

# Introduction to Multilevel Modelling and the software *MLwiN*

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## Review some concepts

The data set used to address the related issues  
in this lecture – *MLwiN* tutorial sample

(see Rasbash, et al, 2005 – A User's Guide to *MLwiN*)

## Basic information of the data set

	Name	n	missing	min	max
1	school	4059	0	1	65
2	student	4059	0	1	198
3	normexam	4059	0	-3.666072	3.666091
4	cons	4059	0	1	1
5	standlrt	4059	0	-2.934953	3.015952
6	girl	4059	0	0	1
7	schgend	4059	0	1	3
8	avslrt	4059	0	-0.7559605	0.6376559
9	schav	4059	0	1	3
10	vrband	4059	0	1	3

Data source: *MLviz*N tutorial sample

Number of schools: 65

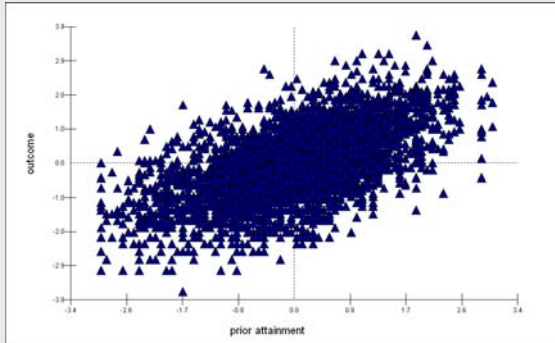
Number of pupils: 4059

Normexam: pupil's exam score  
at age 16

Standlrt: pupil's score at age 11 on  
the London Reading Test

## We are interested in

- Q1: The relationship between 'pupil's exam score at age 16' and 'pupil's score at age 11 on the London Reading Test' - the **effect** of 'standlrt' (prior attainment) on 'normexam' (outcome)

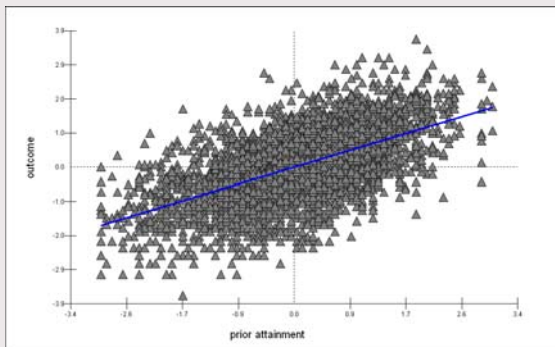


Data source: *MLwiN* tutorial sample

## Simple scatter plot

All 4059 pupils

WJ Peng



Data source: *MLwiN* tutorial sample

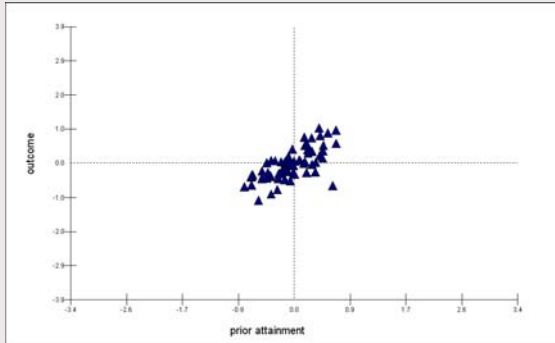
## Ordinary linear regression method

Unit of analysis – pupil

One regression line

WJ Peng



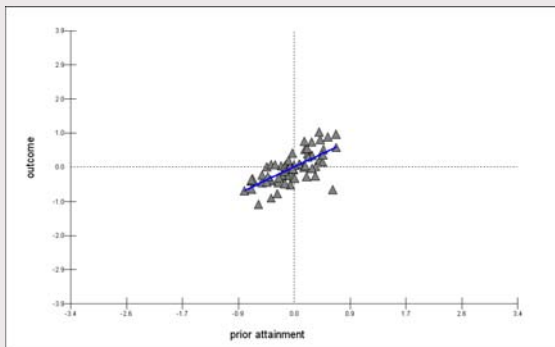


Data source: *MLwiN* tutorial sample

## Simple scatter plot

Aggregated data for 65 schools

WJ Peng



Data source: *MLwiN* tutorial sample

## Ordinary linear regression method

Unit of analysis – school

One regression line

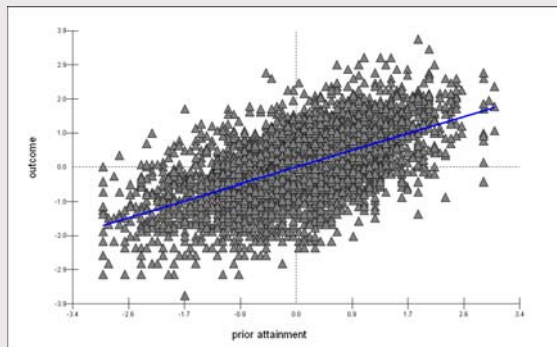
WJ Peng



# What went missing in the analysis?

The hierarchical structure of the data set

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Data source: *MLwiN* tutorial sample

