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Reputation and Ownership of Public Goods

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Reputation and Ownership of Public Goods

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Abstract
This paper analyzes the effect of reputation on ownership of public goods in the Besley and Ghatak (2001) model. We show that in the dynamic setup the optimal ownership depends not only on the relative valuations for the public good but also on technology (elasticity of investment). We also show that joint ownership of public good can be optimal in both the static and repeated game but it emerges for a different parameter range. Our results are applied to the case of return of cultural goods to their country of origin.

Keywords: public goods, property rights, reputation, joint ownership, cultural goods

JEL Classification: D23, H11, H41, L14, L33, Z1

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1 Introduction

The return of cultural goods to their country of origin has always been a contentious issue. One needs to look no further than the cases of the Icelandic manuscripts and of the Parthenon marbles to get an idea of how controversial such restitutions are. The Icelandic manuscripts, the largest restitution of cultural goods to day, were returned to Iceland in their entirety in 1997, eighty years after the initial request. In the case of the Parthenon marbles, the initial request for their return was made more than a century ago and the issue has yet to be resolved.[1]

The issue of restitution of a cultural good is, in essence, a question of ownership. Who should be the owner of the good, the country of origin or the host country? Up to now, the debate over ownership of such goods has been primarily based on legal, historical and moral arguments. Has the host country acquired the cultural good legally? Do colonial powers have a moral obligation in returning cultural goods back to their ex-colonies? Surprisingly, economic considerations seem to be completely absent from this debate. We are addressing this open question in the debate by examining which ownership structure would generate the highest overall investments in cultural goods. Cultural goods are public goods. We take the property rights approach in determining the optimal ownership structure for public goods.

Besley and Ghatak (2001) have been among the first to examine how the allocation of ownership affects incentives in the case of public goods. Their main application is on the experience of NGO involvement in public good provision in developing countries. They build on Grossman and Hart (1986) and Hart and Moore (1990), who developed the property rights theory for private goods, to analyze government versus private ownership of public

[1] For more details and for further examples see Greenfield (2007).

goods. They find that the party with the highest valuation for the public good should be the owner irrespective of the relative importance of the investments. So if the NGO is more caring, then it should own the public good even if it does not have an important investment. This is in stark contrast with the private goods case where the relative importance of investments is one of the main factors determining the allocation of ownership.

In this paper we examine the effect of ownership on incentives for the public good case, but contrary to Besley and Ghatak we are also interested in reputation effects. Indeed, a dynamic model seems the natural way of examining ownership of cultural goods, given the longevity of such projects. To our knowledge this is the first dynamic treatment of Besley and Ghatak (2001).

In a long-term relationship reputation concerns can overcome the hold-up problem. In a repeated game the optimal ownership structure gives the best incentives for cooperation by minimizing the gain from deviation relative to the punishment. We first compare ownership by the high valuation agent, h, and low valuation agent, l, in a setup where only l has an investment. We find that when the investment is elastic, ownership by the noninvesting high valuation agent is optimal – just as in the static game. This is because it minimizes the gain from deviation. When h is the owner only part of l's investment is sunk in the project if they separate and therefore h cannot extract from the full value of l's first best investment. h ownership is optimal although it also minimizes the loss from deviation. Under the optimal ownership structure of the static game punishment path is very attractive. Thus, if the country of origin values the cultural good more, then it should be the owner when the investment is elastic.

When the investment is inelastic the emphasis is on maximizing the punishment – although this results also in maximal gain from deviation – and low valuation ownership of the public good is optimal. Therefore due to technological reasons it can be optimal for the low valuation party (perhaps the host country) to own the cultural good.

The results of the static model are quite different depending on whether we are analyzing private or public goods. With private goods the only investing agent should be the owner while with public goods the high valuation agent should be the owner – even if he does not have an investment. However, when we compare our dynamic results with the case of private goods analyzed by Halonen (2002) the results are surprisingly similar. Dynamic incentives with both private and public goods are driven by how easy it is to generate punishment. When investments are elastic, it is easy to generate punishment and results of the static game hold. While with inelastic investments the only way to have enough punishment power it to choose the most unattractive punishment path which is provided by the worst structure of the static game. This result holds for both private and public goods.

Where the results differ from the private goods case is in relation to joint ownership. We firstly have a new result about joint ownership in the static game. We show that it can be more cost-effective to have two intermediate investments of joint ownership than one very high and one very low investment under single ownership. With investment cost function $c(y) = y^{\gamma}$ this is the case when γ is *high enough*. Secondly, in the repeated game joint ownership of the public good is optimal when it is important to minimize the gain from deviation. This is the case when investments are elastic and accordingly γ is *low enough*. This is why joint ownership arises for a different (but partially overlapping) parameter range in the static and the repeated game.

We apply our results to cultural goods. We examine in detail the case of the Icelandic manuscripts which is an example of return to the high valuation party. We also discuss joint ownership of cultural goods, most notably the statue of Ur-ningirsu. The head of the statue belongs to the Metropolitan Museum of Art in New York and its body to the Louvre in Paris. The parts have been joined since 1974 and the statue rotates between the joint owners for exhibition.

Our results suggest a dynamically shifting optimal owner for a cultural good. When it is very costly to increase the value of cultural good (γ is high) the host country ownership is optimal. When the necessary expertise has been developed and the investments become less costly (γ is low) it is optimal to return the cultural good to the country of origin. Return can either take the form of source country ownership or joint ownership.

Relatively few papers have been exploring directly the model by Besley and Ghatak (2001). Rasul (2006) applies their model to child custody and introduces a continuum of ex post custodial schemes and endogenous probability of marital breakdown. Francesconi and Muthoo (2007) introduce impurity of public goods and make a case for shared control. They also find that ownership by the low valuation party is optimal if the degree of impurity is intermediate and the importance of the parties' investments is similar. In our paper ownership by the low valuation party can be optimal even with pure public goods.

The incomplete contracting literature on privatization (e.g. Hart et al. (1997) and Schmitz (2000)) and on public-private partnerships (e.g. Hart (2003), Bennett and Iossa (2006) and Martimort and Poyet (2008)) are related. We differ in allowing also the private providers to be value driven and modelling explicitly the public good nature of the projects.

The rest of the paper is organized as follows. Section 2 presents our benchmark: the static model of Besley and Ghatak (2001). Section 3 analyzes the repeated game and compares ownership by high valuation and low valuation parties. Section 4 focuses on joint ownership and finds when it can be optimal in both static and repeated game. Section 5 applies the theory to the case of cultural goods. Section 6 concludes.

2 Benchmark

Our benchmark is a simplified version of Besley and Ghatak (2001). There are two players, l and h. Low valuation agent l makes a project-specific investment in human capital, y. In our case of cultural goods we can think of this as any investment that facilitates the restoration, protection, study and display of cultural goods. High valuation agent h's contribution to the project is a fixed value which we can without loss of generality normalize to zero. Our results would not change if both agents had an investment but analysis with one investment is much simplified.

Public good is produced and the benefit from the project is equal to y. The players value the project differently: the low valuation agent's utility from the public good is $\theta_l y$ and the high valuation agent's utility is $\theta_h y$ where $\theta_l < \theta_h$. Investment costs are given by $c(y) = y^\gamma$ where $\gamma > 1$.

Joint surplus is equal to $(\theta_l + \theta_h) y - c(y)$. First best investment is then given by the following first-order condition:

$$(\theta_l + \theta_h) = c'(y^*) \tag{1}$$

But contracts are incomplete and typically holdup problem emerges. Ex ante contracts can only be written on the ownership of the project.

The timing is the following:

- Stage 1. l and h contract on ownership of the project. Either l or h is the owner.
- Stage 2. l invests in project-specific human capital.
- Stage 3. l and h bargain over the completion of the project and produce the public good.

If bargaining breaks down the owner can exclude the non-owner from taking part in the *production* of the public good but cannot exclude him from *consuming* the public good. Therefore non-owner's investment has less effect on the benefit from the project if bargaining breaks down. The benefit from the project when bargaining breaks down and h is the owner is μy where $0 \leq \mu \leq 1$.

Parameter μ measures how much of the non-owner's investment is sunk in the project. The value of μ is high when investment is e.g. about designing and organizing project implementation and plans and already adopted or written down. In this situation when the agent leaves, a large part of his investment is already sunk in the project. The value of μ is low if most of the investment is embedded in the person e.g. charismatic leadership. Then if the agent leaves, he takes the investment with him.

Also in this situation the players value the project differently and agent i's utility is $\theta_i \mu y$ if bargaining breaks down under h ownership. When l owns the public good all of his investment contributes to the value of the project even if bargaining breaks down. Then agent i's utility is $\theta_i y$ if bargaining breaks down under l ownership.

When the high valuation agent owns the public good Nash bargaining leads to the following payoffs u_h^h and u_l^h where superscript denotes the owner and subscript the agent.

$$\begin{aligned} u_h^h &= \theta_h \mu y + \frac{1}{2}(\theta_l + \theta_h)(1-\mu) y \\ &= \frac{1}{2}(\theta_l + \theta_h) y + \frac{1}{2}(\theta_h - \theta_l) \mu y \end{aligned} \tag{2}$$

$$u_l^h = \frac{1}{2}(\theta_l + \theta_h) y + \frac{1}{2}(\theta_l - \theta_h) \mu y - c(y) \tag{3}$$

Optimal investment, denoted by y^h, is then given by:

$$\frac{1}{2}(\theta_l + \theta_h) + \frac{1}{2}(\theta_l - \theta_h) \mu = c'\left(y^h\right) \tag{4}$$

Agent l receives half of the marginal value of his investment plus half of the marginal change in default points. Because the agents are producing a public good, higher investment increases both parties' default points. (Even if bargaining breaks down everybody can consume the public good.) Which increases more depends on the relative valuations. Higher investment by l increases the high valuation agent's default point more than his own and therefore l's bargaining position is worse. This is why the second term in (4) is negative. Note that the second term for h (in equation (2)) is positive. l's higher investment improves h's bargaining position.

When l is the owner, his payoff and investment, denoted by y^l, are given by:

$$u_l^l = \frac{1}{2}(\theta_l + \theta_h)\, y + \frac{1}{2}(\theta_l - \theta_h)\, y - c(y) \tag{5}$$

$$\frac{1}{2}(\theta_l + \theta_h) + \frac{1}{2}(\theta_l - \theta_h) = c'\left(y^l\right) \tag{6}$$

When l is the owner, all of his investment contributes to the public good even if there is disagreement. This is why μ does not appear in equation (6).

