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# Circularity and the Undervaluation of Privatised Companies

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## Abstract
Circularity arises in regulation if the value of a company's assets used to set prices is itself determined by future earning capacity. The view that it is not possible to determine values and prices in these circumstances has become well established in the literature. In this paper we formalise this precise circularity result. We then show that if there is a possibility of future review and the regulator makes symmetric errors in estimating the cost of capital (known to the company) then almost all valuations involve undervaluation of the assets of the privatised company.

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## 1. Introduction

The view that it is not possible to determine regulatory asset values that are based on future earning capacity has become a core notion in the literature on regulation. It was first identified almost a hundred years ago in the US following on from the Supreme Court judgement in *Smyth v. Ames* (1898). In this case the Court suggested that probable earning capacity had to be considered when valuing utilities for regulatory purposes. But then a circularity problem arises since the value of assets in place is to be determined by reference to future rates but these future rates are themselves determined by a fair return on the value. There followed a lengthy, hotly disputed legal and academic debate, which has continued in various forms throughout the twentieth century, as to the appropriated mechanism to value the assets of a utility. This literature is immense and, while diverse on many issues, has been unanimous on the problem of circularity.

The circularity problem has re-appeared in the last decade as a result of the wave of privatisations of public utilities that have taken place around the world. With the general exception of telecommunications, there has been a strong tendency for the market price of privatised utilities to be below their replacement cost at privatisation. That is, undervaluation and associated underpricing is common place. A large literature is developing on undervaluation and underpricing although little of this is concerned with the consequent problem it creates for regulatory policy.[1] The difficulty is to decide what asset base should be used to set prices for these regulated utilities. Textbooks have traditionally focused on replacement cost or book value as

[1] We use the standard terminology of undervaluation to imply assets are purchased below their replacement cost and underpricing to imply they are purchased below their 'equilibrium' value.

the appropriate valuation for regulatory purposes. However, regulatory authorities tend to argue that if these are used as the asset base in privatisations where shareholders have paid less than this for the assets then there appears to be an unfair transfer from customers to shareholders. Typically, the procedure used to avoid this transfer has been the implementation of a market value (debt plus equity) approach as the asset base. However, as is recognised in the literature, this brings the circularity problem back to the fore.

This short paper outlines the circularity and undervaluation problem and then makes two main contributions. One, it formalises, we believe for the first time, the precise circularity result and investigates its robustness. We show that the view that it is never possible to tie down values and prices is not particularly robust but that there are more subtle versions that contradict some of the basic predictions whilst retaining a form of circularity. Second, we show that there is a theoretical relationship between the use of market values (with its inherent circularity) and undervaluation of privatised assets.

Section 2 begins by providing a brief history of asset valuation and the circularity problem in the US and describes the undervaluation problem and how it has been dealt with in the UK privatisation programme. It then formalises the circularity result as it is commonly stated. It is shown that if existing assets are valued according to their earning power, i.e., present value, then there is indeed no unique value to these assets (and consequently no unique regulatory prices). However, to obtain this result we assume that the valuing exercise is conducted once, i.e., there is no possibility of review. Section 3 shows that if there is a possibility of future reviews the situation changes. We allow for reviews that arrive by a Poisson process and assume that the regulatory agency may make errors in their estimation of the cost of capital. It is shown (Proposition 1) that, while there is an infinite set of valuation paths, on each path almost every valuation is close to replacement cost. That is, once the initial valuation is set then all future valuations are fully determined and converge to replacement cost. In this sense the core circularity problem, that it is never possible to tie down the value of the utility now or in the future, no longer holds. However, a far weaker and subtler form of circularity exists since the initial valuation is still indeterminate. Almost all valuations have to be close to replacement cost but there is still scope for indeterminacy at privatisation. Circularity can only be completely avoided if we are willing to introduce stronger assumptions to take advantage of the stationarity of the problem. In particular, if we are willing to impose the restriction that whenever two reviews are identical in all respects save for the time label they should have the same value, there is a unique valuation. This is equal to the replacement cost of the assets.

