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The use of regression analysis for resource allocation by central government

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Abstract. The use of multiple regression models to determine weights for the prediction of UK local authority spending targets is criticised. It is argued that predictions based upon such regression weights are not valid estimates of 'need to spend' and in particular that they should not be used as the basis for decisions about 'capping'.

Introduction

In the United Kingdom, under the old rating system, the short-lived poll tax (community charge) system, and the new council tax system of local government finance, a central feature is the estimation of a target spending figure for each local authority (LA). Known as the Standard Spending Assessment (SSA), this is meant to estimate each LA's spending 'need' (DoE, 1990). The central government grant allocation to each LA is then based upon this figure. Successive governments have made extensive use of 'capping' procedures whereby LAs are forced to reduce their budgets if they exceed the SSA by a certain amount. In 1990/91 the exceedance threshold was set at *either* 12.5% above SSA or £75 per head above SSA per head of population. In 1991/92 the second criterion was removed and further criteria introduced based upon percentage change in budget between 1990/91 and 1991/92. In 1990 those LAs which had been capped appealed on the grounds of the unfair content and unfair application of the formulas used to determine what constituted 'excess' expenditure. The LAs lost that appeal.

One argument which was not pursued by the LAs was whether the government's assessment of spending need could be biased or faulty. Clearly, if the SSA itself were questionable so that it could not be considered a proper estimate of an authority's true spending need, then *any* procedure for capping must also be questionable. In the present paper I analyse the methodology used for arriving at the SSA.

Assessing need

Derbyshire (1987) discusses the problems of assessing need for individual LAs. He deals in detail with the case of children under five 'at risk' for social services purposes. He points out that, in general, direct estimates of current need are difficult to obtain and that surrogates, based upon readily available census and other data, will usually be necessary.

He presents an analysis of survey data and shows how population data *measured at the LA level* can be used to predict the proportion of children at risk. In fact, as the response variable to be predicted and the predictor variables in practice are required to be at the LA level, the appropriate analysis will be at that level too.

In the present paper I study the consequences of using such aggregate-level analyses and in particular analyse the procedures which are used currently by central government to establish spending need estimates, in order to illustrate their inadequacies.

The rationale for regression analysis

I shall make reference to a Department of Environment document (DoE, 1989) which gives the basis for the statistical modelling used in calculating SSAs. This document contains a description of the use of regression analysis to determine the prediction formulas to be used.

Types of prediction

First, it is quite clear that the concern of the DoE is with the prediction of need because this is stated in several places. Second, with the exception of the use of some General Household Survey data to formulate indicators of need among the elderly, all the analyses use aggregate LA data, and this typically was the case under the old rating system too. Most importantly, the response variables in these analyses are not direct estimates of need; rather they are measures of actual expenditure or extent of provision used as surrogates for need.

This use of expenditure or provision measures is clearly convenient and relatively inexpensive, but there appears to be no adequate research which compares the results of using such measures as opposed to direct estimates of need. This problem is recognised in the DoE document, but no objective justification is offered for the use of expenditure as response.

Clearly, actual expenditure and provision by LAs is determined by local definitions and perceptions of need, and these in turn depend upon historical contingencies and political views. These in turn will be associated with objective indicators of need as well as the demographic characteristics of the LA. In general, therefore, we should not expect predictors of expenditure also to be adequate predictors of need.

Although the above issue is clearly important, I shall not address it here directly. Rather my concern is with the stability of aggregate-level analyses, and what I have to say about analyses based upon expenditure measures will apply to some extent also to any which might be carried out using direct measures of need.

Using regression prediction

It is worth noting a curious paradox in the use of social indicators in a prediction of expenditure in order to derive a resource allocation formula. As past expenditure is known accurately, why not simply base future resource allocation directly on that expenditure?

