for example a single manufacturer were selling through competing retailers, it would set wholesale prices high enough to induce retail prices at the monopoly level – and could then recover retail margins through franchise fees. Likewise, if a single retailer were acting as a common agent for several manufacturers, as in Berheim-Whinston (1985), manufacturers would sell at marginal cost, thereby inducing the retailer to adopt monopoly prices, and could recover again profits through franchise fees.

Here, in contrast, the existence of competition at both the upstream and downstream levels maintains retail prices below the monopoly level. This is because, as noted above, manufacturers only take into account the retail margin on their rival's products, and thus fail to account that a reduction in their own prices hurt their rival's upstream profits. If for example retailers are pure Bertrand competitors (that is, assuming away any downstream differentiation), they are both active only if wholesale prices are symmetric ($w_{ij} = w_i$), in which case retail prices simply reflect wholesale ones ($p_{ij} = w_i$) and franchise fees are zero, so that manufacturer i's profit reduces to

$$\Pi_i^r(\mathbf{w}) \equiv (w_i - c - \gamma)\,\hat{D}_i(w_A, w_B),$$

where $\hat{D}_i(p_A, p_B)$ represents the demand for product $i = A, B$ when the price of product A (respectively B) is p_A (respectively p_B). The situation is then formally the same as if the two manufacturers were directly competing against each other.

3.2   Resale Price Maintenance

Suppose now that manufacturers can resort to RPM. Imposing retail prices is then always a dominant strategy for the manufacturers: Whatever the strategy adopted by its rival,

a manufacturer can always replicate with RPM the retail prices that would emerge and the profits it would earn without RPM.

Under RPM, the last stage of the game is straightforward. In the first stage, given the market break-down assumption, if manufacturer h imposes retail prices (p_{h1}, p_{h2}), manufacturer i will choose wholesale prices w_{i1} and w_{i2}, retail prices p_{i1} and p_{i2} and franchises F_{i1} and F_{i2} so as maximize as before its profit, given the retailers' participation constraints:

$$\max_{w_{i1}, w_{i2}, p_{i1}, p_{i2}, F_{i1}, F_{i2}} (w_{i1} - c)D_{i1}(\mathbf{p}) + F_{i1} + (w_{i2} - c)D_{i2}(\mathbf{p}) + F_{i2}$$

$$s.t. \quad \begin{array}{c} (p_{i1} - w_{i1} - \gamma)D_{i1}(\mathbf{p}) - F_{i1} + (p_{h1} - w_{h1} - \gamma)D_{h1}(\mathbf{p}) - F_{h1} \geq 0, \\ (p_{i2} - w_{i2} - \gamma)D_{i2}(\mathbf{p}) - F_{i2} + (p_{h2} - w_{h2} - \gamma)D_{h2}(\mathbf{p}) - F_{h2} \geq 0, \end{array}$$

or, since the participation constraints are clearly binding:

$$\max_{p_{i1}, p_{i2}} \Pi(\mathbf{p}, w_{h_1}, w_{h_2}) \equiv (p_{i1} - c - \gamma)D_{i1}(\mathbf{p}) + (p_{i2} - c - \gamma)D_{i2}(\mathbf{p}) \tag{2}$$
$$+ (p_{h1} - w_{h1} - \gamma)D_{h1}(\mathbf{p}) + (p_{h2} - w_{h2} - \gamma)D_{h2}(\mathbf{p}).$$

As before, each manufacturer fully internalizes (through the franchise fees that it can extract from the retailers) the entire margins on its product, but internalizes only the retail margins on the rival's product. But now, as the program (2) makes clear, since the manufacturer controls retail prices, its wholesale prices have no longer any effect on its profit (without RPM, these wholesale prices had an indirect effect, as they affected retailers' prices); however, these wholesale prices affect the rival's profit (which only account for the retail margins on the manufacturer's product) and thus the equilibrium behavior of the competitor. As a result, there usually exists a continuum of equilibria – one equilibrium for every profile of wholesale prices $w = (w_{A1}, w_{B1}, w_{A2}, w_{B2})$.

If for example manufacturer h sells at cost $(w_{h1} = w_{h2} = c)$, (2) becomes:

$$\max_{p_{i1}, p_{i2}} (p_{i1} - c - \gamma)D_{i1}(\mathbf{p}) + (p_{h1} - c - \gamma)D_{h1}(\mathbf{p})$$
$$+ (p_{i2} - c - \gamma)D_{i2}(\mathbf{p}) + (p_{h2} - c - \gamma)D_{h2}(\mathbf{p}).$$

Manufacturer i then fully internalizes the impact of its retail prices on aggregate profits, and thus sets its prices at the monopoly level if manufacturer h does so; there thus exists an equilibrium in which both manufacturers set wholesale prices to c and retail prices to the monopoly level, and share monopoly profits. RPM can thus prevent the exercise of

interbrand as well as intrabrand competition.[16]

If instead manufacturers adopt wholesale prices above cost, since they do not take into account upstream margins on the rival brand they will tend to choose more aggressive retail prices for their own brand. As a result, one would expect an inverse relation between wholesale and retail prices. The next proposition confirms this intuition under the following regularity condition:

Assumption 2

i) For $w_{h1} = w_{h2} = w_h$ and $p_{h1} = p_{h2} = p_h$, the revenue function Π is single-peaked in (p_{i1}, p_{i2}) and maximal for symmetric prices, $\hat{p}_{i1} = \hat{p}_{i2} = \hat{p}(p_h, w_h)$;

ii) $\hat{p}(.,.)$ satisfies $0 < \partial_1 \hat{p} < 1$ and, for any w, the function $p \to \hat{p}(p, w)$ has a unique fixed point.

This assumption first states that retail price responses are well defined and preserve symmetry; in addition, for any symmetric profile of wholesale prices, there exists a unique, stable, "retail equilibrium" (looking at a reduced strategic game where manufacturers would simply choose retail prices, taking wholesale prices as given). We have:

Proposition 2 *If RPM is allowed then:*

i) There exists a symmetric subgame perfect equilibrium in which wholesale prices are equal to cost ($w^* = c$), retail prices are at the monopoly level $\left(p^* = p^M\right)$, retailers earn zero profit and manufacturers share equally the monopoly profit.

ii) Under Assumption 2, there exists a decreasing function $p^*(.)$ such that, for any w^* there exists a symmetric subgame perfect equilibrium in which wholesale prices are equal to w^*, retailers earn zero profit and retail prices are equal to $p^*(w^*)$.

Proof. See Appendix B. ∎

There is thus a continuum of symmetric equilibria and within this set of equilibria, retail prices are inversely related to wholesale prices. Retail prices are at the monopoly level when wholesale prices are equal to cost – in this equilibrium, manufacturers thus

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[16]The argument still applies when marginal costs are not constant, interpreting c as the marginal cost for monopolistic production levels.

"eliminate" any competition and achieve monopoly profits – while upstream mark-ups sustain lower retail prices.[17]

The monopolistic equilibrium relies on the manufacturers' ability to prevent retail prices from falling despite low wholesale prices. Thus, price floors would suffice to maintain the monopoly outcome, while in contrast, maximum resale prices (price ceilings) would not help the manufacturers to maintain higher prices – in particular, retailer h would then lower its prices below the monopoly level if it expected retailer h set monopoly prices. This analysis thus justifies the more negative attitude often adopted by competition authorities towards minimum RPM (or imposed prices), compared with maximum RPM

In essence, with RPM the situation is one where manufacturers deal with two, non-competing, common agents. Consider for example the polar case where retailers are pure Bertrand competitors (no downstream differentiation). With RPM the manufacturers eliminate retail competition and de facto allocate half of the demand for their products to each retailer; the monopolistic equilibrium then simply mimics the Bernheim and Whinston (1985) common agency equilibrium (without RPM) within each half-market. The above analysis generalizes this insight to the case where retailers are differentiated. Resorting to RPM generates however a coordination problem that does not arise in the context of a single common agent:[18] there exists indeed here (infinitely) many other equilibria, including very competitive ones. The next subsection addresses this coordination issue.

- **Remark:** *bilateral bargaining power.*

While we have assumed here that manufacturers have all the bargaining power and can make take-it or leave-it offers to retailers, the analysis would remain similar if retailers had

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[17]Conversely, negative upstream margins would sustain retail prices above the monopoly level. The range of equilibrium prices depends on the domain of validity of Assumption 2. For example, for the linear demand used in section 5, any retail price from $c + \gamma$ up to the maximal price for which quantities are 0 can be sustained.