Both these results (i.e., the circularity result and Proposition 1) allow for the possibility that the regulatory agency makes an error in the estimate of the cost of capital (we assume that the error generating process has a bounded, symmetric density function with zero mean). However, we assume that, at the time of valuation, the precise error is unknown. In practice, a regulatory agency's choice of rate of return is known, at least within a range. Therefore, we also consider what happens if there is a possibility of future review and at each review the regulatory agency announces the agency's view of the appropriate cost of capital, before the valuation process is conducted. That is, the market knows the error when prices are set. The valuation is unique but, of course, its exact value depends on the error drawn from the error generating process. We can, therefore, determine a mean valuation across all the possible errors. This provides a result similar to Proposition 1 save that in this case on any path almost all these mean valuations will be strictly less than replacement cost (Proposition 2), i.e., we find that on average circularity induces undervaluation of privatised assets.

The paper is implicitly concerned with valuations set in a stock market but, since the model is driven by common present valuation of cash flows, the general principles should carry over to situations where the valuation is determined by auction. Indeed, as long as potential bidders have privately known signals drawn from a common, strictly –increasing, atomless distribution then the revenue equivalence theorem suggests that this should be true for all well known types of auctions (see, for example, Bulow and Klemperer (1996) and Riley and Samuelson (1981)).[2]

## 2. Circularity

### *2.1 The circularity problem and associated literature*

The problem of circularity in regulation goes back to the nineteenth century, arising in the seminal Supreme Court case of *Smyth v. Ames* (1898). The case concerned a dispute about railroad rates in Nebraska. The state of Nebraska's newly created Board of Transportation set recommended rates that the railroads challenged on the basis that they were confiscatory. The background to the dispute was that the

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[2] Of course, the common distribution is central for this (e.g., Maskin and Riley (1985)).

railroads had been constructed at the time of the Civil War, when prices were high, and they argued that they should receive a return on this original cost. In reaching a judgement on the case, the Supreme Court created the notion of fair value (the Court concluded the company was entitled to a 'fair return on fair value'). It argued that value should be approached as follows:

> In order to ascertain that value, the original cost of construction, the amount expended in permanent improvements, the amount and market value of its bonds and stock, the present as compared to the original cost of construction, the probable earning capacity of the property under particular rates prescribed by statute, and the sum required to meet operating expenses, are all matters for consideration, and are to be given such weight as may be just and right in each case. We do not say that there may not be other matters to be regarded in estimating the value of the property.[3]

The presence of (i) the market value of its bonds and stock, and (ii) the probable earning capacity of the property under particular rates prescribed by statute, as elements that had to be considered when valuing utilities raised the spectrum of circularity. The value of assets in place is to be determined by reference to future rates but these future rates are themselves determined by a fair return on the value.

There followed a lengthy, hotly disputed legal and academic debate, which has continued in various forms throughout the twentieth century as to the appropriated mechanism to value the assets of a utility. Until the Second World War, the Supreme Court fluctuated in its views and it was only in 1944 that the practical issue was laid somewhat to rest as a result of the *Hope Natural Gas* case.[4] In this case the Supreme Court put its emphasis on pragmatism and a presumption in favour of the

[3] Smyth v. Ames 169 U.S. 466 (1898).

Commissions - 'he who would upset the rate order under the Act carries the heavy burden of making a convincing showing that it is invalid because it is unjust and unreasonable in its consequences'.[5] Bonbright (1948) described the Hope case as:

> 'One of the most important economic pronouncements in the history of American Law. Unless the Court reverses itself, no longer will it impose upon legislatures or commissions, state or federal, the severe restrictions upon their powers to fix rates that it previously imposed under its doctrine in Smyth vs. Ames. The rule of 'reasonable return on fair value' may still be retained by states that choose to retain it. But it has ceased to be 'law of the land'.

From this time on regulators exercised their power and, in the inflationary era, tended to favour original cost valuations.[6]

Although the awkward notion of fair value remained the 'law of the land' until the Hope Natural Gas case, its problems and, in particular, the circularity issue received heavy criticism by both academics and lawyers (e.g., Bauer (1918, 1924), Bonbright (1926), Southworth (1922), and the series of sessions in the papers and proceedings of American Economic Review in 1924, 1927 and 1928 when the debate was at its height). The inability to value assets in the face of circularity has become an industry standard (e.g., see Kahn's classic text on regulation (Kahn (1988)). Indeed, even by 1918 Bauer pointed out that 'the circle of value and rates has been pointed out perhaps a thousand times'.

