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Passive Industry Interests in a Large Polity: A Large Poisson Game Approach to Endogenous Lobby Formation

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Abstract

In this paper we offer a theory of lobby formation that is consistent with passive industry groups. Our focus is on the variable `political influence' costs associated with direct transfers and the decision to contribute towards these costs in a large polity. Modelling the latter decision as a Large Poisson Game, we show that it can never be an equilibrium for more than one lobby to attempt to solicit policy favours. Rather a single group that does not oppose the legislator's prior preference dominates the policy process. Direct corollaries are that `money never changes hands' and lobbying is purely counteractive.

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1. Introduction

For any particular policy issue it is often easy to identify a number of different interests groups, or sub-sets of citizens likely to share a common policy goal. It is a common observation, however, that such groups seem to differ in the degree to which they participate, collectively, in the political process in an attempt to further these goals. Moreover, in a number of cases, lobby group competition appears to be absent altogether, with just one 'voice' appearing to dominate the lobbying process.

The most obvious example of such a phenomenon is the subject of its own literature, namely 'regulatory capture'. Inspired by Stigler's (1971) re-interpretation of US regulatory policy, this literature suggests that industry groups lack competition to such an extent that they can effectively 'capture' legislators and thus ensure that regulatory policy is supplied exclusively in their favour. The dominance of a single lobby has also been noted in trade policy. For example Krueger (1990) points to the fact that industrial users such as soft drink manufacturers, as well as ordinary consumers, played a surprisingly passive role in the debate between sugar growers and the US government over the issue of sugar price supports. Similarly, in a cross-sectional study of 435 4-digit industries Gawande (1997) notes that there is little evidence to suggest that a protectionist lobby faced effective competition from either a pro-export or downstream group, let alone consumers, when lobbying for non-tariff barriers on Japanese imports.

Naturally this asymmetry has not gone unnoticed by economists and political scientists. Rather, the universal response has been to build on Olson's (1965) work on collective action in which it is suggested that certain groups fail to organise, and thus participate, collectively, in policy debates, because at some level the fixed costs of group formation exceed the expected rewards. Received theory therefore suggests a taxonomy of group formation driven by variations in group size and per member net benefits. That is, if a group sharing a common policy goal contains at least one member for whom the benefits from political action exceeds the associated costs, even if borne alone, it is suggested that the group is privileged and will always find a 'voice' in the policy process. Analogously, if the costs associated with formation exceed the total amount by which a group stands to gain from political participation, collective action is sub-optimal. In such an event the group is thought to be latent: always failing to participate in the policy process.

In the remaining case, however, when aggregate, but no individual, benefit exceeds the associated fixed costs, the outcome is deemed to be dependant on the level of free-riding and thus 'indeterminate'. That is, since the provision of a favourable policy outcome has all the characteristics of a pure public good, depending on the size of the group, members face incentives to take an easy, if not free, ride. Thus to 'form', and hence be intermediate, a group must be sufficiently small to limit the possibility of easy / free-riding and thereby ensure that a coalition is willing to bear the fixed costs of political participation. Once group size becomes large it is conjectured that actions will not be considered pivotal and thus that easy / free-riding will ensure the group remains latent.

Taking the regulatory example above, this theory of lobby formation does indeed offer a plausible explanation as to why groups representing regulated industries lack lobbying competition. Firstly, industry groups often have a pre-existing structure (trade, shareholder or union associations) and are smaller in size, implying that the fixed organisational costs they face are likely to be lower than those of their obvious competitors, consumers. This fixed cost differential obviously increases the likelihood that: a) the industry group faces an aggregate, if not per-member, net gain from political participation and b) the consumer group faces an aggregate net loss that ensures it is always latent. Secondly, in the event that both groups face an aggregate net gain, the larger size of the consumer group increases the probability that free-riding will prohibit the formation of a consumer but not of a producer lobby. In other words group size and per member net benefits combine to suggest that the consumer group will prove to be latent but the industry group intermediate.

However, whilst the above logic fares well as an explanation of patterns of regulatory group formation, it has far less predictive power in other policy spheres. In particular the examples cited above pose two worrying questions. Firstly, as Dixit (1996) has noted, Olsonian theories actually support the existence of an active industrial sugar users group as well as a farming lobby; like wise for pro-export and downstream industry lobbies in addition to active protectionists. Put very simply all these groups represent 'industry interests', implying that the playing field should level, both in terms of the likelihood of an aggregate net gain and any problems stemming from free-riding. Of course it is possible that sufficiently great differences remain to explain the observed lack of competition but, ex ante at least, Olsonian logic seems to have lost its bite. Secondly and more worryingly, in addition to the lack of competition for non-tariff barriers, Gawande (1997) observed that "lobbying by export-oriented industries seems to be directed at instruments that facilitate exports rather than anti-protectionism"[1]. If robust this result surely questions the validity of the whole Olsonian approach. For, if a group has financed the fixed costs associated with collective participation in one policy dimension why would it remain silent in another?

The above examples therefore suggest that received theory offers us an insight into the issue of passive consumer groups, but crucially not passive industry interests. Thus the obvious question, and hence the subject of this paper, is why do some industry interest groups participate in the policy process whilst others of similar size and per member net benefits do not; alternatively, as commentators have often asked, 'where was Coca-Cola in the debate over sugar prices?'

We attempt to shed light on the empirical puzzle posed by 'passive industry groups' by re-formulating current thinking in two ways. Firstly, we focus attention away from the exogenous asymmetries that drive Olsonian theories of group formation. That is, since we seek to explain why variation in participation arises even when 'the playing field is level' in Olsonian terms, we remove exogenous asymmetry by normalising fixed organisational costs to zero[2]. In contrast to existing models, then, our modeling approach focuses on each industry group's ability to raise variable political influence costs, and in particular those costs associated with lobbying by virtue of direct transfers to policy makers. Similarly, rather than looking to exogenous differences in potential membership to explain variations in formation across intermediate groups, we allow for the possibility of groups of approximately equal size.

Secondly, we note that the total number of stakeholding firms, or potential benefactors, across all industry groups is actually often large. For instance in the trade example above potential contributors to any group spanned all pro-export, downstream and protectionist firms in the US. Thus, whilst any one industry group might not be considered large, the population, or rather the expected number of all firms with a policy stake, almost certainly will be. Driven by this observation, we investigate the decision to contribute towards variable political influence costs by adopting recent developments in games with a large number of players.

When modelling interactions in a large polity, ones notices that the standard assumption that every player takes other players' behaviour as given and known

[1] Gawande (1997), pp. 65.

[2] Note the suggestion here is simply that, across the groups we wish to consider, fixed costs might be similar. In normalising these costs to zero the implicit suggestion is either that such organisational costs have been borne for alternative purposes (e.g. union formation), or that a sufficient residual remains from other activities to cover further costs (e.g. hiring professional lobbyists).

seems implausible. Or, as Myerson (1998) puts it, in games with a very large number of players 'it is unrealistic to assume that every player knows all the other players in the game; instead, a more realistic model should admit some uncertainty about the number of players in the game.' In fact Myerson (1998) goes even further, proposing the use of so-called Large Poisson Games as more realistic modeling devices of such situations. We therefore follow his suggestion and model the information structure underlying the contribution decision as a Large Poisson Game.

In essence we show that, in the absence of ex ante asymmetries, adopting a Large Poisson Game approach to endogenous lobby formation does indeed offer an insight into the phenomenon of passive industry groups. Drawing on established results from the literature on policy menu-auctions we demonstrate that it can never be an equilibrium for more than one industry lobby to attempt to solicit policy favours. More intuitively, we show that when the total number of potential contributors across all groups is very large the free-rider problem becomes insurmountable; no group is able to raise sufficient contributions to offer strictly positive transfers. However, rather than eliminating influence seeking altogether, as Olson conjectured, this leaves the political market place open to groups that 'do not need to pay'. In other words to lone lobbies that do not need to compensate legislators for their policy choices. In this sense, then, industry lobbying becomes purely counteractive; a single group may choose to enter the policy process to ensure that participation becomes a costly activity for its competitors.

The remainder of the paper is organised as follows. The next section discusses the relationship between our approach and other models of the lobbying process in more detail. Section 3 describes the details of our model. Sections 4 and 5 present results from the common agency policy selection game and the standard / large Poisson contribution games respectively and Section 6 concludes.

2. Related Literature

This paper relates directly to three strands of literature: a) 'intra-group' models of the strategic interaction between the members of any one group over individual contributions towards the provision of collective lobbying activity; b) 'inter-group' models of the strategic interaction between a given number of lobbies, each able to finance the costs of political participation; c) fully endogenous models that seek to derive simultaneously an equilibrium policy choice and equilibrium level of lobby group participation.

In all of the 'intra-group' models the final policy choice is seen as a public good. Such models therefore focus on the extent to which free-riding causes under-provision of this public good.[3] In every case the relationship between group resources and the supply of the public good, or policy favour, is determined exogenously and thus all such models lack a micro-founded theory of endogenous policy selection.

The first and certainly the most influential contribution in this field was Olson (1965). Abstracting from the effect of group competition on individual contributions, Olson suggested that (own) group size plays an important role in group formation. In particular Olson conjectured that individuals in sufficiently small non-privileged groups could regard their actions as pivotal and thus that collective action was a possibility; whilst "by contrast in a large group in which no single individual's contribution makes a perceptible difference...it is certain that a collective good will not be provided [voluntarily]"[4]. In one sense, then, our paper confirms Olson's conjecture that free-riding may inhibit group formation. However, our point of departure is to highlight that the endogeneity of the policy choice actually implies that a single lobby may still form even if all contributions are considered non-pivotal.

Also relevant in this context are 'inter-group' models of endogenous policy selection that abstract from collective action problems and therefore treat the number and identity of competing lobbies as given. Two basic approaches can be distinguished. The first set of models - the common agency approach - can be seen as an attempt to describe lobbying under (policy maker) certainty. Following pioneering work by Bernheim & Whinston (1986), such models assume that competing groups simultaneously offer the policy maker a 'menu' of bids in an attempt to sway policy in their favour.[5] In contrast, in the second set of models - the strategic information transmission approach - participants face uncertainty

[3] Important contributions to the literature on the private provision of public goods include Olson (1965), Chamberlain (1974), Palfrey & Rosenthal (1985), Bergstrom, Blume & Varian (1986) and Bagnoli & Lipman (1989). For interesting applications to lobbying activities in particular see Stigler (1974), Austen-Smith (1981) and Magee, Brock & Young (1989).

[4] Olson (1965) pp.44.

[5] The application of the menu-auction / common agency approach to models of economic influence seeking was popularised by Grossman & Helpman (1994, 1996) in applications to trade policy and electoral competition respectively. Of the burgenoing literature that has followed Dixit et al (1997) provide a very useful equilibrium characterisation theorem; a result which has been used by Besley & Coate (1997a) to introduce lobby action into a model of representative democracy.

over policy outcomes. Consequently interest groups become competing 'experts' or 'advocates'; incurring information acquisition costs to subsequently offer policy advice[6]. Clearly, given that our intention in this paper is to abstract from fixed costs, we restrict attention to the first - common agency - approach to policy selection.