[18]With a single common retailer, there exists a unique *symmetric* equilibrium in two-part (or non-linear) tariffs, which yields the monopoly outcome; however, introducing RPM in that case would again generate a multiplicity of equilibria, since as above each manufacturer would respond to its rival's wholesale price and be indifferent as to its own wholesale price. Introducing RPM in that case is not helpful and even possibly harmful for the manufacturers.

some bargaining power. Suppose for example that retailers have all the bargaining power. In the first stage, given the prices (p_{Ak}, p_{Bk}) adopted by retailer k, retailer h would then propose wholesale prices (w_{Ah}, w_{Bh}), retail prices (p_{Ah}, p_{Bh}) and franchise fees (F_{Ah}, F_{Bh}) so as to maximize its profit, given the manufacturers' participation constraints:

$$\max \quad (p_{Ah} - w_{Ah} - \gamma) D_{Ah}(\mathbf{p}) - F_{Ah} + (p_{Bh} - w_{Bh} - \gamma) D_{Bh}(\mathbf{p}) - F_{Bh}$$

$$s.t. \quad \begin{aligned} (w_{Ah} - c) D_{Ah}(\mathbf{p}) + F_{Ah} + (w_{Ak} - c) D_{Ak}(\mathbf{p}) + F_{Ak} \geq 0, \\ (w_{Bh} - c) D_{Bh}(\mathbf{p}) + F_{Bh} + (w_{Bk} - c) D_{Bk}(\mathbf{p}) + F_{Bk} \geq 0, \end{aligned}$$

or, since participation constraints are clearly binding:

$$\max_{(p_{Ah}, p_{Bh})} \Pi(\mathbf{p}, w_{Ak}, w_{Bk}) = (p_{Ah} - c - \gamma) D_{Ah}(\mathbf{p}) + (p_{Bh} - c - \gamma)$$

$$+ (w_{Ak} - c) D_{Ak}(\mathbf{p}) + (w_{Bk} - c) D_{Bk}(\mathbf{p})$$

With RPM, there would again exist an equilibrium in which prices are at the monopoly level – although now the retailers rather than the manufacturers would get all the profits. To achieve this, however, instead of removing the upstream margin ($w^* = c$) the retailers would remove the downstream margin $\left(w^* = p^M\right)$, so as to allow each of them to internalize the whole margin on the manufacturers' sales through the other retailer – franchise fees would then be used to extract the manufacturers' expected revenues.

3.3 Effort and Equilibrium Selection

The multiplicity of equilibria stressed above comes from the fact that manufacturers have more control variables than "needed." Retail prices allow a manufacturer to monitor the joint profits earned together with the retailers, while both franchise fees and wholesale prices can be used to recover retailers' profits. The multiplicity of equilibria then derives from the fact that a manufacturer is indifferent with respect to the level of its wholesale prices, which however drive its rival's decisions.

The multiplicity of equilibria generates various types of problems. First, it creates a coordination problem, all the more severe that there are infinitely many equilibria.[19] While the monopolistic equilibrium always exists (even in the absence of Assumption 2) and yields monopoly profits, the manufacturers may end up being locked into a "bad"

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[19]While the previous proposition shows that there exists a continuum of symmetric equilibria, the same logic allows as well to construct equilibria around asymmetric profiles of wholesale prices.

equilibrium. Second, it is difficult to draw policy implications, since some equilibria are better and others worse than the equilibrium that would emerge in the absence of RPM. To circumvent this issue, we now introduce a (non contractible) retail effort which affects the demand and is chosen by the retailers at the same time as they set prices. The level of this effort will be affected by wholesale prices, so that there are no longer more control variables (retail price, franchise and marginal wholesale price) than targets (industry profits, profit sharing and effort level); as a consequence, the multiplicity disappears.

To fix ideas, suppose that the demand for a given product depends on both the retail prices and a retail effort e, as follows:

$$Q_{ij}\left(\mathbf{p}, e_{ij}\right) \equiv D_{ij}\left(\mathbf{p}\right) + \eta\phi(e_{ij}),$$

where $\eta > 0$ is a scaling parameter and ϕ satisfies $\phi' > 0 > \phi''$ and $\phi(0) = 0$. The effort e_{ij} costs $\eta\psi(e_{ij})$ to the retailer, where ψ satisfies $\psi', \psi'' > 0$ and $\psi(0) = 0$. The second stage of game G is then modified as follows:

2. Downstream Competition: if all offers have been accepted, retailers simultaneously set their retail prices (as imposed by the manufacturer under RPM) and choose their effort levels (one for each product they sell); demands are satisfied and payments made according to contracts. Otherwise, no product is sold and all firms earn zero profit.

Under RPM, in this last stage retailer j chooses its efforts e_{ij} and e_{hj} so as to maximize:

$$(p_{ij} - w_{ij} - \gamma)Q_{ij}\left(\mathbf{p}, e_{ij}\right) - F_{ij} - \eta\psi\left(e_{ij}\right) + (p_{hj} - w_{hj} - \gamma)Q_{hj}\left(\mathbf{p}, e_{hj}\right) - F_{hj} - \eta\psi\left(e_{hj}\right).$$

This leads to an effort level e_{ij} which depends on the retail price p_{ij} and on the wholesale price w_{ij}, and which we denote by $e_{ij}^r \equiv e_{ij}^r(p_{ij}, w_{ij})$.

Note that the adjusted monopoly profit

$$\widetilde{\Pi}\left(\mathbf{p}, \mathbf{e}\right) = \sum_{\substack{i=A,B \\ ,j=1,2}} \left[\left(p_{ij} - c - \gamma\right)Q_{ij} - \eta\psi\left(e_{ij}^r\right)\right],$$

is now concave in $(\mathbf{p}, \mathbf{e})$; we denote by $p^M\left(\eta\right)$ the adjusted (symmetric) monopoly price.

In contrast with the previous situation, manufacturers are no longer indifferent as to the choice of their wholesale prices, since they affect retail efforts. To provide adequate

incentives, they must make retailers residual claimants for their efforts, which requires wholesale prices equal to marginal cost. As a result:

Proposition 3 *When RPM is allowed, for any $\eta > 0$ in equilibrium manufacturers use RPM, set wholesale prices equal to the marginal cost $\left(w_{ij}^* = c\right)$ and retail prices at the monopoly level $\left(p_{ij}^* = p^M\left(\eta\right)\right)$, and share the monopoly profits.*

Proof. See Appendix C. ∎

The proposition establishes that, whatever the impact of the effort (even infinitesimal), in equilibrium the wholesale prices are always equal to the marginal cost. Therefore, the only equilibria that are robust to the introduction of retail efforts lead to the monopoly outcome (in particular, $p^* \to p^M$ when $\eta \to 0$).[20] This result reinforces the presumption that RPM has a negative impact on welfare, by allowing firms to eliminate any competition that might otherwise prevail.

4 "Competitive" Retailers

The above "market break-down" assumption imposes double common agency as the equilibrium market structure and moreover implies that manufacturers extract all profits. We now relax this assumption in order to endogenize the market structure and the distribution of profits. While this assumption is clearly ad-hoc and, as such, unrealistic, it captures the essential ingredients of (potential) retail competition. Indeed, if there is no retail bottleneck, in the sense that manufacturers can find equally efficient alternative channels for each relevant retail location, then as in the previous section the following two features are likely to hold:

- retailers have no market power, so that manufacturers extract all profits;

- manufacturers cannot exclude their rivals from any retail location.

The analysis of the precedent section is then likely to prevail: manufacturers are deemed to "accommodate" each other and their best strategy is to maintain monopoly

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[20]As in standard (single) common agency situations, there still exist several equilibria but they only differ on how the manufacturers share the monopoly profit.

prices and share the monopoly profits, which they can indeed achieve by adopting common retailers (rather than marketing their products themselves or through different retailers) and eliminating intrabrand competition between these common retailers through RPM.

To capture the absence of retail bottleneck in a simple way, we now interpret D_{ij} as the demand for brand $i = A, B$ at retail location $j = 1, 2$. and assume that, for each retail location, each manufacturer has access to at least one potential alternative, equally efficient retailer. Manufacturers can thus either distribute their products through the established retailers (who can carry both brands) or bypass them and use instead alternative exclusive retailers. We denote 1_A, 1_B, 2_A and 2_B the alternative retailers and assume that they face the same retail cost γ as the established retailers. In order to stick as much as possible to the above analysis, we assume that manufacturers first try to deal with established retailers and therefore adapt the competitive game G by modifying the second stage only:

2. Downstream competition:

   (a) Whenever a manufacturer has an offer rejected by a retailer, it can offer a contract to its relevant alternative retailer. All such offers to the alternative retailers are made simultaneously and public. Acceptance decisions are also simultaneous and public.

   (b) Retailers having accepted an offer simultaneously set their retail prices (as possibly imposed by the manufacturer under RPM); demands are satisfied and payments made according to the contracts.

The first stage of the game allows the manufacturers to adopt a common retailer at each location, while the second stage captures the absence of retail bottleneck. A manufacturer whose offer is rejected in stage 1 then markets its product through its alternative retailer as long as there is a positive demand for its product. This, in effect, prevents manufacturers from trying to foreclose their rivals' access to consumers; it also ensures that retailers are willing to accept any offer that gives them non-negative profits. More generally, alternative retailers need not be exclusive and might well deal with both manufacturers; conversely, manufacturers could also make offers to alternative retailers

at stage one as well (see the discussion below). This would not affect the essence of the analysis but would however complicate its exposition, by increasing the number of cases to be considered.

We now show that the results of propositions 1, 2 and 3 remain valid here for a reasonably large range of situations. More precisely, in the absence of RPM, proposition 1 still applies to any equilibrium with double common agency, and thus ensures that prices are somewhat competitive; prices are moreover likely to be even more competitive if manufacturers do not adopt common retailers, since in that case manufacturers and retailers no longer internalize the impact of their pricing decisions on their rivals' downstream margins.[21]

When RPM is allowed, the preliminary analysis outlines a candidate equilibrium where manufacturers share the monopoly profit: in this "monopolistic" candidate equilibrium, manufacturers adopt the "established" retailers 1 and 2 as common agents, sell at cost and impose monopolistic retail prices, and extract all profits through franchise fees. By construction, no deviation from this monopolistic equilibrium is profitable for a manufacturer if retailers keep accepting the rival's offers.[22] However, by deviating and opting for a more aggressive behavior, a manufacturer can now discourage a retailer from carrying the rival brand.[23] In essence, such moves allow the deviating manufacturer to act as a Stackelberg leader: imposing a price below the monopoly level forces the rival to deal with the alternative retailers and therefore to set retail prices that "best respond" to the deviating manufacturer's prices. Such deviations will thus be unattractive when, as one may expect, Stackelberg profits – which involve some competition – are lower than monopoly profits.