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[4] Federal Power Commission v. Hope Natural Gas Co. 320 U.S. 591 (1944).
[5] Federal Power Commission v. Hope Natural Gas Co. 320 U.S. 591 (1944), 602.
[6] See Grout and Jenkins (2001) for an assessment of the extent of opportunism displayed by regulators as a result of price changes. By 1991 a study of 53 regulatory commissions revealed that 44 were using

The circularity problem appeared to have disappeared as a practical issue in the post war era and was replaced for many years with a focus on original and reproduction cost. However, as indicated in the introduction, the problem has reappeared on a major scale in the last decade with the wave of privatisations of public utilities that have taken place around the world. The background to the new circularity problem in utility regulation is that, with the general exception of telecommunications, there is a tendency to sell assets below their replacement cost at privatisation. The UK has been at the forefront of privatisation (e.g., in the recent Dewenter and Malatesta (1997) international study of privatisation, over 60% of the offers in their most complete regression come from the UK) so it is instructive to look at the undervaluation of privatised utilities in the UK. This is summarised in Table 1 where the definition of market value is debt plus equity valued at the end of first day of trading.

**Table 1**
**Undervaluation in UK Utility Privatisations**[7]

| **Utility Companies** | **Market value divided by CCA book value** |
|---|---|
| **British Telecom** | **97.3%** |
| **British Gas** | **42.0%** |
| **Water & Sewerage Companies** | **3.6%** |
| **Regional Electricity Companies** | **60.5%** |
| **National Grid Company** | **40.4%** |
| **National Power** | **57.1%** |
| **PowerGen** | **48.9%** |
| **Railtrack** | **68.8%** |

There are likely to be many contributory factors that lead to undervaluation. Here we provide a few examples. First, the one that we model in this paper, arises because the regulatory regime has to estimate the relevant risk adjusted cost of capital. This

historic cost, while 7 still adhered to fair value, and two commissions considered all the evidence, without a predetermined choice of rate base.

[7] Source: Carey et al (1994) and privatisation prospectuses

affects the valuation since it fixes future prices. Errors, or perhaps deliberate policies, by a regulator that lead to an initial return below the cost of capital will depress the price that the market is willing to pay for the assets. If this lower market value is then the basis for asset valuation used to determine future price then the impact on the valuation may be large.[8] A second example arises because the government has a pre-commitment problem that it may mitigate by limiting the transfer of ownership. Perotti (1995) provides a formal model of this effect and a theoretical argument for underpricing of stock.[9] A final example arises if the post privatisation management may be able to provide performance that is more efficient than the performance of the traditional public management and superior to government supported efforts to reorganise prior to privatisation (see for example, Lopez-de-Silanes (1997), and Megginson et al (1994)). If such benefits take time to implement then one would expect assets to be undervalued at privatisation.

The undervaluation of privatised utilities causes a problem for regulatory policy. What asset base should be used for the review of prices? In the face of undervaluation a shift to replacement cost appears to provide a windfall gain to shareholders and a price increase for consumers that governments are reluctant to contemplate. While a solution could be achieved by windfall taxes and lump sum transfers there has been little attempt to follow this route. The standard procedure has

[8] This may not be simply a direct impact on the cost of capital but could be through subtle mechanisms. Jones et al (1999), provide a detailed analysis of the more explicit ways that way government's affect companies around privatisation.

[9] Underpricing of privatised stock is well documented (e.g., Jenkinson and Mayer (1988), Dewenter and Malatesta (1997)). The evidence indicates, for whatever reason, that the scale of 'underpricing' is greater when calculated over a longer period than if it is calculated against the end of first day trading. Therefore, even if the regulatory asset value is set equal to the market valuation at the end of first day trading (when underpricing should have unravelled), if prices continue to rise for some time then underpricing will cause undervaluation.

been an institutionalisation of the market value (debt plus equity) approach as the regulatory solution. The UK experience is informative in this regard.