It is clear from (4) and (6) that agent l has best incentives when the negative second term is minimized. Since $\mu \leq 1$ ownership by the high valuation agent provides the best incentives despite the fact that only the low valuation agent has an investment. Under h ownership l's investment worsens his bargaining position least. The main result of Besley and Ghatak (2001) is that the more caring agent should own the public good. This result holds also when both agents have an investment. Our interest is in examining how the incentives change in a dynamic setup.

3 Repeated game

Now we consider the possibility of cooperation in an infinitely repeated game, where cooperation is supported through the use of trigger strategies with reversion to the Nash equilibrium of the static game as punishment.[2] In such an equilibrium, agent l implicitly agrees to make the efficient investment, y^*, and agent h agrees to pay l a transfer T^* which is such that the individual incentive compatibility constraints are satisfied. Cooperation is maintained as long as both agents follow the efficient behavior. Any deviation from the efficient behavior is observed in the same period and triggers punishment for the rest of the game. The crucial issue is to determine which ownership structure gives the lowest critical discount factor above which first best investment is sustainable.

First best will be supported if and only if the discounted payoff stream from efficient behavior exceeds the payoff stream from deviation for both agents. The incentive compatibility constraints are:

$$\frac{1}{1-\delta}\left(\theta_h y^* - T^*\right) \geq P_h^d + \frac{\delta}{1-\delta} P_h^p \tag{7}$$

$$\frac{1}{1-\delta}\left[\theta_l y^* + T^* - c\left(y^*\right)\right] \geq P_l^d + \frac{\delta}{1-\delta} P_l^p \tag{8}$$

where δ is the discount factor, P_i^d is i's one-shot deviation payoff and P_i^p is i's payoff in the punishment path.

If l deviates in investment, h observes it already in the same period and he will not pay T^* to l but surplus is shared by Nash bargaining. Therefore $P_l^d = P_l^p$ and equation (8) simplifies to:

$$\theta_l y^* + T^* - c\left(y^*\right) \geq P_l^p \tag{9}$$

[2] We do not allow for renegotiation of ownership structure in the punishment path. But Blonski and Spagnolo (2007) show that optimal punishments with renegotiation implement efficient investment for the same discount factors at which trigger strategies support first best investments without renegotiation of ownership structure (that is, in our case).

There is no trade-off from gain today against punishment tomorrow for l. Therefore to provide h with the best incentives we choose the lowest T^* that just satisfies equation (9).

$$T^* = P_l^p + c(y^*) - \theta_l y^* \tag{10}$$

Substituting T^* in (7) we obtain:

$$\frac{1}{1-\delta}\left[(\theta_h + \theta_l) y^* - c(y^*) - P_l^p\right] \geq P_h^d + \frac{\delta}{1-\delta} P_h^p \tag{11}$$

Agent h may default on the promised transfer T^* and instead demand a larger share of the surplus in Nash bargaining. Agent h's deviation payoff under h ownership is:

$$P_h^d = \frac{1}{2}(\theta_l + \theta_h) y^* + \frac{1}{2}(\theta_h - \theta_l)\mu y^* \tag{12}$$

This deviation payoff is obtained by using the Nash bargaining formula and substituting in l's first best investment.

We can further simplify equation (11) by denoting the gain from deviation by $G = \left[P_h^d - (\theta_l + \theta_h) y^* + c(y^*) + P_l^p\right]$ and the loss from deviation by $L = \left[(\theta_l + \theta_h) y^* - c(y^*) - P_l^p - P_h^p\right]$. G shows how much more agent h can obtain by deviating than by cooperation and L denotes how much the surplus in the punishment path drops from the first best level. Then equation (11) is equivalent to:

$$\delta \geq \frac{G}{G+L} \equiv \underline{\delta}$$

As usual cooperation is sustainable for high enough discount factor. If the agents care enough about the future, one-shot gain from deviation is outweighed by long term punishment. Gain and loss from deviation depend on the ownership structure. Our aim is to find an ownership structure for which cooperation is sustainable for the largest range of discount factors. In other words, the optimal ownership structure minimizes $\underline{\delta}$.

We know from the previous section that ownership by the low valuation agent minimizes the surplus in the static game. Therefore punishment path is most unattractive and the loss from deviation is maximized. This is the strength of low valuation ownership in the repeated game. But it turns out that also the gain from deviation is largest when the low valuation agent owns the public good. This is proved by Proposition 1. (All the proofs are in the Appendix.)

Proposition 1 *Both the gain and loss from deviation are higher under ownership by the low valuation agent than under ownership by the high valuation agent.*

Proposition 1 shows that the gain from deviation is largest when the low valuation agent owns the public good. The deviation payoff under l ownership is given by

$$P_h^d = \frac{1}{2}(\theta_l + \theta_h) y^* + \frac{1}{2}(\theta_h - \theta_l) y^* \quad (13)$$

Notice that the second term in (13) is positive. This is because a higher investment by l increases h's default payoff more than l's default payoff and therefore h's bargaining position is improved. Clearly h's deviation payoff under l ownership (given by (13)) is greater than under h ownership (given by (12)). When l owns the public good, his first best investment contributes fully to the project even if there is disagreement. Therefore h can extract from the full value of l's first best investment and his gain is maximal. While under h ownership only the sunk proportion of the investment improves h's bargaining position. This is why gain is maximal under l ownership.

Since both the gain and the loss from deviation are higher under ownership by the low valuation agent, it is not obvious which structure gives the best incentives for cooperation. Therefore we need to analyze which ownership structure gives the lowest critical discount factor, i.e. minimizes the gain relative to the loss. We normalize $\theta_l = 1$ to simplify the proof of Lemma 1 which helps us in determining the lowest critical discount factor.

Assumption 1. $\theta_l = 1$ and $\theta_h > 1$.

Lemma 1 *(i)* $\underline{\delta}^h = \underline{\delta}^l$ *if* $\mu = 1$.
(ii) $\frac{\partial \underline{\delta}^l}{\partial \mu} = 0$.
(iii) $\frac{\partial \underline{\delta}^h}{\partial \mu} > 0$ *if and only if* $\gamma < 2$.

Our main Propositions 2 and 3 follow directly from Lemma 1.

Proposition 2 *Ownership by the high valuation agent provides better incentives for cooperation than ownership by the low valuation agent if and only if* $\gamma < 2$.

We find that for $\gamma < 2$ the agents have the best incentives for cooperation when the gain from deviation is minimal – although at the same time also the loss from deviation is minimized. In this parameter range ownership by the high valuation agent is optimal – just like in the static game.

Proposition 3 *Ownership by the low valuation agent provides better incentives for cooperation than ownership of the high valuation agent if and only if* $\gamma > 2$.

While for $\gamma > 2$ maximal loss from deviation will provide the best incentives for cooperation – although at the same time also the gain from deviation is maximized. Now ownership by the low valuation agent is optimal. In the static game the low valuation agent should never be the owner but in the repeated game it can provide the maximal punishment.

The intuition for our results is the following. The elasticity of investment to surplus share determines how much the surplus drops along the punishment path. This elasticity is equal to $1/(1-\gamma)$. Therefore the investment is inelastic (elastic) when $\gamma > 2$ ($\gamma < 2$). When the investment is inelastic the surplus does not fall much after deviation under high valuation ownership. Then low valuation ownership is needed to provide enough punishment power. While for elastic investment, even high valuation ownership provides large enough punishment, which combined with minimal gain from deviation results in the best incentives.

These results are in line with Halonen (2002) which analyzes private goods. In that paper the results of the static game hold for $\gamma < 2$ because the gain from deviation is minimized – just like here. And for $\gamma > 2$ the worst ownership structure of the static game (joint ownership in the private goods case) is optimal because it provides the maximal punishment. It is surprising that although there is such a stark difference in the private goods and the public goods case in the static game, in the repeated game we obtain similar results.

But where the public goods case is different from the private goods case is in relation to joint ownership. That is the focus of Section 4.

In this section we have assumed that only l has an investment. Propositions 1 - 3 would not change if also h had an investment. This is easy to verify by numerical simulations. One investment case simplifies the proofs greatly.[3]

[3] This is particularly important to take into account as strictly speaking joint ownership would provide even better incentives than high valuation ownership in the static game when only l has an investment. When both agents have an investment, this is no longer true.

4 Joint ownership

In this section we analyze joint ownership both in the static and repeated game. We focus on the case where $\mu = 1$, i.e. all of the non-owners investment is sunk in the project. In this case h and l ownership are equivalent in the static game and we can concentrate on comparing joint ownership and single ownership.

Furthermore, in this section both agents have an investment which is denoted by y_h and y_l. It is more natural to examine joint ownership in a framework where both agents invest. The investments are equally important and have the same cost function. The agents differ only in how they value the public good.

4.1 Static game

Under joint ownership both parties' agreement is needed for the project to go ahead. Since both parties have blocking power, the disagreement payoffs are zero. For example the cultural good could be stored away in the case of disagreement under joint ownership, thus preventing both sides from utilizing it because both parties can veto exhibition. In the static game Nash bargaining leads to the following payoffs u_i^J where superscript J denotes joint ownership:

$$u_i^J = \frac{1}{2} (\theta_l + \theta_h) (y_h + y_l) - c (y_i) \qquad \text{for } i = l, h$$

The agents split the ex post surplus 50:50 which is why the payoffs do not depend on relative valuations. The incentives are given by:

$$\frac{1}{2} (\theta_l + \theta_h) = c' \left(y_i^J \right) \tag{14}$$

Since these investments are equal for both agents, we drop the subscript and denote $y_i^J \equiv y^J$.

While under single ownership (denoted by superscript 1) the agents' incentives are:

$$\frac{1}{2}(\theta_l + \theta_h) + \frac{1}{2}(\theta_l - \theta_h) = c'\left(y_l^1\right) \tag{15}$$

$$\frac{1}{2}(\theta_l + \theta_h) + \frac{1}{2}(\theta_h - \theta_l) = c'\left(y_h^1\right) \tag{16}$$

Just like in Section 3 the second term in (15) is negative. l's higher investment increases h's default payoff by more than his own and puts l in a worse bargaining position. While for the high valuation agent the second term in (16) is positive. His higher investment increases his own default payoff more than l's default payoff and therefore his bargaining position is improved.

