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Mixing Private and Public Service Providers and Specialization

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Mixing Private and Public Service Providers and Specialization

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Abstract
We analyze the reform of public sector welfare services such as education. In this paper we compare a mix of private and a public service provider with full privatization. In both cases the suppliers specialize in serving particular customer types. In the mixed institution the government sets the public fee such that service quality does not deteriorate and the price of the private supplier is anchored at comparatively low level. Under full privatization, however, prices escalate to the highest possible level. As a consequence, consumer welfare is higher with a mixed institution – unless the proportion of low-cost customers is high. The mixed institution can also accommodate wealth constraints of customers to some extent.

Keywords: private and public suppliers, specialization, welfare services, mixed institutions

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1 Introduction

The public sector has been going through considerable reforms recently. Privatization is one of the ways to potentially provide stronger incentives. The focus of this paper is on the question whether competition between private and public suppliers could ever provide better incentives than complete privatization.

This paper has been written with education sector in mind but it can apply to other welfare services as well. One of the main features of welfare services is that final output (educational value added) is nonverifiable and therefore providing incentives by simple contracts is not easy. Although quality is nonverifiable it is observable. People have an idea of which schools are good although contracts on quality cannot be enforced by courts. As quality is observable the higher-quality service providers attract more customers and competition can provide incentives.

The second important feature of welfare services is that they are provided by professionals (teachers) who are not purely motivated by money. These professionals are either internally motivated and get job satisfaction from providing a good quality service – or externally motivated and being associated with a high-quality institution is good for their career concerns. We can model this motivation by private benefits the professionals derive from the quality of the service.

The customers (students) are of different types. Some students are hard to educate while others need to have continuous intellectual challenges to remain motivated. Different types of students therefore need different services and teachers' efforts. Further, different types of students give different levels of private benefits to the teachers one type being more rewarding than the other. There are two types of students: rewarding (or low-cost) and ordinary (or high-cost). We assume that the type of the customer is observable to teachers but not verifiable.

Including these main features of the welfare services we compare full privatization to a mix of a private and a public supplier. We find that the suppliers specialize. Each supplier concentrates on serving one customer type and can therefore tailor their service for the needs of that type and provide first-best quality. The suppliers can soften the competition by specialization rather than competing head to head for both types of customers.

Under privatization the suppliers' profits have to be equal although they are providing different quality levels to different customers. Otherwise the specialization equilibrium does not exist. The equal profits requirement implies that we can only solve for the price difference in the equilibrium. The supplier serving the rewarding type charges a higher price because they are providing higher quality and hence higher cost service. Under privatization the prices are likely to escalate to the highest level and the consumer surplus is low.

In the mixed institution the government sets the public fee so low that the public supplier can only serve the ordinary type. This leaves the private supplier free to concentrate on the rewarding type. The fee the private supplier can charge is limited by the competition from the public supplier and accordingly the private fee remains relatively low.

The strength of the mixed institution is that the low public fee set by the government anchors the private supplier's fee also relatively low while under privatization the fees are likely to escalate. That is why the mixed institution maximizes consumer surplus when the proportion of ordinary types is high. While when the rewarding type has a large majority, privatization is optimal. The equal profit requirement constrains the price for the majority type so much that it is lower than the relatively low price under mixed institution. Although the ordinary type is always better off in the mixed institution, their share is so small that the aggregate consumer welfare is maximized under privatization.

We also analyze mixed institution when a proportion of the population cannot opt out from the public sector because of wealth constraints. We find that a service trap emerges and the public supplier provides minimal service to all its customers. Vouchers to poor customers enable the poor customers to have a choice and restore the merits of the mixed institution.

We build on Halonen and Propper (1999) who analyze quasimarket reforms for welfare services and compare competition between public suppliers to a single public agency. In this paper we include private suppliers in the framework. Related literature on privatization and incomplete contracts are among others Shleifer and Vishny (1994), Hart et al. (1997) and Hart (2003). None of these papers analyzes the mixed structure where public and private suppliers coexist.

Epple and Romano (1998) (ER) also analyse competition between private and public schools and obtain specialization of schools in different ability types. Our paper is complementary to theirs as we analyse quite a different environment. We focus on incentive problems while in ER the school's quality depends on the mean ability of the students and the costs depend on the number of the students (neither depend on the supplier's effort). Public sector is a passive player in ER while in our paper both public and private suppliers are strategic. ER take the mixed structure as given while our focus is on optimal institutional design. In ER student types are completely verifiable and what drives the results is price discrimination between different types of students. Our paper analyzes the other extreme where student types are not at all verifiable. The real world lies between these two extremes.

In many countries, most notably in the UK and US, both private and public suppliers are present in the education sector. Our paper suggests an economic rationale for the coexistence of privately and publicly owned providers of education services.

Our paper belongs to a rapidly growing literature on the optimal organization of the provision of collective goods when agents are motivated by non-pecuniary aspects of motivation (see Akerlof and Kranton (2003), Bénabou and Tirole (2003), Francois (2000), Murdock (2002) and Besley and Ghatak (2005)). Our paper emphasizes that specialization among competing suppliers can accommodate different non-pecuniary benefits from serving customers. While optimal specialization is possible under full privatization and under a mixed institution, the latter avoids price escalation and tends to yield higher consumer welfare.

The rest of the paper is organized as follows. Section 2 describes our model. Section 3 derives the first-best solution. Section 4 analyzes privatization while Section 5 examines competition between public and private suppliers. Section 6 determines the optimal institution. Wealth constraints are discussed in Section 7. Section 8 concludes.

2 The model

There are two suppliers (schools), indexed by $i = 1, 2$, each run by a manager, M_i. The customers (students) of the suppliers are of two types, A and B. Proportion γ^A of the population are of type A and proportion $\gamma^B = \left(1 - \gamma^A\right)$ are of type B. We assume that the type of the customer is observable to M_i but is not verifiable.

The value of the supplier's service to the customers, v_i^A and v_i^B, depends on M_i's effort for each type e_i^A and e_i^B ($e_i^j \geq 0, j = A, B$).[1]

[1] It is by no means obvious or necessary that the value of a unit of effort is the same for both types. Our assumptions put all the heterogeneity across types in the cost differences.

$$v_i^A = \ln\left(e_i^A + 1\right) \text{ for type A}$$

$$v_i^B = \ln\left(e_i^B + 1\right) \text{ for type B}$$

We assume that the type-specific quality of the service v_i^A and v_i^B is observable to the customers but is not verifiable. There are many aspects of school quality that are difficult to verify but are not difficult to observe. Customers have a clear idea of which schools are good but contracts on quality cannot be enforced by courts.

Effort creates a disutility ce_i^A and ce_i^B per customer of each type to M_i where $c > 0$.[2] The managers of the suppliers are risk neutral. Moreover, they derive some private benefit per customer, denoted by b, from their work. They are either internally motivated and get job satisfaction from providing a good quality service – or externally motivated and get more respect when they are associated with a high-quality institution. Private benefits are given by:

$$b\left(e_i^A, e_i^B\right) = \mu_A e_i^A + \mu_B e_i^B$$

with $\mu_A, \mu_B \geq 0$. We assume that one activity is more rewarding than the other. In particular, we assume that $\mu_B > \mu_A$. One interpretation is that type B are the bright students. M_i either enjoys serving type B more or high value service for type B is more important for career concerns. Hence, the net costs for M_i are given by:

$$ce_i^A + ce_i^B - b\left(e_i^A, e_i^B\right) = (c - \mu_A)\, e_i^A + (c - \mu_B)\, e_i^B = c_A e_i^A + c_B e_i^B$$

where c_A and c_B are the net costs per unit of effort of each task. Note that our assumption implies $c_A = c - \mu_A > c_B = c - \mu_B$. Type B is referred to as the *rewarding* type and type A as the *ordinary* type.

We assume $c_A < 1$. Otherwise marginal costs of providing the service to the high-cost customers would be higher than the marginal value for any

By suitable renormalization we can translate value differences into cost differences if value differences can be captured by a scale parameter.

[2] Strictly speaking, ce_i^A and ce_i^B are cost densities since we are working with a continuum of customers. To reduce unnecessary language we will drop the density term in the whole paper.

positive level of effort. We also assume that $\mu_B < c$ so that the costs are positive. It will be essential for our results that c_A and c_B differ. We will develop the precise condition on the cost difference over the course of the paper.

We apply the Hotelling model in which the suppliers are located at the extremes of the unit interval $[0, 1]$. Supplier 1 is located at 0 and supplier 2 at 1. Each supplier has a capacity of 1. The customers of each type are uniformly distributed on the unit interval with density 1. They incur a transportation cost t per unit of length. The distance can be interpreted literally as a geographical distance between two schools. The market shares of the suppliers are denoted by s_i^A and s_i^B, $i = 1, 2$ ($s_2^A = 1 - s_1^A$; $s_2^B = 1 - s_1^B$).

The regulator designs the institution. We analyze two institutions. Either both suppliers are private or we have a mixed structure where one supplier is private and the other is public. The regulator is assumed to be benevolent.

If the suppliers are *privatized*, they are free to choose prices, denoted by p_1 and p_2, to their customers. In this private institution neither suppliers nor customers receive public funds.

The second option is that the regulator chooses a *mixed structure* with one public and one private supplier. Then the regulator pays the public supplier 1 a fee f_1 per customer and a fixed wage w to the manager if the fees are not sufficient to motivate the manager to run the organization. Fees are financed by lump sum taxes. The private supplier 2 is free to charge a price p_2 to customers. The private supplier does not receive any public funds. Neither do the customers of the private supplier.

The third option is that both suppliers are public. We do not examine this third option in detail but we will comment on it in the last section.

In our model, the customer's type is not verifiable and hence the suppliers charge the same price/fee for both types.[3] We assume that the regulator

[3]If there were price discrimination between the types, the supplier would always claim the customer to be of the more expensive type, since the type cannot be verified.

There are legal limits to price discrimination even if the types are verifiable as the Amazon case demonstrates. Uniform pricing may also result from political concerns. We can argue that the poor customers are the high-cost ones. Then price discrimination would result in higher fees for the poor. This may not be politically feasible.