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[21]Due to coordination issues, there always exists an equilibrium where both manufacturers bypass the "established" retailers and use the alternative "exclusive" retailers instead, even if doing so is less profitable than the double common agency outcome: indeed, in such an equilibrium, each manufacturer is indifferent between waiting and dealing with the alternative retailers or relying on the "established" retailers, who would act as exclusive agents anyway.

[22]Since each manufacturer gets half the monopoly profit when its offers are accepted by the two retailers, and retailers will not accept offers that yield negative profits.

[23]Retailers will refuse the manufacturer's offer, which involves a franchise fee equal to the monopoly profit (per product), whenever they expect rival prices below the monopoly level.

The following proposition confirms this intuition. To introduce the relevant conditions, however, we need to consider two hypothetical scenarios of Stackelberg competition: in the first scenario, the leader (respectively, the follower) produces at cost $c + \gamma$ the "products" $A1$ and $A2$ (respectively, $B1$ and $B2$); in the second scenario, the leader produces the three products $A1$, $A2$ and $B1$ while the follower produces $B2$. The first scenario is thus a mere extension of the standard Stackelberg price competition to a symmetric duopoly in which each firm produces and sells two products, while the second scenario involves asymmetric firms.

Assumption 3 *In the two Stackelberg scenarios just described, per product, the leader's average profit is lower than the monopoly profit.*

In the first scenario, the requirement is satisfied whenever prices are strategic complements: Gal-Or (1985) shows indeed that the leader's profit is then lower than the follower's profit,[24] and since the industry-wide profit cannot exceed the monopoly level, the leader's profit is thus less than half the monopoly profit. Amir and Grilo (1994) note that the comparison between the leader's and the follower's profits is more ambiguous when they are in an asymmetric position, as in the second scenario; however, there is still some competition between the two firms, and since the follower sells one product only, it is likely to be even more agressive, so that the above requirement sounds again quite reasonable. Assumption 3 is for example always satisfied in the linear case analyzed in section 5 as well as when prices are strategic complements and there is strong intrabrand or interbrand competition.[25]

Assumption 4 *The revenue function $\pi_{ij}(p) = (p - c - \gamma) D\left(p, p^M, p^M, p^M\right)$ is single-peaked in p.*

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[24]When prices are strategic complements, L is willing to increase its prices in order to encourage F to (partially) follow-up and, as a result, in equilibrium L's prices are higher than F's ones; thus, F "best responds" to L's comparatively higher prices, while L does not even best respond to F's lower prices.

[25]The second, asymmetric Stackelberg scenario boils down to a symmetric Stackelberg duopoly when there is strong intrabrand and/or interbrand competition. Suppose for example that retailers are perfect substitutes (no downstream differentiation); that is, there is a demand $D_i(p_A, p_B)$ for brand $i = A, B$ and perfect Bertrand between stores. Then, in the asymmetric Stackelberg scenario, L anticipates that F will undercut its price for B (that is, $p_{B2} \leq p_{B1}$) and the analysis is the same as for a standard symmetric Stackelberg duopoly between a leader producing A and a follower producing B.

Assumption 4 is simply a regularity condition ensuring that there is a unique price maximizing $\pi_{ij}(p)$;[26] this condition is clearly satisfied in the linear case analyzed in section 5.

Proposition 4 *Under Assumptions 3 and 4, there exists a subgame perfect equilibrium where manufacturers adopt common retailers (double common agency) and RPM, set wholesale prices to marginal cost $(w^c = c)$ and retail prices to the monopoly level $(p^c = p^M)$, and achieve monopoly profits (that is, retail profits are zero).*

Proof. See Appendix D. ∎

The intuition underlying this result is straightforward. It is impossible for a manufacturer to exclude its competitor from any location, since the rival always finds it profitable to deal with its alternative retailer at that location in the second stage. But then, the best way to "accommodate" the rival manufacturer is by adopting RPM and sharing retailers. As noted in the previous section, RPM eliminates competition between the common agents, and common agency "eliminates" competition between the manufacturers.

Two part tariffs have played an important role in the analysis; franchise fees provide an additional instrument for profit-sharing which, in the absence of RPM, avoids double-marginalization problems; with RPM, franchise fees allow manufacturers to extract all retail revenues and thus encourage them to maintain monopoly prices and profits. However, franchise fees are not essential for the argument and other types of contracts would generate a similar analysis. Consider for example royalties instead of franchise fees. In the absence of RPM, they would eliminate double marginalization as well and, together with RPM, asking each retailer to pay back to the manufacturer a percentage of its total profit (almost half of it, say) would still sustain an equilibrium with monopoly prices.

This proposition thus extends Bernheim and Whinston's insights to the case of "double common agency". Our analyses share two essential "ingredients" that derive from some form of potential competition in the downstream market: (i) retailers accept any offer as

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[26]The profit maximizing price is necessarily below the monopoly level p^M, since (will all derivatives of D evaluated at $\mathbf{p}^M$):

$$\pi'_{ij}\left(p^M\right) = D\left(p^M\right) + \left(p^M - c - \gamma\right)\partial_1 D = -\left(p^M - c - \gamma\right)\left(\partial_2 D + \partial_3 D + \partial_4 D\right) < 0.$$

long as their expected profit is non-negative; and (ii) manufacturers cannot exclude their competitors. This derives here from the manufacturers' ability to use alternative exclusive retailers when an offer has been rejected. Other situations sharing the same ingredients (*i*) and (*ii*) would yield the same outcome.

- This would for example be the case if, instead of using alternative (equally efficient) retailers at each location, the manufacturers had the possibility to sell their products directly to the consumers (at the same cost). Although it might not always be easy and/or costless for the manufacturers to set-up their own retail network, it might not always be an unrealistic assumptiom, especially in sectors where internet sales can be a perfect substitute for in-store sales.

- The situation would be similar if there was a competitive supply of potential retailers for each retail location. Indeed, a similar analysis would prevail if we introduced a second round of offers in which manufacturers whose offer has been rejected in the first round could simply turn to another retailer. This is probably one of the most realistic cases: a manufacturer whose offers has being rejected by a retailers is then unlikely not to deal with another retailer in order to be present in each location.

- Yet another possibility would be to extend Bernheim and Whinston's framework to the multiple retail outlets case, and allow manufacturers to make (withdrawable) offers to several retailers (at any location) at the same time.[27]

The admittedly ad-hoc but simplifying "market break-down" of the previous section is thus not crucial and there exists a wide range of situations for which monopoly prices (through the adoption of common retailers and RPM) constitute a likely outcome.

The equilibrium mutiplicity issue still arises in this context. It is however somewhat less acute than before, since some of the above-described equilibria involve low industry profits and would therefore be destabilized by a manufacturer's attempt to convince established retailers to carry only its own brand – thereby placing this manufacturer in the position of a (admittedly constrained) Stackelberg leader. In addition, the introduction of (arbitrarily

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[27]In a previous version of this paper, we obtained indeed a similar result using a framework more directly inspired by Bernheim and Whinston's original analysis of common agency.

small) retail efforts would again single out the equilibrium where retailers are residual claimants – and retail prices are at the monopoly level.

5   Retail Market Power

We now turn on to situations where manufacturers have no alternative to the established retailers. The existence of retail bottlenecks raises two issues. First, a manufacturer can now try to eliminate its rivals, by inducing retailers to carry exclusively its own brand. While this might induce more competitive outcomes, we show that it may also prevent the emergence of any equilibrium where both brands are proposed at both stores – despite the fact that there is demand for each brand at each store. Second, retailers now have some market power and manufacturers must therefore share the profits with them. As a result, while RPM may again allow manufacturers to maintain monopoly prices, they may favor an equilibrium with lower retail prices in order to reduce retail rents – that is, they may prefer more competitive prices, and have a bigger share of a smaller pie.

To fix ideas, we suppose that only the two established retailers (1 and 2) can reach consumers, as in the "intrinsic common agency" framework exposed in section 3, but retailers are now free to refuse a contract. The second stage of the competitive game is thus now as follows:

2. Downstream competition: The two retailers compete in prices (or charge the retail price imposed by the manufacturer under RPM) for the brands they have accepted to carry; demands are satisfied and payments made according to accepted contracts.

In a double common agency situation, manufacturers must now ensure that retailers get at least as much as they could obtain by selling exclusively the rival brand; as we will see, this implies that manufacturers must leave a rent to retailers – that is, they cannot extract all the industry profits, even if they can make take-it-or-leave-it offers.[28]

The existence of these rents – and the fact that they must be evaluated for asymmetric structures too – somewhat complicates the analysis. We could provide a partial characterization of double common agency equilibria for general demand structures but

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[28]They may be able to reduce retailers' rents by making both exclusive and non-exclusive offers; we rule out this possibility, however, in order to better assess the impact of retail market power.

it is difficult to assess the existence of these equilibria and thus to evaluate the impact of RPM on prices and profits. In order to shed some light, we therefore restrict attention in this section to a linear model where costs are normalized to zero:

$$c = \gamma = 0,$$

and demand is given by:[29]

$$D_{ij}(\mathbf{p}) = 1 - p_{ij} + \alpha p_{hj} + \beta p_{ik} + \alpha\beta p_{hk},$$

with $\alpha, \beta \geq 0$. The parameter α measures the degree of interbrand substitutability; the demands for brands A and B are independent when $\alpha = 0$ and the brands become closer substitutes as α increases. Similarly, β measures the degree of intrabrand substitutability.[30] To ensure that demand decreases when all prices increase (condition (1)), we suppose:

$$\alpha + \beta + \alpha\beta < 1.$$

5.1 Two-Part Tariffs

Starting with the case where RPM is not allowed, we first show that retailers' market power ensure that they earn positive rents whenever they carry both brands.