Comparing the incentives under joint ownership and single ownership we notice that joint ownership removes the second terms in (15) and (16). Therefore the high valuation agent has lower incentives under joint ownership and the low valuation agent has better incentives under joint ownership.

Besley and Ghatak (2001) analyze joint ownership in a static game where investments differ in productivities. They find that when the low valuation agent has got relatively more important investment, joint ownership can be optimal as incentives are improved for the agent with the more important investment.

In this paper we analyze a setup where the agents differ only in how they value the public good. We show that joint ownership of the public good can be optimal even when the investments are equally important. Proposition 4 gives our new result for the static game.

Proposition 4 *(i) Joint ownership is optimal in the static game if* $\mu = 1$ *and* $\gamma > 1 + \frac{\theta_h}{\theta_l + \theta_h}$.

(ii) It is optimal to have a single owner in the static game if $\mu = 1$ *and* $\gamma < 1 + \frac{\theta_l}{\theta_l + \theta_h}$.

Under joint ownership both agents have equal, intermediate incentives. While with a single owner the high valuation agent has strong incentives and the low valuation agent has weak incentives. Now if γ is high enough, it is more cost-effective to have two intermediate investments than one very high and one very low. Then joint ownership is optimal. This is proved in Proposition 4. While for γ low enough highest surplus is obtained when the high valuation agent has strong incentives even if it means that the low valuation agent has poor incentives.

4.2 Repeated game

We start analyzing the repeated game by examining the gain and loss from deviation.[4]

Proposition 5 (i) *Joint ownership minimizes (maximizes) the loss from deviation if* $\mu = 1$ *and* $\gamma > 1 + \frac{\theta_h}{\theta_l + \theta_h}$ $\left(\gamma < 1 + \frac{\theta_l}{\theta_l + \theta_h}\right)$.

(ii) *Joint ownership minimizes the gain from deviation if* $\mu = 1$.

Proposition 5 shows that the relative loss from deviation depends on γ. From Proposition 4 we know that joint ownership is optimal in the static game for $\gamma > 1 + \frac{\theta_h}{\theta_l + \theta_h}$. Therefore joint ownership minimizes the loss from deviation in this parameter range. While for $\gamma < 1 + \frac{\theta_l}{\theta_l + \theta_h}$ joint ownership is the worst structure in the static game and therefore provides the maximal punishment in the repeated game.

Proposition 5 also shows that joint ownership minimizes the gain from deviation for $\mu = 1$. Under joint ownership the agents can extract from only half of the value of the other agent's first best investment. While under single ownership they can extract from the full value since even the non-owner's investment is fully sunk in the project. This is why under joint ownership the gain is minimal.

[4] The sharing rule when both agents invest is presented in the Appendix.

Proposition 5 shows that for $\gamma < 1 + \frac{\theta_l}{\theta_l + \theta_h}$ and $\mu = 1$ joint ownership provides both the maximal punishment and the minimal gain from deviation. Then there is no trade-off but joint ownership unambiguously provides the best incentives for cooperation. While for $\gamma > 1 + \frac{\theta_h}{\theta_l + \theta_h}$ there is a familiar trade-off: joint ownership minimizes the gain but also minimizes the punishment. Further examination is therefore needed to determine which ownership structure gives the lowest gain relative to the loss. Lemma 2 compares the critical discount factors for different ownership structures.

Lemma 2 *(i)* $\underline{\delta}^J = \underline{\delta}^1$ *if* $\theta_l = \theta_h$.
(ii) $\partial \underline{\delta}^J / \partial \theta_h = 0$.
(iii) $\partial \underline{\delta}^1 / \partial \theta_h > 0$ *if and only if* $\gamma < 2$.

Proposition 6 follows from Lemma 2.

Proposition 6 *(i) Joint ownership is optimal if* $\mu = 1$ *and* $\gamma < 2$.
(ii) It is optimal to have a single owner if $\mu = 1$ *and* $\gamma > 2$.

Proposition 6 shows that the results again depend on whether γ is smaller or greater than 2. But now also the results of the static game depend on γ and that gives a new twist.

For $\gamma > 2$ the emphasis is on maximizing punishment – as in Section 3 – and the worst structure of the static game guarantees it. In this parameter range single ownership provides the largest punishment because for large γ it is not cost-efficient that the high valuation agent has a very large investment and the low valuation agent has a very low investment.

For $\gamma < 2$ minimizing the gain from deviation provides the best incentives for cooperation. But now joint ownership minimizes the gain from deviation always, whether it is optimal or not in the static game. This is where the results differ from Section 3: minimal gain is no longer equivalent to optimal structure of the static game. The results differ because when we introduce joint ownership, also the results of the static game depend on γ.

It is interesting to compare when joint ownership emerges in the static and in the repeated game. In the static game joint ownership is optimal for large γ while in the repeated game joint ownership arises for small γ. This demonstrates that joint ownership can be optimal in both static and repeated game but it can emerge for different parameter values. But the parameter range is partially overlapping: for $1 + \frac{\theta_h}{\theta_l + \theta_h} < \gamma < 2$ joint ownership is optimal in both static and repeated game.

In the repeated game joint ownership is optimal when it is important to minimize the gain from deviation. In the static game joint ownership is optimal when it is more cost-effective to have two intermediate investments rather than one low and one high investment. Minimizing the gain from deviation is important when the investments are elastic ($\gamma < 2$) while intermediate investments are more cost-effective when the cost function is very steep $\left(\gamma > 1 + \frac{\theta_h}{\theta_l + \theta_h}\right)$. Joint ownership therefore behaves quite differently in static and repeated settings.

We can also compare Proposition 6 to the case of private goods. We note that the result is exactly the opposite. With private goods joint ownership is optimal for $\gamma > 2$ and single ownership for $\gamma < 2$ (Halonen (2002)). This is naturally because joint ownership is never optimal in the static game with private goods.

Finally, we discuss how relaxing the assumption $\mu = 1$ would affect our results. Figure 1 summarizes our findings.

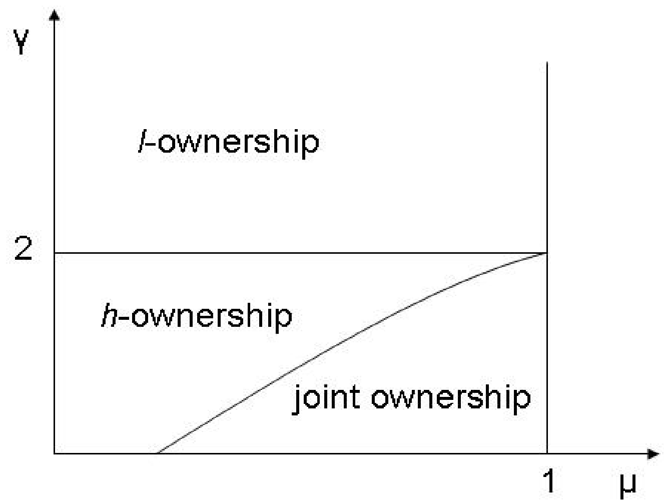

Figure 1

The incentives under joint ownership do not depend on μ. When we lower μ from 1 $\underline{\delta}^l$ and $\underline{\delta}^h$ diverge. For $\gamma > 2$ it is important to maximize punishment and ownership by the low valuation agent provides the most unattractive punishment path. Therefore ownership of the low valuation agent would be optimal for $\gamma > 2$ and $\mu < 1$.

While for $\gamma < 2$ the emphasis is on minimizing the gain. Proposition

5 shows that joint ownership minimizes the gain for $\mu = 1$ but it turns out that this result does not hold for all values of μ. From Proposition 1 we know that the gain under h ownership is always lower than the gain under l ownership. Therefore we are comparing joint ownership and h ownership to find minimal gain. The gain under h ownership is increasing in μ and it is maximal for $\mu = 1$; when all of the non-owner's first best investment is sunk in the project, the owner can extract from the full value of the first best investment. Then joint ownership provides the minimal gain. While for low values of μ the gain under h ownership is lower than the gain under joint ownership (which does not depend on μ). This is why joint ownership is optimal for $\gamma < 2$ and μ large enough while ownership by the high valuation agent is optimal for $\gamma < 2$ and μ low.

5 Cultural goods

Ownership of cultural goods is a fiercely debated topic where emotions run high. Source countries, armed with historical and/or moral reasons have requested the return of cultural property. On the other hand, host countries have in many cases rejected such requests based on legal and/or historical grounds. For example, in the case of the Icelandic manuscripts, Iceland's request for their return was primarily based on historical and moral grounds. The manuscripts were seen as a central part of Icelandic cultural tradition and their return as an issue of utmost importance. In the words of a leading modern Icelandic historian, "next to the issues of fishing boundaries around and the defence of Iceland itself, the return of the manuscripts [was] the biggest and most serious problem in the foreign relations of independent Iceland." (Nielsen (2002), p.5) Furthermore, Iceland claimed that Denmark had a moral obligation to return the manuscripts, especially after the ending of the monarchical union with Denmark in December 1944. Opponents of return argued that the manuscripts constituted a pan-Scandinavian heritage and that Iceland had no legal claim to them. They also claimed that Iceland lacked both technical resources to conserve the manuscripts and scholarly resources to study and publish them while Copenhagen was a recognized centre for Old Norse studies. (Greenfield (2007))

Similar arguments have also been presented in the most famous amongst the cultural restitution claims, the case of the Parthenon marbles located in the British Museum. The main argument of the proponents of return has been that the marbles are an important part of Greek cultural heritage that were removed by Lord Elgin at a time when Greece was under Ottoman control and under dubious circumstances. The British Museum's response to the restitution calls has been that the Parthenon marbles had been legally acquired by Lord Elgin and furthermore, the museum's trustees do not have the right to dispose of any objects.

Such lines of arguments are by no means unique in the cultural restitution literature. Modelling cultural goods as public goods and following a property rights approach provides a new perspective to the restitution question. It enables us to abstract from legal, historical and moral considerations and concentrate on what is best for the cultural good itself, i.e. allocate ownership in such a way so that investments in restoration, protection, study and display of the cultural good are maximized.

5.1 Cultural goods and property rights theory

Our framework applies well to cultural property and restitution. UNESCO defines cultural property as "historical and ethnographical objects and documents including manuscripts, works of the plastic and decorative arts, paleontological and archaeological objects and zoological, botanical and mineralogical specimens" (UNESCO (2001), p.9). In essence, cultural goods are public goods.