The first major utility privatisation in the UK was British Telecom, which was initially regulated on an historical cost basis, and then later on a reproduction cost basis. As appears common for telecommunications, the government was able to extract the full replacement value of the assets. The situation, however, was very different for the other privatisations. In the case of British Gas, the allowed rate of return was set at just over 60 per cent of the true risk adjusted cost of capital. This mark-down in the allowed return was specifically to reflect the difference between the market value of the assets and replacement cost. In the water industry the water regulator, Ofwat, explicitly used a market value of the company as the asset base for regulatory purposes (based on share prices averaged over the first 200 days' trading from the date of privatisation). In the electricity sector market values at end of first day trading are also the standard asset valuation approach with a mark-up on end of first day trading value ranging between 15% and 0% depending on the company. Most recently, Railtrack's asset value is set at market value based on the share price at the end of the first trading day with no uplift.[10]

Privatisation and the market value approach to asset valuation, which we might think of as the UK standard, has brought the circularity problem once more to the fore. Not surprisingly, the conventional view, that circularity negates the use of market values, is also present in the new privatisation and regulation debate.[11] For example, the

[10] Railtrack have now ceased trading having failed in the face of an unexpected massive increase in cost.

[11] For example, the leading text on UK privatisation and regulation states: 'Using market values makes some sense because shareholders need to be convinced that they will not be expropriated at review

leading text on UK privatisation and regulation states 'Using market values makes some sense because shareholders need to be convinced that they will not be expropriated at review time. But this runs the risk of circularity, and the use of market values is not feasible for firms about to be privatised or that are subsidiaries and not separately quoted.' Armstrong et al (1994)

*2.2 The basic model and the circularity result*

In this section we first provide what we believe to be the first formalisation of the circularity result. The problem arises, not from the valuation of assets that will be purchased in the future, but from the valuation of existing assets. If there is a one off valuation of a business, that will not be revised at some later date, then all assets that will be purchased in the future have a value that is set by their market price.[12] In particular, if the market for supply of physical capital is competitive then future assets will be available at their reproduction cost. The core problem is the assets that are already in place at the time of the review. In one sense, of course, the market also sets their price but this leads directly to the problem of circularity. That is, the market price will be based on earning power but, for a regulated utility, the earning power depends on the valuation used to determine future returns. As Rose (1962) points out, 'The cost to the utility of facilities previously employed is dependent upon anticipated earnings and is inadmissible as the rate base'.

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time. But this runs the risk of circularity, and the use of market values is not feasible for firms about to be privatised or that are subsidiaries and not separately quoted.' Armstrong et al (1994). We assume throughout that the regulator does not renage on policy; otherwise we run into a time consistency problem (see, for example, Grout (1984)).

[12] In Section 3 we address the situation where the valuation of assets is updated in the future.

To analyse the circularity problem we assume that the regulatory agency values the assets in place at the current time at their market value (i.e. their earning power). We denote this valuation by $V$. We assume that these assets will depreciate over time at a rate $\delta$ ($\delta > 0$), so, if the value of the assets is set at the market value, then at any point $t$ in the future the current assets in place will have a value of $Ve^{-\delta t}$ since $V(1-e^{-\delta t})$ will have disappeared due to depreciation. These will be replaced with new assets that, in aggregate, cost $A$.[13] Hence looking forward to period $t$ the value of the company's assets will be

$$Ve^{-\delta t} + A(1- e^{-\delta t}).$$

The company will be allowed to earn a return on these assets which, if there are no errors, will be set at the relevant risk adjusted cost of capital. That is the company is allowed to set prices to achieve the allowed cost of capital and to cover depreciation.

The true cost of capital is $\rho$ ($\rho > 0$). However, the regulator has to estimate the risk adjusted cost of capital and may make an error, $\varepsilon$, in assessing the true cost of capital. Therefore, the current value of the stream of earnings can be expressed as

$$\int_0^\infty (\rho + \delta + \varepsilon)\left(Ve^{-\delta t} + A(1 - e^{-\delta t})\right)e^{-\rho t}\,dt\ .$$

[13] There is no inflation in the model so there is no distinction between original cost and replacement cost; both are equal to A.