Epple and Romano (1998) analyze competition between private and public schools when the student ability types are completely verifiable. Price discrimination does all the work in their model. We can view our paper as exploring the other extreme: types are not at all verifiable. The real world lies between these two extreme cases.

maximizes the overall sum of the utility of customers by choosing an appropriate institution and fee levels if a supplier is public.

The timing of the model is given as follows. First, the regulator chooses the institution. Second, under the mixed structure the regulator chooses the fee for the public supplier. Third, the rest of the decisions are made simultaneously: the managers choose the efforts and the prices and the customers choose their supplier.

3 First-best solution

In this section we derive the first-best solution under the assumption that the regulator can verify the quality of the service. The regulator is assumed to maximize the aggregate welfare of the customers in the sense of a utalitarian welfare function under the condition that the regulator needs to motivate the manager to participate. We assume that the manager can earn zero wages outside. It is obvious that the first-best solution implies equal effort levels for each type of customers across suppliers $\left(e^j \equiv e_1^j = e_2^j \text{ for } j = A, B\right)$ and equal market shares $\left(s_i^j = \frac{1}{2} \text{ for } i = 1, 2, \ j = A, B\right)$ because travelling costs are minimized in this case. Then the regulator's problem of maximizing consumer welfare is given by:

$$\underset{\{w, e^A, e^B\}}{Max} \left\{ \gamma^A v^A + \gamma^B v^B - \frac{1}{4} t - 2w \right\}$$

$$\text{s.t. } w - \frac{1}{2} \gamma^A e^A c_A - \frac{1}{2} \gamma^B e^B c_B \geq 0$$

where $v^A = \ln\left(e^A + 1\right)$ and $v^B = \ln\left(e^B + 1\right)$. Aggregate welfare is defined as the sum of the customers' valuation of the services minus transportation costs and wages paid for the managers. The term $\frac{1}{4}t$ is the average transportation cost. The manager's participation constraint is binding and therefore the first-order conditions are given by:

$$\frac{1}{e^A + 1} - c_A = 0 \tag{1}$$

$$\frac{1}{e^B + 1} - c_B = 0 \tag{2}$$

From equations (1) and (2) we obtain the first-best solutions denoted by e^{A*} and e^{B*}.

Proposition 1 *The first-best solution is given by*
$e^{A*} = \frac{1-c_A}{c_A}$,
$e^{B*} = \frac{1-c_B}{c_B}$ *and*
$w^* = \frac{1}{2}\left(\gamma^A(1-c_A) + \gamma^B(1-c_B)\right)$.

The first-best solution simply equates marginal costs (c_A and c_B) to marginal utility $\left(\frac{1}{e^A+1}\right.$ and $\left.\frac{1}{e^B+1}\right)$ for each type of customer. Suppose that task A is much less rewarding than task B $(c_A >> c_B)$. Then the first-best solution also requires $e^{A*} << e^{B*}$. The first-best solution does not only depend on the production costs but also on the private benefits of the managers. If task A is not rewarding in the sense of private benefits, the manager has to be compensated by higher monetary wage to undertake this effort. This increases the regulator's marginal cost for type A and therefore a lower quality level for this type is implemented.

The first-best solution in both cases shows that the optimal effort and quality levels for one type are independent of the optimal level for the other type. The optimal effort level e^{A*} only depends on the marginal costs c_A. This is because both the values and the costs are separable across types.

In what follows we examine alternative institutions when quality levels are observable but not verifiable. Because of unverifiability contracts on quality cannot be written. But because quality is observable, competition between the suppliers can provide incentives to increase service quality in order to attract more customers.

4 Competition between private suppliers

We start by analyzing competition between private suppliers. The industry is fully privatized: the suppliers are private and the service is funded privately. The customers pay the fees themselves and do not receive subsidies from the government.

The managers maximize supplier profits taking their own effort costs into account. The manager's profit maximization problem is:

$$\underset{\left\{p_i, e_i^A, e_i^B\right\}}{Max}\ \Pi_i = p_i\left(\gamma^A s_i^A + \gamma^B s_i^B\right) - c_A\gamma^A e_i^A s_i^A - c_B\gamma^B e_i^B s_i^B$$

where

$$s_i^k = \frac{v_i^k - v_j^k + t + p_j - p_i}{2t} \text{ for } i, j = 1, 2, \ i \neq j, \ k = A, B$$

The demand functions are given by the Hotelling model. Customer of type A compares the value for money he receives from supplier 1, $\left(v_1^A - p_1\right)$, to what he can get from supplier 2, $\left(v_2^A - p_2\right)$, taking into account the transportation costs.

Profit maximization gives the following first-order conditions:

$$\frac{\partial \Pi_i}{\partial e_i^A} = \left(p_i - c_A e_i^A\right) \frac{1}{2t\left(e_i^A + 1\right)} - c_A s_i^A = 0 \tag{3}$$

$$\frac{\partial \Pi_i}{\partial e_i^B} = \left(p_i - c_B e_i^B\right) \frac{1}{2t\left(e_i^B + 1\right)} - c_B s_i^B = 0 \tag{4}$$

$$\frac{\partial \Pi_i}{\partial p_i} = \left(\gamma^A s_i^A + \gamma^B s_i^B\right) - \left(p_i \gamma^A - c_A \gamma^A e_i^A\right) \frac{1}{2t} - \left(p_i \gamma^B - c_B \gamma^B e_i^B\right) \frac{1}{2t} = 0 \tag{5}$$

Higher effort for type A increases the value of the service for type A and the market share accordingly. This marginal benefit is the first term in equation (3) while the second term gives the marginal cost: higher effort is exerted for all the type A customers served by this supplier. Equation (5) is the first-order condition with respect to the service price, p_i. As the customer type is not verifiable a uniform price is set for both types by an individual supplier. Increasing the price marginally results in higher profits from the market served by this supplier but lowers the market shares putting pressure on profits. This trade-off determines the optimal price. Solving from the first-order conditions (3) – (5) we obtain:

$$\widetilde{e}^A = \frac{1 + t - c_A \gamma^A - c_B \gamma^B - t c_A}{c_A (1 + t)}$$

$$\widetilde{e}^B = \frac{1 + t - c_A \gamma^A - c_B \gamma^B - t c_B}{c_B (1 + t)}$$

$$\widetilde{p} = 1 + t - c_A \gamma^A - c_B \gamma^B$$

As the equilibrium is symmetric we drop the subscripts and denote the values by $\widetilde{e}^A$, $\widetilde{e}^B$ and $\widetilde{p}$. In what follows we, however, show that this symmetric equilibrium does not exist as the suppliers have an incentive to specialize in serving only one type. We prove this for the symmetric equilibrium that we obtain for $t \to 0$. By continuity of the first-order conditions with respect to t, the argument holds for small values of t. For $t \to 0$ the symmetric equilibrium amounts to:

$$\widetilde{e}^A = \frac{1 - c_A\gamma^A - c_B\gamma^B}{c_A} \tag{6}$$

$$\widetilde{e}^B = \frac{1 - c_A\gamma^A - c_B\gamma^B}{c_B} \tag{7}$$

$$\widetilde{p} = 1 - c_A\gamma^A - c_B\gamma^B \tag{8}$$

Given supplier 1 chooses $\widetilde{e}^A$, $\widetilde{e}^B$ and $\widetilde{p}$ as determined by equations (6) – (8), is it optimal for supplier 2 to deviate and specialize in type B? Supplier 2's profit maximization problem in this case is:

$$\underset{\{p_2, e_2^B\}}{Max} \left\{\gamma^B \left(p_2 - c_B e_2^B\right)\right\}$$

$$\text{s.t. } v_2^B - p_2 \geq \widetilde{v}^B - \widetilde{p}$$

Supplier 2 maximizes profits from serving only type B subject to the constraint that type B will select supplier 2. Supplier 2 gives just enough value for money to type B to attract him so that the constraint is binding:

$$p_2 = v_2^B - \widetilde{v}^B + \widetilde{p} \tag{9}$$

Substituting equation (9) in the profit function we have:

$$\underset{\{e_2^B\}}{Max} \left\{\gamma^B \left(v_2^B - \widetilde{v}^B + \widetilde{p} - c_B e_2^B\right)\right\}$$

This profit maximization problem gives the following optimal effort level for type B.

$$e_2^B = \frac{1 - c_B}{c_B} \tag{10}$$

Supplier 2 offers first-best value of service to type B. We can solve the price by inserting (10) in (9).

$$p_2 = \ln\left(\frac{1}{c_B}\right) - \widetilde{v}^B + \widetilde{p}$$

Supplier 2's profits are then:

$$\Pi_2 = \gamma^B\left(\ln\left(\frac{1}{c_B}\right) - \widetilde{v}^B + \widetilde{p} - (1 - c_B)\right) \tag{11}$$

Profits in the symmetric equilibrium are zero. From $(6) - (8)$ we see that the price just covers the costs: $\widetilde{p} = c_A\widetilde{e}^A = c_B\widetilde{e}^B$. Therefore if the profits given by equation (11) are positive, it is optimal for supplier 2 to deviate and the symmetric equilibrium does not exist. We can rewrite equation (11) as:

$$\begin{aligned}\Pi_2 &= \gamma^B\left[\left(v^{B*} - c_B e^{B*}\right) - \left(\widetilde{v}^B - \widetilde{p}\right)\right] \\ &= \gamma^B\left[\left(v^{B*} - c_B e^{B*}\right) - \left(\widetilde{v}^B - c_B\widetilde{e}^B\right)\right] > 0 \end{aligned} \tag{12}$$

We have taken into account in (12) that $\widetilde{p} = c_B\widetilde{e}^B$ as per equations (7) and (8). Then supplier 2's specialization profits depend on the difference between the surplus from the first-best effort, e^{B*}, and the surplus from the proposed symmetric equilibrium effort $\widetilde{e}^B$. By definition the surplus from the first-best effort is greater than from any other level of effort, including $\widetilde{e}^B$. Therefore the difference and the profits from specialization are positive and the symmetric equilibrium does not exist.

The intuition for the result runs as follows. By competing head to head for both types of customers the suppliers would drive the profits down to zero. When each supplier specializes in one type, they can soften the competition and earn higher profits. This is why the symmetric equilibrium does not exist.

We have yet to prove that the specialization equilibrium exists.[4] We proceed in two steps. In the first proposition we characterize specialization equilibria. In the second proposition we prove their existence.