Furthermore, the investments are to a large extent project-specific and in human capital. A good example are the Dunhuang manuscripts. In 1900, a total of 40,000 documents were discovered in a cave near Dunhuang, China. Roughly a quarter were taken to each of Beijing, London, Paris and St. Petersburg. Although the manuscripts were discovered more than a century ago, conservation and cataloguing has been very limited. Apart from the size of the undertaking, the limited number of individuals with the necessary expertise to deal with such a unique find has been chiefly responsible for the slow progress. (Whitfield (2001))

Another example was raised in an interview of archeologist Nicoletta Momigliano. Excavations are often dated by the type of pottery found. An archeologist specialized in the pottery sequence in Knossos will find this knowledge of very limited value in closeby South Western Turkey. In other words, this is project-specific human capital.

5.2 Return of cultural good to the high valuation party

The value that different agents place on the cultural good is an important determinant of the optimal ownership structure in our model. This is consistent with the reasoning of UNESCO's Intergovernmental Committee for the Return of Cultural Property, the main body dealing with restitution claims from source countries. It takes an active role in resolving restitution claims by mediating between the source and host countries. The committee's role is to evaluate the claim and recommend return if the cultural good is "highly charged with cultural (or natural) significance ... the removal of [such an] object from its original cultural context irrevocably divests that culture of one of its dimension".[5] We can safely interpret this condition as meaning that recommendation for return will only be granted for goods highly valued by the source country. The recommendation has been made on moral grounds. Property rights theory can show that ownership by the high valuation party can also generate the highest investments in the cultural good.

[5] Greenfield (2007), p. 365.

Greenfield (2007), a leading authority, uses a similar argument when calling for the return of cultural goods to their country of origin. According to her "... cultural property is most important to the people who created it or for whom it was created or whose particular identity and history it is bound with. This cannot be compared with the scholastic or even inspirational influence on those who merely acquire such objects or materials".[6] Greenfield argues for return of (*i*) historic records or manuscripts of a nation, (*ii*) objects torn from immovable property and (*iii*) paleontological materials.

A good example of return of cultural good to a high valuation party is the Icelandic manuscripts. Made of vellum or paper, these documents held the medieval saga literature of Iceland, and were first collected for the most part by Icelander Árni Magnússon in the early 18th century. A professor at the University of Copenhagen (then the only university serving Iceland, being part of the Danish kingdom), Magnússon was sent to Iceland to compile a register of its farms and estates. Being a keen antiquarian, he used his spare time in this period to search the country for manuscripts, and on his return to Copenhagen brought back fifty-five crates full. Over his remaining life he continued to add to this collection, and though two thirds were destroyed by fire in 1728, the collection was still large on being left to the university after his death in 1730.

Beginning in the 19th century, requests were made for the manuscripts' return to Iceland, and on the country's independence in 1944 the campaign became an uppermost priority. Finally in 1971, after much wrangling, a Danish law was ratified which required that all manuscripts held to be 'Icelandic cultural property' would be returned to Iceland. These were generally defined as works composed or translated by an Icelander, whose content was wholly or chiefly concerned with Iceland. A committee of two Danish and two Icelandic scholars decided which manuscripts satisfied these conditions.

Iceland was clearly the country which valued the manuscripts most. De-

[6] Greenfield (2007), p. 411.

sire for their return had been a running theme throughout Iceland's path to independence, and when the first manuscripts finally arrived in the country it was a national event. "Shops and schools were closed. The whole nation ... was listening to the radio or watching television for a live account of the historic event which was taking place." (Greenfield (2007), p. 1)

There are many other examples of repatriation: the 1930s return from Britain to Sri Lanka of the shrine, scepter and orb of the last king of Kandy; the 1964 return from Britain to Burma of the Mandalay Regalia; the 1982 return from Italy to Ethiopia of the throne of Emperor Menelek II; the 2007 return of the Venus de Cyrène from Italy to Libya. Even fragments of the Parthenon were returned to Greece in 2008 from an individual in Sweden, from the Salinas Museum in Palermo and from the Vatican Museum. Repatriation of cultural property continues to occur, and is increasingly accepted by western nations as being, in many cases, the correct course of action.

5.3 Joint ownership

There are some examples of joint ownership of cultural goods. The prime example is the statue of Ur-ningirsu. Until 1974 the 22nd century BC statue of a Mesopotamian ruler was in two parts: its head resided in and belonged to the Metropolitan Museum of Art in New York, and its body resided in and belonged to the Louvre in Paris. After 20 years of discussion an agreement was reached whereby each party would retain ownership of its respective part, but the statue would be exhibited in its entirety, alternating between the two museums every three years. This arrangement, since then altered to allow longer stays at each museum and out of schedule transfers for special occasions, has continued to the present day. (Greenfield (2007), p. 401)

Another arrangement resembling joint ownership may be found with the bronze statue 'Saint Christopher Carrying the Christ Child with the Globe of the World'. The statue of the saint holding in his outstretched hand the baby Christ, who in turn holds the world, was until 1970 in two parts. Saint

Christopher was held by the Louvre, and thought to be Hercules or Atlas, and the Christ Child was held by Washington D.C.'s National Gallery of Art, thought just to be a 'Boy with a Ball'. That year their true roles were discovered, and from 1973 it was agreed that they be exhibited together at the Louvre, with the Christ Child on permanent loan from Washington. In exchange, the Louvre made permanent loan of a different bronze to Washington. (Greenfield (2007), p. 401)

A further example of joint ownership is the proposed joint acquisition of two Titian paintings, Diana and Actaeon, and Diana and Callisto, by the National Gallery and the National Galleries of Scotland. Both paintings were secured by the Duke of Bridgewater in 1798 and have been on long-term loan to the National Galleries of Scotland since 1945. If successful, the two Titians will be displayed in London and Edinburgh, alternating between the galleries every five years.[7]

5.4 Changing optimal owner

Our results suggest that ownership by the high valuation party is not always optimal. On the contrary, under certain conditions the low valuation, host country, should own the cultural good. More specifically, this is the case when it is very costly to increase the value of the cultural good ($\gamma > 2$) as then it is important to maximize the punishment. This is likely to hold at a time when relevant expertise is limited and for more unique finds. The case of the Dunhuang manuscripts illustrates this point. Due to the uniqueness of these manuscripts and the lack of sufficient numbers of individuals with the necessary expertise, any increase in the value is achieved at a considerable cost. (Whitfield (2001))

Investments are likely to become less costly later in the life of the project after the necessary expertise has been developed. This suggests that, although host ownership is initially optimal, as investments become less costly

[7] http://www.nationalgallery.org.uk/campaign-titians.htm

($\gamma < 2$), return becomes optimal as then it is most important to minimize the gain from deviation. Return can be either full (source country ownership) or partial (joint ownership). In our model return of cultural good to the source country is optimal when not much of the host country's investment is sunk in the project (low μ and $\gamma < 2$). This is consistent with the case of Icelandic manuscripts where the Arnamagnean Institute in University of Copenhagen still holds much expertise, which is witnessed by continuing active cooperation with the Arnagarður Institute in Reykjavik (e.g. by exchanging members of staff and summer schools). While when investments are largely sunk in the project (high μ and $\gamma < 2$) joint ownership is optimal. For a statue of Ur-ningirsu investments (largely in protection) are mainly sunk.

5.5 Rescuing cultural property?

One of the arguments in the current debate has been that many of the objects were removed in the past for safe keeping. In effect this is an argument about changing relative valuations. In the past host country was the high valuation party while now the roles are reversed, which triggers the requests for return.

We give an alternative explanation to changes in ownership which is rooted in a technological change. When expertise has been developed, it becomes optimal to shift ownership from the low valuation host country to the country of origin.

There are instances where a rescue argument is valid, most notably during the Cultural Revolution in China in the late 1960s when cultural destruction was intentional. Other upheavals and wars have destroyed cultural treasures. Some cultural treasures would no longer exist if they had not been removed. China's largest encyclopaedia has both suffered from upheavals and been rescued. The encyclopaedia was commissioned by the Ming Dynasty emperor Yung Lo and finished in 1407. Over 2000 scholars worked more than four years to complete 22,937 volumes. Two copies were made in addition to the original in 1567. But only one copy survived the fall of the Ming Dynasty: the original and one copy were destroyed. Then during the Boxer Rebellion in 1900 fire was set to the building where the only remaining copy was kept. But it turned out that a number of the volumes had been removed to Russia and the United Kingdom and were thus rescued.[8]

The destruction of the Buddhas of Bamiyan in Afghanistan by the Taliban in 2001 illustrates a further instance where a rescue argument would have been valid. The two seventh-century Buddhas, one nearly 175 feet tall and the other 120 feet, were destroyed by the Taliban with the excuse that "these idols have been [the] gods of the infidels", despite the offers by various international organisations to rescue the statues.[9]

But rescue argument has also been used as an excuse. Most blatantly by André Malraux when he claimed he was on a mission of rescue when he removed carvings embedded in the walls of the temple of Bantea-Srei of the Angkor complex in Cambodia in 1923. He said his aim was to put into circulation what has been 'lost' in the 'abandoned' temple in the jungle. His group was arrested in Phnom Penh on a ship carrying the carvings. The carvings were restored to the temple. Malraux was sentenced to three years imprisonment, an appeal reduced it to suspended sentence of one year and 8 months – and in the 1960s Malraux became the French Minister of Cultural Affairs.[10]

6 Conclusion

In this paper we analyze the effect of reputation on ownership of public goods. We show that in the dynamic setup the optimal ownership of a public good depends not only on the parties' relative valuations for the good but also on

[8] Greenfield (2007), p. 404.

[9] *The New York Times,* March 4th, 2001. For further information see also *The New York Times,* March 3rd and 19th, 2001

[10] Greenfield (2007), p. 392-395.

technology (elasticity of investment).

In the static game optimal ownership depends importantly on whether we are analysing private or public goods. The ownership of private goods is largely determined by technology (e.g. importance of investment) while relative valuations drive the results with public goods. This is because with private goods higher investment increases only agent's own default payoff. Ownership guarantees high default payoff and good incentives for the agent with important investment. With public goods everyone's default payoff is increased by higher investment and the agent with high valuation gains most and obtains stronger bargaining position while low valuation agent's bargaining position is weaker. Ownership by high valuation agent both maximizes the positive effect on himself and minimizes the negative effect on the low valuation agent.