The strategic information literature does however have some interesting implications for group formation. In particular, abstracting from any collective action problems, Austen-Smith & Wright (1992) note that "legislators are often lobbied by just one of two competing groups"[7] and hence they conclude that "groups do not always compete by clashing directly over a single legislator's vote"[8]. In contrast to our results, however, the prediction is that such lobbying will not always be counteractive. In particular, the suggestion in that paper is that it is the group that disagrees with the legislator's voting predisposition that will typically lobby alone.

The final class of models that are relevant in this context are those that seek to endogenise both the process of policy selection and lobby group formation. To date only Mitra (1999) has attempted to combine a fully micro-founded model of policy selection with an individual contribution game[9]. In fact Mitra's model may be seen as mutually compatible with our own. That is, whilst both approaches precede the benchmark common agency model with an individual contribution game, the focus of Mitra's work is on the ability of different interest groups to cover the fixed rather than variable costs associated with lobbying activities.

By assuming, firstly that pre-play communication resolves Olson's 'indeterminacy' when aggregate, but no individual, benefit exceeds the costs associated with political participation, and secondly that "once the lobby is formed, the lobby machinery can enforce perfect coordination among members of that group in the collection of political contributions"[10] Mitra ensures that players can credibly com-

[6] See in particular Austen-Smith & Wright (1992). Contributions in a similar vein include Austen-Smith (1993), Lohmann (1998), Dewatripont & Tirole (1999) and Krishna & Morgan (1999).

[7] Austen-Smith & Wright (1992) pp. 230.

[8] Austen-Smith & Wright (1992) pp. 245.

[9] Since the first version of this paper was written a number of other papers have addressed the issue of endogenous lobby formation. Contributions include Damania & Fredrikson (2000) who also model the policy selection stage as a menu-auction but focus on the implications for free-riding in a repeated game; and Felli & Merlo (2000) who endogenise lobby formation in the context of the citizen candidate model developed by Besley & Coate (1997a).

[10] Mitra (1999) pp. 1122.

mit to making contributions. Thus, by ensuring that it is only those groups that face an aggregate net loss that remain latent, Mitra is able to establish a unique equilibrium: only those groups with fixed costs less than an endogenously determined critical value form. Thus, so this story goes, pro-exporters and downstream firms failed to lobby over non-tariff barriers and industrial sugar users kept silent over sugar prices, because their fixed costs ultimately proved too high. It would appear, then, that the relative merits of a variable rather than a fixed cost approach to industry group participation is an empirical question and is one that is left to future research.

3. The Model

Our polity is populated by N agents[11] each endowed with y units of a private good. Each agent is of 'type' $t \in T = \{1; 2\}$, where t denotes the preference for a public policy $g \in G = \{1; 2\}$. Let $r(t) > 0$ denote the fixed proportion of each type in the polity. We denote the gross utility of a representative agent of type t by $U(g; t; y) = u_t(g) + y$; where $u_t(g) = H_t > 0$ if $t = g$ and $u_t(g) = 0$ if $t \neq g$. We assume that $r(t)$; $u_t(g)$; G and the type set T are all common knowledge.

We define 'lobbying activity' as an attempt to influence the choice of $g \in G$ by virtue of non-negative transfers to the policy making authority. We will consider a group to have formed a lobby if and only if sufficient contributions of y have been raised from individual agents for the lobby to demonstrate an ability to offer strictly positive transfers to the policy maker. The number of such lobbies is given by L, where $L \leq 2$ and is endogenously determined. To avoid unnecessary complications we assume the pre-existence of two 'manifestos', indexed by $t = \{1; 2\}$; such that it is common knowledge that a lobby with manifesto t will, by virtue of direct transfers to the policy maker, seek to secure the policy choice favoured by agents of type t (i.e. $g = t$), regardless of the actual distribution of its benefactors' policy preferences.

We denote the number of contributions of y to any lobby t by $x(y_t)$ and thus its gross utility by $U(g; t; y; x(y_t)) = x(y_t)\, u_t(g) + x(y_t) y$: Thus, given the above definition of lobby formation, we consider a group to have formed a lobby if and only if $x(y_t) > 0$: Denoting the set of lobbies by $\mathcal{L}$, there are clearly four possible lobbying configurations: no lobbies, $\mathcal{L} = \{\emptyset\}$; if $x(y_t) = 0 \;\forall\; t = 1; 2$; only lobby

[11] Since our focus is competition between industry groups the appropriate unit of analysis is the individual firm. We use the term agent to reflect the generality of our approach.

1, $L = \{1\}$; if $x(y_1) > 0$ and $x(y_2) = 0$; only lobby 2, $L = \{2\}$; if $x(y_1) = 0$ and $x(y_2) > 0$; or lobbying competition $L = \{1;2\}$ if $x(y_t) > 0 \ \forall t$.

The final player in our model is a policy maker responsible for the policy choice $g \in G = \{1;2\}$. In taking this decision we assume that the policy maker considers both the utility she derives from the policy outcome itself[12], $u_p(g)$, as well as any conditional transfers from the L lobbies, which we denote by $\{B_t(g)\}_{g \in G}$. The policy maker's objective function is therefore given by

$$\pi_p = u_p(g) + \sum_{t \in L} B_t(g); \tag{3.1}$$

where, without any loss of generality, we assume $u_p(1) = H_p \geq u_p(2) = 0$, implying that the policy maker shares a preference ordering with lobby 1.[13] Consequently, in the absence of transfers, we assume that the policy maker always implements $g = 1$.

In light of the above, it should be clear that the remaining action spaces and pay-offs are as follows. Every lobby $t \in L$ (with resources $x(y_t)y$ available for influence seeking) must decide on a schedule of payments that it will offer to the policy maker in an attempt to secure a policy choice $g = t$. We denote this choice by $B_t(g) \in \mathcal{B}_t = [0; x(y_t)y] \ \forall g \in G$, where the bounds on the feasible set $\mathcal{B}_t$ reflect the fact that, by assumption, no lobby can offer more than it has received from individual agents or demand payments from the policy maker. Lobby t's objective function is therefore given by

$$\pi_t = U(g; t; y; x(y_t)) - B_t(g). \tag{3.2}$$

Each individual agent must decide between contributing their endowment y to lobby 1; lobby 2 and not contributing at all. We denote this choice by $c \in C = \{y_1; y_2; \;\}$; the resulting outcome, or action profile, by the vector $x = \{x(c)\}_{c \in C}$ and the set of possible action profiles by $Z(C)$[14], where x determines L. Assuming that agents are aware that the lobbies will return an equal share of any 'unused'

[12] Such preferences could easily be explained by virtue of the representative democracy model of Besley & Coate (1997a).

[13] In an attempt to preserve generality we make no assumptions concerning relative preference intensity between any of the players. In particular we consider a policy maker with no policy preferences (i.e. $H_p = 0$) as a special case. In what follows we will make extensive use of the notion of a lobby opposing the policy maker's preferences. Note this refers only to the case where $H_p > 0$.

[14] To aid the exposition of the Large Poisson Game we follow Myerson's (1997, 1998) notation throughtout our discussion of the contribution stage game.

contributions, or $x(y_t)y - B_t(g)$, to their benefactors[15], the objective function for a representative agent of type t is given by,

$$\pi_t = u_t(g(c, x_{-i})) + y - \phi(c, x_{-i}); \qquad (3.3)$$

where

$$\phi(c, x_{-i}) = \begin{cases} \frac{1}{x(y_1)} B_1(g(c, x_{-i})) & \text{if } c = y_1 \\ \frac{1}{x(y_2)} B_2(g(c, x_{-i})) & \text{if } c = y_2 \\ 0 & \text{if } c = \emptyset \end{cases};$$

x_{-i} is the action profile of the other players and $Z(C)_{-i}$ is the set of possible corresponding action profiles.

At this point we also impose the stability condition

$$y \geq \max_{t \in \{1,2\}} [H_t]; \qquad (3.4)$$

which simply says that endowments are such that every individual agent can contribute an amount greater than or equal to their 'willingness to pay' for a policy favour. It is of interest to note that such an assumption will always be necessary to rule out non-existence of equilibria in menu-auctions when payment schedules are bounded from above as well as below.

From above it should be clear that there are two stages, or sub-games, to the lobbying process. The first is the contribution stage in which every individual agent decides which lobby, if any, to contribute to. The second is the policy selection stage in which every lobby, that has formed in the first stage game, selects a payment schedule in an attempt to influence the policy choice. Our aim, then, is to establish equilibrium lobbying configurations L^o[16]. That is, the number and identity of industry groups able to raise sufficient contributions to demonstrate an ability to offer equilibrium direct transfers $\{\{B_t^o(g)\}_{g \in G}\}$. We establish such equilibria by analysing the two stages in reverse order.

[15] Obviously the notion that lobbies return 'unused' contributions is not universally applicable, although it has been used by Bagnoli & Lipman (1989) and Felli & Merlo (2000) among others. Note that our refund assumption is formally equivalent to the case where, ex ante, benefactors (credibly) commit to making a contribution and thus, ex post, only make the necessary equilibrium payment. Real world examples of such a mechanism abound. For instance, close to home a scheme called on the residents of Kingsdown, Bristol, to pledge a willingness to pay towards a campaign against parking restrictions.

[16] Throughout we use the supercript o to denote an equilibrium value.

4. Policy Selection Stage Game

We follow Bernheim and Whinston (1986), and numerous others, in modelling influence seeking as a two stage menu-auction. We assume then that the policy selection game begins with each 'formed' lobby $t \in L$ choosing a payment schedule to maximise group utility, taking as given similar optimising behaviour by the other lobbies and anticipating an optimising policy choice. It is completed when, in light of all these 'simultaneous and non-cooperative offers', the policy maker selects a policy g^o that maximises her utility. An equilibrium of this policy selection game is therefore any pair $[g^o; \{\{B_t^o(g)\}_{g\in G}\}_{t\in L(x)}]$ that satisfies the requirement that: a) for each lobby t, $\{B_t^o(g)\}_{g\in G}$ is optimal (maximising 3.2 above) given the offers $\{B_s^o(g)\}_{s\in L(x)/\{t\}}$ of the other L_{-t} lobbies[17] and the policy choice induced by all L offers and b) g^o is optimal (maximising 3.1) given all such L offers.

Since, for the purposes of the contribution stage, we require the policy choices and actual payments made in all such equilibria we proceed as follows. Using the equilibrium characterisation theorem for common agency games from Dixit, Grossman and Helpman (1997) we first examine the two cases when only one lobby forms before proceeding to the two lobby scenario.