[4] To derive Bertrand equilibria in our setup we employ a flexible tie-breaking rule. That is, to which supplier indifferent customers go is determined by equilibrium requirements.

Proposition 2 *A specialization equilibrium is characterized by:*

(i) $\overline{v}_1^A = \ln\left(\frac{1}{c_A}\right)$, $\overline{s}_1^A = 1$

(ii) $\overline{v}_1^B = \ln\left(\frac{1}{c_B}\right) - \Theta$, $\overline{s}_1^B = 0$

(iii) $\overline{v}_2^A = \ln\left(\frac{1}{c_A}\right) + \Theta$, $\overline{s}_2^A = 0$

(iv) $\overline{v}_2^B = \ln\left(\frac{1}{c_B}\right)$, $\overline{s}_2^B = 1$

(v) $\gamma^B \overline{p}_2 - \gamma^A \overline{p}_1 = \gamma^B (1 - c_B) - \gamma^A (1 - c_A)$,

where $\Theta \equiv \overline{p}_2 - \overline{p}_1 = \frac{\gamma^B}{\gamma^A}(1 - c_B) + \overline{p}_2 \left(1 - \frac{\gamma^B}{\gamma^A}\right) - (1 - c_A)$

Proposition 3 *For every set of parameters* $\{\gamma^A, c_A, c_B\}$ *there exists a pair* $\{p_1^h, p_2^h\}$ *with* $p_1^h \geq 1 - c_A$ *and* $p_2^h \geq 1 - c_B$ *so that*

(i) For every price $p_1 \in [1 - c_A, p_1^h]$ *there exists a price* $p_2 \in [1 - c_B, p_2^h]$ *such that a specialization equilibrium exists as characterized by Proposition 2.*

(ii) The upper limits p_1^h *and* p_2^h *are given by*

$p_1^h = 1 - c_A + \Delta^0$ *and* $p_2^h = 1 - c_B + \frac{\gamma_A}{\gamma_B}\Delta^0$

where $\Delta^o = min\{\tilde{\Delta}, ln(\frac{1}{c_A}) - (1 - c_A), \frac{\gamma_B}{\gamma_A}(ln(\frac{1}{c_B}) - (1 - c_B))\}$

and $\tilde{\Delta}$ *is the minimal solution of*

$-ln(1 - \gamma^A(1 - e^{\Theta})) + \gamma^A(c_A - c_B) + \frac{(\gamma_A)^2}{1-\gamma_A}\Delta = 0$

with $\Theta = p_2 - p_1 = c_A - c_B - \frac{\gamma_B - \gamma_A}{\gamma_B}\Delta.$

The proofs are given in the Appendix. Propositions 2 and 3 have the following implications. Supplier 1 specializes in the ordinary type A and supplier 2 in the rewarding type B. The rewarding customers receive a higher level of service than the ordinary customers, each receiving the quality level according to the first-best solution. Even with full specialization the suppliers provide potential competition for each other. Effort is also planned for the type that in the end is not served. This keeps the serving supplier's quality up.

Profits from specialization in type A and B have to be equal although the service levels and costs are different for the two types. Otherwise specialization equilibrium does not exist: the low-profit supplier would capture the market of the high-profit supplier by providing the same level of service with $\varepsilon-$lower price.

An important feature of Proposition 3 is that there is indeterminacy of equilibria. Any pair of prices such that the suppliers have equal profits, $\gamma^B \overline{p}_2 - \gamma^A \overline{p}_1 = \gamma^B (1 - c_B) - \gamma^A (1 - c_A)$, can occur in equilibrium as long as the prices cover the suppliers' costs, prices are not larger than the customer's valuation for the service and suppliers have no incentive to attract all customers. The lowest prices in equilibrium are given by $\overline{p}_1 = 1 - c_A$ and $\overline{p}_2 = 1 - c_B$. In this case the suppliers' profits are zero. The highest prices in equilibrium are such that at least one customer type is left with zero surplus from this service.

In Proposition 3 the upper limits of the prices are not only related to the customers' valuation of the service but also to $\tilde{\Delta}$. $\tilde{\Delta}$ is defined so that any $p_1 > 1 - c_A + \tilde{\Delta}$ and a corresponding p_2 that gives supplier 2 equal profits cannot be sustained in specialization equilibrium because the temptation to capture both types is too high. When prices are very high undercutting the rival slightly and capturing both types is profitable and this breaks the specialization equilibrium. But if $1 - c_A + \tilde{\Delta}$ is greater than type A's valuation of the service, $\tilde{\Delta}$ does not play any role in equilibrium. Even at a price which leaves no surplus to the customer there is no incentive to deviate from the specialization equilibrium.

The intuition for the indeterminacy runs as follows. In order to attract the customer served by the competitor that makes positive profits, a firm must offer a price and service level combination that is more appealing for the customer. However, this requires that the price for the customers that are currently served will change too because price discrimination is not possible. In equilibrium prices and service levels are determined such that trying to capture both customer types will not be profitable.

In the following we discuss the properties of the specialization equilibrium in more detail. Our first observation shows that indeterminacy rests on the existence of cost and share differences. From Proposition 3 we obtain:

Corollary 1 *Suppose that* $\gamma^A = \gamma^B$ *and* $c_A = c_B$. *Then* $p_1^h = 1 - c_A$ *and* $p_2^h = 1 - c_B$. *Hence, there exists a unique equilibrium with* $\overline{p}_1 = \overline{p}_2 = 1 - c_A = 1 - c_B$.

The preceding Corollary shows that we obtain a Bertrand type equilibrium if all customers are completely homogenous. From Proposition 3 we obtain that the indeterminacy is increasing in the cost difference and that for large cost differences $\tilde{\Delta}$ may not be binding.[5]

The following Corollary gives some further properties of the specialization equilibrium.

Corollary 2 *(i)* $\overline{\Pi}_1 = \overline{\Pi}_2 \geq 0$
(ii) $\overline{\Pi}_1 = \overline{\Pi}_2 = 0$ *if and only if* $\overline{p}_1 = 1 - c_A$ *and* $\overline{p}_2 = 1 - c_B$
(iii) Suppose $\gamma^A = \gamma^B = \frac{1}{2}$. *Then* $\overline{p}_2 - \overline{p}_1 = c_A - c_B$.

Property (i) in Corollary 2 states that the profits of the suppliers are equal in this specialization equilibrium. The minimum value for the profits is zero and according to property (ii) it occurs when the prices are equal to the lowerbound given in Proposition 3, i.e. when the price just covers the costs. For any other prices higher than the lowerbound the suppliers' profits are positive.

Finally, property (iii) shows that when there are equal proportions of the two types in the population we can solve for the price difference in equilibrium. Supplier 2 charges a higher price than supplier 1. This is because supplier 2 is serving the rewarding type who receives a higher level of service than the ordinary type served by supplier 1. The price difference reflects the different production costs.

The indeterminacy raises the question about the selection of equilibria. A standard selection criterion is Payoff dominance under which strategic players coordinate on equilibrium that yields higher payoffs than any other equilibria. Applying this criterion leads to the selection of equilibrium that yields the highest profits for the firms. From the profit equations in equilibrium

$$\Pi_1 = \gamma^A \left(p_1 - (1 - c_A)\right) = \gamma^B \left(p_2 - (1 - c_B)\right) = \Pi_2 \tag{13}$$

we immediately conclude that Payoff dominance (among firms) yields the equilibrium with the highest prices. This is examined in the following Corollary where we assume that indeterminacy is sufficiently large.

[5] Whether the constraint $\tilde{\Delta}$ is binding or not depends also in a complicated way on the share of type A and type B customers. Details are available upon request.

Corollary 3 *Suppose* $\tilde{\Delta} \geq \min\{\ln(\frac{1}{c_A} - (1 - c_A), \frac{\gamma_B}{\gamma_A}\left(\ln(\frac{1}{c_B}) - (1 - c_B)\right)\}$.

Payoff dominance among firms yields the equilibrium with the following highest prices:

$\overline{p}_1 = \ln\left(\frac{1}{c_A}\right)$ *and* $\overline{p}_2 = (1 - c_B) + \frac{\gamma^A}{\gamma^B}\left[\ln\left(\frac{1}{c_A}\right) - (1 - c_A)\right]$ *if and only if* $\gamma^A \leq \overline{\gamma}^A$

$\overline{p}_1 = (1 - c_A) + \frac{\gamma^B}{\gamma^A}\left[\ln\left(\frac{1}{c_B}\right) - (1 - c_B)\right]$ *and* $\overline{p}_2 = \ln\left(\frac{1}{c_B}\right)$ *if and only if* $\gamma^A > \overline{\gamma}^A$

where $\overline{\gamma}^A = \frac{\left(\ln\left(\frac{1}{c_B}\right) - (1 - c_B)\right)}{\ln\left(\frac{1}{c_A}\right) - (1 - c_A) + \ln\left(\frac{1}{c_B}\right) - (1 - c_B)}$ *and* $\frac{\partial \overline{\gamma}^A}{\partial (c_A - c_B)} > 0$.

The proof is given in the Appendix. Corollary 3 shows that on the knife-edge ($\gamma^A = \overline{\gamma}^A$) both suppliers charge a price equal to the customer's valuation of the service. But typically one customer type pays less than his valuation. When $\gamma^A > \overline{\gamma}^A$ (resp. $\gamma^A < \overline{\gamma}^A$) it is type A (resp. B) that receives a positive surplus from this service. When the proportion of the type is high in the population, this type receives a surplus. This is driven by the equal profit requirement. The supplier of the majority type has to earn a lower price-cost margin than the supplier of the minority type to make equal profits. This constrains the price for the majority type and leaves him with a positive surplus.

Corollary 3 also shows that the critical value $\overline{\gamma}^A$ is increasing in the cost difference between the two types. When the cost difference increases, the gap between the first best service levels of the two types increases. Since type A's valuation is lower it becomes more likely that A's valuation becomes the binding constraint for the highest prices. Then type A has to have an even higher majority to receive positive surplus.

Notice that the critical value $\overline{\gamma}^A$ is greater than half. That is, type A has to have a significant majority to receive surplus from the service. While even equal proportions of the types in the population imply that type B receives surplus. This is because type A's valuation is lower and it is more likely to be the binding constraint for the prices.