Given a vector of wholesale prices $w = (w_{ij})_{i,j}$ (with the convention $w_{ij} = \emptyset$ if retailer j does not carry brand i), at the last stage retail competition leads to a vector of equilibrium prices $\mathbf{p}^r(\mathbf{w}) = \left(p_{ij}^r(\mathbf{w})\right)_{i;j}$ (with $p_{ij}^r = \bar{p}_{ij}$ if $w_{ij} = \emptyset$ – see footnote 29) and quantities $D_{ij}^r(\mathbf{w}) = D\left(p_{ij}^r, p_{hj}^r, p_{ik}^r, p_{hk}^r\right)$. At the second stage, retailer 1, say, will accept to carry both brands if, by doing so, it earns profits that are not only non-negative, but also higher than the profit it could derive from selling only one brand. Therefore in any equilibrium

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[29]The expression of the demand is valid as long as all four products are effectively sold. When product ij is not sold (e.g., when the above demand would be negative or when retailer j refuses to carry brand i), the demand for the other products must be evaluated by replacing the price of that product with a virtual price $\bar{p}_{ij}$, computed by equating D_{ij} to zero (i.e., $\bar{p}_{ij} = 1 + \alpha p_{hj} + \beta p_{ik} + \delta p_{hk}$).

[30]For simplicity, we moreover assume that the parameter that measures the effect of an increase in one price on the demand for the rival brand at the rival store is simply the product of the intrabrand and interbrand parameters.

where both retailers carry both products, the contract between A and 1 must satisfy the following three constraints:

$$(p^r_{A1} - w_{A1})D^r_{A1} - F_{A1} + (p^r_{B1} - w_{B1})D^r_{B1} - F_{B1} \geq 0, \tag{3}$$

$$\text{"} \qquad \geq (\tilde{p}_{B1} - w_{B1})\tilde{D}_{B1} - F_{B1}, \tag{4}$$

$$\text{"} \qquad \geq (\hat{p}_{A1} - w_{A1})\hat{D}_{A1} - F_{A1}, \tag{5}$$

where $\tilde{p}_{B1} = p^r_{B1}(\emptyset, w_{B1}, w_{A2}, w_{B2})$ and $\tilde{D}_{B1} = D^r_{B1}(\emptyset, w_{B1}, w_{A2}, w_{B2})$ (respectively $\hat{p}_{A1}$ and $\hat{D}_{A1}$) denote the prices and quantities that result from retail competition when 1 carries only brand B (respectively, brand A).

Since removing one brand from one store eliminates one of the "products" available, it leads to higher per product profits; retailer 1 can therefore guarantee itself a positive profit:

Lemma 1 *Whenever a retailer carries both brands, this retailer earns positive profits.*

Proof. Suppose, say, that retailer 1 refuses to carry brand A and consider the impact on the profits achieved by 1 on B. First, removing product $A1$ increases the demand for all other products. Keeping the prices for the other products fixed, this gives each retailer j an incentive to raise the price for product ij. The nature of the retail price equilibrium in this linear model (strategic complementarity of prices, stability of the equilibrium) then implies that, in the new equilibrium, all retail prices are higher. Finally, in the new equilibrium, retailer 1 faces a higher demand for product $B1$ (both because of the report from product $A1$ and from the increase in the price for the other products) and therefore achieves a greater profit on this product.

This implies $(\tilde{p}_{B1} - w_{B1})\tilde{D}_{B1} > (p^r_{B1} - w_{B1})D^r_{B1}$; therefore (4) implies

$$(p^r_{A1} - w_{A1})D^r_{A1} - F_{A1} > 0. \tag{6}$$

The same argument shows that $(\hat{p}_{A1} - w_{A1})\hat{D}_{A1} > (p^r_{A1} - w_{A1})D^r_{A1}$, and thus (5) implies

$$(p^r_{B1} - w_{B1})D^r_{B1} - F_{B1} > 0. \tag{7}$$

Combining (6) and (7) implies that retailer 1 gets positive rents and that (3) is not binding. ∎

The next Proposition shows that, due to retailers' market power, it may be the case that no symmetric equilibrium exists where both retailers carry both brands.

Proposition 5 *For $\alpha = 0.1$ and $\beta = 0.3$, without RPM there exists no symmetric equilibrium with double common agency.*

Proof. See Appendix E. ∎

Even though there is a positive demand for each brand at each store, there does not always exist an equilibrium where both retailers sell both products. Because manufacturers must now rely on the established retailers to distribute their products, manufacturers have now increased incentives to eliminate competitors by signing up the retailers into exclusive relationships. In addition, manufacturers must now leave a rent to the retailers, to convince them to carry their products. However, they will seek to minimize this rent, which means that, in equilibrium, each retailer is almost indifferent between accepting or refusing to carry each particular brand; this implies that it is indeed quite easy, for a deviating manufacturer, to break this indifference and sign up one or both retailers into an exclusive dealing arrangement. As a result, a manufacturer can actually deviate in a many different ways: it can try to eliminate the rival completely, by signing up both retailers into exclusive relationships, but it can also deviate with only one exclusive dealer (and continue to use the second retailer as a common agent). In addition, in contrast with the standard single common agent case (i.e., two producers selling their products through a single retailer), the rent that manufacturer i must leave to retailer k now depends on the tariff offered to retailer h; this implies that, when deviating towards exclusive deals, a manufacturer can affect the rent it has to leave to each retailer. As a result of this richness of possible deviations, there does not always exist a "double common agency" equilibrium, in contrast to the single agent case, in which there always exists a common agency equilibrium.

5.2 Resale Price Maintenance

When manufacturers impose retail prices, in any symmetric equilibrium where both retailers carry both brands, the contract (w, p, F) must meet the following two constraints

(where $D\left(p_{ij}, \emptyset, p_{hj}, p_{hk}\right)$ denotes the demand for brand i at retailer j when this retailer carries only that brand):

$$(p-w)D - F + (p-w)D - F \;\geq\; 0 \tag{8}$$

$$'' \;\geq\; (p-w)D\left(p, \emptyset, p, p\right) - F \tag{9}$$

Since removing a product increases the demand for the remaining ones, (9) is the relevant constraint and retailers therefore earn a positive rent whenever the imposed retail price is higher than the wholesale price. The next proposition shows that such equilibria exist and describe some of their properties:

Proposition 6 *There exist ranges of values for α and β such that, with RPM, there exists a continuum of symmetric equilibria with double common agency, with contracts of the form $w_{ij} = w^*$, $p_{ij} = p^*$ such that:*

- *$p^* \in \left[\underline{p}\left(\alpha, \beta\right), \overline{p}\left(\alpha, \beta\right)\right]$;*

- *$w^* \in \left[\underline{w}\left(\alpha, \beta\right), \underline{p}\left(\alpha, \beta\right)\right]$ and is inversely related to p^*:*

$$p^* = p^*\left(w^*\right) = \frac{1 - \alpha(1-\beta)w^*}{2(1 - \alpha - \beta)},$$

  so that $p^*\left(.\right)$ decreases from $\overline{p} > p^M$ for $w^* = \underline{w}$ to $p^* = \underline{p} < p^M$ for $w^* = \underline{p}$;

- *retailers' profits are equal to $(p^* - w^*)\left[D\left(p^*, \emptyset, p^*, p^*\right) - D^*\right]$, increase in p^* as long as $p^* \leq p^M$;*

- *manufacturers' profits are a decreasing function of p^*.*

Proof. See Appendix F. ∎

Note that proposition 6 only provides sufficient conditions for the existence of symmetric equilibria with double common agency. There may exist other equilibria, including other symmetric double common agency equilibria. Figure 1 represents the range of values for which the existence result of proposition 6 applies.

Despite the presence of retail rents, the equilibrium retail price is still inversely related to the equilibrium wholesale price. Two effects are now at work. First, as in the absence of
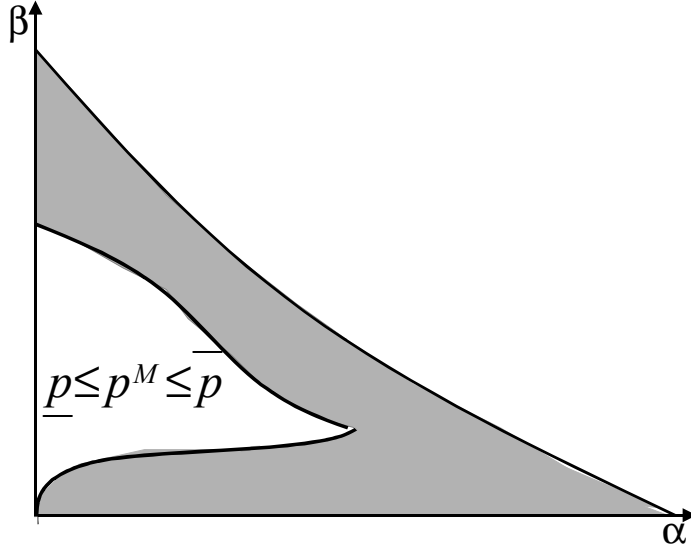
Figure 1: Existence of a symmetric double common agency equilibria

retail rents, raising manufacturer h's wholesale prices reduces manufacturer i's incentives to increase sales of h's products, thereby inducing manufacturer i to lower its retail prices. However, increasing manufacturer h's wholesale prices also reduces each retailer's rent, which is given by:

$$(p_h - w_h)D\left(p_h, \emptyset, p_h, p_i\right) - F_h.$$

This second effect tends to mitigate manufacturer i's incentives to keep low prices in order to reduce retail rents, but is dominated by the first one in this linear model.