This point appears to be recognised by the DoE because in section 16 (iii) and (iv) of the document it is pointed out that both the choice of indicators and the final coefficient values or weights used rely on professional judgment as well as the specific estimates from the regression analysis. Coefficients that are not 'plausible' are modified, although we are not told how and in the DoE (1990) document which gives details of allocations it appears that, on the whole and apart from the occasional negative coefficient, the regression analysis results are used without further adjustment. Nevertheless, it will be the case, in general, that a number of different sets of values can all be considered plausible. The fact that a particular set is plausible does not imply that it is the most appropriate. This point is elaborated below.

The logic of the procedure seems dubious. It would be possible to view the regresion analyses simply as a starting point for a debate about resource allocation formulas, with no special significance attached to the coefficient estimates. Because this is not the case, and the actual coefficients are used in many of the final formulas, the end result will depend strongly on the particular regression equation which happens to have been used. I shall discuss the consequences of using these regression predictions.

The principal measure used by the DoE to judge the usefulness of the regression prediction is the proportion of the total variance in the response variable which is explained (DoE, 1989, paragraph 11). This proportion varies considerably, as high as 0.8 and sometimes much lower. Even where these values are high, however, there is still an important amount of variation unaccounted for; that is, the prediction cannot account for all the variation between authorities. Thus the formulas used, based upon the regression weights, can only approximately predict expenditure, and hence the need for which expenditure is a surrogate. This uncertainty, however, is not embodied in the final predictions in a way which would provide any kind of uncertainty interval to be computed.

Before I turn to the details of the regression analyses, it is worth mentioning related approaches suggested by several authors (Mayston and Smith, 1990; Taylor, 1989). These authors variously propose the use of regression-type models to predict examination performance or to estimate cost and demand functions. The approaches are based upon the use of aggregate LA data and hence suffer from the same technical limitations as the models I now examine in more detail.

The adequacy of regression prediction

There is very little discussion of the adequacy of the regression model in the DoE (1989) document, except in paragraph 24 where the possibility of nonlinear and interactive models is mentioned but dismissed as unimportant. In the document it is claimed that to consider such elaborations would be "more complex and would therefore lead to more complicated needs assessments". It is difficult to understand this statement. To carry out such analyses is no more difficult than carrying out multiple regression analyses of the kind which have already been done and is readily accomplished within standard statistical computer packages. Nor would it complicate the needs assessments in any real way. On the other hand, if such nonlinear or interactive relationships are in fact present, any analysis which ignores them runs the risk seriously of biasing the results.

A related point is that of response-variable transformations. Especially where proportions are used, it is standard practice to apply a nonlinear transformation. This has the function both of tending to linearise an otherwise nonlinear relationship and to stabilise the residual variance. The latter is important in providing a more efficient analysis, and the former in avoiding biases.

It appears, though, that response-variable transformations, nonlinear and interaction models have not been investigated in any serious way. Without access to the detailed analyses it is difficult to suggest where biases might occur. To understand some of the possible consequences, however, suppose that in high-need LAs there is a stronger relationship with a continuous predictor variable than in low-need LAs. In this case a linear relationship will bias the prediction against the high-need authorities as can be seen in figure 1. If the nonlinear curve here gives a more accurate estimate than the linear predictor as shown by the straight line, then the use of the latter will overpredict for low-need LAs and underpredict for high-need LAs.

To illustrate this and further problems I have chosen a data set from the Department of Education and Science (DES), relating educational expenditure to various social indicators. The data are similar in kind to those upon which need estimates are based and serve to exemplify the general point. The point I am making, therefore, is a general methodological one and I am not seeking to comment upon the particular analyses carried out by the DoE.

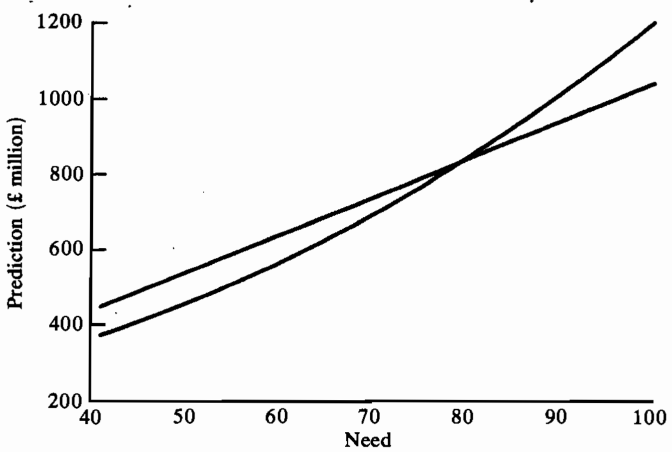

Figure 1. Linear and nonlinear predictions.