We immediately observe that the equilibrium selected by the Payoff dominance criterion among the suppliers is welfare minimal for the consumers. At least one type of customer pays for his service a price equal to his valuation and therefore receives zero surplus from this service. One type of customer may get a positive but small surplus.

In equilibrium the customers receive first best service ($\overline{v}_1^A = \ln\left(\frac{1}{c_A}\right)$ and

$\overline{v}_2^A = \ln\left(\frac{1}{c_B}\right)$) and therefore the service levels are independent of prices. Hence, high prices simply amount to a transfer from the customers to the suppliers. Note that service levels $\overline{v}_1^B$ and $\overline{v}_2^A$ depend on prices but are not demanded in equilibrium by the customers.

In the next section we examine how the outcome will differ when one of the suppliers is public. We maintain the assumption that prices under private competition are selected according to the Payoff dominance criterion.

5 Competition between public and private suppliers

In this section we analyse the mixed institution with public supplier 1 and private supplier 2. We consider directly the case $t = 0$. The maximization problem for the public manager 1 is given by:

$$\underset{\{e_1^A, e_1^B\}}{Max}\ f_1\left(\gamma^A s_1^A + \gamma^B s_1^B\right) - c_A\gamma^A e_1^A s_1^A - c_B\gamma^B e_1^B s_1^B$$

The public supplier's fee f_1 is set by the regulator and financed by lump sum taxation while the manager chooses the effort levels for each type.

The maximization problem of the private manager 2 is given by:

$$\underset{\{p_2, e_2^A, e_2^B\}}{Max}\ p_2\left(\gamma^A s_2^A + \gamma^B s_2^B\right) - c_A\gamma^A e_2^A s_2^A - c_B\gamma^B e_2^B s_2^B$$

The private supplier is free to choose the price for the service in addition to the effort levels.

Government pays the fees for the public supplier 1's customers while the private supplier 2's customers pay the fees themselves. Therefore, when choosing which supplier to go to, the customers compare the quality difference $(v_2^i - v_1^i)$ to the private fee. The quality of the private service has to be high enough compared to the quality of the public service to convince some customers to opt out and pay the private fees.

We assume in the following that the fee revenues are sufficient to motivate the manager to participate and thus the regulator does not need to pay an additional fixed wage.[6]

Proposition 4 gives our main result for the mixed institution.

[6]The extension to the case $w > 0$ is straightforward and omitted.

Proposition 4 *Assume* $1 - c_B < -\ln(1 - c_A + c_B)$. *Suppose that the regulator sets* $\widehat{f}_1 = 1 - c_A$. *Then there exists a specialization equilibrium with*

(*i*) $\widehat{v}_1^A = \ln\left(\frac{1}{c_A}\right)$, $\widehat{s}_1^A = 1$

(*ii*) $\widehat{v}_1^B = \ln\left(\frac{1-c_A+c_B}{c_B}\right)$, $\widehat{s}_1^B = 0$

(*iii*) $\widehat{v}_2^A = \ln\left(\frac{1}{c_A(1-c_A+c_B)}\right)$, $\widehat{s}_2^A = 0$

(*iv*) $\widehat{v}_2^B = \ln\left(\frac{1}{c_B}\right)$, $\widehat{s}_2^B = 1$

(*v*) $\widehat{p}_2 = \ln\left(\frac{1}{1-c_A+c_B}\right)$

The proof is given in the Appendix. When the regulator sets the public fee $\widehat{f}_1 = (1 - c_A)$ the public supplier specializes in the ordinary type A and the private supplier in the rewarding type B. Each type of customer receives first-best service level.

In equilibrium the private supplier offers higher quality service for both types. Type A, however, opts for the lower quality public service to avoid paying the private fee. While for type B the quality difference is high enough to justify paying the private fee.

Why does not the private supplier also try to capture type A customers? Type A is comparing the value of public service, v_1^A, to the surplus from the private service, $\left(v_2^A - p_2\right)$. To attract type A – who already receives first best quality from the public supplier – the private supplier has to offer A more value for money. The best way to do this is to lower both price and quality (see part (*v*) in the proof of Proposition 4) so that the price falls more than the quality. (This lower quality is still higher than first best.) Even with this best deviation the private supplier would be making a loss on type A customers. The private supplier therefore does not wish to serve the high-cost customers but concentrates on the low-cost customers. This is related to the concern in the health literature about the effects of competition on high-cost customers (e.g. Newhouse (1996) and Ellis (1998)). However, in our model dumping by the private supplier does not harm type A customers as they are served by the public supplier and receive first best quality there.

Why does not the public supplier capture type B customers? To attract type B customers the public supplier has to offer them higher quality. But the regulator has set the public fee so low that it does not cover the cost of increased quality. The public supplier would make a loss on type B and accordingly optimally specializes in type A. Note that the first best quality for the high-cost type A is lower than the first best quality for type B. The public fee is just sufficient to cover the cost of first best quality for type A.

The assumption $1 - c_B < -\ln(1 - c_A + c_B)$ requires that the cost difference is sufficiently large. The assumption does not hold for $c_A = c_B$ as the customer types have to be sufficiently different in order to get separation in equilibrium. In the remainder of the paper we assume that the assumption $1 - c_B < -\ln(1 - c_A + c_B)$ holds.

The fee level $\widehat{f_1} = (1 - c_A)$ leaves the public supplier with zero profits while the private supplier is able to obtain positive profits by its service and price differentiation. This is stated in the following Corollary.

Corollary 4 *The profits in equilibrium are given by*
(i) $\widehat{\Pi}_1 = 0$
(ii) $\widehat{\Pi}_2 = \gamma^B [-\ln(1 - c_A + c_B) - (1 - c_B)] > 0.$

Proof. Straightforward using Proposition 4.

The higher is the cost difference, $(c_A - c_B)$, the higher are the private supplier's profits and the price it charges for type B customers. The intuition runs as follows. A larger cost difference makes the public supplier's offers to type B customers rather unattractive as it cannot charge a higher fee level. The private supplier can exploit this by offering high prices and more attractive service levels which generates profits.

Under privatization the specialization equilibrium exists only if the profits of the two suppliers are equal. In the mixed structure we do not have the same requirement because the regulator sets so low fee for the public supplier that they cannot capture type B who requires a high-value service.

6 Optimal institution

In Sections 4 and 5 we have shown that the first best service level emerges under both privatization and the mixed institution. However, the institutions differ in the fee levels and – as we will illustrate – for some parameter values the mixed institution can protect the customers from excessive prices.

Let us first examine the fees for the ordinary type A. In the mixed institution the public supplier 1 charges a fee $\widehat{f_1} = (1 - c_A)$ for type A.

Under privatization the lowerbound for the price type A pays is $(1-c_A)$ whereas in the equilibrium selected by Payoff dominance it is equal to A's valuation for the service, $\overline{p}_1 = \ln\left(\frac{1}{c_A}\right)$, when γ^A is low or somewhat lower than A's valuation, $\overline{p}_1 = (1-c_A) + \frac{\gamma^B}{\gamma^A}\left[\ln\left(\frac{1}{c_B}\right) - (1-c_B)\right]$, for high γ^A (provided that $\tilde{\Delta}$ is sufficiently large). Under privatization type A clearly pays a higher fee for the same service level.

Type B is served by a private supplier in both institutions. In the mixed institution the rewarding type pays a price $\widehat{p}_2 = \ln\left(\frac{1}{1-c_A+c_B}\right)$ while under privatization the price is equal to B's valuation for the service, $\overline{p}_2 = \ln\left(\frac{1}{c_B}\right)$, for high γ^A or somewhat lower $\overline{p}_2 = (1-c_B) + \frac{\gamma^A}{\gamma^B}\left[\ln\left(\frac{1}{c_A}\right) - (1-c_A)\right]$ for low γ^A. The price in the mixed institution is clearly lower than the maximal price $\overline{p}_2 = \ln\left(\frac{1}{c_B}\right)$. What requires more analysis is the case where type B receives a positive surplus under privatization. We do that in the following Corollary.

Corollary 5 *Suppose* $\tilde{\Delta} \geq \min\{\ln\left(\frac{1}{c_A}\right) - (1-c_A), \frac{\gamma_B}{\gamma_A}\left(\ln(\frac{1}{c_B}) - (1-c_B)\right)\}$.
(i) $\widehat{f}_1 < \overline{p}_1$
(ii) $\widehat{p}_2 < \overline{p}_2$ *if and only if* $\gamma^A > \widetilde{\gamma}^A$
where $\widetilde{\gamma}^A = \frac{\ln\left(\frac{1}{1-c_A+c_B}\right)-(1-c_B)}{\ln\left(\frac{1}{c_A}\right)-(1-c_A)+\ln\left(\frac{1}{1-c_A+c_B}\right)-(1-c_B)}$ *and* $\frac{\partial\widetilde{\gamma}^A}{\partial(c_A-c_B)} > 0$.

The proof is given in the Appendix. In the mixed structure the regulator sets the public fee so low that it just covers the costs of serving type A. This fee is clearly lower than the fee type A pays under privatization. The low public fee also constrains through competition the fee level the private supplier can charge. Therefore also the private fee paid by type B is relatively low in the mixed institution.

Under privatization there is no anchor for the prices and the prices escalate to the highest possible level. The upperbound for the price is the customers' valuation for the service. But the price is also constrained by the equal profit condition. It is this equal profit condition that can work powerfully when type B has a significant majority. The supplier of the minority type A has to earn a higher price-cost margin than the supplier of the majority type B to make equal profits. Therefore supplier 2's price has to be much lower than B's valuation for the service. From Corollary 5 we know that for $\gamma^A < \widetilde{\gamma}^A$ the equal profit condition constrains the price for type B so much that it is actually lower than the relatively low price in the mixed institution.

Corollary 5 also states that the critical value $\widetilde{\gamma}^A$ is increasing in the cost difference. This is because the price $\widehat{p}_2$ under mixed institution is increasing in the price difference while the price $\overline{p}_2$ under privatization (when type B receives a positive surplus) is decreasing in the cost difference. When the cost difference increases (i.e. c_A increases) the price-cost margin for supplier 1 serving type A is reduced. To keep the profits equal also supplier 2's price-cost margin has to decrease and therefore the price type B pays becomes lower under privatization. Since privatization becomes more favourable to type B, the range of parameter values for which B's price is lower in the mixed institution is reduced, i.e. the critical $\widetilde{\gamma}^A$ increases.