Adapted to our model the characterisation theorem yields:

Proposition 4.1. (Dixit, Grossman, Helpman 1997) *A vector of payment schedules $\{\{B_i^o(g)\}_{g\in G}\}_{t\in L(x)}$ and a policy choice g^o constitute an equilibrium of the policy selection game for any $L \in \{1,2\}$ if and only if:*

(a) for all $t \in L$:

$$B_t^o(g) \in \mathcal{B}_t \;\; \forall g \in \{1,2\}; \tag{4.1}$$

(b)

$$g^o = \arg\max_{g\in\{1,2\}} [u_p(g) + \sum_{t\in L} B_t^o(g)] \;\; \forall g \in \{1,2\}; \tag{4.2}$$

(c) for every $t \in L$, $[g^o; B_t^o(g^o)]$ solves the following problem:

$$\max_{(g;B_t)\in G\times \mathcal{B}_t} x(y_t)\,u_t(g) + x(y_t)y - B_t \tag{4.3}$$

[17] Note that $L_{-t} = \# L_{/\{t\}}$.

subject to

$$u_p(g) + \sum_{s \in L-\{t\}} B_s^o(g) + B_t \geq \max_{g' \in \{1,2\}} [u_p(g') + \sum_{s \in L-\{t\}} B_t^o(g')] \qquad (4.4)$$

Explanation[18]: Condition (a) simply corresponds to the requirement that payment schedules must be feasible, while condition (b) follows from optimisation by the policy maker. Condition (c) is more complicated and results from viewing each lobby's 'best response' calculation as a constrained maximisation problem.

In essence, each lobby t is aware that if the policy maker is to choose a particular policy g she must have no incentive to deviate to her outside option g'; where g' is the policy she would choose if lobby t offered nothing and the other L_{-t} lobbies maintained their equilibrium payment schedules $\{B_s^o(g)\}_{g \in G}$. Hence condition 4.4. Therefore, if lobby t satisfies this constraint, it knows that it effectively has the policy maker 'on side' and thus that it can propose a policy g^o and corresponding payment $B_t^o(g^o)$ that maximises its own utility. Hence condition 4.3. ■

4.1. Equilibria with One Lobby

From proposition 4.1 we have,

Proposition 4.2. If $L = \{1\}$ then $g^o = 1$ is the unique equilibrium policy choice and $B_1^o(1) = 0$ is the unique equilibrium payment.

Explanation: Since preferences are common knowledge lobby 1 knows that, in the absence of competition from another group, the policy maker will set $g^o = 1$ even if it makes a zero contribution. In other words, regardless of its size, lobby 1 finds it can 'free ride' on the back of sharing a preference ordering with the policy maker. ■

Proposition 4.3. If $L = \{2\}$ then:

(a) If the policy maker has 'stronger' policy preferences than lobby 2 (i.e. $H_p > x(y_2)H_2 > 0$); then $g^o = 1$ is the unique equilibrium policy choice and $B_2^o(1) = 0$

[18] We offer verbal explanations in place of formal proofs in the text. Such proofs may be found in the appendix.

is the unique equilibrium payment;

(b) If the policy maker and lobby 2 have 'equal' policy preferences (i.e. $x(y_2)H_2 = H_p > 0$); then either $g^o = 2$ and $B_2^o(2) = H_p$ or $g^o = 1$ and $B_2^o(1) = 0$;

(c) If the policy maker has 'weaker' policy preferences than lobby 2 (i.e. $x(y_2)H_2 > H_p \geq 0$), then $g^o = 2$ is the unique equilibrium policy choice and $B_2^o(2) = H_p$ is the unique equilibrium payment.

Explanation: Since preferences are common knowledge lobby 2 knows that, in the absence of competition from another group, a policy maker will set $g = 1$ if it makes a zero contribution. It therefore also knows that to secure its preferred outcome it must offer such a policy maker an amount that at least compensates her for any utility lost by choosing $g = 2$ rather than $g = 1$; that is H_p. Clearly, in equilibrium, this payment will only be made if it is lower than or equal to the lobby's 'willingness to pay' for the policy favour, or rather $H_p \leq x(y_2)H_2$. Should the group prove too small and/or preference intensity too 'weak' the equilibrium payment will clearly be zero and $g = 1$. ■

4.2. Equilibria with Two Lobbies

As Besley and Coate (1997b) note, whilst a discrete policy choice ensures a unique equilibrium policy choice, the equilibrium characterisation theorem given above fails to tie down equilibrium payment schedules when the number of principals is greater than one. Thus, since in this context we must specify every policy choice and equilibrium payment that any agent might expect when making her contribution decision, we invoke the 'truthfulness refinement' for common agency games developed by Bernheim and Whinston (1986)[19]. A definition of a globally truthful payment schedule is given below.

Definition 4.4. (Dixit, Grossman and Helpman (1997)). A feasible payment schedule $\tilde{B}_t \in \mathcal{B}_t$ for lobby t is said to be truthful relative to the constant u_t^o if

$$\tilde{B}_t(g) = \max[0; \min\{(x(y_t)u_t(g) - u_t^o); x(y_t)y\}] \;\; \forall g \in \{1; 2\};$$

[19] It is now standard to justify the use of the 'truthfulness' refinement by noting: firstly that truthful strategies are extremely simple and thus possibly 'focal'; and secondly that truthful equilibria are both efficient (maximising the sum of gross utilities) and coalition-proof. See Bernheim & Whinston (1986). However, truthful strategies very often result in complex payments schedules. An alternative set of strategies - so-called natural strategies - that do not exhibit this problem have been suggested by Kirchsteiger & Prat (1999). Nevertheless, in the case of a binary policy choice, truthful and natural equilibria coincide.

where $u_t^o = x(y_t)\,u_t(g^o) - B_t^o(g^o)$ or equilibrium utility net of any payment[20].

It should therefore be clear that replacing condition 4.1 with

$$B_t^o(g) = \tilde{B}_t(g) \ \forall g \in \{1;2\}$$

in proposition 4.1 above yields a theorem (hereafter proposition 4.1′) which may be used to characterise all the truthful equilibria of this policy selection game when $L = 2$.

We first establish two useful lemmas.

Lemma 4.5. In any truthful equilibrium $\tilde{B}_t^o(t) \geq \tilde{B}_t^o(s)$ with $t \neq s \in \{1;2\}$:

Explanation: No lobby will offer more for its worst outcome than for its best outcome. It should be clear from above that 'global truthfulness' implies that lobbies offer their true (net) willingness to pay for any policy choice relative to the equilibrium policy choice. Therefore, if a lobby anticipates that its best (worst) option will be chosen in equilibrium, it cannot offer more (less) for the remaining option than it will pay in this equilibrium. Hence $\tilde{B}_1^o(1) \geq \tilde{B}_1^o(2)$ and $\tilde{B}_2^o(2) \geq \tilde{B}_2^o(1)$: ■

Lemma 4.6 In any truthful equilibrium of the policy selection game, when $L = 2$; $\tilde{B}_t^o(s) = 0$ with $t \neq s \in \{1;2\}$:

Explanation: No lobby will offer a positive payment for its worst outcome. Since preferences are common knowledge, lobby 2 knows that the policy maker will set $g = 1$ if it makes a zero contribution and thus that it must provide the policy maker with at least the utility she receives under this outside option if $g = 2$ is ever to be chosen. Clearly positive payments for $g = 1$ (lobby 2's worst outcome) simply serve to increase the payment needed to realise $g = 2$ (lobby 2's best outcome). The same logic applies if lobby 1 anticipates that the policy maker will set $g = 2$ when it makes a zero contribution.

In contrast, if lobby 1 anticipates that the policy maker will set $g = 1$ when it makes a zero contribution, then it knows that it can effectively 'free ride' and offer a zero payment even for its best outcome ($g = 1$). Thus, since all payments are

[20] Note that equilibrium payments are truthful by construction and thus $B_t^o(g^o) \equiv \tilde{B}_t(g^o)$: A proof of such 'local truthfulness' is given in Dixit, Grossman & Helpman (1997).

'truthful', we know lobby 1 must also offer a zero payment for its worst outcome ($g = 2$). ■

We are now in a position to establish our central proposition for the policy selection stage game concerning possible equilibrium policy choices and corresponding equilibrium payments:

Proposition 4.7. If $L = \{1; 2\}$ then:

(a) If the policy maker has 'stronger' policy preferences than lobby 2, or if the policy maker and lobby 2 have 'equal' policy preferences (i.e. $H_p \geq x(y_2)H_2 > 0$); then $g^o = 1$ is the unique equilibrium policy choice and $\tilde{B}_1^o(1) = 0$; $\tilde{B}_2^o(1) = 0$ are the unique equilibrium payments;

(b) If the policy maker has 'weaker' policy preferences than lobby 2 (i.e. $x(y_2)H_2 > H_p \geq 0$), then:

(i) If lobby 1 is more 'powerful'[21] (i.e. $x(y_1)H_1 > x(y_2)H_2 - H_p$); then $g^o = 1$ is the unique equilibrium policy choice and $\tilde{B}_1^o(1) = x(y_2)H_2 - H_p$; $\tilde{B}_2^o(1) = 0$ are the unique equilibrium payments;

(ii) If lobby 2 is more 'powerful' (i.e. $x(y_1)H_1 < x(y_2)H_2 - H_p$); then $g^o = 2$ is the unique equilibrium policy choice and $\tilde{B}_1^o(2) = 0$; $\tilde{B}_2^o(2) = x(y_1)H_1 + H_p$ are the unique equilibrium payments;

(iii) If the lobbies are equally 'powerful' (i.e. $x(y_1)H_1 = x(y_2)H_2 - H_p$); then either $g^o = 1$ and $\tilde{B}_1^o(1) = x(y_2)H_2 - H_p(y_1)$; $\tilde{B}_2^o(1) = 0$,
or $g^o = 2$ and $\tilde{B}_1^o(2) = 0$; $\tilde{B}_2^o(2) = x(y_1)H_1 + H_p$ are the equilibrium payments.

Explanation: The intuition behind this result is as follows. Consider the case of a policy maker with 'stronger' policy preferences than lobby 2 and begin by supposing that $g^o = 2$. Since preferences are common knowledge lobby 2 knows that when $g^o = 2$ lobby 1 will have offered the policy maker its true willingness to pay ($x(y_1)H_1 > 0$) for $g = 1$. Lobby 2 also knows that if it offers nothing for $g = 2$ the policy maker will choose $g = 1$; deriving a utility of $H_p + x(y_1)H_1$ under this 'outside option'. Using this information lobby 2 therefore knows that it must offer the policy maker $x(y_1)H_1 + H_p$ if $g^o = 2$ is indeed to be an equilibrium policy choice. Clearly lobby 2 will only be prepared to make this payment if it is lower than or equal to the group's benefit from the policy favour, implying

[21] In this context a lobby's 'power' refers to its willingness to pay with an adjustment to reflect the policy maker's preferences.

$x(y_2)H_2 \geq x(y_1)H_1 + H_p$. However, the fact that lobby 1 has formed is evidence that $x(y_1)H_1 > 0$ and thus we can be certain that lobby 2 will not be prepared to make the necessary payment since $H_p > x(y_2)H_2$. It therefore follows that $g = 2$ cannot be an equilibrium policy choice when the policy maker has 'stronger' policy preferences than lobby 2.

Our alternative is thus to suppose $g^o = 1$. Again since preferences are common knowledge lobby 1 knows that when $g^o = 1$ lobby 2 will have offered the policy maker its true willingness to pay $(x(y_2)H_2)$ for $g = 2$. Lobby 1 also knows that if it offers nothing for $g = 1$ the policy maker will choose $g = 1$, deriving a utility of H_p under this 'outside option'. Using this information lobby 1 therefore knows that it can offer the policy maker nothing and still secure an equilibrium policy choice of $g^o = 1$ since $H_p > x(y_2)H_2$. Finally we can be certain that lobby 1 will be prepared to make the necessary payment since $x(y_1)H_1 > 0$ when lobby 1 has formed.