Because of this second effect, however, the equilibrium retail price is below the monopoly level when wholesale prices are equal to marginal cost: for $w^* = 0$,

$$p^*\left(0\right) = \frac{1}{2(1 - \alpha - \beta)} < p^M.$$

However, since $\bar{p} > p^M$, there exists $w^M \in [\underline{w}, 0]$ such that $p^*\left(w^M\right) = p^M$: manufacturers can sustain monopoly prices, but to do so they must set wholesale prices below their marginal cost of production.

Subsidizing wholesale prices increases retail rents, however. In equilibrium, this rent

(per retailer and per brand) is equal to:[31]

$$\pi_R^* = (p^* - w^*)\left[D\left(p^*, \emptyset, p^*, p^*\right) - D\left(p^*, p^*, p^*, p^*\right)\right]$$
$$= \alpha(p^* - w^*)D^*.$$

Therefore,

$$\frac{1}{\alpha}\frac{d\pi_R^*}{dp^*} = \frac{d\left(p^* - w^*\right)}{dp^*}D^* + (p^* - w^*)\frac{dD^*}{dp^*}.$$

Given the inverse relationship between p^* and w^*, the mark-up $(p^* - w^*)$ increases with p^* and this effect dominates when p^* is small (namely here, as long as p^* remains below the monopoly level), since then $(p^* - w^*)$ is small and D^* is large.

Manufacturers' profits (per retailer) are of the form:

$$\pi_P^* = \underbrace{p^*D^*}_{\text{Industry Profit}} - \underbrace{\pi_R^*.}_{\text{Rent to be left to the retailer}}$$

Hence, starting from $p^* = \underline{p}$, manufacturers face a trade-off between increasing industry profits (by raising retail prices to the monopoly level) and reducing retail rents (by maintaining low retail prices). Proposition 6 shows that in this linear model, the rent effect dominates; therefore:

Corollary 1 *Among the equilibria with double common agency described in proposition 6, the most profitable one for the manufacturers is the equilibrium with the lowest retail price (and thus the highest wholesale price).*

In contrast, the most profitable candidate equilibrium for the retailers entails a retail price exceeding the monopoly level –and if only lower prices can actually be sustained in equilibrium, retailers will prefer the equilibrium with the highest price.

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[31]By definition, $D\left(p^*, \emptyset, p^*, p^*\right) = D\left(p^*, \tilde{p}, p^*, p^*\right)$, where $\tilde{p}$ is such that $D\left(\tilde{p}, p^*, p^*, p^*\right) = 0$, that is, $\tilde{p} = 1 + (\alpha + \beta + \alpha\beta)\,p^*$. Hence:

$$D\left(p^*, \emptyset, p^*, p^*\right) = 1 - p^* + \alpha\tilde{p} + \beta p^* + \alpha\beta p^*$$
$$= (1 + \alpha)\,D\left(p^*, p^*, p^*, p^*\right).$$

6   Conclusion

This paper provides a basis for competition authorities' tough attitude towards RPM. In a context of interlocking relationships where competing retailers carry several competing brands, and as long as retailers' rents are eliminated and that manufacturers are prevented from trying to exclude their rival, RPM allows firms to maintain monopoly prices and thus defeat both upstream and downstream competition. This is the case even when retail prices are set independently for each retailer, and vertical contracts are negotiated bilaterally and independently from each other (purely "vertical" RPM).

The situation is however more complex when imperfect competition among retailers generate rents downstream. First, equilibria where competing retailers carry competing brands may no longer exist, even if demand conditions would make this outcome desirable; RPM may still allow firms to generate prices closer to the monopoly level – as well as lower than without RPM. Second, which price level will prevail depends on how firms coordinate their equilibrium behavior; retailers favor high prices while manufacturers prefer low prices in order to minimize retailers' rents.

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A   Proof of Proposition 1

We first show that equilibrium upstream margins are positive ($w^e > c$). The conclusion then follows from the fact that manufacturers fail to account for (and thus "free-ride" on) their rivals' upstream margins.

- $w^e > c$. At a symmetric equilibrium of the form ($p_{ij} = p^e, w_{ij} = w^e$), manufacturer i must find it optimal to choose $w_{i1} = w_{i2} = w^e$ when its rival adopts $w_{h1} = w_{h2} = w^e$; $w = w^e$ must therefore maximise

$$2 \left[(\widetilde{p}(w, w^e) - c - \gamma)\widetilde{D}(w, w^e) + (\widetilde{p}(w^e, w) - w^e - \gamma)\widetilde{D}(w^e, w) \right].$$

The first-order condition yields (with D evaluated at p^e and the derivatives of $\widetilde{D}$ and $\widetilde{p}$ evaluated at (w^e, w^e)):

$$(\partial_1\widetilde{p} + \partial_2\widetilde{p}) D + (p^e - c - \gamma) \partial_1\widetilde{D} + (p^e - w^e - \gamma) \partial_2\widetilde{D} = 0, \tag{10}$$

implying

$$(\partial_1\widetilde{p} + \partial_2\widetilde{p}) D + \left(\partial_1\widetilde{D} + \partial_2\widetilde{D} \right)(p^e - w^e - \gamma) = -(w^e - c) \partial_1\widetilde{D}. \tag{11}$$

Note that

$$\partial_1\widetilde{D} = \lambda_M \partial_1\widetilde{p} + \hat{\lambda}_M \partial_2\widetilde{p} \text{ and } \partial_2\widetilde{D} = \lambda_M \partial_2\widetilde{p} + \hat{\lambda}_M \partial_1\widetilde{p},$$

where $\lambda_M \equiv \partial_1 D + \partial_3 D$ represents the marginal impact on the demand for "product" ij of a uniform increase in the retail prices for manufacturer i, while $\hat{\lambda}_M \equiv \partial_2 D + \partial_4 D$ represents instead the impact of the rival manufacturer's retail prices. Therefore, (11) can be rewritten as

$$(\partial_1\widetilde{p} + \partial_2\widetilde{p}) [D + \lambda (p^e - w^e - \gamma)] = -(w^e - c) \partial_1\widetilde{D}, \tag{12}$$

where $\lambda \equiv \lambda_M + \hat{\lambda}_M$ represents the impact on demand of a uniform increase in all retail prices and is thus negative. But a symmetric retail equilibrium is characterized by the first-order condition:

$$D = -\lambda_R (p^e - w^e - \gamma), \tag{13}$$

where $\lambda_R \equiv \partial_1 D + \partial_2 D$ represents the impact on the demand for "product" ij of a uniform increase in retailer j's prices. Combining (12) and (13) yields

$$(\partial_1 \widetilde{p} + \partial_2 \widetilde{p}) \, \hat{\lambda}_R \, (p^e - w - \gamma) = - (w^e - c) \, \partial_1 \widetilde{D}, \tag{14}$$

where $\hat{\lambda}_R \equiv \partial_3 D + \partial_4 D = \lambda - \lambda_D$ represents the marginal impact on demand of a simultaneous increase in the rival retailer's prices and is thus positive. Note that $\lambda_R < 0$ (since $\lambda < 0 < \hat{\lambda}_R$), and thus (13) implies $p^e \geq w^e + \gamma$. But then, since $\partial_1 \widetilde{p} + \partial_2 \widetilde{p} > 0$ and $\partial_1 \widetilde{D} < 0$ from Assumption 1, (14) implies $w^e > c$.