The service levels are first best in both institutions. Therefore analyzing the consumer welfare boils down to comparing the prices. The ordinary customer is better off in the mixed institution. The fee is lower and furthermore it is the government that pays the fee. In the end type A of course bears his share of the tax burden. The cost of this service to him in the mixed structure is $\gamma^A \widehat{f}_1$ which is definitely lower than $\overline{p}_1$ under privatization.

The rewarding customer also pays a lower price in the mixed institution for $\gamma^A > \widetilde{\gamma}^A$ but he has the additional tax burden of $\gamma^A \widehat{f}_1$. While for low γ^A type B is better off under privatization since he pays a lower price and avoids the tax burden of the mixed institution. In general type B can be better or worse off in the mixed institution.[7] The following Proposition gives the welfare effects.

Proposition 5 *A change from the private to the mixed institution*
(i) increases welfare of type A customers,
(ii) decreases welfare of type B customers if $\gamma^A \leq \widetilde{\gamma}^A$ *and*
(ii) increases aggregate consumer welfare if and only if $\gamma^A > \underline{\gamma}^A$ *where*

$$\underline{\gamma}^A = \frac{\left[\ln\left(\frac{1}{1-c_A+c_B}\right)-(1-c_B)\right]}{\left[\ln\left(\frac{1}{1-c_A+c_B}\right)-(1-c_B)\right]+2\left[\ln\left(\frac{1}{c_A}\right)-(1-c_A)\right]} \quad \text{and} \quad \frac{\partial \underline{\gamma}^A}{\partial(c_A-c_B)} > 0.$$

[7] Due to complexity of calculations Proposition 5 gives only a sufficient condition for type B's welfare effect.

When γ^A is large, the aggregate consumer welfare is higher in the mixed institution. Type A's fee is always lower in the mixed institution and additionally also type B pays a lower price in the mixed institution when it is a small minority. Therefore clearly the mixed institution maximizes the consumer surplus when γ^A is large.

When type B is a large majority (γ^A is small) the equal profit requirement determines that B pays a lower price under privatization. Although type A's fee is lower in the mixed institution, their share of the population is so small that the positive effect for type B dominates and surprisingly privatization maximizes consumer surplus for $\gamma^A < \underline{\gamma}^A$.

Proposition 5 also states that the critical $\underline{\gamma}^A$ is increasing in the cost difference. The larger is the cost difference, the smaller is the parameter range where the mixed institution maximizes consumer welfare. We know from Corollary 5 that a higher cost difference makes privatization more favourable for type B. This is because the price in the mixed institution increases while the price under privatization decreases. When examining type A we find that his price decreases in both institutions as a response to a higher cost difference. Under privatization type A pays a price equal to his valuation of the service while in the mixed structure his fee is equal to the costs of the service. When the cost difference increases (i.e. c_A increases), the level of service provided for type A is reduced resulting in lower value and lower costs. Supplier 1's price-cost margin is reduced, which means that the price decreases more under privatization. Accordingly, both type A and B are relatively better off under privatization when the cost difference is higher and $\underline{\gamma}^A$ increases.

When the regulator sets the public fee low in the mixed institution, it also anchors the competing private supplier's fee relatively low. Under competition there is no anchor for the prices and they escalate to the highest level. The mixed institution therefore protects the consumers from excessive pricing – unless type B has a majority. Then the equal profit requirement constrains the price for type B so much that surprisingly privatization maximizes consumer welfare – although a move from the mixed structure to privatization is not a Pareto improvement for all the customers.

Finally, we note that total welfare that includes the producer and consumer surplus is equal in both institutions. Customers receive the same quality level of service and effort costs are the same. Potentially high prices under private competition are simply a redistribution from the customers to the firms.

Excessive pricing under private competition is not only a transfer problem but becomes a total welfare (and social) problem if there are poor or wealth constrained customers. We take this up in the next Section.

7 Wealth constraints

In this Section we discuss the consequences of wealth constraints. Suppose that a fraction α^A of type A customers are poor. Similarly, a fraction α^B of type B customers are poor. Being poor could have two meanings. First, customers cannot pay any positive price for the services. Second, poor customers can pay a smaller price. As an illustration suppose that poor customers cannot pay a higher price than $(1 - c_A)$ if they are type A customers and not more than $\ln\left(\frac{1}{1-c_A+c_B}\right)$ if they are type B customers. Rich customers are not wealth constrained. We focus on the case where $\gamma^A > \widetilde{\gamma}^A$, i.e. the prices for both types are lower in the mixed institution.

7.1 Privatization versus mixed institution

Under full privatization poor customers are not served at all if they cannot pay a positive price. When they can pay at most the smallest possible price, it is straightforward to see that prices under privatization will tend to be higher and at least poor customers of type A are not served.[8]

The mixed institution can alleviate the customers' wealth constraints in two ways. Obviously, everybody can obtain the services of the public supplier at no cost. The second effect is more subtle. By fixing the fee at $\widehat{f}_1 = (1 - c_A)$, the price of the private supplier is also kept down. This allows

[8] At the price $(1 - c_A)$ for type A customers, private suppliers would make zero profits. Although poor customers will drop out of the market when prices are raised, the profits of private suppliers will become positive. Payoff dominance implies that prices will be at the highest possible level. Hence, poor type A customers are not served. Since $(1 - c_B)$ is smaller than $\ln\left(\frac{1}{1-c_A+c_B}\right)$ it depends on the parameter values and in particular on the share of poor type B customers whether these customers will be served or not. A higher share of poor type B customers makes it more attractive to set the highest price at $\ln\left(\frac{1}{1-c_A+c_B}\right)$.

type B customers who can afford $\widehat{p}_2 = \ln\left(\frac{1}{1-c_A+c_B}\right)$ but not the higher prices under privatization to benefit from the private supplier which may not be possible when both suppliers are private.

However, the mixed institution faces problems on its own as the public supplier may have an incentive to lower the quality of its services when very poor customers are present. The quality problem is illustrated by the following proposition.

Proposition 6 *Suppose that the regulator chooses the fee level $\widehat{f}_1 = (1 - c_A)$ and that there are poor type A customers who cannot pay any positive price. Then the equilibrium of Proposition 4 does not exist anymore as the public supplier would deviate to $e_1^A = e_1^B = 0$.*

Proof. Suppose that the private supplier selects $\widehat{e}_2^A$, $\widehat{e}_2^B$ and $\widehat{p}_2$. If the public supplier chooses $\widehat{e}_1^A = \frac{1-c_A}{c_A}$ and $\widehat{e}_1^B = \frac{1-c_A}{c_B}$ its profits will be zero. By choosing $e_1^A = e_1^B = 0$ the profits will be

$$\Pi_1 = f_1\left(\gamma^A\alpha^A + \gamma^B\alpha^B\right) > 0$$

as poor customers cannot switch to the private supplier. Hence the equilibrium of Proposition 4 does not exist anymore. Q.E.D.

In the equilibrium of the mixed institution in Section 5 the public supplier makes zero profits. Now the public supplier can profit from the poor customers who have no choice. The public supplier can reduce the quality of the service to zero and earn the fixed fee paid for the poor customers. The important insight of Proposition 6 is the existence of a *service trap* if the government sets the remuneration for serving customers at the level $(1 - c_A)$. The public supplier offers minimal service levels in such cases. Note that even an arbitrarily small share of poor customers destroys the existence as the public supplier is always better off to switch to $e_1^A = e_1^B = 0$. [9]

The government might try to overcome the service trap by raising the fee levels. However, this is only works if the share of poor customers is small. The problem is that higher fee levels raise not only the incentives of the supplier to provide better services to attract rich customers, but also the gains from

[9] Service trap exists even when the poor customers have a choice between two public suppliers as long as a fraction of customers are immobile (e.g. for geographical reasons) and must go to one supplier.

serving the poor customers with the lowest quality level. The latter effect dominates if the share of poor customer is sufficiently high. Moreover, by raising the fee level, the regulator triggers higher prices for type B customers which causes poor customers of those type to switch to the public supplier who offers lower services. Finally, rich customers at the private supplier will face higher prices.

To sum up, in contrast to full privatization the mixed institution can accommodate wealth constraints as long as the poor customers can pay minimal prices or the share of very poor customers is sufficiently small. If wealth constraints are tight and widespread, however, the regulator faces a severe service trap problem.

7.2 Vouchers

As discussed above wealth constraints may introduce undesirable features to the mixed institution. In order to preserve the merits of the mixed institution, the poor customers can be offered service vouchers which enable them to pay a certain amount of money to the service provider and thus the wealth constraint would be eliminated. In particular, the poor students could be offered the voucher with value $\widehat{p}_2 = \ln\left(\frac{1}{1-c_A+c_B}\right)$ in order to be able to choose between the public and the private supplier. It is obvious that the voucher solution dominates the mixed solution under wealth constraints in terms of welfare as long as tax distortions are sufficiently small.

8 Conclusions

In this paper we have shown that a mix of a public and a private supplier can be superior to full privatization. In the mixed institution government sets the public fee so low that it also anchors the private fee relatively low. While under privatization the prices escalate to the highest level.

In this paper we did not analyse an institution in which both suppliers are public. A complete public supply cannot improve on the provision of services because the service level already reaches first best. Additional tax distortions would occur and therefore we do not expect that refraining from privatization is socially desirable. Moreover, in the mixed institution, the regulator only needs to determine one fee level and can leave the remaining pricing to the private supplier. A complete public organization of service delivery would require that different public suppliers obtain different fee levels in order to achieve service differentiation comparable to that under the mixed institution. This would increase the complexity of the regulatory task.

We have assumed that the values and the costs of different types are separable. In reality there are externalities between students learning from each other (and perhaps being disturbed by other students). Teacher's effort costs for different types of students are also not independent as the teacher is allocating a limited time between different tasks. Relaxing separability assumptions remains as an open question.

Appendix

Proof of Proposition 2

Suppose supplier 1 specializes in type A customers and supplier 2 in type B customers.

(i) In equilibrium it must hold that

$$v_1^A - p_1 = v_2^A - p_2 = d$$

$$v_1^B - p_1 = v_2^B - p_2 = f$$

Otherwise suppose $v_1^A - p_1 > v_2^A - p_2$. Then supplier 1 could raise profits by raising p_1.