We assume that the error generating process has a zero mean, symmetric density function $\phi : \Re \to \Re$ with support (-$\lambda$, $\lambda$), $\lambda$>0.[14] We further assume, for current purposes, that the realisation of this error is unknown when the valuation of the company's assets takes place. The expected stream of earnings has a present value of

$$E(\int_0^\infty (\rho + \delta + \varepsilon)\left(Ve^{-\delta t} + A(1 - e^{-\delta t})\right)e^{-\rho t}dt) .$$

The above formula represents the earning power of the company save for the fact that the company has to fund the replacement of assets. Regardless of the regulatory agency's decision that the assets are only worth (and only allowed to earn) their market value, the true cost of replacing the assets is $A$ and hence the company has to pay this to meet depreciation over the life of the assets. This is:

$$\delta \int_0^\infty Ae^{-\rho t}dt .$$

Therefore, the future earning power of the company is

$$E(\int_0^\infty (\rho + \delta + \varepsilon)\left(Ve^{-\delta t} + A(1 - e^{-\delta t})\right)e^{-\rho t}dt - \delta \int_0^\infty Ae^{-\rho t}dt) \quad . \qquad (1)$$

The circularity arises since the expected value of this future earning stream (1), based on $V$, determines the present market value, i.e.,

[14] The interpretation of $\lambda$ is given in Section 3.

$$V = E(\int_0^\infty (\rho + \delta + \varepsilon)\left(Ve^{-\delta t} + A(1 - e^{-\delta t})\right)e^{-\rho t}\,dt - \delta \int_0^\infty Ae^{-\rho t}\,dt).$$

Solving the equation for market value $V$ we obtain

$$V = \int_0^\infty (\rho + \delta + E(\varepsilon))\left(Ve^{-\delta t} + A(1 - e^{-\delta t})\right)e^{-\rho t}\,dt - \delta \int_0^\infty Ae^{-\rho t}\,dt$$

$$V = \int_0^\infty (\rho + \delta)\left(Ve^{-\delta t} + A(1 - e^{-\delta t})\right)e^{-\rho t}\,dt - \delta \int_0^\infty Ae^{-\rho t}\,dt$$

$$V = \frac{\delta + \rho}{-(\delta + \rho)}(V - A)(0 - 1) + \frac{\rho}{-\rho}A(0 - 1)$$

$$V = V - A + A$$

$$V = V.$$

That is, any value of $V$ satisfies the equation. Since $V$ can take on any value, then the prices that the utility charges are themselves indeterminate. This is the circularity problem identified in the literature which, to avoid confusion with later propositions, we call the ‘circularity result’.

## 3. Undervaluation

A feature of the circularity model given in the previous section is that the market value is determined and set once and for all. More generally, it is reasonable to assume that the valuation is reviewed at certain periods and another new asset base set, albeit along exactly the same lines as the previous one. That is, at the *n*th review the market value, $V_n$, and earning power of the assets is reassessed and this new value is used for regulatory purposes. We thus need to establish a timing relationship for reviews to be conducted. We assume that the life of the current agreement and each future review is limited but the exact timing of the next review is unknown. We assume that the timing of the reviews is described by a Poisson process, where *N(t)* is the number of reviews that occur at or prior to time *t*, and $E(N(1)) = \lambda$ . The market value of the asset at each review depends on the expected value at the next review. Let V denote a consistent time path of valuations and $\vartheta$ the set of all the consistent time paths of valauations.

In this section, we consider two different cases concerning what is known at the initial valuation and at the time of the future reviews about the regulatory error. At the initial valuation and each review we assume that the regulatory agency makes a fresh estimate of the cost of capital, i.e. a fresh draw from the same distribution of $\varepsilon$.

**Case 1.** We say that $\varepsilon$ is *unknown at review* if at the initial valuation and each review the relevant regulatory agency's estimate of the cost of capital is revealed to the market after the valuation has taken place.

**Case 2**. We say that $\varepsilon$ is *known at review* if at the initial valuation and each review the regulatory agency's estimate of the cost of capital (for that review) is known before the valuation has taken place.

Of course, at each current review in case 2 the participants will not know the $\varepsilon$ that is relevant for the next review, although it will become known before the valuation at the next review. We would like to stress that when $\varepsilon$ is 'known at review' the actual value will depend on the relevant $\varepsilon$. In this case it is possible to calculate an expected value before the relevant $\varepsilon$ is revealed and the actual valuation determined. To avoid confusing terminology we always refer to this expected valuation as the 'mean valuation' and use this terminology only in this context.

The following proposition deals with the case when $\varepsilon$ is 'unknown at review'. This is also the assumption in the previous section so the model in Proposition 1 is identical to that used to derive the circularity result save that we now allow the possibility of review.