It therefore follows that $g^o = 1$ is the unique equilibrium policy choice and $\hat{B}^o_1(1) = 0$; $\hat{B}^o_2(1) = 0$ are the unique equilibrium payments when the policy maker has 'stronger' policy preferences than lobby 2. Note that the intuition behind the remaining results is exactly analogous. ■

Following this logic we therefore see that the policy selection game, for any given number of lobbies, has a unique equilibrium in every case except $x(y_2)H_2 = x(y_1)H_1 + H_p$. For the purpose of the contribution stage game, we make the (purely) simplifying assumption that in such an event agents anticipate that the equilibrium with the policy maker choosing her 'default' policy ($g = 1$) will prevail.

Finally it will prove useful to emphasise the following results which, in part, drive the main result of our paper. That is, we have shown that any lobby that does not oppose the preference ordering of the policy maker will never pay to secure a policy favour, unless it competes for that favour with another group exhibiting stronger preferences than those of the policy maker. In contrast a lobby that opposes the policy maker's preferences will always face a cost to persuade the authority to implement its preferred policy.

5. Contribution Stage Game

Armed with the results of the policy selection game we can now move to the first stage of the lobbying process, or contribution stage, in which every individual agent decides whether to contribute the endowment y to lobby 1, lobby 2 or not to contribute at all. In other words, having established the equilibrium policy choice g^o and payments $\{B_t^o(g^o)\}_{t\in L(x)}$ for every action profile $x \in \mathcal{Z}(C)$; we now analyse the contribution decision $c \in C = \{y_1; y_2; \ldots\}$. Assuming all agents choose c to maximise 3.3 above, we may establish equilibrium action profiles x^o and thus complete our characterisation of all possible equilibrium lobbying configurations.

In standard voluntary-contribution games it is assumed that the total number of players N in any game is common knowledge, and thus by implication that every player takes other agents' behaviour (i.e. x^{-i}) as given and known when contemplating her best response. In any modern polity, however, the total number of potential benefactors across all groups will often be large - an assertion certainly borne out in the trade examples cited in the Introduction. As a result, the assumption that players condition their strategies on x^{-i} begins to look unrealistic. We therefore follow Myerson's (1998) suggestion and model the contribution stage game as a Large Poisson Game. A consequence of this approach, as Myerson (1997, 1998) has shown, is that players form 'beliefs' over x^{-i} when contemplating their best response and thus we establish an equilibrium action profile x^o when it is indeed optimal for every player to behave according to these 'beliefs'.[22]

Nevertheless, it will aid the understanding of the Large Poisson Contribution Game to bear in mind the implications of modelling the contribution game under the conventional assumption that N is common knowledge. The next sub-section therefore briefly discusses the standard voluntary-contribution game adapted to our model.

5.1. Standard Voluntary-Contribution Game

When N is common knowledge, players are aware of every possible element x^{-i} of the set $\mathcal{Z}(C)^{-i}$. Each player therefore resolves their contribution decision by calculating which elements of $C = \{y_1; y_2; \ldots\}$ constitute a best response to any action profile x^{-i}. Formally, each player solves the problem

$$\max_{c\in C}\left[\pi_t(c, x^{-i}) \equiv u_t(g(c, x^{-i})) + y - \phi(c, x^{-i})\right] \quad \forall x^{-i} \in \mathcal{Z}(C)^{-i}. \tag{5.1}$$

In such a game we assert that any action profile x from which no player has an incentive to deviate if no other player deviates is an equilibrium action profile x^o.

In such a framework, it can be shown after some tedious but standard calculations, that at least one pure strategy Nash equilibrium exists for each possible

[22] In this respect, our approach to lobby formation echoes that of Austen-Smith (1981). However, in that paper, 'beliefs' over x^{-i} were imposed a priori and thus lacked the requirement that these beliefs should be consistent with equilibrium.

lobby configuration. Intuitively, this result rests on the fact that agents have two possible motives for deviation. Firstly, if the equilibrium policy choice is 'unfavourable', or $g^o \neq t$, there is an incentive to deviate to another action $c \neq c^o$ if this alternative choice is pivotal in securing $g^o = t$ and results in a higher net pay-off given $\phi(c, x^o_{-i})$. Secondly, if the equilibrium choice is 'costly', or $\phi(c^o; x^o_{-i}) > 0$; there is an incentive to deviate to another action $c \neq c^o$ if this alternative choice is less costly and results in a higher net payoff given $u_t(g^o)$. Note that the second motive includes the incentive to free ride as a special case; that is, when the equilibrium policy choice is 'favourable', or $g = t$, but another action $c \neq c^o$ results in $\phi(c, x^o_{-i}) = 0$.

With this logic in mind it should be clear that the results of the policy selection game allow us to distinguish between two classes of equilibria. On the one hand, there are those in which agents perceive an alternative action to be pivotal yet result in a lower net pay-off. Alternatively, there are those in which equilibrium actions are neither pivotal nor costly. In either case no agent has an incentive to save on costs by free-riding. Note that in the first class of equilibria the configuration will always feature lobby 2. Intuitively, if lobby 2 has not formed, contributions to lobby 1 cannot be pivotal since lobby 1 does not oppose the preference ordering of the policy maker. By the same logic, in the second class of equilibria the configuration will never result in strictly positive transfers to the policy maker. The above results clearly echo Olson's notion that if the total number of players is common knowledge and actions possibly pivotal, the contribution stage game will have multiple equilibria - in particular one of these industry lobbies "might form but it might not"[23].

In solving the standard contribution game, however, we notice that many of the equilibria are supported by conditions that seem somewhat 'implausible' in a game where the total number of potential contributors is large. Intuitively, would an agent really be willing to believe that the policy maker could have 'strong' preferences or that his contribution might be pivotal if he expected the total number of other contributors to be very large? If not then clearly the first class of equilibria will disappear.

In the next section we therefore model the contribution stage as a Large Poisson Game, allowing us to explore Olson's notion that in large groups "it is certain that a collective good will not be provided [voluntarily]"[24]. In effect we see that Olson was correct to conjecture that in large games free-riding will limit the num-

[23] See Olson (1965) pp. 43.

[24] Olson (1965) pp. 44.

ber of groups that form. However, armed with an endogenous policy choice we see that this effect will not be symmetric across groups as, one, but not both groups will fail to form. Thus, by demonstrating that our large Poisson contribution game exhibits only the latter - non-pivotal and non-costly - class of equilibria, we establish that a lone industry group can be part of a unique equilibrium in a non-trivial way.

5.2. Large Poisson Voluntary-Contribution Game

For the reasons given above we now relax the assumption that N is common knowledge. More specifically we assume that the total number of players is a random variable. However, as Myerson (1997) shows, the use of a Poisson distribution allows such games to be simplified considerably. We therefore assume hereafter that N is a Poisson random variable (PRV) with mean n. Note that by the decomposition property of the Poisson distribution (see Myerson (1997)) the total number of players with type t is a also a PRV, with mean $nr(t)$.

Introducing such uncertainty implies that players can no longer employ a strategy profile that assigns a strategy to each specific individual in the game, simply because they are not aware of who they all are. Instead we assume that players use the only information they do have, namely the type set T. We follow Myerson (1998) in describing the strategic behaviour of players in this Poisson game in terms of a distributional strategy τ, defined as any probability distribution over the set $C \times T$ such that the marginal distribution on T is equal to r. Note that any $\tau(c;t)$ should be interpreted as the 'beliefs' players choose to hold over the likely behaviour of any type, or rather the perceived probability that a randomly sampled player will have type t and choose action c.[25] Given r, any τ is also associated with a unique strategy function $\sigma(c\,|\,t)$, or conditional probability that a randomly sampled player will choose c if he is of type t.

Since the total number of players is a PRV with mean n, it follows that, when the players behave according to τ, the number of players of any type that choose any action c in C is a PRV with mean $n\tau(c;T) \equiv n\tau(c)$[26], and thus that the

[25] "...each player is only aware of the possible types of the other players, and so players can only form perceptions about how the strategic behaviour of other players is likely to depend on their types...That is, going to a model of population uncerainty requires us to specify a probability distribution over actions for each type of player, rather than for each individual", Myerson (1997) pp. 4,5.

[26] Any τ also induces a marginal probability distribution on C and thus $\tau(c) = \tau(c;T)$. Hereafter we use $\tau(c)$ to denote the marginal probability of any action.

expected action profile in such a game is $n\tau \equiv \{n\tau(c)\}_{c \in C}$. Therefore, since it can be shown that the number of players in a Poisson game who choose the action c is independent of the number of players who choose all other actions[27], we establish that, for any x in $Z(C)$, the probability that x is the action profile of the players in the game is

$$P(x|n\tau) = \prod_{c \in C} \left[\frac{e^{-n\tau(c)}(n\tau(c))^{x(c)}}{x(c)!} \right]. \tag{5.3}$$

Then, using the fact that any player in the Poisson game assesses the same probabilities for the action profile of the other players in the game (not counting himself)[28], we may re-write the optimisation problem 5.1 as

$$\max_{c^i \in C} \left[E\left[\pi_t(c, x^{-i})\right] \equiv \sum_{x^{-i} \in Z(C)^{-i}} P(x^{-i}|n\tau) \left[u_t(g(c, x^{-i})) + y - \phi(c, x^{-i}) \right] \right]. \tag{5.4}$$

In other words each player resolves his 'contribution decision' for a game of size n by formulating 'beliefs' τ over the likely behaviour of each type and then calculating which elements of $C = \{y_1, y_2, \ldots\}$ maximise his expected utility given the resulting expected action profile $n\tau$.

Following Myerson (1998), we let $S(b; n\tau) \subseteq T$ denote the set of all types for whom choosing action $b \in C$ would maximise 5.4 when n is the expected number of players and τ is the distributional strategy. Then we may assert that a distributional strategy τ is an equilibrium of the Poisson voluntary-contribution game Γ defined by $\{T, n, r, C, E[\pi_t(\cdot)]\}$ if and only if

$$\tau(b; S(b; n\tau)) = \tau(b) \quad \forall b \in C.$$

That is, if "all the probability of choosing action b comes from types for whom b is an optimal action, when everyone else is expected to behave according to this distributional strategy"[29]. Alternatively any τ, such that no type has an incentive to deviate when the other type does not deviate, is an equilibrium distributional strategy τ^{o}.

In light of the above discussion we proceed by attempting to establish whether, in a large game of size n, an equilibrium distributional strategy exists to support

[27] A proof of what has been termed the 'independent actions' property of Poisson games can be found in Myerson (1997).

[28] This result has been termed the 'environmental equivalence' property of Poisson games and, again, a proof can be found in Myerson (1997).