- $p^e < p^M$. The first-order condition (10) can also be rewritten as:

$$(\partial_1 \widetilde{p} + \partial_2 \widetilde{p}) D + \left(\partial_1 \widetilde{D} + \partial_2 \widetilde{D} \right) (p^e - c - \gamma) = (w^e - c) \, \partial_2 \widetilde{D}.$$

Given $w^e > c$ and $\partial_1 \widetilde{D} + \partial_2 \widetilde{D} = \lambda (\partial_1 \widetilde{p} + \partial_2 \widetilde{p})$, with $\partial_1 \widetilde{p} + \partial_2 \widetilde{p} > 0$, this implies:

$$D + \lambda (p^e - c - \gamma) > 0,$$

which in turn implies that, starting from $p = p^e$, a uniform increase in all prices increases the monopoly profit. By assumption the monopoly profit is single-peaked at $p^{\mathbf{M}}$ and thus, $p^e < p^M$. ∎

B   Proof of Proposition 2

If manufacturer h adopts $w_{h1} = w_{h2} = w^*$ and $p_{h1} = p_{h2} = p^*$, from Assumption 2, manufacturer i 's revenue function Π is single-peaked in (p_{i1}, p_{i2}) and maximal for symmetric prices, $\hat{p}_{i1} = \hat{p}_{i2} = \hat{p}(p^*, w^*)$; this price maximizes $\Pi(p, p^*, p, p^*, w^*, w^*)$ and thus solves:

$$\hat{p}(p^*, w^*) = \arg\max_p f(p, p^*, w^*) \equiv (p - c - \gamma) D(p, p^*, p, p^*) + (p^* - w^* - \gamma) D(p^*, p, p^*, p). \tag{15}$$

Obviously, $p^M = \hat{p}(p^M, c)$; thus $(w^* = c, p^* = p^M)$ always constitutes an equilibrium. In addition, for any wholesale price w^* there exists a price p^* satisfying $p^* = \hat{p}(p^*, w^*)$; this price is characterized by the first-order equation:

$$D + \lambda_M (p^* - c - \gamma) + \hat{\lambda}_M (p^* - w^* - \gamma) = 0, \tag{16}$$

with λ_M and $\hat{\lambda}_M$ as defined in the previous section. To establish that p^* decreases when w^* increases, note first that $\partial_{13}^2 f = -\hat{\lambda}_M < 0$. Therefore, a standard revealed preference argument leads to $\partial_2 \hat{p} < 0$. From Assumption 2, $0 < \partial_1 \hat{p} < 1$ the fixed point to $p \to \hat{p}(p, w^*)$ then decreases when w^* increases. ∎

C   Proof of Proposition 3

At the last stage, retailer j chooses e_{ij} and e_{hj} so as to maximize its profit:

$$
\begin{aligned}
\pi_j^D = \ & (p_{ij} - w_{ij} - \gamma)\left(D_{ij}(\mathbf{p}) + \eta\phi\left(e_{ij}^r\right)\right) - F_{ij} - \eta\psi(e_{ij}) \\
& + (p_{hj} - w_{hj} - \gamma)\left(D_{hj}(\mathbf{p}) + \eta\phi\left(e_{hj}^r\right)\right) - F_{hj} - \eta\psi(e_{hj}).
\end{aligned}
$$

This profit function is concave in the effort levels and the adopted effort level $e_{ij}^r \equiv e_{ij}^r(p_{ij}, w_{ij})$ is thus characterized by the first order condition:

$$
\psi'(e_{ij}^r) = (p_{ij} - w_{ij} - \gamma)\phi'(e_{ij}^r). \tag{17}
$$

This reaction function e_{ij}^r does depend on the wholesale price w_{ij}, since

$$
\partial_{w_{ij}e_{ij}}^2 \pi_j^D = -\eta\phi'\left(e_{ij}\right) < 0.
$$

Given the franchise fees it can impose on retailers, manufacturer i seeks to maximize:

$$
\sum_{j=1,2} (p_{ij} - c - \gamma)\left(D_{ij} + \eta\phi\left(e_{ij}^r\right)\right) - \eta\psi\left(e_{ij}^r\right) + (p_{hj} - w_{hj} - \gamma)\left(D_{hj} + \eta\phi\left(e_{hj}^r\right)\right) - \eta\psi\left(e_{hj}^r\right).
$$

The wholesale price w_{ij} affects this profit through only through its impact on e_{ij}^r; furthermore, this profit is strictly concave in e_{ij}^r and maximal for e_{ij}^r such that:

$$
\psi'(e_{ij}^r) = (p_{ij} - c - \gamma)\phi'(e_{ij}^r).
$$

Given the retailers' behavior characterized by (17), the manufacturer now cares about the level of its wholesale price and optimally chooses: $w_{ij} = c$. Thus, in equilibrium all wholesale prices are set to the marginal cost. Taking into account (17), this implies that manufacturer's variable profit coincides with the monopoly profit. Hence, under our assumptions on $\widetilde{\Pi}$, there exists a unique symmetric equilibrium, which is such that $w_{ij}^* = c$ and $p_{ij}^* = p^M(\eta)$. ∎

D Proof of Proposition 4

The proof is constructive and based on the following candidate equilibrium path: both manufacturers offer the contract $C^c = \left(w^c = c, p^c = p^M, F^c = \left(p^M - c - \gamma \right) D \left(\mathbf{p}^M \right) \right)$ to the "established" retailers 1 and 2 and all four offers are accepted at stage 1. Retailers thus make zero profits and manufacturers share the monopoly profit.

No profitable deviation for the retailers

Let us first show that it is actually an equilibrium for the two retailers to accept both offers. It cannot be profitable for a retailer to reject both offers since it would then get zero profit. The only deviation to consider is thus one in which retailer j ($j = 1, 2$) rejects the offer made by manufacturer i (but accepts manufacturer h's offer; $i \neq h \in \{A, B\}$). In this case, because $w^c = c$, manufacturer i will always find it profitable to deal with the alternative retailer j_i, and, under Assumption 4 sets a retail price $p_{ij} = p^{**}$ below the monopoly level:

$$p^{**} = \arg\max_{p} \left(p - c - \gamma \right) D \left(p, p^M, p^M, p^M \right) < p^M.$$

This means that retailer j would then sell a quantity product hj lower than $D \left(\mathbf{p}^M \right)$ and would therefore achieve a negative profit.

No profitable deviation for the manufacturers

If manufacturer i deviates from the equilibrium path at stage 1, this affects the set of contracts that are accepted by the retailers in this first round. We therefore analyze the possible effect of a deviation by manufacturer i on its profit depending on the market structure at the end of stage 1 (that is, on the set of accepted contracts). To be profitable a deviation must be such that manufacturer i achieves a profit strictly larger than $\frac{\pi^M}{2}$. The deviations fall into three categories:

Manufacturer h's offers have both been accepted

We can easily rule out any such deviation. If this is the case, retailers j and k have accepted to pay $F^c = \frac{\pi^M}{4}$ each to the manufacturer h. Moreover, the total profit generated by the

sales of all products cannot be larger than π^M. Since a retailer would never accept an offer if it expects to make losses, the maximum profit that manufacturer i will be able to achieve is $\pi^M - 2F^c = \frac{\pi^M}{2}$.

Manufacturer h's offers have both been rejected

At stage 2, manufacturer h thus deals with the alternative retailers (1_h and 2_h) and thus chooses the prices p_{h1} and p_{h2} that are its best replies to the prices p_{i1} and p_{i2} that have either been accepted by the retailer(s) at stage 1 or that are set by manufacturer i (dealing with retailers 1_i and 2_i) at stage 2. The prices p_{h1} and p_{h2} are therefore equal to the prices that the follower of our first Stackelberg scenario when the leader sets prices p_{i1} and p_{i2}.[32] Given that manufacturer i's profit can only come from the sales of products $i1$ and $i2$ (through either the "established" or the "alternative" retailers), the highest profit it can achieve is the profit of the leader of this first Stackelberg scenario which, by Assumption 3, is lower than $\frac{\pi^M}{2}$.

Only one of manufacturer h's offers has been accepted (say, by retailer j)

At stage 2, manufacturer h thus sells products hk through the "alternative" retailer k_h. Given that $w_{hj} = c$, it chooses the price p_{hj} that maximizes the profit made on the sales of this product. This price is thus the best response of the follower of our second Stackelberg scenario when the leader sets prices p_{i1}, p_{i2} and $p_{h1} = p^M$.[33]

We now have two possibilities to consider depending on whether the deviation is such that retailer j accepts manufacturer i's offer or not.

- Suppose first that the deviation is such that the offer ij is accepted. In this case, through the franchise F_{ij}, manufacturer i can expect to recover the retail profit made by retailer j on product hj minus the franchise $F^C = \frac{\pi^M}{4}$ that retailer j has to pay to manufacturer h. Given that $w_{hj} = c$, the retail margin is in this case equal to the total margin. Manufacturer i thus sets prices p_{i1} and p_{i2} (simultaneously or

---

[32]In this scenario, the leader (respectively, the follower) produces and sells at cost $c + \gamma$ the "products" $A1$ and $A2$ (respectively, $B1$ and $B2$).

[33]In this scenario, the leader produces and sells at cost $c + \gamma$ the "products" $A1$, $A2$ and $B1$, while the follower produces and sells at cost $c + \gamma$ the "products" $B2$.

not) taking into account the total margin (wholesale plus retail) on products $i1$ and $i2$, but also on product hj. Remember however that manufacturer i cannot set the price of this last product (this price is necessarily $p_{hj} = p^M$) and that a share equal to $\frac{\pi^M}{4}$ of the profit made on product hj has to be paid to manufacturer h. The highest profit that manufacturer i can achieve with a such deviation is therefore lower than the leader's profit of the second Stackelberg scenario minus $\frac{\pi^M}{4}$. Under Assumption 3, this profit is lower than $\frac{3\pi^M}{4} - \frac{\pi^M}{4} = \frac{\pi^M}{2}$.

- Suppose finally that the deviation is such that the offer ij is rejected. For such a situation to arise at the end of stage 1, the contracts must be such that retailer j expects its retail profit (on product hj) to cover the franchise to be paid to manufacturer h. This means that the profit generated by product hj has to be larger than $\frac{\pi^M}{4}$. However if this is the case, manufacturer i would rather make an offer to retailer j (rather than distributing the product through the alterntive retailer j_i) to recover all the profit generated above $\frac{\pi^M}{4}$ on product hj. ∎

E Proof of Proposition 5

Set $\alpha = 0.1$ and $\beta = 0.3$. First, if there exists a symmetric equilibrium with double common agency (with $w_{ij} = w, F_{ij} = F(w)$), then retail prices and quantities are respectively given by (with $w = (w, w, w, w)$):

$$p^r(\mathbf{w}) = 0.6802 + 0.6122w,$$
$$q^r(\mathbf{w}) = D(\mathbf{p}^r(\mathbf{w})) = 0.6122 - 0.3489w = 0.3489\,(1.7544 - w)\,.$$

Therefore, necessarily, $w \leq 1.7544$.