***Proposition 1.*** *If $\varepsilon$ is 'unknown at review' and $\lambda$ is finite, then almost all $V_n \in V \subset \vartheta$ are arbitrary close to replacement cost A.*

***Proof.***

First, let us presume that the first review is known to happen at time *T*. Then, the market value of the business is

$$\int_0^T (\delta + \rho + E(\varepsilon))(Ve^{-\delta t} + (1 - e^{-\delta t})A)e^{-\rho t} dt - \int_0^T \delta A e^{-\rho t} dt + \tilde{V} e^{-\rho T},$$

where $\tilde{V}$ is the expected value of the company's assets set as a result of the review at time *T*. *T* is a random variable with an exponential distribution and the expected value of regulator's error is zero, thus the value of the company at time 0 is

$$V = \int_0^\infty \left[ \int_0^T (\delta + \rho)(Ve^{-\delta t} + (1 - e^{-\delta t})A)e^{-\rho t}\, dt - \int_0^T \delta A e^{-\rho t}\, dt + \tilde{V} e^{-\rho T} \right] \lambda e^{-\lambda T}\, dT$$

$$V = \lambda \int_0^\infty (-V(e^{-(\delta+\rho)T} - 1) + A(e^{-(\delta+\rho)T} - e^{-\rho T}) + \tilde{V} e^{-\rho T})e^{-\lambda T}\, dT$$

$$V = V \frac{\rho + \delta}{\rho + \delta + \lambda} - A \frac{\delta\lambda}{(\rho + \delta + \lambda)(\rho + \lambda)} + \tilde{V} \frac{\lambda}{\rho + \lambda}.$$

Solving this equation for *V* we obtain:

$$V = \frac{-\delta}{\rho + \lambda} A + (1 + \frac{\delta}{\rho + \lambda})\tilde{V}. \tag{2}$$

To distinguish the time path of the valuations let us redefine $V = V_{n-1}$ and $\tilde{V} = V_n$, where *n =1, 2, …* denotes the position in the sequence of reviews and *n* = 0 denotes the initial valuation. Then the above equation becomes

$$V_{n-1} = -\frac{\delta}{\rho + \lambda} A + (1 + \frac{\delta}{\rho + \lambda})V_n,$$

or

$$V_n = \frac{\delta}{\rho + \lambda + \delta} A + \frac{\rho + \lambda}{\rho + \lambda + \delta} V_{n-1}.$$

In other words

$$V_n = \left(1 - \left(\frac{\rho + \lambda}{\rho + \lambda + \delta}\right)^n\right) A + \left(\frac{\rho + \lambda}{\rho + \lambda + \delta}\right)^n V_0. \tag{3}$$

It immediately follows that because $\frac{\rho+\lambda}{\rho+\lambda+\delta}<1$, the sequence $\{V_n\}$ has limit:

$$\lim_{n\to\infty} V_n = A.$$

More technically, for any $\xi > 0$, there exists such $n'$, that for all $n > n'$, we have $|V_n - A| < \xi$. Moreover, from equation 3, it is easy to calculate that $n'$ is determined due to the formula

$$n' = \frac{\ln(\xi) - \ln(|V_0 - A|)}{\ln(\rho+\lambda) - \ln(\rho+\lambda+\delta)}.$$

To complete the proof we need to show that when the time horizon is sufficiently large the probability of having almost all valuations in the $\xi$- neighbourhood of $A$ is one. More precisely, we need to show that when $t$ goes to infinity the (probability) measure of the ratio of the set of valuations that lie within the $\xi$-precision interval to the set of all expected valuations that would occur in the time interval $[0, t]$ is equal to one. To demonstrate this we prove that, when $t$ goes to infinity, for any arbitrary $\alpha > 0$ the probability that $\frac{n'}{N(t)} < \alpha$ is 1, i.e.

$$\lim_{t\to\infty} P(N(t) > \frac{n'}{\alpha}) = 1,$$

The above is true as

$$\lim_{t\to\infty} P(N(t) \le \tfrac{n}{\alpha}) = \sum_{k=0}^{\left[\frac{n'}{\alpha}\right]} \frac{(\lambda t)^k}{k!} e^{-\lambda t} \le \lim_{t\to\infty} e^{-\lambda t} \left[\frac{n'}{\alpha}\right] \frac{(\lambda t)^{\left[\frac{n'}{\alpha}\right]}}{\left(\left[\frac{n'}{\alpha}\right]\right)!} = 0, \qquad (4)$$