[29] Myerson (1998), pp. 7.

each lobbying configuration. That is, we seek to verify the conditions, if any, under which: a) $\tau(\emptyset; S(\emptyset; n\tau)) = \tau(\emptyset) = 1$ which would support $L^o = \{\emptyset\}$; b) $\tau(y_1; S(y_1; n\tau)) = \tau(y_1) > 0$ and $\tau(y_2; S(y_2; n\tau)) = \tau(y_2) = 0$ supporting $L^o = \{1\}$; c) $\tau(y_1; S(b; n\tau)) = \tau(y_1) = 0$ and $\tau(y_2; S(y_2; n\tau)) = \tau(y_2) > 0$ supporting $L^o = \{2\}$; and finally $\tau(y_1; S(b; n\tau)) = \tau(y_1) > 0$ and $\tau(y_2; S(y_2; n\tau)) = \tau(y_2) > 0$ supporting $L^o = \{1; 2\}$.

It appears then, that we must find $P(x^{-i} \mid n\tau)$ for every x^{-i} in $\mathcal{Z}(\mathcal{C})^{-i}$. Fortunately Myerson (1998) has shown that this is not actually the case, proving that it is possible to find a solution to optimisation problems such as 5.4, for any given set of 'beliefs', by following a two stage procedure[30].

The first step is to simplify the optimisation problem by partitioning $\mathcal{Z}(\mathcal{C})^{-i}$ into events, or pay-off equivalent sub-sets of individual action profiles x^{-i}; relevant to each action $c \in \mathcal{C}$. Taking $c = y_1$ as an example, the results of the policy selection suggest that there are three relevant events:[31]

(i) "The policy maker has 'stronger' policy preferences than lobby 2":

$$W_1 = \{x^{-i} \in \mathcal{Z}(\mathcal{C})^{-i} \mid H_p \geq x(y_2)H_2\}$$

implying

$$\begin{aligned}\pi_t(y_1; x^{-i}) &= u_t(g(y_1; x^{-i})) + y - \phi(y_1; x^{-i}) \\ &= u_t(1) + y.\end{aligned}$$

(ii) "The policy maker has 'weaker' policy preferences than lobby 2 and lobby 1 is more 'powerful'":

$$W_2 = \{x^{-i} \in \mathcal{Z}(\mathcal{C})^{-i} \mid (x(y_1)^{-i} + 1)H_1 + H_p \geq x(y_2)H_2 > H_p; x(y_2) > 0\}$$

implying

$$\begin{aligned}\pi_t(y_1; x^{-i}) &= u_t(g(y_1; x^{-i})) + y - \phi(y_1; x^{-i}) \\ &= u_t(1) + y - \frac{\tilde{B}_1^o(1)}{x(y_1)^{-i} + 1},\end{aligned}$$

where $\tilde{B}_1^o(1) = x(y_2)H_2 - H_p$.

[30] In the text we give only a brief outline of the steps invloved in solving a large Poisson contribution game. A fuller explananation and proof of our subsequent proposition may be found in the appendix.

[31] Note that in this case $x(y_2)^{-i} = x(y_2)$ and $x(\emptyset)^{-i} = x(\emptyset)$.

(iii) "The policy maker has 'weaker' policy preferences than lobby 2 and lobby 2 is more 'powerful' ":

$$W_3 = \{x^i \in Z(C)^i \mid x(y_2)H_2 > (x(y_1)^i + 1)H_1 + H_p, x(y_2) > 0\}$$

implying

$$\begin{aligned} \pi_t(y_1, x^i) &= u_t(g(y_1, x^i)) + y - \phi(y_1, x^i) \\ &= u_t(2) + y. \end{aligned}$$

The second step is to estimate the probabilities of these events. Here we make use of the concept of the magnitude μ of an event W, or rather the rate at which the probability of this event goes to zero in the equilibria of large Poisson games where the size parameter n tends to infinity. That is, we first derive the magnitude of each event using Myerson's "Magnitude Theorem" which implies that, for any event $W \in Z(C)^i$,

$$\mu = \lim_{n \to \infty} \log \frac{(P(W \mid n\tau_n))}{n} = \lim_{n \to \infty} \max_{x_n^i \in W} \sum_{c \in C} \tau_n(c) \psi \left(\frac{x_n^i(c)}{n\tau_n(c)} \right);$$

where the function $\psi : \Re_+ \to \Re$ is defined by the equations

$$\begin{aligned} \psi(\theta) &= \theta(1 - \log(\theta)) - 1 \leq 0, \ \forall \theta > 0 \\ \psi(0) &= \lim_{\theta \to 0} \psi(\theta) = -1; \end{aligned}$$

and the convention that

$$\begin{aligned} \text{if } \tau_n(c) &= 0 \text{ and } x_n^i(c) = 0 \text{ then } \tau_n(c)\psi[x_n^i(c)/n\tau_n(c)] = 0 \\ \text{if } \tau_n(c) &= 0 \text{ and } x_n^i(c) > 0 \text{ then } \tau_n(c)\psi[x_n^i(c)/n\tau_n(c)] = -\infty \end{aligned}$$

is adopted.

In essence the "Magnitude Theorem" tells us that the rate at which the probability of an event W tends to zero as $n \to \infty$ is determined by the rate at which the probability of its elements, or individual action profiles x_n^i, tend to zero as $n \to \infty$. More specifically, the magnitude of any event W, is equal to the greatest magnitude of any vector $x_n^i \in W$. Note that μ cannot be strictly positive since the log of a probability is always less than or equal to zero. Further, this value

μ is effectively determined by the difference between the actual number of contributions $x_n^i(c)$ and the expected number $n\tau_n(c)$, given the size parameter n and beliefs τ.

Having established the magnitude of every event for a given set of 'beliefs' τ, the final task is to solve 5.4. Here we may exploit the fact that the "Magnitude Theorem" implies as a corollary that, for any two events W_i and W_j $i \neq j$ in $Z(C)^i$; if $\mu_i > \mu_j$ then

$$\lim_{n \to \infty} \frac{P(W_j | n\tau_n)}{P(W_i | n\tau_n)} = 0:$$

Or rather that, as the size of the game n tends to infinity and for any given set of 'beliefs' τ; players' perception of the probability of events with negative magnitude becomes infinitesimally small relative to those events with zero magnitude. Then, repeating this procedure for each set of 'beliefs' we may verify whether actions that are consistent with these 'beliefs' maximise 5.4.[32]

To prove the main proposition we make use of the following lemma:

Lemma 5.1. *As $n \to \infty$:*

(a) If $\tau : \tau(y_t) = 0 \ \forall t$ each player anticipates that: if $H_p \geq H_2$, the policy maker will always implement $g^o = 1$ and thus that no action will be pivotal; but if $H_p < H_2$; that the policy maker will implement $g^o = 2$ if and only if $c = y_2$:

(b) If $\tau : \tau(y_1) < \tau(y_2)\frac{H_2}{H_1}$ each player anticipates that the policy maker will always implement $g^o = 2$ and thus that no action will be pivotal:

(c) If $\tau : \tau(y_1) > \tau(y_2) = 0$ or $\tau(y_1) \geq \tau(y_2)\frac{H_2}{H_1}$ and $\tau(y_2) > 0$ each player anticipates that the policy maker will always implement $g^o = 1$ and thus that no action will be pivotal.

Explanation. When beliefs are such that no other player is expected to contribute ($\tau(y_t) = 0 \ \forall t$) and $c = y_2$ lobby 2 will form and act alone ($L = \{2\}$):

[32] Myerson (1998) also proves the so-called "Offset Theorem" which is very usefull for comparing the probabilities of events that differ by a finite additive translation. In fact, this theorem is essential for estimating pivot probabilities in voting games. However, in our model we work with events that do not differ by adding a fixed vector of integers and thus the use of the "Magnitude Theorem" and its corollary suffices for our purpose.

Proposition 4.3 therefore implies that $g^o = 2$ should be expected with certainty only if $H_p < H_2$. In all other cases - i.e. if $c \in \{y_1, \emptyset\}$ - lobby 2 will not form and thus Proposition 4.2 implies that any player will expect $g^o = 1$ with certainty.

When beliefs are such that some contributions are expected to be made ($\tau(y_t) \neq 0$ for some t), then, given that the expected total number of players is large, no player is willing to believe that he is pivotal. More specifically if the relative probability that a randomly selected player will contribute to lobby 1, $\frac{\tau(y_1)}{\tau(y_2)}$, is high enough each player anticipates that the policy maker will always implement $g^o = 1$: Consequently contributions to either lobby are never expected to be pivotal. Analogously if $\frac{\tau(y_1)}{\tau(y_2)}$ is low enough players anticipate that the policy maker will always implement $g^o = 2$. Again contributions to either lobby are never expected to be pivotal. ■

We are now finally in a position to state our central proposition concerning the endogenous formation of lobby groups given an endogenous binary policy choice:

Proposition 5.2. In the Poisson voluntary-contribution game Γ:

(a) If the policy maker has 'stronger' policy preferences than agents of type 2, or if the policy maker and agents of type 2 have equal policy preferences, (i.e. $H_p \geq H_2 > 0$) then the set of equilibrium distributional strategies is such that $L^o = \{\emptyset\}$ and $L^o = \{1\}$ are the only possible equilibrium lobbying configurations.

(b) If the policy maker has positive but 'weaker' policy preferences than agents of type 2 (i.e. $H_2 > H_p > 0$) then the set of equilibrium distributional strategies is such that $L^o = \{1\}$ is the unique equilibrium lobbying configuration.

(b) If the policy maker has no policy preferences (i.e. $H_p = 0$) then the set of equilibrium distributional strategies is such that $L^o = \{1\}$ and $L^o = \{2\}$ are the only possible equilibrium lobbying configurations.

Explanation: $L = \{2\}$ cannot be an equilibrium when $H_p > 0$: To see this, firstly note that Proposition 4.3 implies that, even when acting alone, lobby 2 must offer a strictly positive transfer H_p to secure $g^o = 2$: However, also note that Lemma 5.1 implies that if $\tau(y_2) > 0$ contributions to lobby 2 are never expected to be pivotal when the expected total number of potential contributors is large. Free-riding is therefore complete and thus lobby 2 fails to form.

$L = \{1; 2\}$ cannot be an equilibrium. This follows from the fact that Proposition 4.6 implies that all potential contributors to lobby t anticipate that strictly positive, or costly, transfers will be necessary to secure $g^o = t$; whilst Lemma

5.1 implies that when $\tau(y_t) > 0 \ \forall t$ contributions towards these variable political influence costs are never expected to be pivotal. All agents therefore have an incentive to free-ride.

$L = \{\emptyset\}$ cannot be an equilibrium when $H_2 > H_p > 0$: Proposition 4.3 and Lemma 5.1 imply that when $\tau(y_t) = 0 \ \forall t$ and $H_2 > H_p > 0$ all agents of type 2 have an incentive to 'deviate' from this distributional strategy and contribute to lobby 2 in order to secure $g^o = 2$. In contrast, $L = \{\emptyset\}$ can be an equilibrium when $H_2 \leq H_p$; since Lemma 5.1 implies that no actions are expected to be pivotal.