(ii) Choices of e_1^A and e_2^B.

Supplier 1

$$\underset{\{p_1, e_1^A\}}{Max} \{p_1 - c_A e_1^A\}$$

$$s.t. \ln\left(e_1^A + 1\right) - p_1 = d$$

Substituting in the constraint we get

$$\underset{\{e_1^A\}}{Max} \left\{\ln\left(e_1^A + 1\right) - d - c_A e_1^A\right\}$$

which implies

$$e_1^A = \frac{1 - c_A}{c_A}$$

and similarly

$$e_2^B = \frac{1 - c_B}{c_B}$$

(iii) The suppliers' profits must be equal in equilibrium. Suppose $\Pi_1 < \Pi_2$. Then if supplier 1 sets $e_1^B = e_2^B$, $p_1 = p_2 - \varepsilon$ and $e_1^A = 0$ it will capture all type B customers and make a higher profit (arbitrarily close to Π_2).

$$\Pi_1 = \gamma^A \left[p_1 - (1 - c_A)\right] = \Pi_2 = \gamma^B \left[p_2 - (1 - c_B)\right]$$

From the equal profit condition we can solve for p_1.

$$p_1 = \frac{\gamma^B \left[p_2 - (1 - c_B)\right] + \gamma^A (1 - c_A)}{\gamma^A}$$

Note that we can only solve for p_1 as a function of p_2.

(iv) Then we solve for the remaining efforts:

$$v_1^B = v_2^B - p_2 + p_1 = \ln\left(\frac{1}{c_B}\right) - \left[\frac{\gamma^B}{\gamma^A}(1 - c_B) + p_2\left(1 - \frac{\gamma^B}{\gamma^A}\right) - (1 - c_A)\right]$$

Define $\Theta \equiv \frac{\gamma^B}{\gamma^A}(1 - c_B) + p_2\left(1 - \frac{\gamma^B}{\gamma^A}\right) - (1 - c_A)$

$$\begin{aligned} v_2^A &= v_1^A - p_1 + p_2 \\ &= \ln\left(\frac{1}{c_A}\right) + \Theta \end{aligned}$$

Notice that $\Theta = p_2 - p_1$. Q.E.D.

Proof of Proposition 3

Naturally the price has a lowerbound and an upperbound as the price has to cover the costs ($p_1 \geq 1 - c_A$ and $p_2 \geq 1 - c_B$) and the price cannot be higher than the customer's valuation for the service ($p_1 \leq \ln\left(\frac{1}{c_A}\right)$ and $p_2 \leq \ln\left(\frac{1}{c_B}\right)$). We next explore the other bounds on equilibrium prices.

Suppose in the specialization equilibrium supplier 1 is charging $p_1 = 1 - c_A + \Delta$ where $0 \leq \Delta \leq \ln\left(\frac{1}{c_A}\right) - (1 - c_A)$. From equal profit condition $p_2 = 1 - c_B + \frac{\gamma_A}{\gamma_B}\Delta$. Is it profitable for supplier 1 to deviate and capture both types?

$$\underset{\left\{p_1, e_1^A, e_1^B\right\}}{Max} \; p_1 - \gamma^A c_A e_1^A - \gamma^B c_B e_1^B \tag{14}$$

$$s.t. \; v_1^A - p_1 \geq v_2^A - p_2$$

$$v_1^B - p_1 \geq v_2^B - p_2$$

Substituting in the constraints:

$$\ln\left(e_1^A+1\right)- \ p_1 \geq \ln\left(\frac{1}{c_A}\right)+\Theta-(1-c_B)-\frac{\gamma_A}{\gamma_B}\Delta \tag{15}$$

$$\ln\left(e_1^B+1\right)- \ p_1 \geq \ln\left(\frac{1}{c_B}\right)-(1-c_B)-\frac{\gamma_A}{\gamma_B}\Delta \tag{16}$$

Subtracting (16) from (15) we obtain:

$$\ln\left(e_1^A+1\right)-\ln\left(e_1^B+1\right)=\ln\left(\frac{c_B}{c_A}\right)+\Theta$$

$$\Leftrightarrow \frac{\left(e_1^A+1\right)c_A}{\left(e_1^B+1\right)c_B}=e^{\Theta}$$

$$\Leftrightarrow \left(e_1^A+1\right)=\frac{c_B}{c_A}e^{\Theta}\left(e_1^B+1\right) \tag{17}$$

And from the constraint (16) we have:

$$p_1=\ln\left(e_1^B+1\right)-\ln\left(\frac{1}{c_B}\right)+(1-c_B)+\frac{\gamma_A}{\gamma_B}\Delta \tag{18}$$

Substitute (17) and (18) in (14) :

$$\underset{\left\{e_1^B\right\}}{Max} \ \ln\left(e_1^B+1\right)-\ln\left(\frac{1}{c_B}\right)+(1-c_B)+\frac{\gamma_A}{\gamma_B}\Delta-\gamma^A c_B e^{\Theta}\left(e_1^B+1\right)+\gamma^A c_A-\gamma^B c_B e_1^B$$

The first-order condition is:

$$\frac{1}{e_1^B+1}-\gamma^A c_B e^{\Theta}-\gamma^B c_B=0$$

$$\Leftrightarrow e_1^B+1=\frac{1}{c_B\left(\gamma^B+\gamma^A e^{\Theta}\right)} \tag{19}$$

To solve for p_1 substitute (19) in (18).

$$\begin{aligned} p_1 &= \ln\left(\frac{1}{c_B\left(\gamma^B+\gamma^A e^{\Theta}\right)}\right)-\ln\left(\frac{1}{c_B}\right)+(1-c_B)+\frac{\gamma_A}{\gamma_B}\Delta \\ &= \ln\left(\frac{1}{\gamma^B+\gamma^A e^{\Theta}}\right)+(1-c_B)+\frac{\gamma_A}{\gamma_B}\Delta \end{aligned} \tag{20}$$

We can solve for e_1^A by substituting (19) in (17).

$$\left(e_1^A + 1\right) = \frac{e^\Theta}{c_A\left(\gamma^B + \gamma^A e^\Theta\right)}$$

The deviation profits are then:

$$\begin{aligned}
\Pi &= \ln\left(\frac{1}{\gamma^B + \gamma^A e^\Theta}\right) + (1 - c_B) + \frac{\gamma_A}{\gamma_B}\Delta - \gamma^A c_A\left(\frac{e^\Theta}{c_A\left(\gamma^B + \gamma^A e^\Theta\right)} - 1\right) \\
&\quad -\gamma^B c_B\left(\frac{1}{c_B\left(\gamma^B + \gamma^A e^\Theta\right)} - 1\right) \\
&= \ln\left(\frac{1}{\gamma^B + \gamma^A e^\Theta}\right) + (1 - c_B) + \frac{\gamma_A}{\gamma_B}\Delta + \gamma^A c_A + \gamma^B c_B - \frac{\gamma^B + \gamma^A e^\Theta}{\gamma^B + \gamma^A e^\Theta} && (21) \\
&= \ln\left(\frac{1}{\gamma^B + \gamma^A e^\Theta}\right) - c_B + \gamma^A c_A + \gamma^B c_B + \frac{\gamma_A}{\gamma_B}\Delta && (22) \\
&= \ln\left(\frac{1}{1 - \gamma^A\left(1 - e^\Theta\right)}\right) + \gamma^A\left(c_A - c_B\right) + \frac{\gamma_A}{\gamma_B}\Delta && (23)
\end{aligned}$$

It is instructive to consider the case $\Delta = 0$ first. Note that in the lowest price equilibrium $\Theta = c_A - c_B$ and in the specialization equilibrium profits are zero.

$$\Pi(\Theta) = \ln\left(\frac{1}{1 - \gamma^A\left(1 - e^\Theta\right)}\right) + \gamma^A\Theta = -\ln\left[1 - \gamma^A\left(1 - e^\Theta\right)\right] + \gamma^A\Theta$$

$$\Pi(0) = 0$$

$$\begin{aligned}
\Pi'(\Theta) &= -\frac{\gamma^A e^\Theta}{1 - \gamma^A\left(1 - e^\Theta\right)} + \gamma^A \\
&= \gamma^A\left[\frac{-e^\Theta}{1 - \gamma^A\left(1 - e^\Theta\right)} + 1\right] \\
&= \gamma^A\left[\frac{-e^\Theta + 1 - \gamma^A\left(1 - e^\Theta\right)}{1 - \gamma^A\left(1 - e^\Theta\right)}\right] \\
&= \gamma^A\left(1 - e^\Theta\right)\left[\frac{1 - \gamma^A}{1 - \gamma^A\left(1 - e^\Theta\right)}\right] < 0 && (24)
\end{aligned}$$

Equation (24) is negative since $e^{\Theta} > 1$ for $\Theta > 0$. The deviation from the specialization equilibrium would give supplier 2 negative profits. Therefore specialization is an equilibrium.

We next look at the case $\Delta > 0$. The profits in the specialization equilibrium are $\gamma^A \Delta$. Therefore deviation does not pay if and only if

$$\ln\left(\frac{1}{1-\gamma^A\left(1-e^{\Theta}\right)}\right) + \gamma^A\left(c_A - c_B\right) + \frac{\gamma_A}{\gamma_B}\Delta < \gamma^A \Delta$$

$$\ln\left(\frac{1}{1-\gamma^A\left(1-e^{\Theta}\right)}\right) + \gamma^A\left(c_A - c_B\right) + \frac{\left(\gamma_A\right)^2}{1-\gamma_A}\Delta < 0$$

We have shown that this holds for $\Delta = 0$. We know that $\frac{\partial \Theta}{\partial \Delta} > 0$ if type A is majority and therefore the first term is decreasing in Δ while the last term is increasing in Δ. When B is majority both terms are increasing in Δ. Let $\tilde{\Delta}$ be the minimal solution of $ln(\frac{1}{1-\gamma^A(1-e^{\theta})}) + \gamma^A(c_A - c_B) + \frac{(\gamma_A)^2}{1-\gamma_A}\Delta = 0$ where we allow that the solution may become infinite when type A is a majority.