In the repeated game the results are surprisingly similar to the private goods case. This is because dynamic incentives with both private and public goods are driven by how easy it is to generate punishment. With inelastic investments the only way to have enough punishment power is to choose the most unattractive punishment path. Ownership structure that performs worst in the static game guarantees largest punishment. In the public good case it is ownership by the low valuation agent. While when investments are elastic we can concentrate on minimizing the gain from deviation, which in both cases is obtained by the optimal ownership stucture of the static game: ownership by the high valuation agent for public goods.

We also find that the joint ownership of public good can emerge in both static and repeated setup but for a different parameter range. In the static setup the benefit of joint ownership is that incentives are equalized which is particularly important when γ is high. In the repeated setup the benefit of joint ownership is that the gain from deviation is minimized[11] which is paramount when γ is low.

[11] When μ is high enough.

Our key findings are applied to return of cultural goods to the country of origin. Property rights theory shows that both moral and economic arguments can go hand in hand: ownership by the high valuation party can also generate the highest investments in the cultural good. But in the dynamic context also ownership by the low valuation party can provide the best incentives. Ownership by the low valuation host country is optimal when relevant expertise is limited and return of the cultural good becomes optimal when the necessary expertise is developed.

Valuation for the public good is a key driving force in our analysis. We follow Besley and Ghatak (2001) in assuming that the parties' valuations are common knowledge. Optimal ownership with asymmetric information about the valuations remains as an open question.

7 Appendix

Firstly we give the explicit forms of the investments. For the one investment case of Sections 2 and 3 we have:

$$y^* = \left(\frac{\theta_l + \theta_h}{\gamma}\right)^{\frac{1}{\gamma-1}}$$

$$y^l = \left(\frac{\theta_l}{\gamma}\right)^{\frac{1}{\gamma-1}}$$

$$y^h = \left(\frac{(\theta_l + \theta_h) + \mu(\theta_l - \theta_h)}{2\gamma}\right)^{\frac{1}{\gamma-1}}$$

And for the two investment case of Section 4 (where $\mu = 1$) we have:

$$y^* = \left(\frac{\theta_l + \theta_h}{\gamma}\right)^{\frac{1}{\gamma-1}}$$

$$y_i^1 = \left(\frac{\theta_i}{\gamma}\right)^{\frac{1}{\gamma-1}}$$

$$y^J = \left(\frac{\theta_l + \theta_h}{2\gamma}\right)^{\frac{1}{\gamma-1}}$$

Proof of Proposition 1.

The gain from deviation under h ownership is equal to:

$$
\begin{aligned}
G^h &= P_h^d - (\theta_l + \theta_h) y^* + c(y^*) + P_l^p \\
&= \frac{1}{2}(\theta_l + \theta_h) y^* + \frac{1}{2}(\theta_h - \theta_l)\mu y^* - (\theta_l + \theta_h) y^* + c(y^*) \\
&\quad + \frac{1}{2}(\theta_l + \theta_h) y^h + \frac{1}{2}(\theta_l - \theta_h)\mu y^h - c\left(y^h\right) \\
&= \left[\frac{1}{2}(\theta_l + \theta_h) y^h + \frac{1}{2}(\theta_l - \theta_h)\mu y^h - c\left(y^h\right)\right] \\
&\quad - \left[\frac{1}{2}(\theta_l + \theta_h) y^* + \frac{1}{2}(\theta_l - \theta_h)\mu y^* - c(y^*)\right]
\end{aligned} \tag{17}
$$

While under l ownership we have:

$$
G^l = \left[\frac{1}{2}(\theta_l + \theta_h) y^l + \frac{1}{2}(\theta_l - \theta_h) y^l - c\left(y^l\right)\right] - \left[\frac{1}{2}(\theta_l + \theta_h) y^* + \frac{1}{2}(\theta_l - \theta_h) y^* - c(y^*)\right] \tag{18}
$$

For $\mu = 1$ $y^h = y^l$ (see equations (4) and (6)) and therefore $G^h = G^l$. Now lower μ from 1. Note that $\partial y^*/\partial\mu = 0$. We first differentiate G^h with respect to μ and obtain:

$$
\frac{\partial G^h}{\partial \mu} = \frac{1}{2}(\theta_l - \theta_h)\left(y^h - y^*\right) + \left[\frac{1}{2}(\theta_l + \theta_h) + \frac{1}{2}(\theta_l - \theta_h)\mu - c'\left(y^h\right)\right]\frac{\partial y^h}{\partial \mu}
$$

Using the envelope theorem this is equivalent to:

$$
\frac{\partial G^h}{\partial \mu} = \frac{1}{2}(\theta_l - \theta_h)\left(y^h - y^*\right) > 0 \tag{19}
$$

$\partial G^h/\partial\mu > 0$ since by definition $\theta_l < \theta_h$ and $y^h < y^*$ due to holdup problem.

While from (18) we have:

$$\frac{\partial G^l}{\partial \mu} = 0 \tag{20}$$

Since $G^h = G^l$ for $\mu = 1$, equations (19) and (20) prove that $G^h < G^l$ for $\mu < 1$.

It is obvious that the loss from deviation is higher under l ownership as in the static game h ownership is optimal. Q.E.D.

Proof of Lemma 1.

(*i*) For $\mu = 1$ $y^h = y^l$ (see equations (4) and (6)) and therefore $G^h = G^l$, $L^h = L^l$ and $\underline{\delta}^h = \underline{\delta}^l$.

(*ii*) In the proof of Proposition 1 we showed that $\partial G^l/\partial\mu = 0$ (equation (20)). Furthermore, it is obvious that also L^l is also independent of μ since $\partial y^l/\partial\mu = 0$ and $\partial y^*/\partial\mu = 0$. Therefore $\partial\underline{\delta}^l/\partial\mu = 0$.

(*iii*) By definition

$$\underline{\delta}^h = \frac{G^h}{G^h + L^h}. \tag{21}$$

Substituting (17) and

$$L^h = \left[(\theta_h + \theta_l)\, y^* - c\,(y^*)\right] - \left[(\theta_h + \theta_l)\, y^h - c\left(y^h\right)\right]$$

in (21) and simplifying gives:

$$\underline{\delta}^h = \frac{\left[\frac{1}{2}(\theta_h + \theta_l) + \frac{1}{2}(\theta_l - \theta_h)\,\mu\right] y^h - c\left(y^h\right) - \left[\frac{1}{2}(\theta_h + \theta_l) + \frac{1}{2}(\theta_l - \theta_h)\,\mu\right] y^* + c\,(y^*)}{\left[\frac{1}{2}(\theta_h + \theta_l) + \frac{1}{2}(\theta_h - \theta_l)\,\mu\right](y^* - y^h)} \tag{22}$$

Denote $\nu = \frac{(\theta_l+\theta_h)+\mu(\theta_l-\theta_h)}{2\gamma}$, $\psi = \frac{(\theta_l+\theta_h)+\mu(\theta_h-\theta_l)}{2\gamma}$ and $\eta = \frac{(\theta_l+\theta_h)}{\gamma}$. Then $y^* = \eta^{\frac{1}{\gamma-1}}$ and $y^h = \nu^{\frac{1}{\gamma-1}}$. Substituting these in (22) gives

$$\underline{\delta}^h = \frac{\gamma\nu\nu^{\frac{1}{\gamma-1}} - \nu^{\frac{\gamma}{\gamma-1}} - \gamma\nu\eta^{\frac{1}{\gamma-1}} + \eta^{\frac{\gamma}{\gamma-1}}}{\gamma\psi\left(\eta^{\frac{1}{\gamma-1}} - \nu^{\frac{1}{\gamma-1}}\right)}$$

Next we differentiate $\underline{\delta}^h$ with respect to μ. Note that $\partial\nu/\partial\mu = \frac{(\theta_l-\theta_h)}{2\gamma}$, $\partial\psi/\partial\mu = -\frac{(\theta_l-\theta_h)}{2\gamma}$ and $\partial\eta/\partial\mu = 0$.

$$\begin{aligned}\frac{\partial\underline{\delta}^h}{\partial\mu} &\overset{s}{=} \gamma\psi\left(\eta^{\frac{1}{\gamma-1}} - \nu^{\frac{1}{\gamma-1}}\right)\left[\gamma\frac{(\theta_l-\theta_h)}{2\gamma}\nu^{\frac{1}{\gamma-1}} + \frac{1}{\gamma-1}\gamma\frac{(\theta_l-\theta_h)}{2\gamma}\nu\nu^{\frac{1}{\gamma-1}-1}\right.\\
&\left.-\frac{\gamma}{\gamma-1}\frac{(\theta_l-\theta_h)}{2\gamma}\nu^{\frac{\gamma}{\gamma-1}-1} - \gamma\frac{(\theta_l-\theta_h)}{2\gamma}\eta^{\frac{1}{\gamma-1}}\right]\\
&-\left(\gamma\nu\nu^{\frac{1}{\gamma-1}} - \nu^{\frac{\gamma}{\gamma-1}} - \gamma\nu\eta^{\frac{1}{\gamma-1}} + \eta^{\frac{\gamma}{\gamma-1}}\right)\left[-\gamma\frac{(\theta_l-\theta_h)}{2\gamma}\left(\eta^{\frac{1}{\gamma-1}} - \nu^{\frac{1}{\gamma-1}}\right)\right.\\
&\left.+\gamma\psi\left(-\frac{1}{\gamma-1}\frac{(\theta_l-\theta_h)}{2\gamma}\nu^{\frac{1}{\gamma-1}-1}\right)\right] \qquad (23)\end{aligned}$$

After manipulations (detailed steps in the Supplementary Appendix) we find that (23) implies that

$$\frac{\partial\underline{\delta}^h}{\partial\mu} \overset{s}{=} \frac{(\theta_l-\theta_h)}{2\nu(\gamma-1)}\left[f(\eta) - f(\nu)\right] \qquad (24)$$

where $f(x) = x\left[\frac{1}{\gamma-1} - x^{\frac{2-\gamma}{\gamma-1}}(\nu\eta)^{-\frac{2-\gamma}{2(\gamma-1)}}\right]$. Since $(\theta_l-\theta_h) < 0$, $\frac{\partial\underline{\delta}^h}{\partial\mu} > 0$ if and only if $f(\eta) < f(\nu)$.