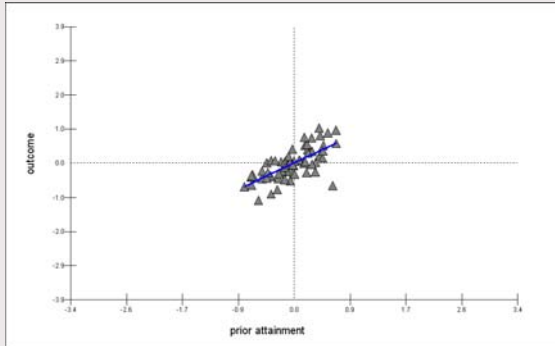
- The fact that those  
4059 pupils were  
from 65 schools was  
ignored

Ordinary linear regression method

Unit of analysis – pupil  
One regression line

WJ Peng





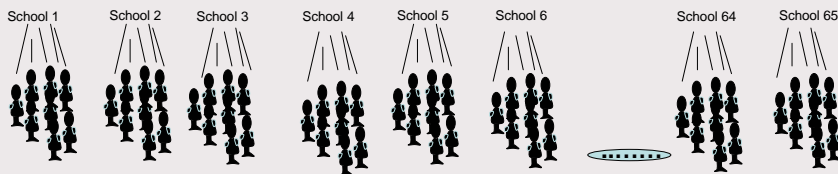
Data source: *MLwiN* tutorial sample

- The information about individual pupils was discarded

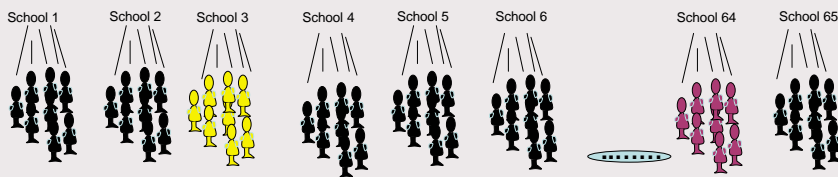
## Ordinary linear regression method

Unit of analysis – school  
One regression line

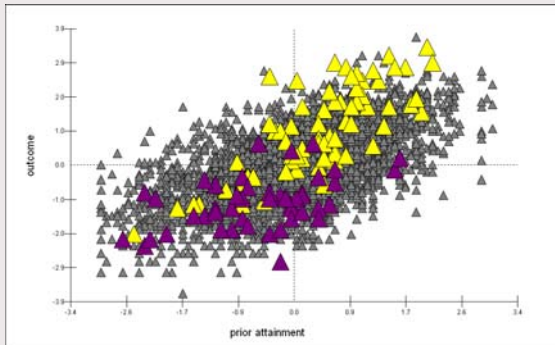
## How this structure affects the measurement of interest?



## How this structure affects the measurement of interest?



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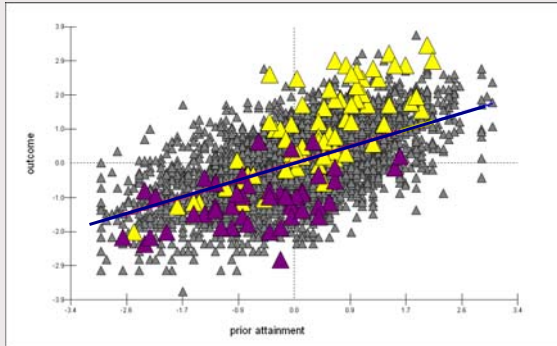
Data source: *MLwiN* tutorial sample

### Simple scatter plots

For School 3 and  
School 64

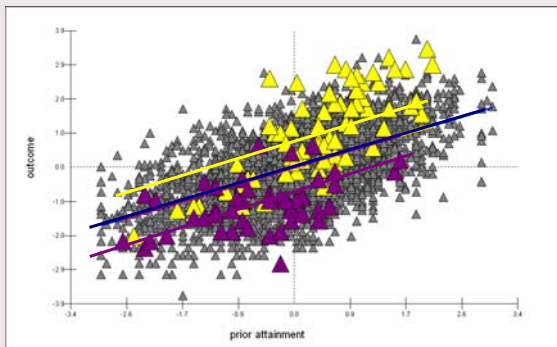
WJ Peng





Data source: *MLwiN* tutorial sample

WJ Peng

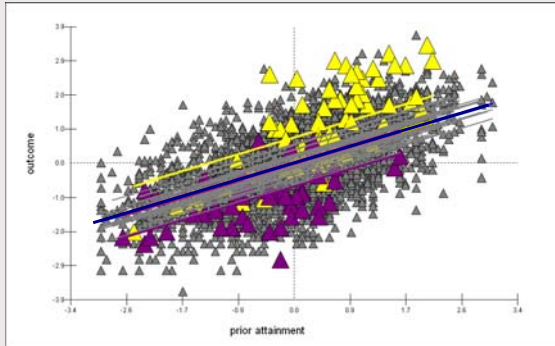


Data source: *MLwiN* tutorial sample

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Data source: *MLwiN* tutorial sample