where [x] denotes the largest integer number in interval [0,x]. Because the probability of any event is always non-negative, we conclude from (4) that

$$\lim_{t\to\infty} P(N(t) \le \tfrac{n}{\alpha}) = 0 \,.$$

■

Proposition 1 partially breaks the circularity results. Once any one valuation is given then all future valuations are fully determined by the unique equilibrium path. In this sense the core circularity problem, that it is impossible to tie down the value of the utility now or in the future, no longer holds. Of greater significance, it must be the case that virtually all of the valuations of the utility will be approximately equal to replacement cost. However, the circularity is not fully avoided in that any possible initial valuation is consistent with some time path of equilibrium valuations. Thus, having introduced the possibility of future reviews we have a weaker and subtler form of circularity.

Circularity can only be completely avoided if we are willing to introduce stronger assumptions to take advantage of the stationarity of the problem. Note, at any given review, the model at the next review will be identical to the current review. That is the market value at the next review will depend on the earning power at the review, say at time *T*' in exactly same way that *V* depends on the earning power at the current review, say at time *T*. If we are willing to impose the restriction that if two reviews are identical in all respects, save for the time label, they should have the same value, then the only equilibrium is that all valuations must be equal to replacement cost.

The exception to this occurs when $\lambda$ goes to infinity. For example, from equation 2,

$$V = \lim_{\lambda \to \infty} \frac{-\delta}{\rho + \lambda} A + (1 + \lim_{\lambda \to \infty} \frac{\delta}{\rho + \lambda}) \tilde{V}$$

$$V = \tilde{V}.$$

Of course, this essentially implies that reviews take place infinitely often in any small interval of time.

We now turn to the case when $\varepsilon$ is 'known at review'.

***Proposition 2***. *If $\lambda$ is finite and $\varepsilon$ is 'known at review', then there exists a market valuation B such that (i) B is strictly less than replacement cost A and (ii) almost all market valuations of the company's assets $V_n \in V \subset \vartheta$ are arbitrary close to B.*

***Proof.***

If the exact timing, $T$, of the first review is unknown, then the present value of the company's assets in the investment period [0,$T$] is determined by the following equation

$$V = \int_0^\infty \left[ \int_0^T (\delta + \rho + \varepsilon)(Ve^{-\delta t} + (1 - e^{-\delta t})A)e^{-\rho t} dt - \int_0^T \delta A e^{-\rho t} dt + \tilde{V} e^{-\rho T} \right] \lambda e^{-\lambda T} dT ,$$

or

$$\frac{\lambda-\varepsilon}{\rho+\delta+\lambda}V=\frac{(\varepsilon-\lambda)\delta}{(\rho+\lambda)(\rho+\delta+\lambda)}A+\frac{\lambda}{\rho+\lambda}\tilde{V}.$$

Because $\varepsilon<\lambda$ there exists a unique finite solution

$$V=-\frac{\delta}{\rho+\lambda}A+(\frac{\lambda}{\lambda-\varepsilon})(1+\frac{\delta}{\rho+\lambda})\tilde{V}. \qquad (5)$$

Equation 5 will be true for any following investment period assuming that the other parameters of the model, i.e. $\rho$, $\delta$, $\lambda$ are unchanged. Taking the mean value across all $\varepsilon$ such that $|\varepsilon|<\lambda$ we obtain

$$E(V)=E(-\frac{\delta}{\rho+\lambda}A+(\frac{\lambda}{\lambda-\varepsilon})(1+\frac{\delta}{\rho+\lambda})\tilde{V})=$$

$$=-\frac{\delta}{\rho+\lambda}A\int_{-\lambda}^{\lambda}\phi(\varepsilon)d\varepsilon+(1+\frac{\delta}{\rho+\lambda})\tilde{V})\int_{-\lambda}^{\lambda}\frac{\lambda}{\lambda-\varepsilon}\phi(\varepsilon)d\varepsilon$$

$$=-\frac{\delta}{\rho+\lambda}A+(1+\frac{\delta}{\rho+\lambda})\tilde{V})\int_{-\lambda}^{\lambda}\frac{\lambda}{\lambda-\varepsilon}\phi(\varepsilon)d\varepsilon,$$

where $\phi$ is a density function of $\varepsilon$ defined in Section 2.2.