Unfortunately $f(x)$ is not monotonic. We will now examine the properties of $f(x)$.

$$
\begin{aligned}
f'(x) &= \left[\frac{1}{\gamma-1} - x^{\frac{2-\gamma}{\gamma-1}} (\nu\eta)^{-\frac{2-\gamma}{2(\gamma-1)}}\right] + x\left[-\frac{2-\gamma}{\gamma-1} x^{\frac{2-\gamma}{\gamma-1}-1} (\nu\eta)^{-\frac{2-\gamma}{2(\gamma-1)}}\right] \\
&= \left[\frac{1}{\gamma-1} - x^{\frac{2-\gamma}{\gamma-1}} (\nu\eta)^{-\frac{2-\gamma}{2(\gamma-1)}}\right] + \left[-\frac{2-\gamma}{\gamma-1} x^{\frac{2-\gamma}{\gamma-1}} (\nu\eta)^{-\frac{2-\gamma}{2(\gamma-1)}}\right] \\
&= \frac{1}{\gamma-1}\left[1 - x^{\frac{2-\gamma}{\gamma-1}} (\nu\eta)^{-\frac{2-\gamma}{2(\gamma-1)}}\right] \qquad (25)
\end{aligned}
$$

Evaluate $f'(x)$ for $x = \nu$. According to equation (25) $f'(\nu) > 0$ if and only if

$$
\nu^{\frac{2-\gamma}{\gamma-1}} (\nu\eta)^{-\frac{2-\gamma}{2(\gamma-1)}} < 1 \qquad (26)
$$

Take logs from equation (26)

$$
\frac{2-\gamma}{\gamma-1} \ln \nu < \frac{2-\gamma}{2(\gamma-1)} \ln (\nu\eta)
$$

$$
\Leftrightarrow (2-\gamma) \ln \nu < (2-\gamma) \ln \eta \qquad (27)
$$

Since $\eta > \nu$ equation (27) holds and $f'(\nu) > 0$ if and only if $\gamma < 2$.

Similarly we can prove that $f'(\eta) > 0$ if and only if $\gamma > 2$.

Finally, we derive the second derivative of $f(x)$.

$$
f''(x) = \frac{\gamma-2}{(\gamma-1)^2} x^{\frac{3-2\gamma}{\gamma-1}} (\nu\eta)^{-\frac{2-\gamma}{2(\gamma-1)}} > 0 \text{ if and only if } \gamma > 2
$$

Accordingly, we have shown that for $\gamma > 2$ $f(x)$ is decreasing at $x = \nu$, increasing at $x = \eta$ and convex. While for $\gamma < 2$ $f(x)$ is increasing at $x = \nu$, decreasing at $x = \eta$ and concave. Figure 2 illustrates $f(x)$.

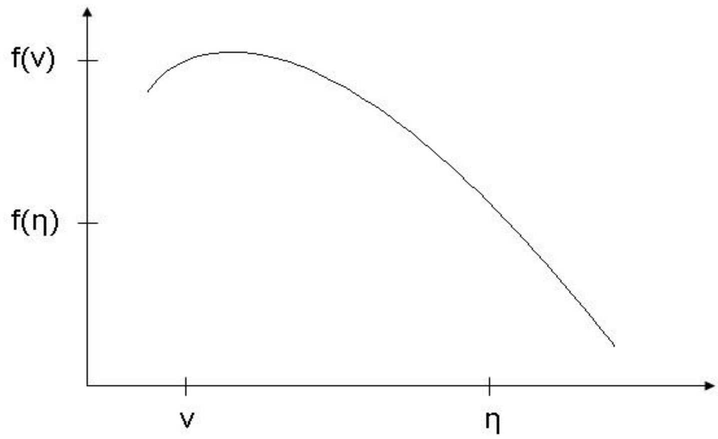

Figure 2a: $\gamma < 2$

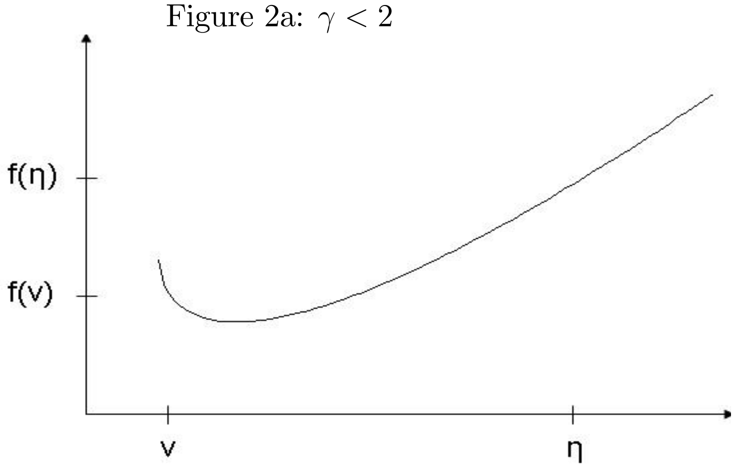

Figure 2b: $\gamma > 2$

Our aim is to prove that $f(\eta) > f(\nu)$ if and only if $\gamma > 2$. That is,

$$\eta\left[\frac{1}{\gamma-1} - \eta^{\frac{2-\gamma}{\gamma-1}}(\nu\eta)^{-\frac{2-\gamma}{2(\gamma-1)}}\right] > \nu\left[\frac{1}{\gamma-1} - \nu^{\frac{2-\gamma}{\gamma-1}}(\nu\eta)^{-\frac{2-\gamma}{2(\gamma-1)}}\right] \text{ if and only if } \gamma > 2 \tag{28}$$

As Figure 2 illustrates, it is most difficult to satisfy $f(\eta) > f(\nu)$ for $\mu = 1$ when $\gamma > 2$, since ν is decreasing in μ (and η does not depend on μ). If $f(\eta) > f(\nu)$ for $\mu = 1$, it holds for all $0 \leq \mu \leq 1$. Similarly it is most difficult to satisfy $f(\eta) < f(\nu)$ for $\mu = 1$ when $\gamma < 2$. Now let us assume that $\mu = 1$ and thus $\nu = \theta_l/\gamma$.

Substitute $\nu = \theta_l/\gamma$ and $\eta = (\theta_l + \theta_h)/\gamma$ in (28) and also take into account Assumption 1 ($\theta_l = 1$).

$$\frac{(1+\theta_h)}{\gamma}\left[\frac{1}{\gamma-1} - \left(\frac{1+\theta_h}{\gamma}\right)^{\frac{2-\gamma}{\gamma-1}}\left(\frac{1+\theta_h}{\gamma^2}\right)^{-\frac{2-\gamma}{2(\gamma-1)}}\right]$$
$$> \frac{1}{\gamma}\left[\frac{1}{\gamma-1} - \left(\frac{1}{\gamma}\right)^{\frac{2-\gamma}{\gamma-1}}\left(\frac{1+\theta_h}{\gamma^2}\right)^{-\frac{2-\gamma}{2(\gamma-1)}}\right]$$

$$\Leftrightarrow (1+\theta_h)\left[\frac{1}{(\gamma-1)} - (1+\theta_h)^{\frac{2-\gamma}{\gamma-1}}(1+\theta_h)^{-\frac{2-\gamma}{2(\gamma-1)}}\right] > \frac{1}{(\gamma-1)} - (1+\theta_h)^{-\frac{2-\gamma}{2(\gamma-1)}}$$

$$\Leftrightarrow (1+\theta_h)^{\frac{\gamma}{2(\gamma-1)}} - (1+\theta_h)^{\frac{\gamma-2}{2(\gamma-1)}} - \frac{\theta_h}{(\gamma-1)} < 0 \tag{29}$$

Denote $\alpha = \frac{\gamma}{2(\gamma-1)}$. This gives $\gamma = \frac{2\alpha}{2\alpha-1}$. Note that $\lim_{\gamma\to1}\alpha = \infty$ and $\lim_{\gamma\to\infty}\alpha = \frac{1}{2}$. Therefore the relevant range for us is $\alpha > \frac{1}{2}$. We have to prove that

$$(1+\theta_h)^{\alpha} - (1+\theta_h)^{1-\alpha} - (2\alpha - 1)\,\theta_h < 0 \text{ if and only if } \frac{1}{2} < \alpha < 1 \quad (30)$$

This implies that $\frac{\partial \delta^h}{\partial \mu} < 0$ if and only if $\frac{1}{2} < \alpha < 1$, or if and only if $\gamma > 2$.

Define $x = 1 + \theta_h > 2$. Substitute in (30).

$$x^{\alpha} - x^{1-\alpha} < (2\alpha - 1)(x - 1) \quad (31)$$

Denote $y(\alpha) = x^{\alpha}$ and $z(\alpha) = x^{1-\alpha}$.

$$y'(\alpha) = y(\alpha)\ln x > 0$$

$$z'(\alpha) = -z(\alpha)\ln x < 0$$

Therefore the left-hand side of (31) is increasing in α since $y'(\alpha) - z'(\alpha) > 0$.

Let us examine further the properties of the left-hand side of (31).

$$y''(\alpha) = y(\alpha)(\ln x)^2 > 0$$

$$z''(\alpha) = z(\alpha)(\ln x)^2 > 0$$

$$y''(\alpha) - z''(\alpha) = (\ln x)^2 [y(\alpha) - z(\alpha)] > 0 \text{ if and only if } \alpha > 1/2$$

Accordingly the left-hand side of (31) is increasing in α, is concave up to $\alpha = 1/2$ and then convex. We will further evaluate it for some values for α.

$$y(0) - z(0) = 1 - x < 0$$

$$y\left(\frac{1}{2}\right) - z\left(\frac{1}{2}\right) = 0$$

$$y(1) - z(1) = x - 1 > 0$$

Then we will examine the right-hand side of (31). Denote $g(\alpha) = (2\alpha - 1)(x - 1)$. We have $g'(\alpha) = 2(x - 1) > 0$ and $g''(\alpha) = 0$. Furthermore,

$$g(0) = 1 - x < 0$$
$$g\left(\frac{1}{2}\right) = 0$$
$$g(1) = x - 1 > 0$$

Therefore the left-hand side and the right-hand side of (31) are equal for $\alpha = \left\{0, \frac{1}{2}, 1\right\}$.

We have established that both the left-hand side and the right-hand side of (31) are increasing in α. Since the right-hand side is linear and the left-hand side is concave between 0 and $\frac{1}{2}$ and convex after $\frac{1}{2}$ and the right-hand side and left-hand side are equal for $\alpha = \left\{0, \frac{1}{2}, 1\right\}$, it has to be true that (31) holds if and only if $\frac{1}{2} \leq \alpha \leq 1$. (See Figure 3).