Finally, $L = \{1\}$ can always be an equilibrium, whilst $L = \{2\}$ can be an equilibrium when $H_p = 0$. In both cases Propositions 4.2 and 4.3 and Lemma 5.1 imply that no player has an incentive to deviate from the corresponding distributional strategy since contributions are unnecessary to secure the preferred policy choice. In other words in these cases the incentive to free-ride has disappeared. ■

In essence, then, industry lobbying is purely counteractive. To see this recall from above that large game population uncertainty ensures that a lobby opposing the policy maker's preferences, will never form. An immediate corollary to this result is that in equilibrium lobbying bears no variable political influence cost. However, this does not mean that lobbying exists in a non-trivial way. In fact, if a lobby forms, demonstrating an ability to offer strictly positive transfers along the equilibrium path, it will only be when an opposing group poses a credible threat, or rather is privileged, on an off-equilibrium path. In effect, then, an industry lobby forms simply to ensure that its opponent faces competition and thus a collective action problem.

As an illustration, consider the case $H_2 > H_p > 0$: In this case, $L = \{\emptyset\}$ cannot be an equilibrium since lobby 2 poses a credible threat to form if no contributions are expected to be made. If, on the other hand, lobby 1 is expected to form, then lobby 2 will inevitably face costs to influence policy and hence, given that the expected size of the polity is large, an insurmountable collective action problem. Accordingly, lobby 1 forms to ensure that lobby 2 never forms.[33] In fact, in this case, $L = \{1\}$ is the unique lobbying configuration. The above consideration is reinforced if we consider what happens if $H_2 \leq H_p$: In this case, lobby 2 no longer poses a credible threat to form on any off-equilibrium path and therefore either $L = \{\emptyset\}$ or $L = \{1\}$ can be part of the equilibrium.

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[33] The same intuition holds when $H_p = 0$: Note, however, that in this case lobby 2 can also deter lobby 1 from forming. Therefore either $L = \{2\}$ or $L = \{1\}$ can be part of the equilibrium.

6. Conclusion

Given the empirical observation that certain industry lobby groups appear to have played a surprisingly passive role in policy debates, this paper offers a theory of lobby formation that, in contrast to the Olsonian approach, is not driven by asymmetries in fixed organisational costs or group size. Rather, we look to other factors to explain patterns of group formation, namely the relationship between population size and the free-rider problem associated with the procurement of variable 'political influence' costs. More specifically, we build on Olson's (1965) point that in large groups in which "no single individual's contribution makes a perceptible difference...it is certain that a collective good will not be provided [voluntarily]'. However, in contrast to Olson, we explore the relationship between population size and free-riding in the context of lobby group competition over an endogenous policy choice determined by direct transfers.

In essence we show that, when the policy choice is endogenous, ex ante asymmetries absent and the expected size of the polity is large, only one of two industry groups will be beset by free-rider problems and thus fail to form. Furthermore, we suggest that if a group opposes the policy maker's policy preference we can be certain that, at least on this policy issue, it will always remain passive. The cornerstone of this result is simply that in a menu-auction, a group's equilibrium transfer must cover the welfare loss it imposes on others participating in the political process. Thus, if a group shares a preference ordering with the policy maker and lacks competition from another lobby, its equilibrium transfer will actually be zero. By implication then, in an environment where all contributions are expected to be non-pivotal and thus the incentive to free-ride on costly transfers complete, we see that, if lobbying exists, it will only come from the group that shares a preference ordering with the policy maker.

In short, by abstracting from the endogeneity of the policy choice, existing theories of lobby formation have overlooked the fact that free-rider problems need not be symmetric, even when groups are of approximately equal size. Rather, in light of this observation, this paper shows that it is possible to explain variations in industry group formation, without recourse to asymmetries in fixed organisational costs and group size. More importantly perhaps, we offer a reason why Gawande (1997) observed that pro-export groups preferred to direct effort at instruments that facilitate exports rather than compete with anti-protectionist groups over the level of non-tariff barriers[34]. Our approach suggests that, if free-riding limits the

[34] See Gawande (1997) pp. 65.

extent to which individual firms contribute towards the variable political influence costs of lobbying, it is indeed rational for lobbies to specialise in areas where policy maker's either do not have a prior preference or are already 'on-side'. We therefore avoid the worrying question posed earlier: if a group has formed by virtue of financing the fixed costs associated with collective participation in one policy dimension why would it remain silent in another?

Finally we note that a corollary of our results is that 'money never changes hands', invalidating an often heard criticism of the common agency approach to economic influence seeking. But this, then, raises the question what do lobbies do? A common answer has been that lobbies exist, among other reasons, to actively change the voting disposition of a legislator or policy maker. However, our analysis shows that, in a large polity, such groups will be beset by free-rider problems and fail to form. We therefore suggest that all industry lobbying by virtue of direct transfers, is purely counteractive; such lobbies do not exist to change the default policy outcome but rather to ensure that other groups cannot.

Our analysis also offers some interesting peripheral results. By endogenising industry lobby formation, we develop a model that - in contrast to the existing literature on policy formation in the presence of lobbies - allows us to predict equilibria more precisely. We are therefore able to suggest that a non-benevolent policy maker (i.e. $u_p(g) = 0 \ \forall g$) will always choose a socially sub-optimal policy choice. The reason being that one group will always lack representation and thus such a policy maker will fail to maximise the gross utilities of all groups in society. Similarly, our results imply that the choice by a semi-benevolent policy maker (i.e. $u_p(g) > 0$ for some g) will always be intended to maximise social welfare, since no group will ever form to oppose this choice. Or, rather, a semi-benevolent policy maker will always implement the socially optimal choice, even if a group lacks representation.

Finally, reflecting on the fact that our results are potentially mutually compatible with those of Mitra (1999) we see two directions for future research. Namely, both an empirical and a theoretical investigation of the importance of variable political influence costs, relative to fixed organisational costs, in explaining the variation in industry group participation.

7. Appendix

Proof of Proposition 4.1. A formal proof of this proposition is given in Dixit, Grossman and Helpman (1997). ■

In this form the characterisation theorem clearly aids intuitive understanding of the common agency problem and is therefore presented as such in the text. However for the purpose of proving subsequent propositions it is useful to re-state proposition 4.1. as follows.

Proposition 4.1.* A vector of payment schedules $\{\{B_t^o(g)\}_{g \in G}\}_{t \in L}$ and a policy choice g^o constitute an equilibrium of the policy selection game for any $L = \{1, 2\}$ if and only if:

(a) for all $t \in L$:

$$B_t^o(g) \in B_t \; \forall g \in \{1, 2\}; \tag{4.1*}$$

(b)

$$u_p(g^o) + \sum_{t \in L} B_t^o(g^o) \geq u_p(g) + \sum_{t \in L} B_t^o(g) \; \forall g \in \{1, 2\}; \tag{4.2*}$$

(c) for every $t \in L$, $t \neq s$:

(i) $\forall g \in \{1, 2\}$

$$x(y_t)\, u_t(g^o) + u_p(g^o) + \sum_{s \in L = \{t\}} B_s^o(g^o) \geq x(y_t)\, u_t(g) + u_p(g) + \sum_{s \in L = \{t\}} B_s^o(g) \tag{4.3*}$$

(ii)

$$B_t^o(g^o) = \max_{g' \in \{1, 2\}} [u_p(g') + \sum_{s \in L = \{t\}} B_s^o(g')] - [u_p(g^o) + \sum_{s \in L = \{t\}} B_s^o(g^o)]: \tag{4.4*}$$

Explanation: Condition (a) remains unchanged. Condition (b) simply re-expresses optimisation by the policy maker in a more convenient form. In the case of condition (c), optimisation in 4.3 by lobby t clearly implies that the constraint will bind with equality, hence condition 4.4*; that is, each lobby is aware that, for any proposed policy choice, it is never in its interest to provide the policy maker with more than her reservation utility. Further it should be clear that condition 4.3* follows directly from substituting condition 4.4* into the objective function in 4.3. More intuitively, the existence of an 'outside option' on the part of the policy-maker implies that the lobby must take into account any utility the policy maker derives from the policy choice itself, together with any offers from other lobbies, when contemplating its actions.

Proof of Proposition 4.2. Suppose $g^o = 2$. Then when $L = \{1\}$ condition 4.3* of the characterisation theorem implies

$$0 \geq x(y_1)H_1 + H_p;$$

which cannot be the case. So $g^o = 1$ is the unique equilibrium policy choice.

Letting

$$\hat{g}_1^{o} = \arg\max_{g^o \in \{1,2\}} [u_p(g^o) + 0]$$

we have $\hat{g}_1^o = 1$; and thus when $L = \{1\}$ condition 4.4* implies $B_1^o(1) = 0$. ■

Proof of Proposition 4.3. Part (a): suppose $g^o = 2$. Then when $L = \{2\}$ condition 4.3* of the characterisation theorem implies

$$x(y_2)H_2 \geq H_p;$$

which cannot be the case. So $g^o = 1$ is the unique equilibrium policy choice.

Letting

$$\hat{g}_2^{o} = \arg\max_{g^o \in \{1,2\}} [u_p(g^o) + 0]$$

we have $\hat{g}_2^o = 1$; and thus when $L = \{2\}$ condition 4.4* implies $B_2^o(1) = 0$. Parts (b) and (c) can be proved in an analogous way. ■

Proof of Lemma 4.5. This follows immediately from Definition 4.4. ■

At this point, note that by invoking the 'truthfulness' refinement to Proposition 4.1* we end up with

$$B_t^o(g) \in \tilde{B}_t(g) \quad \forall g \in \{1,2\}; \tag{4.1*′}$$

instead of 4.1*.

Proof of Lemma 4.6. Let,

$$\hat{g}_1^{o} = \arg\max_{g^o \in \{1,2\}} [u_p(g^o) + \hat{B}_2^o(g^o)]$$

and

$$\hat{g}_2^{o} = \arg\max_{g^o \in \{1,2\}} [u_p(g^o) + \hat{B}_1^o(g^o)]:$$

Lemma 4.5 implies that $\tilde{\pi}^o_1(1) \geq \tilde{\pi}^o_1(2)$ and thus $\hat{g}^o_2 = 1$. Hence, we have from 4.4* that

$$u_p(g^o) + \sum_{t \in L} \tilde{\pi}^o_t(g^o) = H_p + \tilde{\pi}^o_1(1) \;\; \forall g^o \in \{1;2\} \tag{6.1}$$

and

$$H_p + \tilde{\pi}^o_1(1) = u_p(\hat{g}^o_1) + \tilde{\pi}^o_2(\hat{g}^o_1): \tag{6.2}$$

Suppose that $g^o = 1$. Then clearly 6.1 implies that $\tilde{\pi}^o_2(1) = 0$. Alternatively suppose that $g^o = 2$. Then 6.1 and 4.2* imply respectively,

$$\sum_{t \in L} \tilde{\pi}^o_t(2) = H_p + \tilde{\pi}^o_1(1)$$

and

$$\sum_{t \in L} \tilde{\pi}^o_t(2) \geq H_p + \sum_{t \in L} \tilde{\pi}^o_t(1);$$

together implying that $\tilde{\pi}^o_2(1) \leq 0$. Thus, given 4.1*′, $\tilde{\pi}^o_2(1) = 0$.