The prediction of secondary school expenditure

The data set is that published by the DES (1984) which presents data on expenditure for different parts of the education service in ninety-six English local educational authorities (LEAs) together with various social and demographic indices. The period covered is 1980–83. For present purposes the expenditure per pupil at secondary school is used as the response variable and the percentage of each LEAs population in the 'high' socioeconomic group (SEG) category (mean = 37%) and a social index (mean = 26%) consisting of the average of the percentage on supplementary benefit and the percentage in one-parent families, are chosen as explanatory or predictor variables. All the analyses use the number of secondary school pupils in each LEA as weights.

The first analysis is a linear regression of expenditure on the social index and the percentage in the high SEG. The results are given in the appendix. The second analysis additionally incorporates a nonlinear predictor, the square of the social index and the results are also given in the appendix. To show the difference between the two analyses, predicted estimates are obtained for each LEA for each model. For simplicity these are each scaled so that the mean expenditure is 100 units. The difference between the nonlinear and linear predictions is obtained and this is plotted against SEG.

Figure 2 shows clearly that there are two LEAs (they are Knowlsley and Manchester) which have very low percentages of the high SEG and which have very much higher estimates on the quadratic model than the linear—between 15 and 20 points. There is a general tendency for the advantage associated with the quadratic model to decrease with increasing percentages in the high SEG up to about the overall mean SEG. Interestingly, the rank order of the above two LEAs does not change under the two models, although other rankings do. Nevertheless, because the allocation of grant is a linear function of the predicted value, the actual allocation will be changed by the inclusion of the quadratic term.

Thus, it is clear that the choice of prediction model can change the predictions, in some cases markedly. In this analysis the quadratic model is actually a statistically significant improvement over the linear model and provides a more accurate prediction. In the needs estimation this would imply a reallocation of resources between LAs if the regression coefficients weights were used as they stand, and the quadratic model ones chosen instead of the linear model ones.

Of course, this is not how the final formulas are derived by the DoE. The weights are required to be 'plausible' and the final formula has to reflect this. As already pointed out, however, equally plausible looking models may lead to quite different estimates. In fact, in the quadratic analysis, the results do not seem to be 'plausible' because the linear coefficient as given in the appendix is negative which implies that the prediction does not increase steadily with increasing values of the index. The point, however, is that in statistical terms this analysis is actually a better predictor of expenditure than the linear model. Thus, although the linear model provides a plausible model for directing resources on the basis of social factors, it is not the best available predictor. This further underlines the dubious logic of attempting to predict expenditure as accurately as possible while at the same time being constrained by presentational considerations.

To illustrate further the point about alternative 'plausible' models, two more models are analysed. One uses the percentage of nonwhite children in the LEA and the other the density, expressed as the number of 16 - 18 year olds per hectare. The results are given in the appendix. Both analyses explain about the same percentage of variance (42% and 48%) and both analyses give plausible results.

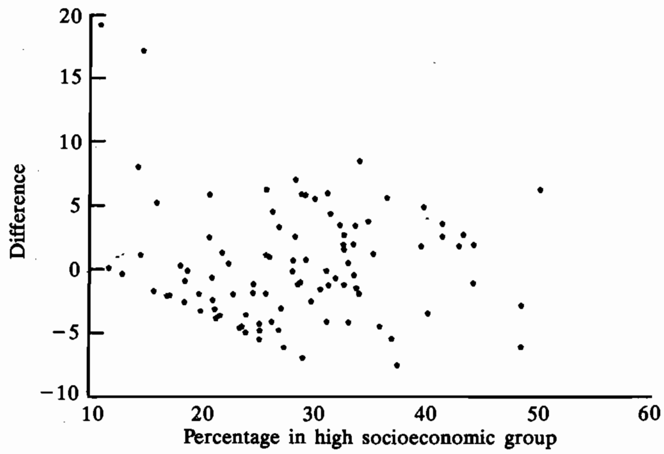

Figure 2. Quadratic - linear prediction difference.