When we repeat the above analysis for supplier 2 we find it is obvious that supplier 2 charges the same deviation price and chooses the same deviation efforts as supplier 1. Therefore also the deviation profits are equal. Since the profits in the specialization equilibrium are equal, neither supplier has an incentive to deviate and therefore specialization equilibrium exists. Q.E.D

Proof of Corollary 3

(*i*) Suppose $\tilde{\Delta} \geq \min\{\ln(\frac{1}{c_A} - (1-c_A), \frac{\gamma_B}{\gamma_A}\left(\ln(\frac{1}{c_B}) - (1-c_B)\right)\}$. Then we know that the upper limits p_1^h and p_2^h are bounded only by the customers' valuation of the service. We first examine whether the maximal prices, $\overline{p}_1 = \ln\left(\frac{1}{c_A}\right)$ and $\overline{p}_2 = \ln\left(\frac{1}{c_B}\right)$, satisfy the equal profit condition:

$$\Pi_1 = \gamma^A\left(\ln\left(\frac{1}{c_A}\right) - (1-c_A)\right) = \gamma^B\left(\ln\left(\frac{1}{c_B}\right) - (1-c_B)\right) = \Pi_2 \quad (25)$$

This equality holds for a unique value of γ^A which is denoted by $\overline{\gamma}^A$:

$$\overline{\gamma}^A = \frac{\left(\ln\left(\frac{1}{c_B}\right) - (1-c_B)\right)}{\left(\ln\left(\frac{1}{c_A}\right) - (1-c_A) + \ln\left(\frac{1}{c_B}\right) - (1-c_B)\right)} > \frac{1}{2} \quad (26)$$

If $\gamma^A = \overline{\gamma}^A$, the suppliers's profits are equal at the maximal prices and the highest prices selected are $\overline{p}_1 = \ln\left(\frac{1}{c_A}\right)$ and $\overline{p}_2 = \ln\left(\frac{1}{c_B}\right)$.

If $\gamma^A > \overline{\gamma}^A$, then the right-hand-side of equation (25) is smaller than the left-hand-side. To equalize the profits $\overline{p}_1$ has to be lower than $\ln\left(\frac{1}{c_A}\right)$. The suppliers have equal profits when $\overline{p}_1 = (1 - c_A) + \frac{\gamma^B}{\gamma^A}\left[\ln\left(\frac{1}{c_B}\right) - (1 - c_B)\right]$ and $\overline{p}_2 = \ln\left(\frac{1}{c_B}\right)$.

If $\gamma^A < \overline{\gamma}^A$, then the right-hand-side of equation (25) is greater than the left-hand-side. Now it is $\overline{p}_2$ that has to be lowered from its maximum value of $\ln\left(\frac{1}{c_B}\right)$ to $(1 - c_B) + \frac{\gamma^A}{\gamma^B}\left[\ln\left(\frac{1}{c_A}\right) - (1 - c_A)\right]$ to equalize profits while $\overline{p}_1 = \ln\left(\frac{1}{c_A}\right)$.

(ii) Denote $c_A = c_B + \delta$.

Then

$$\overline{\gamma}^A = \frac{\left(\ln\left(\frac{1}{c_B}\right) - (1 - c_B)\right)}{\left(\ln\left(\frac{1}{c_B+\delta}\right) - (1 - c_B - \delta) + \ln\left(\frac{1}{c_B}\right) - (1 - c_B)\right)}.$$

Differentiating $\overline{\gamma}^A$ with respect to the cost difference, δ, we obtain:

$$\frac{\partial\overline{\gamma}^A}{\partial\delta} = \frac{\frac{1-c_B-\delta}{c_B+\delta}\left(\ln\left(\frac{1}{c_B}\right) - (1 - c_B)\right)}{\left(\ln\left(\frac{1}{c_B+\delta}\right) - (1 - c_B - \delta) + \ln\left(\frac{1}{c_B}\right) - (1 - c_B)\right)^2} > 0.$$

Q.E.D.

Proof of Proposition 4

(i) Let us first describe profits in equilibrium. The public supplier's profits are given by:

$$\begin{aligned}\Pi_1 &= f_1\left(\gamma^A s_1^A + \gamma^B s_1^B\right) - c_A\gamma^A e_1^A s_1^A - c_B\gamma^B e_1^B s_1^B \\ &= \gamma^A s_1^A\left(f_1 - c_A e_1^A\right) + \gamma^B s_1^B\left(f_1 - c_B e_1^B\right)\end{aligned}$$

In equilibrium the public supplier's profits are zero since $\widehat{f}_1 = 1 - c_A = c_A \widehat{e}_1^A$ and $\widehat{f}_1 = c_B \widehat{e}_1^B$.

The private supplier's profits are:

$$\Pi_2 = \gamma^A s_2^A \left(p_2 - c_A e_2^A\right) + \gamma^B s_2^B \left(p_2 - c_B e_2^B\right)$$

In equilibrium private supplier's profits are given by

$$\widehat{\Pi}_2 = \gamma^B \left(\ln \left(\frac{1}{1 - c_A + c_B} \right) - (1 - c_B) \right) \tag{27}$$

which is positive by assumption.

(*ii*) We first show that all type A customers go to the public supplier and all type B customers go to the private supplier *given* the service levels offered in equilibrium. Assuming an effort level that is larger than $\widehat{e}_1^A$ by an arbitrarily small ε the public supplier offers better value for type A.

$$\widehat{v}_1^A = \ln \left(\frac{1}{c_A} \right) \geq \widehat{v}_2^A - \widehat{p}_2 = \ln \left(\frac{1}{c_A (1 - c_A + c_B)} \right) - \ln \left(\frac{1}{1 - c_A + c_B} \right) = \ln \left(\frac{1}{c_A} \right)$$

All type B customers go to the private supplier (we use a flexible tie-breaking rule)

$$\widehat{v}_1^B = \ln \left(\frac{1 - c_A + c_B}{c_B} \right) \leq \widehat{v}_2^B - \widehat{p}_2 = \ln \left(\frac{1}{c_B} \right) - \ln \left(\frac{1}{1 - c_A + c_B} \right) = \ln \left(\frac{1 - c_A + c_B}{c_B} \right)$$

and therefore $\widehat{s}_1^A = 1$, $\widehat{s}_2^A = 0$, $\widehat{s}_1^B = 0$ and $\widehat{s}_2^B = 1$.

(*iii*) We next show that the public supplier cannot improve its profits given the efforts and the price chosen by the private supplier.

Raising e_1^A is not profitable since $\widehat{s}_1^A = 1$ and the public supplier already receives all type A customers. Lowering e_1^A is not optimal either because $\widehat{v}_1^A = \widehat{v}_2^A - \widehat{p}_2$ and therefore supplier 1 would lose all the customers.

Because $\widehat{f}_1 = c_B \widehat{e}_1^B$, raising e_1^B to attract B type customers would generate negative profits. Decreasing e_1^B is not worthwhile either because $\widehat{s}_1^B$ is already zero. Note that $\widehat{v}_1^B = \widehat{v}_2^B - \widehat{p}_2$.

(*iv*) Given the service level offered by the public supplier, we derive the best response of the private supplier in terms of efforts and price if he wants to attract only customers of type B.

Accordingly quality levels and price must fulfill

$$v_2^B - p_2 = \ln\left(e_2^B + 1\right) - p_2 \geq v_1^B = \ln\left(\frac{1 - c_A + c_B}{c_B}\right)$$

Therefore supplier 2's profit maximization problem is:

$$\underset{\left\{p_2, e_2^B\right\}}{Max}\ \Pi_2 = \gamma^B\left(p_2 - c_B e_2^B\right)$$

$$\text{s.t. } \ln\left(e_2^B + 1\right) - p_2 = \ln\left(\frac{1 - c_A + c_B}{c_B}\right)$$

Inserting p_2 from the constraint yields:

$$\underset{\left\{e_2^B\right\}}{Max}\Pi_2 = \gamma^B\left(\ln\left(e_2^B + 1\right) - \ln\left(\frac{1 - c_A + c_B}{c_B}\right) - c_B e_2^B\right)$$

The first-order condition is

$$\gamma^B\left(\frac{1}{e_2^B + 1} - c_B\right) = 0 \tag{28}$$

and accordingly

$$\widehat{e}_2^B = \frac{1 - c_B}{c_B}. \tag{29}$$

Next we have to check $\widehat{e}_2^A$. Decreasing e_2^A would not affect profits as $\widehat{s}_2^A = 0$. Increasing e_2^A by a small margin would generate $s_2^A = 1$ and additional profits:

$$\Delta\Pi = \gamma^A\left(\ln\left(\frac{1}{1 - c_A + c_B}\right) - \frac{1}{1 - c_A + c_B} + c_A\right) \tag{30}$$

which is negative since $\ln(x) < x - 1$ for any $x \neq 1$. (Remember that $c_A < 1$.)

(v) Given the service level offered by the public supplier, we derive the best response of the private supplier if he wants to attract customers of both types.