We only sketch the proof here; a more detailed proof is available upon request. First, it is shown that the relevant participation constraint is indeed binding in equilibrium. Second, it is shown that eliminating profitable deviations rules out all values $w \leq 1.7544$.

Determination of $F(w)$

Lemma 2 *In any symmetric, double common agency equilibrium, of the form $(w, F(w))$:*

$$F(w) = 2(p^r(\mathbf{w}) - w)q^r(\mathbf{w}) - (p^r_{B1}(w, \emptyset, w, w) - w)D^r_{B1}(w, \emptyset, w, w).$$

Proof. We have already shown in lemma 5 that the only relevant participation constraint is

$$F \leq 2(p^r(\mathbf{w}) - w)q^r(\mathbf{w}) - (p^r_{B1}(w, \emptyset, w, w) - w)D^r_{B1}(w, \emptyset, w, w). \qquad (18)$$

It thus suffices to show that the constraint is indeed binding in equilibrium.

Note first, that if both manufacturers offer (w, F) satisfying (18) to both retailers, it is a continuation equilibrium for the two retailers to accept both offers. This is actually the only continuation equilibrium: clearly, whenever one retailer refuses to carry one or both brands, the other retailer becomes more profitable and is thus willing to carry both brands rather than none; in addition, it can be checked that the other retailer still prefers to carry both brands rather than only one.

Since double common agency is the only continuation equilibrium following any offer of the form $(w_{ij} = w, F_{ij} = F)$ with F satisfying (18), it is now easy to show that there exists a profitable deviation for the manufacturers, if the condition (18) is not binding. If this constraint is not binding, it suffices for manufacturer i, $i = A, B$, to increase the franchise fees

$$F_{i1} = F_{i2} = F(w) + \varepsilon, \text{ with } \varepsilon > 0 \text{ and sufficiently small.}$$

This deviation does not modify the continuation equilibrium and strictly increases manufacturer i's profit. ∎

Under our hypothesis that $\alpha = 0.1$ and $\beta = 0.3$, this condition writes as:

$$F(w) = 0.1179 \left(1.7544 - w\right)^2$$

If there exists a symmetric, double common agency equilibrium of the form $(w, F(w))$, manufacturers' profits are equal to:

$$
\begin{aligned}
\pi_i(w) &= 2\left(wq^r(\mathbf{w}) + F(w)\right) = 2\left(0.3489w(1.7544 - w) + 0.1179(1.7544 - w)^2\right) \\
&= 0.4621\left(1.7544 - w\right)\left(0.8953 + w\right)
\end{aligned}
$$

This profit being necessarily positive, such an equilibrium may exist only if $w \in [-0.8953, 1.7544]$.

Profitable Deviations for the Manufacturers

To show that there exists no symmetric, double common agency equilibrium, we build, for any value of $w \in [-0.8953, 1.7544]$, a profitable deviation for one of the manufacturers.

Lemma 3 *There exists no symmetric double common agency equilibrium for $w < \underline{w} = 0.2435$.*

Proof. To show this, we find a profitable symmetric deviation for manufacturer i (it offers the same contract (v, G) to both retailers), which does not modify the continuation equilibrium (both retailers carry both brands). This deviation must thus satisfy the following three constraints, ensuring that double common agency is a continuation equilibrium:

$$\left(p^r\left(v, w, v, w\right) - v\right) D^r\left(v, w, v, w\right) - G + \left(p^r\left(w, v, w, v\right) - w\right) D^r\left(w, v, w, v\right) - F(w) \geq ...$$

$$... \geq 0 \tag{19}$$

$$... \geq \left(p_{ij}^r\left(w, \emptyset, w, v\right) - w\right) D_{ij}^r\left(w, \emptyset, w, v\right) - F(w) \tag{20}$$

$$... \geq \left(p_{ij}^r\left(v, \emptyset, v, w\right) - v\right) D_{ij}^r\left(v, \emptyset, v, w\right) - G \tag{21}$$

We then assume, that manufacturer i sets a franchise fee $G(v, w)$ saturating condition (20), that is:

$$\begin{aligned} G(v, w) &= \left(p^r\left(v, w, v, w\right) - v\right) D^r\left(v, w, v, w\right) + \left(p^r\left(w, v, w, v\right) - w\right) D^r\left(w, v, w, v\right) \\ &\quad - \left(p_{ij}^r\left(w, \emptyset, w, v\right) - w\right) D_{ij}^r\left(w, \emptyset, w, v\right) \end{aligned}$$

Suppose finally that manufacturer i chooses the wholesale price $v^*(w)$ that maximizes its profit, that is:

$$v^*(w) = \arg\max\left[v D^r\left(v, w, v, w\right) + G(v, w)\right].$$

We then check that this strategy is profitable for any $w < \underline{w} = 0.2435$ (notice that $\underline{w}$ is such that $v^*(\underline{w}) = \underline{w}$), and that double common agency is the unique continuation equilibrium (since the deviation is symmetric, if it is optimal to accept both offers when

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the competitor accepts both, it is a dominant strategy for each retailer to accept both offers). This shows that there exists no symmetric, double common agency equilibrium for $w < 0.2435$. ∎

Lemma 4 *There exists no symmetric double common agency equilibrium for $w > \overline{w} = 0.6423$.*

Proof. Let us again consider a symmetric deviation of the form (v, G). We now look for profitable deviation for manufacturer i such that manufacturer h is completely excluded from the market (that is, such that both retailers accept manufacturer i's offer only). The deviation must therefore satisfy:

$$\left(p_{ij}^r \left(v, \emptyset, v, \emptyset \right) - v \right) D_{ij}^r \left(v, \emptyset, v, \emptyset \right) - G \geq ...$$

$$... \geq 0 \tag{22}$$

$$... \geq \left(p_{hj}^r \left(w, \emptyset, \emptyset, v \right) - w \right) D_{hj}^r \left(w, \emptyset, \emptyset, v \right) - F(w) \tag{23}$$

$$
\begin{aligned}
... \geq & \left(p_{ij}^r \left(v, w, v, \emptyset \right) - v \right) D_{ij}^r \left(v, w, v, \emptyset \right) - G \\
& + \left(p_{hj}^r \left(w, v, \emptyset, v \right) - w \right) D_{hj}^r \left(w, v, \emptyset, v \right) - F(w)
\end{aligned} \tag{24}
$$

Suppose now that manufacturer i chooses the wholesale price $v^*(w)$ and the franchise fee $G^*(w)$ such that constraints (23) and (24) are binding.

We then check that this deviation is profitable for manufacturer i and that the unique continuation equilibrium is such that both retailers accept manufacturer i's offer only for $w > \overline{w} = 0.6423$. ∎

Lemma 5 *There exists no symmetric double common agency equilibrium for $w \in [\underline{w}, \widehat{w}]$, where $\widehat{w} = 0.4973$.*

Proof. To show this, we consider an asymmetric deviation by manufacturer i such that the continuation equilibrium is such that manufacturer h is partially excluded (let

say only product hk is not active). We analyze the following deviation: $w_{ij} = 0.4$ and $w_{ik} = 0$, and the franchise fees (F_{ij} and F_{ik}) are such that:[34]

$$\pi_1\left[(ij, ik), (hj)\right] = \max\left(\pi_1\left[(\emptyset), (hj)\right], \pi_1\left[(ik), (hj)\right]\right)$$

$$\pi_2\left[(ij, ik), (hj)\right] = \max\left(\pi_2\left[(ij, ik), (\emptyset)\right], \pi_2\left[(ij, ik), (hk)\right]\right)$$

where all profits are evaluated at $w_{ij} = 0.4$, $w_{ik} = 0$ and $w_{hj} = w_{hk} = w$.

We then check that the unique continuation equilibrium is such that only product hk is not active, and that this deviation increases manufacturer $i's$ profits for any w between 0.2435 and 0.4973. ∎

Lemma 6 *There exists no symmetric, double common agency equilibrium for $w \in [\widehat{w}, \overline{w}]$.*

Proof. The proof is identical to the proof of lemma 5, but we now consider $w_{ij} = 0.6$ and $w_{ik} = 0$. ∎

This shows that, for $\alpha = 0.1$ and $\beta = 0.3$, there exists no symmetric double common agency equilibrium. ∎

F   Proof of Proposition 6

We look for sufficient conditions on w^* to ensure that $C^* = (w^*, p^*, F^*)$, where

$$p^* = \frac{1 - \alpha(1 - \beta)w^*}{2(1 - \alpha - \beta)} \text{ and } F^* = (1 - \alpha)(p^* - w^*)D\left(\mathbf{p}^*\right),$$

is the equilibrium wholesale contract of a symmetric double common agency equilibrium. We only sketch the proof here; a more detailed proof is available upon request.