Since we have $\tilde{\pi}^o_2(1) = 0$, if $\hat{g}^o_1 = 1$ then clearly 6.2 implies $\tilde{\pi}^o_1(1) = 0$ and thus, by Lemma 4.5 and condition 4.1*′; $\tilde{\pi}^o_1(2) = 0$. However, if $\hat{g}^o_1 = 2$ then 6.2 becomes

$$H_p + \tilde{\pi}^o_1(1) = \tilde{\pi}^o_2(2): \tag{6.3}$$

Suppose $g^o = 1$; then $\tilde{\pi}^o_2(1) = 0$; 4.2* and 6.3 together imply

$$\tilde{\pi}^o_2(2) \geq \tilde{\pi}^o_2(2) + \tilde{\pi}^o_1(2)$$

and thus, given 4.1*′, $\tilde{\pi}^o_1(2) = 0$. Alternatively suppose $g^o = 2$, then 6.1 becomes

$$\tilde{\pi}^o_2(2) + \tilde{\pi}^o_1(2) = H_p + \tilde{\pi}^o_1(1)$$

and thus, given 6.3, $\tilde{\pi}^o_1(2) = 0$. ■

Proof of Proposition 4.7. Part (a): Suppose $g^o = 2$. Then when $L = \{1;2\}$ condition 4.3* of the characterisation theorem implies

$$x(y_2)H_2 + \tilde{\pi}^o_1(2) \geq H_p + \tilde{\pi}^o_1(1): \tag{6.4}$$

Note that Lemma 4.5 implies $\tilde{\pi}^o_1(2) = 0$, which together with condition 4.1*′ of the characterisation theorem (i.e. 'truthfulness') implies

$$\tilde{\pi}^o_1(1) = \min\{x(y_1)H_1; x(y_1)y\}:$$

Then, given the stability condition

$$y \geq \max_{t \in \{1,2\}} [H_t];$$

we have $\tilde{B}^o_1(1) = x(y_1)H_1$. Then 6.4 clearly becomes

$$x(y_2)H_2 \geq H_p + x(y_1)H_1$$

or, since $x(y_1)H_1 > 0$ when $L = \{1;2\}$,

$$x(y_2)H_2 > H_p;$$

which cannot be the case. So $g^o = 1$ is the unique equilibrium policy choice.

Then from Lemma 4.5 we have $\tilde{B}^o_2(1) = 0$; which, together with condition 4.1$^{*\prime}$ of the characterisation theorem and the stability condition above, implies $\tilde{B}^o_2(2) = x(y_2)H_2$. Again let

$$\hat{g}'_1 = \arg \max_{g' \in \{1,2\}} [u_p(g') + \tilde{B}^o_2(g')]$$

and note that

$$H_p \geq x(y_2)H_2.$$

Thus $\hat{g}'_1 = 1$ and thus from condition 4.4* of the characterisation theorem we have $\tilde{B}^o_1(1) = 0$.

Part (b)(i): Suppose $g^o = 2$. Then, in a similar way to that above, 6.4 becomes

$$x(y_2)H_2 \geq H_p + x(y_1)H_1;$$

which cannot be the case. So $g^o = 1$ is the unique equilibrium policy choice. Similarly, then $\tilde{B}^o_2(2) = x(y_2)H_2$. Defining $\hat{g}'_1$ as in part (a), and noting

$$H_p < x(y_2)H_2$$

we have $\hat{g}'_1 = 2$. Thus clearly condition 4.4* of the characterisation theorem implies $\tilde{B}^o_1(1) = x(y_2)H_2 - H_p$. Hence the result. Parts (ii) and (iii) can be proved in an analogous way. ■

Proof of Lemma 5.1. Following the procedure outlined in the text, we begin by using the "Magnitude Theorem" to derive the magnitude of every event relevant to each $c \in C$.

(a) Events relevant to $c = y_1$ (note that in this case $x(y_2) = x(y_2)^{i}$ and $x(\emptyset) = x(\emptyset)^{i}$):

(i) $\mathcal{W}_1 = \{x^{i} \in \mathcal{Z}(C)^{i} \mid x(y_2) \le \frac{H_p}{H_2}\}$ implying $\pi_t(y_1; x^{i}) = u_t(1) + y$:

To find the magnitude for this event we have to solve the following maximisation problem

$$\max_{x_n^{i} \in \mathcal{W}_1} \left[\tau_n(\emptyset)\varphi\left(\frac{x_n(\emptyset)}{n\tau_n(\emptyset)}\right) + \tau_n(y_1)\varphi\left(\frac{x(y_1)^{i}}{n\tau_n(y_1)}\right) + \tau_n(y_2)\varphi\left(\frac{x_n(y_2)}{n\tau_n(y_2)}\right)\right] : \tag{65}$$

Note that the vector $x^{i} \in \mathcal{W}_1$ does not feature any constraints with respect to the feasible choices for $x_n(\emptyset)$ and $x_n(y_1)^{i}$: Thus, given that the function φ reaches its maximum value at $\varphi(1) = 0$; we see that $x_n(\emptyset) = n\tau_n(\emptyset)$ and $x_n(y_1)^{i} = n\tau_n(y_1)$ are part of the solution to the above problem.

Define with m_{n1} the maximum value of the objective function in 65. Then it follows directly that

$$m_{n1} = \begin{cases} 0 & \text{if } n\tau_n(y_2)H_2 \le H_p \\ \alpha_{n1} & \text{if } n\tau_n(y_2)H_2 > H_p \end{cases} ;$$

where $\alpha_{n1} = \tau_n(y_2)\varphi\left(\frac{x(y_2)}{n\tau_n(y_2)}\right) < 0$ with the inequality following on $n\tau_n(y_2)H_2 > H_p \ge 0$; $x_n(y_2)H_2 \le H_p$ and the properties of $\varphi()$. From the 'Magnitude Theorem' we know that the magnitude of $\mathcal{W}_1$, μ_1, is the limit of m_{n1} as $n \to \infty$: Hence, it is clear that

$$\mu_1 = \begin{cases} 0 & \text{if } \tau(y_2) = 0 \\ \alpha_1 < 0 & \text{if } \tau(y_2) > 0 \end{cases} ;$$

where $\alpha_1 = \lim_{n \to \infty} \alpha_{n1}$:

By following analogous steps we find the magnitudes of all remaining events as stated below.

(ii) $\mathcal{W}_2 = \{x^{i} \in \mathcal{Z}(C)^{i} \mid (x(y_1)^{i} + 1)H_1 + H_p \ge x(y_2)H_2 > H_p\}$ implying $\pi_t(y_1; x^{i}) = u_t(1) + y - \frac{\bar{B}_1^{o}(1)}{x(y_1)^{i}+1}$; where $\bar{B}_1^{o}(1) = x(y_2)H_2 - H_p$:

The magnitude of this event is

$$\mu_2 = \begin{cases} 0 & \text{if } \tau(y_2) \in (0; \tau(y_1)\frac{H_1}{H_2}] \\ \alpha_2 < 0 & \text{otherwise} \end{cases} :$$

(iii) $W_3 = f x^i \; 2 Z (C)^i \; j x(y_2) H_2 > (x(y_1)^i + 1) H_1 + H_p g$ implying $¼_t(y_1; x^i) = u_t(2) + y:$

The magnitude of this event is

$$¹_3 = \left(\begin{array}{l} 0 \text{ if } ¿(y_2) > ¿(y_1)\frac{H_1}{H_2} \\ ®_3 < 0 \text{ otherwise} \end{array} \right) :$$

Note that the above events are pairwise adjoint and such that $\mathbf{S}_{i=1} W_i = Z(C)^i$. Hence $\mathbf{P}^3_{i=1} \Pr(W_i) = 1:$ This holds analogously for the following cases.

(b) Events relevant to $c = y_2$ (note that in this case $x(y_1) = x(y_1)^i$ and $x(;) = x(;)^i$) if $H_2 \cdot H_p$:

(i) $W_4 = f x^i \; 2 Z (C)^i \; j x(y_1) H_1 + H_p \, ¸ \, (x(y_2)^i + 1) H_2 g$ implying $¼_t(y_2; x^i) = u_t(1) + y:$

The magnitude of this event is

$$¹_4 = \left(\begin{array}{l} 0 \text{ if } ¿(y_2) \; 2 \; [0; ¿(y_1)\frac{H_1}{H_2}] \\ ®_4 < 0 \text{ otherwise} \end{array} \right) :$$

(ii) $W_5 = f x^i \; 2 Z (C)^i \; j (x(y_2)^i + 1) H_2 > x(y_1) H_1 + H_p g$ implying $¼_t(y_2; x^i) = u_t(2) + y \; i \; \frac{B_2^o(2)}{x(y_2)^i + 1}$ where $B_2^o(2) = x(y_1) H_1 + H_p:$

The magnitude of this event is

$$¹_5 = \left(\begin{array}{l} 0 \text{ if } ¿(y_2) > ¿(y_1)\frac{H_1}{H_2} \\ ®_5 < 0 \text{ otherwise} \end{array} \right) :$$

(c) Events relevant to $c = y_2$ if $H_2 > H_p$:

(i) $W_{4a} = f x^i \; 2 Z (C)^i \; j x(y_1) H_1 + H_p \, ¸ \, (x(y_2)^i + 1) H_2; x(y_1) > 0 g$ implying $¼_t(y_2; x^i) = u_t(1) + y:$

The magnitude of this event is

$$¹_{4a} = \left(\begin{array}{l} 0 \text{ if } ¿(y_1) > 0 \text{ and } ¿(y_2) \; 2 \; [0; ¿(y_1)\frac{H_1}{H_2}] \\ ®_{4a} < 0 \text{ otherwise} \end{array} \right) :$$

(ii) $W_{5a} = \{x^i \in Z(C)^i \mid x(y_1) = 0$ or $x(y_1) > 0$ and $(x(y_2)^i + 1)H_2 > x(y_1)H_1 + H_p; x(y_2)^i > 0\}$ implying $\pi_t(y_2; x^i) = u_t(2) + y - \frac{B_2^o(2)}{x(y_2)^i+1}$ where $B_2^o(2) = x(y_1)H_1 + H_p$.

The magnitude of this event is

$$\mu_{5a} = \left\{ \begin{array}{c} 0 \text{ if } \tau(y_1) = 0 \text{ or } \tau(y_2) > \tau(y_1)\frac{H_1}{H_2} \\ \alpha_{5a} < 0 \text{ otherwise} \end{array} \right\}.$$

(d) Events relevant to $c = \emptyset$ (note that in this case $x(y_1) = x(y_1)^i$ and $x(y_2) = x(y_2)^i$):

(i) $W_6 = \{x^i \in Z(C)^i \mid x(y_1)H_1 + H_p \geq x(y_2)H_2\}$; implying $\pi_t(\emptyset; x^i) = u_t(1) + y$. The magnitude of this event is

$$\mu_6 = \left\{ \begin{array}{c} 0 \text{ if } \tau(y_2) \in [0; \tau(y_1)\frac{H_1}{H_2}] \\ \alpha_6 < 0 \text{ otherwise} \end{array} \right\}.$$

(ii) $W_7 = \{x^i \in Z(C)^i \mid x(y_2)H_2 > x(y_1)H_1 + H_p\}$; implying $\pi_t(\emptyset; x^i) = u_t(2) + y$. The magnitude of this event is

$$\mu_7 = \left\{ \begin{array}{c} 0 \text{ if } \tau(y_2) > \tau(y_1)\frac{H_1}{H_2} \\ \alpha_7 < 0 \text{ otherwise} \end{array} \right\}.$$

We are now ready to prove Lemma 5.1. We proceed as follows. First, for each possible configuration of beliefs τ we compare the magnitudes of the events relevant to each $c \in C$. Then, using the 'corollary' to the 'Magnitude Theorem', we determine the events the probability of which will disappear to zero as $n \to \infty$.