The problem in this case is given by:

$$\underset{\left\{p_2, e_2^A, e_2^B\right\}}{Max}\ \Pi_2 = \gamma^A\left(p_2 - c_A e_2^A\right) + \gamma^B\left(p_2 - c_B e_2^B\right)$$

$$\text{s.t. } \ln\left(e_2^B + 1\right) - p_2 = \ln\left(\frac{1 - c_A + c_B}{c_B}\right)$$

$$\ln\left(e_2^A + 1\right) - p_2 = \ln\left(\frac{1}{c_A}\right)$$

Satisfying both constraints yields:

$$\left(e_2^B + 1\right)\frac{c_B}{1 - c_A + c_B} = \left(e_2^A + 1\right)c_A \tag{31}$$

$$\left(e_2^B + 1\right) = \left(e_2^A + 1\right)\frac{c_A\left(1 - c_A + c_B\right)}{c_B} \tag{32}$$

Moreover, p_2 is equal to

$$p_2 = \ln\left(e_2^A + 1\right) - \ln\left(\frac{1}{c_A}\right) \tag{33}$$

Inserting both relationships (32) and (33) into Π_2 we obtain

$$\Pi_2 = \ln\left(e_2^A + 1\right) - \ln\left(\frac{1}{c_A}\right) - \gamma^A c_A e_2^A - \gamma^B\left(e_2^A + 1\right)c_A\left(1 - c_A + c_B\right) - \gamma^B c_B \tag{34}$$

The first-order condition amounts to

$$\frac{1}{e_2^A + 1} - \gamma^A c_A - \gamma^B c_A\left(1 - c_A + c_B\right) = 0 \tag{35}$$

From (35) we can solve for the optimal effort for type A:

$$e_2^A + 1 = \frac{1}{c_A\left(1 - \gamma^B c_A + \gamma^B c_B\right)} \tag{36}$$

Inserting (36) in (32) and (33) we find the optimal price and effort for type B:

$$p_2 = \ln\left(\frac{1}{1 - \gamma^B c_A + \gamma^B c_B}\right) \tag{37}$$

$$e_2^B + 1 = \frac{1 - c_A + c_B}{c_B\left(1 - \gamma^B c_A + \gamma^B c_B\right)} \tag{38}$$

Profits are given by:

$$\Pi_2 = \gamma^A \left(\ln\left(\frac{1}{1-\gamma^B c_A + \gamma^B c_B} \right) - \frac{1}{1-\gamma^B c_A + \gamma^B c_B} + c_A \right) \qquad (39)$$
$$+\gamma^B \left(\ln\left(\frac{1}{1-\gamma^B c_A + \gamma^B c_B} \right) - \frac{1-c_A+c_B}{1-\gamma^B c_A + \gamma^B c_B} + c_B \right)$$

(vi) Finally, we prove that attracting both types of customers is less profitable for the private supplier than serving only type B customers.

The profits from type A when attracting both types (the first term in (39)) are negative because $\ln(x) + 1 < x$ for any $x \neq 1$. Furthermore, profits from type B (the second term in (39)) are lower than the private supplier's profits when it specializes in type B. Indeed, the comparison amounts to:

$$\gamma^B \left(\ln\left(\frac{1}{1-\gamma^B c_A + \gamma^B c_B} \right) - \frac{1-c_A+c_B}{1-\gamma^B c_A + \gamma^B c_B} + c_B \right) \qquad (40)$$
$$< \gamma^B \left(\ln\left(\frac{1}{1-c_A+c_B} \right) - (1-c_B) \right) = \widehat{\Pi}_2$$

which is equivalent to

$$\ln\left(\frac{y}{x}\right) + 1 < \frac{y}{x}$$

where $y = 1 - c_A + c_B$ and $x = 1 - \gamma^B c_A + \gamma^B c_B$. Again the property holds as long as $y \neq x$. Therefore it is indeed optimal for supplier 2 to specialize in type B. Q.E.D.

Proof of Corollary 5 *(i)* It is obvious that $\widehat{f}_1 = (1-c_A) < \overline{p}_1 = \ln\left(\frac{1}{c_A}\right)$ for $\gamma^A \leq \overline{\gamma}^A$ and
$\widehat{f}_1 = (1-c_A) < \overline{p}_1 = (1-c_A) + \frac{\gamma^B}{\gamma^A}\left[\ln\left(\frac{1}{c_B}\right) - (1-c_B)\right]$ for $\gamma^A > \overline{\gamma}^A$.

(ii) For $\gamma^A \geq \overline{\gamma}^A$ clearly $\overline{p}_2 = \ln\left(\frac{1}{c_B}\right) > \widehat{p}_2 = \ln\left(\frac{1}{1-c_A+c_B}\right)$.

In the case when $\gamma^A < \overline{\gamma}^A$ the price under privatization is higher if and only if

$$(1-c_B) + \frac{\gamma^A}{\gamma^B}\left[\ln\left(\frac{1}{c_A}\right) - (1-c_A)\right] > \ln\left(\frac{1}{1-c_A+c_B}\right)$$

$$\gamma^A \left[\ln\left(\frac{1}{c_A}\right) - (1-c_A) \right] > \gamma^B \left[\ln\left(\frac{1}{1-c_A+c_B}\right) - (1-c_B) \right] \qquad (41)$$

$$\gamma^A > \frac{\ln\left(\frac{1}{1-c_A+c_B}\right) - (1-c_B)}{\ln\left(\frac{1}{c_A}\right) - (1-c_A) + \ln\left(\frac{1}{1-c_A+c_B}\right) - (1-c_B)} = \widetilde{\gamma}^A$$

By the assumption in Proposition 4 $\widetilde{\gamma}^A > 0$. Furthermore, it is straightforward to show that $\widetilde{\gamma}^A < \overline{\gamma}^A$.[10] Putting these together we have found three cases:

(a) The relevant prices are $\overline{p}_2 = \ln\left(\frac{1}{c_B}\right) > \widehat{p}_2 = \ln\left(\frac{1}{1-c_A+c_B}\right)$ if and only if $\gamma^A \geq \overline{\gamma}^A$.

(b) The relevant prices are $\overline{p}_2 = (1-c_B) + \frac{\gamma^A}{\gamma^B}\left[\ln\left(\frac{1}{c_A}\right) - (1-c_A)\right] > \widehat{p}_2 = \ln\left(\frac{1}{1-c_A+c_B}\right)$ if and only if $\widetilde{\gamma}^A < \gamma^A < \overline{\gamma}^A$.

(c) The relevant prices are $\overline{p}_2 = (1-c_B) + \frac{\gamma^A}{\gamma^B}\left[\ln\left(\frac{1}{c_A}\right) - (1-c_A)\right] < \widehat{p}_2 = \ln\left(\frac{1}{1-c_A+c_B}\right)$ if and only if $\gamma^A < \widetilde{\gamma}^A$.

Therefore $\widehat{p}_2 < \overline{p}_2$ if and only if $\gamma^A > \widetilde{\gamma}^A$.

The comparative statics with respect to the cost difference $\delta = c_A - c_B$ is the following. Substituting in δ we have $\widehat{p}_2 = \ln\left(\frac{1}{1-\delta}\right)$ and $\overline{p}_2 = (1-c_B) + \frac{\gamma^A}{\gamma^B}\left[\ln\left(\frac{1}{c_B+\delta}\right) - (1-c_B-\delta)\right]$.

$$\frac{\partial \widehat{p}_2}{\partial \delta} = \frac{1}{1-\delta} > 0$$

$$\frac{\partial \overline{p}_2}{\partial \delta} = -\frac{\gamma^A}{\gamma^B}\left[\left(\frac{1}{c_B+\delta}\right) - 1\right] < 0$$

Clearly $\frac{\partial \widetilde{\gamma}^A}{\partial \delta} > 0$.

Q.E.D.

[10] This follows from $\frac{b}{a+b} > \frac{b'}{a+b'}$ for $a, b > 0$ and $b > b'$.

Proof of Proposition 5

(i) Obvious from Corollary 4 and taking into account that type A only pays $\gamma^A \widehat{f}_1$ in the mixed institution.

(ii) For $\gamma^A \leq \widetilde{\gamma}^A$ type B pays a lower price under privatization and further avoids the tax burden of the mixed institution.

(iii) Aggregate consumer welfare is clearly higher in the mixed institution if $\gamma^A > \widetilde{\gamma}^A$ since both prices are lower.

For $\gamma^A < \widetilde{\gamma}^A$ type B pays a lower price under privatization and there is a tradeoff. From Corollary 2 the relevant prices under privatization are (since $\widetilde{\gamma}^A < \overline{\gamma}^A$):

$$\overline{p}_1 = \ln\left(\frac{1}{c_A}\right)$$

$$\overline{p}_2 = (1 - c_B) + \frac{\gamma^A}{\gamma^B}\left[\ln\left(\frac{1}{c_A}\right) - (1 - c_A)\right]$$

Therefore the aggregate consumer welfare is higher in the mixed institution for the case $\gamma^A < \widetilde{\gamma}^A$ if and only if:

$$\gamma^A (1 - c_A) + \gamma^B \ln\left(\frac{1}{1 - c_A + c_B}\right) < \gamma^A \ln\left(\frac{1}{c_A}\right) + \gamma^B (1 - c_B) + \gamma^A \left[\ln\left(\frac{1}{c_A}\right) - (1 - c_A)\right]$$

$$\gamma^B \left[\ln\left(\frac{1}{1 - c_A + c_B}\right) - (1 - c_B)\right] < 2\gamma^A \left[\ln\left(\frac{1}{c_A}\right) - (1 - c_A)\right]$$

$$\gamma^A > \frac{\left[\ln\left(\frac{1}{1-c_A+c_B}\right) - (1 - c_B)\right]}{\left[\ln\left(\frac{1}{1-c_A+c_B}\right) - (1 - c_B)\right] + 2\left[\ln\left(\frac{1}{c_A}\right) - (1 - c_A)\right]} = \underline{\gamma}^A$$

It is easy to see that $\underline{\gamma}^A < \widetilde{\gamma}^A$. Therefore the aggregate consumer welfare is higher in the mixed institution if and only if $\gamma^A > \underline{\gamma}^A$.

$$\underline{\gamma}^A = \frac{\left[\ln\left(\frac{1}{1-c_A+c_B}\right) - (1 - c_B)\right]}{\left[\ln\left(\frac{1}{1-c_A+c_B}\right) - (1 - c_B)\right] + 2\left[\ln\left(\frac{1}{c_A}\right) - (1 - c_A)\right]}$$

For comparative statics with respect to the cost difference we substitute in $\delta = c_A - c_B$.

$$\underline{\gamma}^A = \frac{\left[\ln\left(\frac{1}{1-\delta}\right) - (1 - c_B)\right]}{\left[\ln\left(\frac{1}{1-\delta}\right) - (1 - c_B)\right] + 2\left[\ln\left(\frac{1}{c_B+\delta}\right) - (1 - c_B - \delta)\right]}$$

$$\frac{\partial \underline{\gamma}^A}{\partial \delta} = \frac{\frac{2}{1-\delta}\left[\ln\left(\frac{1}{c_B+\delta}\right) - (1 - c_B - \delta)\right] + \frac{2(1-c_B-\delta)}{c_B+\delta}\left[\ln\left(\frac{1}{1-\delta}\right) - (1 - c_B)\right]}{\left\{\left[\ln\left(\frac{1}{1-\delta}\right) - (1 - c_B)\right] + 2\left[\ln\left(\frac{1}{c_B+\delta}\right) - (1 - c_B - \delta)\right]\right\}^2} > 0$$

Q.E.D.

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