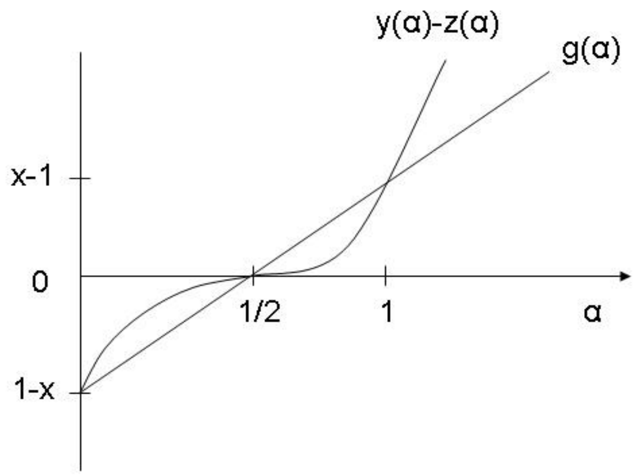

Figure 3

Since the relevant range for us is $\alpha > \frac{1}{2}$ this proves equation (30) and shows that $\frac{\partial \delta^h}{\partial \mu} < 0$ if and only if $\gamma > 2$.

Q.E.D.

Proof of Proposition 2.

It is clear from Lemma 1 that $\underline{\delta}^h < \underline{\delta}^l$ if and only if $\gamma < 2$. Q.E.D.

Proof of Proposition 3.

It is clear from Lemma 1 that $\underline{\delta}^h > \underline{\delta}^l$ if and only if $\gamma > 2$. Q.E.D.

Sharing rule when both agents invest

When both agents invest the incentive compatibility constraints are:

$$\frac{1}{1-\delta}\left[\theta_h\left(y^*+y^*\right)-T^*-c\left(y^*\right)\right]\geq P_h^d+\frac{\delta}{1-\delta}P_h^p \tag{32}$$

$$\frac{1}{1-\delta}\left[\theta_l\left(y^*+y^*\right)+T^*-c\left(y^*\right)\right]\geq P_l^d+\frac{\delta}{1-\delta}P_l^p \tag{33}$$

The agents can always find a suitable T^* that satisfies both agents' incentive compatibility constraints as long as the aggregate incentive compatibility constraint holds. Summing up equations (32) and (33) we obtain:

$$\frac{1}{1-\delta}\left[\left(\theta_l+\theta_h\right)\left(y^*+y^*\right)-2c\left(y^*\right)\right]\geq\left(P_h^d+P_l^d\right)+\frac{\delta}{1-\delta}\left(P_h^p+P_l^p\right) \tag{34}$$

Denoting $G=\left[\left(P_h^d+P_l^d\right)-\left(\theta_l+\theta_h\right)\left(y^*+y^*\right)+2c\left(y^*\right)\right]$ and $L=\left[\left(\theta_l+\theta_h\right)\left(y^*+y^*\right)-2c\left(y^*\right)-P_h^p-P_l^p\right]$ we can again express (34) as

$$\delta\geq\frac{G}{G+L}.$$

Proof of Proposition 4.

Joint surplus under single ownership is larger than under joint ownership if and only if

$$2\left(\theta_l+\theta_h\right)y^J-2c\left(y^J\right)>\left(\theta_l+\theta_h\right)\left(y_l^1+y_h^1\right)-c\left(y_l^1\right)-c\left(y_h^1\right) \tag{35}$$

Substituting the investment in (35) we obtain

$$\begin{aligned} & 2(\theta_l+\theta_h)\left(\frac{\theta_l+\theta_h}{2\gamma}\right)^{\frac{1}{\gamma-1}} - 2\left(\frac{\theta_l+\theta_h}{2\gamma}\right)^{\frac{\gamma}{\gamma-1}} \\ < \ & (\theta_l+\theta_h)\left(\frac{\theta_l}{\gamma}\right)^{\frac{1}{\gamma-1}} + (\theta_l+\theta_h)\left(\frac{\theta_h}{\gamma}\right)^{\frac{1}{\gamma-1}} - \left(\frac{\theta_l}{\gamma}\right)^{\frac{\gamma}{\gamma-1}} - \left(\frac{\theta_h}{\gamma}\right)^{\frac{\gamma}{\gamma-1}} \end{aligned} \quad (36)$$

Define $A = \frac{\theta_l+\theta_h}{2\gamma}$, $B = \frac{\theta_h}{\gamma}$ and $C = \frac{\theta_l}{\gamma}$. Note that $C < A < B$ and A is the average of C and B. (36) is then equivalent to

$$\left[(\theta_l+\theta_h)A^{\frac{1}{\gamma-1}} - A^{\frac{\gamma}{\gamma-1}} - (\theta_l+\theta_h)C^{\frac{1}{\gamma-1}} + C^{\frac{\gamma}{\gamma-1}}\right] <$$

$$\left[(\theta_l+\theta_h)A^{\frac{1}{\gamma-1}} + A^{\frac{\gamma}{\gamma-1}} - (\theta_l+\theta_h)B^{\frac{1}{\gamma-1}} - B^{\frac{\gamma}{\gamma-1}}\right] \quad (37)$$

Define $F(x) = (\theta_l+\theta_h)x^{\frac{1}{\gamma-1}} - x^{\frac{\gamma}{\gamma-1}}$. Equation (37) is equivalent to

$$[F(A) - F(C)] < [F(B) - F(A)] \quad (38)$$

First show that $F'(x) > 0$ for $x \leq B$.

$$\begin{aligned} F'(x) > 0 \ \Leftrightarrow \ & \frac{1}{\gamma-1}(\theta_l+\theta_h)x^{\frac{2-\gamma}{\gamma-1}} > \frac{\gamma}{\gamma-1}x^{\frac{1}{\gamma-1}} \\ & \Leftrightarrow x < \frac{(\theta_l+\theta_h)}{\gamma} \end{aligned} \quad (39)$$

The maximum x for which we are analyzing $F(x)$ is $B = \frac{\theta_h}{\gamma}$. Clearly (39) is satisfied in the relevant range and we have that $F(B) > F(A) > F(C)$. Note furthermore that $B - A = A - C$ since A is the average of C and B. Therefore (38) holds if $F(x)$ is convex. We derive the second derivative of $F(x)$.

$$F''(x) = \frac{2-\gamma}{(\gamma-1)^2}(\theta_l+\theta_h)x^{\frac{3-2\gamma}{\gamma-1}} - \frac{\gamma}{(\gamma-1)^2}x^{\frac{2-\gamma}{\gamma-1}}$$

$$F''(x) > 0 \text{ if and only if } x < (2-\gamma)\frac{(\theta_l+\theta_h)}{\gamma} \tag{40}$$

Evaluate (40) for $x = A$.

$$\frac{\theta_l+\theta_h}{2\gamma} < (2-\gamma)\frac{(\theta_l+\theta_h)}{\gamma}$$

$$\Leftrightarrow \gamma < 1\frac{1}{2}$$

Similarly evaluating (40) for $x = B$ and $x = C$ we find that $F(x)$ is convex at $x = B$ if and only if $\gamma < 1 + \frac{\theta_l}{\theta_l+\theta_h}$ and $F(x)$ is convex at $x = C$ if and only if $\gamma < 1 + \frac{\theta_h}{\theta_l+\theta_h}$.

Therefore $F(x)$ is convex for $x \in [C, B]$ if $\gamma < 1 + \frac{\theta_l}{\theta_l+\theta_h}$ while $F(x)$ is concave for $x \in [C, B]$ if $\gamma > 1 + \frac{\theta_h}{\theta_l+\theta_h}$.

We are evaluating whether $F(x)$ increases more when x increases from C to A than when x increases from A to B where A is the average of C and B. We have proved that $F(x)$ is increasing. Furthermore $F(x)$ is convex in the relevant range if $\gamma < 1 + \frac{\theta_l}{\theta_l+\theta_h}$. For convex $F(x)$ we have that $F(A) - F(C) < F(B) - F(A)$. This is when single ownership is optimal. While when $F(x)$ is concave in the relevant range, i.e. $\gamma > 1+\frac{\theta_h}{\theta_l+\theta_h}$, $F(A) - F(C) > F(B) - F(A)$ and joint ownership dominates.

Q.E.D.

Proof of Proposition 5.

(i) Proposition 4 shows that joint ownership is optimal in the static game for $\mu = 1$ and $\gamma > 1 + \frac{\theta_h}{\theta_l+\theta_h}$. Therefore joint ownership minimizes the loss from deviation in this parameter range. For $\mu = 1$ and $\gamma < 1 + \frac{\theta_l}{\theta_l+\theta_h}$ single ownership dominates joint ownership in the static game. Therefore joint ownership maximizes the loss from deviation in this parameter range.