As in the proof of Proposition 1, let us redefine $V=V_{n-1}$ and $\tilde{V}=V_n$, where *n =1, 2, …* denotes the position in the sequence of reviews and *n = 0* denotes the initial valuation. Then the above equation becomes

$$V_{n-1} = -\frac{\delta}{\rho+\lambda} A + \left(1+\frac{\delta}{\rho+\lambda}\right) V_n \int_{-\lambda}^{\lambda} \frac{\lambda}{\lambda-\varepsilon} \phi(\varepsilon) d\varepsilon \,.$$

Equivalently we can write

$$\begin{aligned} V_n &= \frac{\rho+\lambda}{(\rho+\lambda+\delta)Q} V_{n-1} + \frac{\delta}{(\rho+\lambda+\delta)Q} A \\ &= \frac{\delta}{(\rho+\lambda+\delta)Q-(\rho+\lambda)} \left(1-\left(\frac{\rho+\lambda}{(\rho+\lambda+\delta)Q}\right)^n\right) A + \left(\frac{\rho+\lambda}{(\rho+\lambda+\delta)Q}\right)^n V_0 \end{aligned}$$

where $Q = \int_{-\lambda}^{\lambda} \frac{\lambda}{\lambda-\varepsilon} \phi(\varepsilon) d\varepsilon$ .

From the above representation it is obvious that, because Jensen's inequality guarantees $Q > 1$, the series $\{V_n\}_{n\in\aleph}$ has a finite limit:

$$\lim_{n\to\infty} V_n = \frac{\delta}{(\rho+\lambda+\delta)Q-(\rho+\lambda)} A.$$

Denoting

$$B = \frac{\delta}{(\rho+\lambda+\delta)Q-(\rho+\lambda)} A,$$

we conclude that

$$\lim_{n\to\infty} V_n = B < A.$$

This is because

$$\frac{\delta}{(\rho+\lambda+\delta)Q-(\rho+\lambda)} < \frac{\delta}{(\rho+\lambda+\delta)-(\rho+\lambda)} = 1.$$

To complete the proof and show that almost all valuations, $V_n$, of any valuation path $V$ lie in any arbitrary small $\xi$-neighbourhood of valuation B we need to repeat the reasoning we presented in the proof to Proposition 1, where now

$$n' = \frac{\ln(\xi) - \ln(|V_0 - B|)}{\ln(\rho+\lambda) - \ln(\rho+\lambda+\delta)Q)}.$$

These calculations are absolutely dual to those presented in the proof of Proposition 2 and are not presented here.

■

Proposition 2 shows that on average almost all valuations exhibit undervaluation. Indeed if we are willing to assume that if two reviews are identical in all respects, save for the time label, they should have the same value, then the initial valuation and every review will on average exhibit undervaluation.

The intuition for this undervaluation arises from the exponential structure. For example, a present value calculation or an arrival rate of the Poisson process take the form $1/\rho$ or $1/\lambda$. The regulatory error epsilon has the effect of reducing $\rho$ or $\lambda$, e.g., $1/(\lambda-\varepsilon)$. Therefore, the present value at the current review is a convex function of $\varepsilon$. It follows that if the anticipated value of the next review is replacement cost $A$, then the current mean valuation must be greater than $A$. Therefore $A$ cannot be the steady state solution, which must be less than $A$, hence undervaluation.

## 4. Summary

The paper has outlined the history of the circularity problem and the scale of undervaluation of privatised assets. Despite the fact that both have attracted large literatures this is the first paper to formalise and investigate the received wisdom concerning circularity and to provide a theoretical relationship between the two.

We have shown that the view that it is not possible to tie down values and prices when future returns form the valuation base is not particularly robust. If there is a possibility of future review then almost all valuations are determined. However, there is still scope for indeterminacy at privatisation. That is, there are more subtle versions that contradict some of the basic predictions whilst retaining a form of circularity. We also consider the situation where at each review the regulatory agency announces the agency's view of the appropriate cost of capital before the valuation process is conducted. That is, the market knows the error when prices are set. We show that on any valuation path almost all mean valuations will be strictly less than replacement cost. That is, we find that on average circularity induces undervaluation of privatised assets. This raises a core problem. The use of market values instead of replacement cost as the asset base is justified solely on the basis of undervaluation but announcing in advance that this is the regulatory procedure itself increases the likelyhood of undervaluation.

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