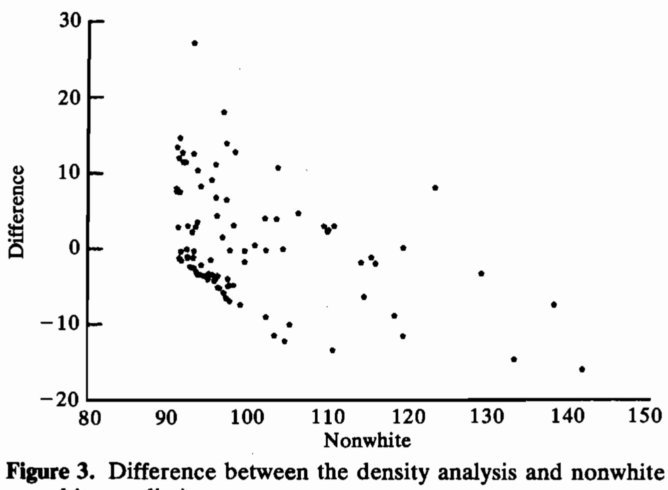

Figure 3. Difference between the density analysis and nonwhite prediction plotted against the nonwhite prediction.

As before, the results of the analyses (see appendix) are standardised to the same scale and the difference between the prediction from the nonwhite children analysis and the density analysis is plotted against the prediction from the former in figure 3. We can see clearly here how those LEAs who have a low predicted value on the basis of the nonwhite analysis tend to do relatively better on the density analysis.

Conclusions

The use of regression analysis to determine the weights in the components of the needs prediction formulas, fails to provide a unique objective prediction. The chosen examples serve to illustrate this point, which is a very general one found typically when the unit of analysis, the local authority, and the unit of interest are the same. Alternative statistical models may be equally 'plausible' yet lead to different results for different kinds of LAs. A further discussion of this issue in a related context is given by Woodhouse and Goldstein (1988).

It is clear that the very many different possible combinations of alternative analyses will give rise to a wide variety of final predicted estimates. In addition, the residual variation in such regression analyses is often large, so that predictions based upon the results of the analyses may be quite inaccurate.

It may be reasonable to use simple combinations of social indicators to allocate resources, although there is clearly room for debate about the precise way in which the allocation functions are constructed. There may also be scope for more sophisticated procedures, for example based upon latent variable modelling, to refine the construction of the indicators used in such functions. It is not reasonable to suppose, however, that there can be a single objective estimator which is both accurate and unbiased. The current methodology, therefore, provides no objective measure of need and hence no basis for capping LAs on the grounds that their spending exceeds their (predicted) target by a fixed amount.

In this paper I have been critical of current methodology. By contrast, it should be possible to obtain good estimates of spending need by carrying out detailed surveys or by using existing administrative data in novel ways. Thus, for example, it is possible to estimate the numbers of children with special educational needs or low achievement at the time of starting school by means of assessment surveys. No doubt this will be expensive, but there would seem to be a strong case for exploring such alternatives to the present procedures. At the very least, research should be carried out into examining the actual relationship between the current target estimates and some more objectively determined measures of true need.

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APPENDIX

Table A1. Results of regression analysis for secondary school expenditure for ninety-six English local education authorities.

Coefficient	Value	Standard error
(a) Related to social index (linear)		
Constant	311.0	
High socioeconomic group	8.61	1.59
Social index	32.26	3.04
(b) Related to social index (nonlinear)		
Constant	1014.0	
High socioeconomic group	4.11	1.71
Social index	−43.37	14.51
(Social index)^2	0.53	0.10
(c) Related to percentage of nonwhite children		
Constant	869.5	
Nonwhite children (%)	9.43	0.66
(d) Related to density		
Constant	879.6	
Density	120.2	7.03