(ii) We will first work out the aggregate gain from deviation under single ownership.

$$
\begin{aligned}
G^1 &= \left[\frac{1}{2}(\theta_l+\theta_h)\left(y^*+y_h^1\right)+\frac{1}{2}(\theta_h-\theta_l)\left(y^*+y_h^1\right)-c\left(y_h^1\right)\right] \\
&\quad+\left[\frac{1}{2}(\theta_l+\theta_h)\left(y^*+y_l^1\right)+\frac{1}{2}(\theta_l-\theta_h)\left(y^*+y_l^1\right)-c\left(y_l^1\right)\right] \\
&\quad-\left[2(\theta_l+\theta_h)y^*-2c(y^*)\right] \\
&= \left[\theta_h y_h^1-c\left(y_h^1\right)\right]+\left[\theta_l y_l^1-c\left(y_l^1\right)\right]-\left[(\theta_l+\theta_h)y^*-2c(y^*)\right] \quad (41)
\end{aligned}
$$

In the same way we can work out the aggregate gain under joint ownership.

$$
\begin{aligned}
G^J &= 2\left[\frac{1}{2}(\theta_l+\theta_h)\left(y^*+y^J\right)-c\left(y^J\right)\right]-2\left[(\theta_l+\theta_h)y^*-c(y^*)\right] \\
&= 2\left[\frac{1}{2}(\theta_l+\theta_h)y^J-c\left(y^J\right)\right]-2\left[\frac{1}{2}(\theta_l+\theta_h)y^*-c(y^*)\right] \quad (42)
\end{aligned}
$$

The ownership structures are equivalent for $\theta_h=\theta_l$ because the second terms cancel out in (15) and (16). Differentiate G^1 and G^J with respect to θ_h. Note that $\partial y_l^1/\partial\theta_h=0$ when $\mu=1$ and use the envelope theorem.

$$
\begin{aligned}
\frac{\partial G^1}{\partial\theta_h} &= \left(y_h^1-y^*\right)+\left[\theta_h-c'\left(y_h^l\right)\right]\frac{\partial y_h^1}{\partial\theta_h}-\left[(\theta_l+\theta_h)-2c'(y^*)\right]\frac{\partial y^*}{\partial\theta_h} \\
&= \left(y_h^1-y^*\right)-\left[(\theta_l+\theta_h)-2c'(y^*)\right]\frac{\partial y^*}{\partial\theta_h} \quad (43)
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial G^J}{\partial\theta_h} &= \left(y^J-y^*\right)+2\left[\frac{1}{2}(\theta_l+\theta_h)-c'\left(y^J\right)\right]\frac{\partial y^J}{\partial\theta_h}-\left[(\theta_l+\theta_h)-2c'(y^*)\right]\frac{\partial y^*}{\partial\theta_h} \\
&= \left(y^J-y^*\right)-\left[(\theta_l+\theta_h)-2c'(y^*)\right]\frac{\partial y^*}{\partial\theta_h} \quad (44)
\end{aligned}
$$

Equations (43) and (44) show that

$$\frac{\partial G^J}{\partial \theta_h} < \frac{\partial G^1}{\partial \theta_h} \Leftrightarrow y^J < y_h^1$$

which is satisfied (see equations (14) and (16)).

We know that for $\theta_l = \theta_h$ $G^J = G^1$. Now increase θ_h from $\theta_h = \theta_l$. Equations (43) and (44) show that the change in G^1 is greater than the change in G^J when $\mu = 1$. Therefore joint ownership provides the minimal gain $\left(G^J < G^1\right)$ for $\theta_h > \theta_l$ and $\mu = 1$.

Q.E.D.

Proof of Lemma 2.

(i) The ownership structures are equivalent if $\theta_l = \theta_h$ because the second terms in the incentives under single ownership (equations (15) and (16)) cancel out.

(ii) Equation (42) gives the gain from deviation under joint ownership. Substituting in the investments we obtain

$$
\begin{aligned}
G^J &= 2\left[\frac{1}{2}(\theta_l+\theta_h)\left(\frac{\theta_l+\theta_h}{2\gamma}\right)^{\frac{1}{\gamma-1}} - \left(\frac{\theta_l+\theta_h}{2\gamma}\right)^{\frac{\gamma}{\gamma-1}}\right] \\
&\quad -2\left[\frac{1}{2}(\theta_l+\theta_h)\left(\frac{\theta_l+\theta_h}{\gamma}\right)^{\frac{1}{\gamma-1}} - \left(\frac{\theta_l+\theta_h}{\gamma}\right)^{\frac{\gamma}{\gamma-1}}\right] \\
&= 2\left[\left(\frac{1}{2}\right)^{\frac{\gamma}{\gamma-1}}\left(\frac{1}{\gamma}\right)^{\frac{1}{\gamma-1}}(\theta_l+\theta_h)^{\frac{\gamma}{\gamma-1}} - \left(\frac{1}{\gamma}\right)^{\frac{\gamma}{\gamma-1}}\left(\frac{1}{2}\right)^{\frac{\gamma}{\gamma-1}}(\theta_l+\theta_h)^{\frac{\gamma}{\gamma-1}}\right] \\
&\quad -2\left[\frac{1}{2}\left(\frac{1}{\gamma}\right)^{\frac{1}{\gamma-1}}(\theta_l+\theta_h)^{\frac{\gamma}{\gamma-1}} - \left(\frac{1}{\gamma}\right)^{\frac{\gamma}{\gamma-1}}(\theta_l+\theta_h)^{\frac{\gamma}{\gamma-1}}\right] \\
&= 2\left(\frac{1}{2}\right)^{\frac{\gamma}{\gamma-1}}\left(\frac{1}{\gamma}\right)^{\frac{1}{\gamma-1}}(\theta_l+\theta_h)^{\frac{\gamma}{\gamma-1}}\left(\frac{\gamma-1}{\gamma}\right) - 2\left(\frac{1}{\gamma}\right)^{\frac{1}{\gamma-1}}(\theta_l+\theta_h)^{\frac{\gamma}{\gamma-1}}\left(\frac{\gamma-2}{2\gamma}\right) \\
&= 2\left(\frac{1}{\gamma}\right)^{\frac{1}{\gamma-1}}(\theta_l+\theta_h)^{\frac{\gamma}{\gamma-1}}\left[\left(\frac{1}{2}\right)^{\frac{\gamma}{\gamma-1}}\left(\frac{\gamma-1}{\gamma}\right) - \left(\frac{\gamma-2}{2\gamma}\right)\right] \\
&= \left(\frac{\theta_l+\theta_h}{\gamma}\right)^{\frac{\gamma}{\gamma-1}}\left[\left(\frac{1}{2}\right)^{\frac{1}{\gamma-1}}(\gamma-1) - (\gamma-2)\right]
\end{aligned}
$$

The sum of gain and loss from deviation is

$$
\begin{aligned}
G^J + L^J &= 2\left[\frac{1}{2}(\theta_l+\theta_h)\,y^J - c\left(y^J\right)\right] - 2\left[\frac{1}{2}(\theta_l+\theta_h)\,y^* - c\left(y^*\right)\right] \\
&\quad +2\left[(\theta_l+\theta_h)\,y^* - c\left(y^*\right)\right] - 2\left[(\theta_l+\theta_h)\,y^J - c\left(y^J\right)\right] \\
&= (\theta_l+\theta_h)\left(y^* - y^J\right) \\
&= (\theta_l+\theta_h)\left[\left(\frac{\theta_l+\theta_h}{\gamma}\right)^{\frac{1}{\gamma-1}} - \left(\frac{\theta_l+\theta_h}{2\gamma}\right)^{\frac{1}{\gamma-1}}\right] \\
&= (\theta_l+\theta_h)^{\frac{\gamma}{\gamma-1}}\left(\frac{1}{\gamma}\right)^{\frac{1}{\gamma-1}}\left[1 - \left(\frac{1}{2}\right)^{\frac{1}{\gamma-1}}\right]
\end{aligned}
$$

Therefore the critical discount factor is

$$
\begin{aligned}
\underline{\delta}^J &= \frac{G^J}{G^J + L^J} \\
&= \frac{\left(\frac{\theta_l+\theta_h}{\gamma}\right)^{\frac{\gamma}{\gamma-1}} \left[\left(\frac{1}{2}\right)^{\frac{1}{\gamma-1}}(\gamma-1) - (\gamma-2)\right]}{(\theta_l+\theta_h)^{\frac{\gamma}{\gamma-1}} \left(\frac{1}{\gamma}\right)^{\frac{1}{\gamma-1}} \left[1 - \left(\frac{1}{2}\right)^{\frac{1}{\gamma-1}}\right]} \\
&= \frac{\left[\left(\frac{1}{2}\right)^{\frac{1}{\gamma-1}}(\gamma-1) - (\gamma-2)\right]}{\gamma\left[1 - \left(\frac{1}{2}\right)^{\frac{1}{\gamma-1}}\right]}
\end{aligned}
$$

Clearly $\partial\underline{\delta}^J/\partial\theta_h = 0$.

(iii) From the proof of Lemma 1 we know that

$$
\underline{\delta}^1 = \frac{\gamma\nu\nu^{\frac{1}{\gamma-1}} - \nu^{\frac{\gamma}{\gamma-1}} - \gamma\nu\eta^{\frac{1}{\gamma-1}} + \eta^{\frac{\gamma}{\gamma-1}}}{\gamma\psi\left(\eta^{\frac{1}{\gamma-1}} - \nu^{\frac{1}{\gamma-1}}\right)}
$$

Since $\mu = 1$ we have $\nu = \theta_l/\gamma$, $\psi = \theta_h/\gamma$ and $\eta = \frac{(\theta_l+\theta_h)}{\gamma}$.

Differentiate $\underline{\delta}^1$ with respect to θ_h. Note that $\partial\nu/\partial\theta_h = 0$ and $\partial\psi/\partial\theta_h = \partial\eta/\partial\theta_h = 1/\gamma$.

$$
\begin{aligned}
\frac{\partial\underline{\delta}^1}{\partial\theta_h} &\overset{s}{=} \psi\gamma\left[\eta^{\frac{1}{\gamma-1}} - \nu^{\frac{1}{\gamma-1}}\right]\left[-\nu\gamma\frac{1}{\gamma-1}\frac{1}{\gamma}\eta^{\frac{1}{\gamma-1}-1} + \frac{\gamma}{\gamma-1}\frac{1}{\gamma}\eta^{\frac{\gamma}{\gamma-1}-1}\right] \qquad (45) \\
&- \left[\nu\gamma\nu^{\frac{1}{\gamma-1}} - \nu^{\frac{\gamma}{\gamma-1}} - \nu\gamma\eta^{\frac{1}{\gamma-1}} + \eta^{\frac{\gamma}{\gamma-1}}\right]\left[\eta^{\frac{1}{\gamma-1}} - \nu^{\frac{1}{\gamma-1}} + \psi\gamma\frac{1}{\gamma-1}\frac{1}{\gamma}\eta^{\frac{1}{\gamma-1}-1}\right]
\end{aligned}
$$

After manipulations (detailed steps can be found in the Supplementary Appendix) we find that (45) implies that

$$\frac{\partial \underline{\delta}^1}{\partial \theta_h} \stackrel{s}{=} (1+\theta_h)^{\frac{\gamma}{2(\gamma-1)}} - (1+\theta_h)^{\frac{\gamma-2}{2(\gamma-1)}} - \frac{\theta_h}{(\gamma-1)}$$

This is equivalent to equation (29) in the proof of Lemma 1. Therefore $\partial \underline{\delta}^1 / \partial \theta_h < 0$ if and only if $\gamma > 2$.

Q.E.D.

Proof of Proposition 6.

It is clear from Lemma 2 that $\underline{\delta}^J < \underline{\delta}^1$ if and only if $\gamma < 2$. Q.E.D.

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