Note that we have constrained the retail price p^* to be higher than the marginal wholesale price w^*, therefore imposing $w^* \leq w^{\max} = \frac{1}{2 - \alpha - 2\beta - \alpha\beta}$. Moreover, quantities must be positive, thereby constraining w^* to be such that:

$$q^* = D\left(\mathbf{p}^*\right) \geq 0 \Leftrightarrow w^* \geq w^{\min} = -\frac{1 - \alpha}{\alpha(1 - \alpha - \beta - \alpha\beta)}.$$

---

[34]We define in brackets the set of contracts that are accepted by the retailers. E.g., $[ij, ik, hj]$ means that retailer k has rejected manufacturer h's offer.

The idea of this proof is now to analyze any possible deviation for manufacturer i, and find sufficient conditions to ensure that these deviations are never profitable. Depending on the contracts C_{i1} and C_{i2} offered by manufacturer i, the equilibrium market structure (that is, the set of contracts accepted by the retailers) differs. We therefore analyze the effect of a deviation on manufacturer i's profits depending on the type of the continuation equilibrium. 16 different structures are possible, but a symmetry argument reduces this number to 10. Moreover, three of them can easily be removed: it is indeed never profitable for manufacturer i to induce a continuation equilibrium in which its contracts are both rejected. Structures $(\emptyset)$, $(h1)$ et $(h1-h2)$ are therefore excluded. We then have 7 possible structures to analyze.

Structure 0 : Double common agency, i.e. all the offers are accepted

In order to obtain a continuation equilibrium where both retailers carry both brands, manufacturer i has to propose contracts C_{i1} and C_{i2} such that, for $j \neq k \in \{1,2\}$

$$(p_{ij} - w_{ij})D\left(p_{i1}, p^*, p_{ik}, p^*\right) - F_{ij} + (p^* - w^*)D\left(p^*, p_{ij}, p^*, p_{ik}\right) - F^* \geq ...$$

$$... \geq 0 \tag{25}$$

$$... \geq (p^* - w^*)D\left(p^*, \emptyset, p^*, p_{ik}\right) - F^* \tag{26}$$

$$... \geq (p_{ij} - w_{ij})D\left(p_{ij}, \emptyset, p_{ik}, p^*\right) - F_{ij} \tag{27}$$

If we only consider constraints $(26)_{j=1}$ and $(26)_{j=2}$, the maximal franchises i can set are such that:

$$F_{ij} = (p_{ij} - w_{ij})D\left(p_{ij}, p^*, p_{ik}, p^*\right) + (p^* - w^*)\left(D\left(p^*, p_{ij}, p^*, p_{ik}\right) - D\left(p^*, \emptyset, p^*, p_{ik}\right)\right),$$

and its maximal profit is therefore:

$$\pi_i(p_{ij}, p_{ik}) = \sum_{j \neq k = 1,2} \left(p_{ij}D\left(p_{ij}, p^*, p_{ik}, p^*\right) + (p^* - w^*)\left(D\left(p^*, p_{ij}, p^*, p_{ik}\right) - D\left(p^*, \emptyset, p^*, p_{ik}\right)\right)\right).$$

However, this profit is maximized for $p_{ij} = p_{ik} = p^*$. Such a deviation can never be strictly profitable for manufacturer i.

Structure $1: (ij - ik - hj)$, contract C_{hk} is rejected.

In order to ensure that this structure can be a continuation equilibrium, contracts C_{ij} and C_{ik} must satisfy the following constraints:

$$(p_{ij} - w_{ij}) D(p_{ij}, p^*, p_{ik}, \emptyset) - F_{ij} + (p^* - w^*)D(p^*, p_{ij}, \emptyset, p_{ik}) - F^* \geq ...$$

$$... \geq 0 \tag{28}$$

$$... \geq (p^* - w^*)D(p^*, \emptyset, \emptyset, p_{ik}) - F^* \tag{29}$$

$$... \geq (p_{ij} - w_{ij}) D(p_{ij}, \emptyset, p_{ik}, \emptyset) - F_{ij} \tag{30}$$

and

$$(p_{ik} - w_{ik}) D(p_{ik}, \emptyset, p_{ij}, p^*) - F_{ik} \geq ...$$

$$... \geq 0 \tag{31}$$

$$... \geq (p^* - w^*)D(p^*, \emptyset, p^*, p_{ij}) - F^* \tag{32}$$

$$... \geq (p_{ik} - w_{ik}) D(p_{ik}, p^*, p_{ij}, p^*) - F_{ik} + (p^* - w^*)D(p^*, p_{ik}, p^*, p_{ij}) - F^* \tag{33}$$

Wholesale prices w_{ij} and w_{ik} can be set so that constraints (30) and (33) are satisfied. If manufacturer i sets the maximal possible fixed fees, its profit is:

$$
\begin{aligned}
\pi_{S1}(p_{ij}, p_{ik}) &= p_{ij}D(p_{ij}, p^*, p_{ik}, \emptyset) - \max\left[0, (p^* - w^*)\left(D(p^*, \emptyset, \emptyset, p_{ik}) - (1 - \alpha)q^*\right)\right] \\
&\quad + p_{ik}D(p_{ik}, \emptyset, p_{ij}, p^*) - \max\left[0, (p^* - w^*)\left(D(p^*, \emptyset, p^*, p_{ij}) - (1 - \alpha)q^*\right)\right] \\
&\quad + (p^* - w^*)\left[D(p^*, p_{ij}, \emptyset, p_{ik}) - (1 - \alpha)q^*\right]
\end{aligned}
$$

It is now sufficient to compare the maximal value of this profit with $\pi_P^*(w^*)$, that is, consider the sign of the expression:

$$\Delta_1(w^*) = \max_{p_{ij}, p_{ik}} \pi_{S1}(p_{ij}, p_{ik}) - \pi_P^*(w^*)$$

We then check that there exist two critical values $\underline{w}_1(\alpha, \beta)$ and $\overline{w}_1(\alpha, \beta)$ such that

$$w^{\min} < \underline{w}_1(\alpha, \beta) < \overline{w}_1(\alpha, \beta) < w^{\max}$$

and $\Delta_1(w^*) \leq 0$ for $w^* \in [\underline{w}_1(\alpha, \beta), \overline{w}_1(\alpha, \beta)]$. This imply that such a deviation is never profitable for $w^* \in [\underline{w}_1(\alpha, \beta), \overline{w}_1(\alpha, \beta)]$.

Structure $2 : (ij - hk)$, only contracts C_{ij} and C_{hk} are accepted

A comparable analysis leads to consider the sign of the expression:

$$\Delta_2(w^*) = \max_{p_{ij} \geq p^{\max}(w^*)} \pi_{S2}(p_{ij}) - \pi_P^*(w^*),$$

where

$$\pi_{S2}(p_{ij}) = p_{ij}D(p_{ij}, \emptyset, \emptyset, p^*) - (p^* - w^*)\left[D(p^*, \emptyset, p^*, \emptyset) - (1 - \alpha)q^*\right]$$

We then check that there exists a critical value $w_2(\alpha, \beta)$ such that $\Delta_2(w^*) \leq 0$, for $w^* \in \left[w^M, w_2(\alpha, \beta)\right]$. Therefore, such a deviation is never profitable for $w^* \in \left[w^M, w_2(\alpha, \beta)\right]$.

Structure $3 : (i1 - i2)$, only contracts C_{i1} and C_{i2} are accepted

A comparable analysis leads to consider the sign of the expression:

$$\Delta_3(w^*) = \max_{p_{i1}, p_{i2}} \pi_{S3}(p_{i1}, p_{i2}) - \pi_P^*(w^*),$$

where

$$\pi_{S3}(p_{i1}, p_{i2}) = \sum_{j \neq k = 1,2} \left(p_{ij}D(p_{ij}, \emptyset, p_{ik}, \emptyset) - \max\left[0, (p^* - w^*)\left(D(p^*, \emptyset, \emptyset, p_{ik}) - (1 - \alpha)q^*\right)\right]\right).$$

We then check that there exist two critical values $\underline{w}_3(\alpha, \beta)$ and $\overline{w}_3(\alpha, \beta)$ such that

$$w^{\min} < \underline{w}_3(\alpha, \beta) < \overline{w}_3(\alpha, \beta) < w^{\max},$$

and $\Delta_3(w^*) \leq 0$ for $w^* \in [\underline{w}_3(\alpha, \beta), \overline{w}_3(\alpha, \beta)]$. This implies that such a deviation is never profitable for $w^* \in [\underline{w}_3(\alpha, \beta), \overline{w}_3(\alpha, \beta)]$.

Other Structures

Doing the same type of analysis for the other structures (that is, (ij), $(ij - hj)$ and $(ij - hj - hk)$), we check that these types of deviation are never profitable for $w^* \in \left[w^M, w^{\max}\right]$.

Conclusions

Let us now define the following values:

$$\underline{w}(\alpha, \beta) = \max\left[w^M, \underline{w}_1(\alpha, \beta), \underline{w}_3(\alpha, \beta)\right]$$

$$\text{and } \overline{w}(\alpha, \beta) = \min\left[\overline{w}_1(\alpha, \beta), w_2(\alpha, \beta), \overline{w}_3(\alpha, \beta)\right] < 0.$$

We then verify that, for the values of the parameters α and β given by figure 1, we have:

$$\underline{w}(\alpha, \beta) = w^M \leq \overline{w}(\alpha, \beta) \Leftrightarrow \underline{p}(\alpha, \beta) \leq p^M \leq \overline{p}(\alpha, \beta),$$

and the contract $C^* = (w^*, p^*, F^*)$ is the equilibrium contract of a symmetric double common agency equilibrium. ∎