It follows, then, by simple inspection that:

(i) When $\tau : \tau(y_1) = 0$ and $\tau(y_2) = 0$ and $H_p \geq H_2$ only $\mu_1 = \mu_4 = \mu_6 = 0$; with all remaining magnitudes less than zero. Thus the 'corollary' to the 'Magnitude Theorem' implies $P(W_1 \mid n\tau) = P(W_4 \mid n\tau) = P(W_6 \mid n\tau) = 1$ as $n \to \infty$. Alternatively, if $H_2 > H_p$; only $\mu_1 = \mu_5 = \mu_6 = 0$; and thus $P(W_1 \mid n\tau) = P(W_5 \mid n\tau) = P(W_6 \mid n\tau) = 1$ as $n \to \infty$.

(ii) When $\tau: \tau(y_1) > 0$ and $\tau(y_2) = 0$ and $H_p \geq H_2$ only $\mu_1 = \mu_4 = \mu_6 = 0$ and thus $P(W_1 \mid n\tau) = P(W_4 \mid n\tau) = P(W_6 \mid n\tau) = 1$ as $n \to \infty$. Alternatively, if $H_2 > H_p$, only $\mu_1 = \mu_{4a} = \mu_6 = 0$, and thus $P(W_1 \mid n\tau) = P(W_{4a} \mid n\tau) = P(W_6 \mid n\tau) = 1$ as $n \to \infty$.

(iii) When τ: if $0 \leq \tau(y_1) < \tau(y_2)\frac{H_2}{H_1}$ and $H_p \geq H_2$ only $\mu_3 = \mu_5 = \mu_7 = 0$, and thus $P(W_3 \mid n\tau) = P(W_5 \mid n\tau) = P(W_7 \mid n\tau) = 1$ as $n \to \infty$. Alternatively, if $H_2 > H_p$, only $\mu_3 = \mu_{5a} = \mu_7 = 0$, and thus $P(W_3 \mid n\tau) = P(W_{5a} \mid n\tau) = P(W_7 \mid n\tau) = 1$ as $n \to \infty$.

(iv) When $\tau: \tau(y_t) > 0\ \forall t$ and $\tau(y_1) \geq \tau(y_2)\frac{H_2}{H_1}$ then: a) if $H_p \geq H_2$ only $\mu_2 = \mu_4 = \mu_6 = 0$, and thus $P(W_2 \mid n\tau) = P(W_4 \mid n\tau) = P(W_6 \mid n\tau) = 1$ as $n \to \infty$; b) if $H_p < H_2$ only $\mu_2 = \mu_{4a} = \mu_6 = 0$, and thus $P(W_2 \mid n\tau) = P(W_{4a} \mid n\tau) = P(W_6 \mid n\tau) = 1$ as $n \to \infty$.

Lemma 5.1, as stated in the text, follows directly from these probabilities. ■

Proof of Proposition 5.2. The distributional strategy $\tau: \tau(y_t) > 0\ \forall t$ supporting $L = \{1, 2\}$, is never an equilibrium. Suppose not. Then, given n and τ, we require $\tau(y_1; S(y_1; n\tau)) = \tau(y_1) > 0$ and $\tau(y_2; S(y_2; n\tau)) = \tau(y_2) > 0$. Note Lemma 5.1 implies that under these beliefs, as $n \to \infty$, either $\tau(y_1) \geq \tau(y_2)\frac{H_2}{H_1}$ and $P(W_2 \mid n\tau) = P(W_4 \mid n\tau) = P(W_6 \mid n\tau) = 1$ or $\tau(y_1) < \tau(y_2)\frac{H_2}{H_1}$ and $P(W_3 \mid n\tau) = P(W_5 \mid n\tau) = P(W_7 \mid n\tau) = 1$. Thus in the first case 5.4 becomes:

$$E[\pi_t(c, x^{-i})] = u_t(1) + y$$

for all $c \in \{y_2; \;\}$ and

$$E[\pi_t(c, x^{-i})] = u_t(1) + y - \frac{B_1^o(1)}{x(y_1)^{-i} + 1}$$

when $c = y_1$. Therefore, given $B_1^o(1) = x(y_2)H_2 - H_p > 0$, $c = y_1$ is a strictly dominated strategy for all t and thus $\sigma(y_1 \mid t) = 0\ \forall t$. The result is $S(y_1; n\tau) = \{\;\}$ and thus $\tau(y_1; S(y_1; n\tau)) \neq \tau(y_1) > 0$; the probability of a contribution to lobby 1 cannot 'come from types for whom choosing y_1 is an optimal action'. Likewise, in the second case, $c = y_2$ is a strictly dominated strategy for all t and thus $\sigma(y_2 \mid t) = 0\ \forall t$ and $S(y_2; n\tau) = \{\;\}$, again implying $\tau(y_2; S(y_2; n\tau)) \neq \tau(y_2) > 0$. Analogously, it can be shown that when $H_p > 0$, the fact that $B_2^o(2) > 0$, implies $c = y_2$ is a strictly dominated strategy for all t under the distributional strategy

¿ : ¿$(y_1) = 0$, ¿$(y_2) > 0$ supporting $L = \{2\}$: Thus again $S(y_2; n$¿$) = \{\emptyset\}$ and ¿$(y_2; S(y_2; n$¿$)) \neq$ ¿$(y_2) > 0$:

The existence of equilibria for the remaining distributional strategies supporting the lobbying configurations $L = \{\emptyset\}$, $L = \{1\}$ and $L = \{2\}$ if $H_p = 0$ can be verified in an exactly analogous manner. ■

References

[1] Austen-Smith, David, (1981), "Voluntary Pressure Groups", Economica, 48, pp. 143-153.

[2] Austen-Smith, David, (1993), "Information and Influence: Lobbying for Agendas and Votes", American Journal of Political Science, 37, pp. 799-833.

[3] Austen-Smith, David and John R. Wright, (1992), "Competitive Lobbying for a Legislator's Vote" Social Choice and Welfare, 9, pp.229-257.

[4] Austen-Smith, David and John R. Wright, (1994), "Counteractive Lobbying", American Journal of Political Science, 38, (1994) pp. 25-44.

[5] Bergstrom, Theodore, Lawrence Blume and Hal Varian, (1986), "On the Private Provision of Public Goods", Journal of Public Economics, 29, pp.25-49.

[6] Besley, Timothy and Stephen Coate (1997a), "An Economic Model of Representative Democracy", Quarterly Journal of Economics, 112, 85-114.

[7] Besley, Timothy and Stephen Coate, (1997b), "Lobbying and Welfare in a Representative Democracy", Working Paper, LSE STICERD TE/97/334.

[8] Bernheim, B. Douglas and Michael Whinston, (1986), "Menu Auctions, Resource Allocation and Economic Influence", Quarterly Journal of Economics, 101, pp. 1-31.

[9] Bagnoli, Mark and Barton Lipman, (1989) "Provision of Public Goods: Fully Implementing the Core through Private Contributions", Review of Economic Studies, 56, pp. 583-601.

[10] Chamberlin, John, (1974), "Provision of Collective Goods as a Function of Group Size", American Political Science Review, 68, pp. 707-716.

[11] Damania, Richard and Per G. Fredriksson, (2000), "On the Formation of Industry Lobby Group", Journal of Economic Behaviour and Organisation, 41, pp315-335.

[12] Dewatripont, Mathias and Jean Tirole, (1999), "Advocates", Journal of Political Economy, 107, pp.1-39.

[13] Dixit, Avinash, (1996), The Making of Economic Policy: A Transaction-Cost Politics Perspective, Cambridge, MA: MIT Press.

[14] Dixit, Avinash, Gene Grossman and Elhanan Helpman, (1997), "Common Agency and Coordination: General Theory and Application to Government Policy Making", Journal of Political Economy, 105, pp. 752-769.

[15] Felli, Leonardo and Antonio Merlo (2000), "Endogenous Lobbying", Working Paper, New York University.

[16] Gawande, Kishore, (1997), "US Non-tariff Barriers as Privately Provided Public Goods", Journal of Public Economics, , pp.61-81.

[17] Grossman, Gene and Elhanan Helpman, (1994), "Protection for Sale", American Economic Review, 84, pp. 833-850.

[18] Grossman, Gene and Elhanan Helpman, (1996), "Electoral Competition and Special Interest Politics", Review of Economic Studies, 63, pp. 265-286.

[19] Kirchsteiger, Georg and Andrea Prat, (1999), "Common Agency and Computational Complexity: Theory and Experimental Evidence", Working Paper 9936, CentER, Tilburg University.

[20] Krishna, Vijay and John Morgan, (1999), "A Model of Expertise", Working Paper, Princeton University.

[21] Krueger, Anne, (1990), "The Political Economy of Controls: American Sugar" in Public Policy and Economic Development, eds. Maurice Scott and Deepak Lal, Oxford: Clarendon Press.

[22] Laffont, Jean-Jaques, (1998), Incentives and Political Economy, forthcoming Oxford: Oxford University Press.

[23] Lohmann, Susan, (1998), "An Information Rationale for the Power of Special Interests", American Political Science Review, 92, pp. 809-827.

[24] Magee, Stephen P., William A. Brock and Leslie Young (1989) Black Hole Tari¤s and Endogenous Policy Theory, Cambridge University Press, Cambridge.

[25] Mitra, Devashish, (1999), "Endogenous Lobby Formation and Endogenous Protection: A Long-Run Model of Trade Determination", American Economic Review, 89, pp. 1116-1134.

[26] Myerson, Roger, (1997), "Population Uncertainty and Poisson Games", Working Paper 1102R, Center for Mathematical Studies in Economics and Management Science.

[27] Myerson, Roger, (1998), "Large Poisson Games", Working Paper 1189, Center for Mathematical Studies in Economics and Management Science.

[28] Olson, Mancur, (1965), The Logic of Collective Action, Cambridge, MA: Harvard University Press.

[29] Persson, Torsten and Guido Tabellini, (1999), "Political Economics and Public Finance", prepared for the Handbook of Public Economics, vol III, eds. Alan Auerbach and Martin Feldstein.

[30] Palfrey, Thomas R. and Howard Rosenthal (1985), "Voter Participation and Strategic Uncertainty", American Political Science Review, 79, pp 62-78.

[31] Sandler, Todd, (1992), Collective Action, Hemel Hempstead: Harvester Wheatsheaf.

[32] Stigler, George, (1971), "The Theory of Economic Regulation", The Bell Journal of Economics and Management Science, 3, pp. 3-21.

[33] Stigler, George, (1974), "Free Riders and Collective Action: An Appendix to Theories of Economic Regulation", The Bell Journal of Economics and Management Science, 5, pp. 359-365.