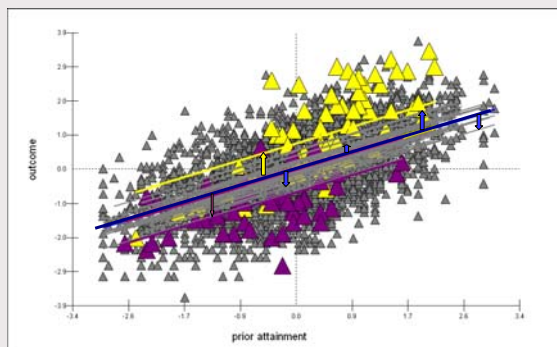
## Underlying Meaning

**In this case, pupils within a school will be more alike, on average, than pupils from different schools.**

## We are also interested in

- Q1: The relationship between 'pupil's exam score at age 16' and 'pupil's score at age 11 on the London Reading Test' - the **effect** of 'standlrt' (prior attainment) on 'normexam' (outcome)
- Q2: How different the relationship is across schools? - the **variability** of the effect of 'standlrt' (prior attainment) on 'normexam' (outcome) across schools

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Data source: *MLwN* tutorial sample

## The variation between schools

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## Can ordinary linear regression method estimate the variation between schools?

It is possible that “The variation between schools could be modelled by incorporating separate terms for each school...”

(Rasbash, et al., 2005)

For example, to fit 64 school dummy variables in a model using school 1 as the reference school

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$$\begin{aligned} \text{normexam}_i = & \alpha + \beta_1 \text{standlit}_i + \beta_2 \text{school}_2_i + \beta_3 \text{school}_3_i + \beta_4 \text{school}_4_i + \beta_5 \text{school}_5_i + \\ & \beta_6 \text{school}_6_i + \beta_7 \text{school}_7_i + \beta_8 \text{school}_8_i + \beta_9 \text{school}_9_i + \beta_{10} \text{school}_{10}_i + \\ & \beta_{11} \text{school}_{11}_i + \beta_{12} \text{school}_{12}_i + \beta_{13} \text{school}_{13}_i + \beta_{14} \text{school}_{14}_i + \beta_{15} \text{school}_{15}_i + \\ & \beta_{16} \text{school}_{16}_i + \beta_{17} \text{school}_{17}_i + \beta_{18} \text{school}_{18}_i + \beta_{19} \text{school}_{19}_i + \beta_{20} \text{school}_{20}_i + \\ & \beta_{21} \text{school}_{21}_i + \beta_{22} \text{school}_{22}_i + \beta_{23} \text{school}_{23}_i + \beta_{24} \text{school}_{24}_i + \beta_{25} \text{school}_{25}_i + \\ & \beta_{26} \text{school}_{26}_i + \beta_{27} \text{school}_{27}_i + \beta_{28} \text{school}_{28}_i + \beta_{29} \text{school}_{29}_i + \beta_{30} \text{school}_{30}_i + \\ & \beta_{31} \text{school}_{31}_i + \beta_{32} \text{school}_{32}_i + \beta_{33} \text{school}_{33}_i + \beta_{34} \text{school}_{34}_i + \beta_{35} \text{school}_{35}_i + \\ & \beta_{36} \text{school}_{36}_i + \beta_{37} \text{school}_{37}_i + \beta_{38} \text{school}_{38}_i + \beta_{39} \text{school}_{39}_i + \beta_{40} \text{school}_{40}_i + \\ & \beta_{41} \text{school}_{41}_i + \beta_{42} \text{school}_{42}_i + \beta_{43} \text{school}_{43}_i + \beta_{44} \text{school}_{44}_i + \beta_{45} \text{school}_{45}_i + \\ & \beta_{46} \text{school}_{46}_i + \beta_{47} \text{school}_{47}_i + \beta_{48} \text{school}_{48}_i + \beta_{49} \text{school}_{49}_i + \beta_{50} \text{school}_{50}_i + \\ & \beta_{51} \text{school}_{51}_i + \beta_{52} \text{school}_{52}_i + \beta_{53} \text{school}_{53}_i + \beta_{54} \text{school}_{54}_i + \beta_{55} \text{school}_{55}_i + \\ & \beta_{56} \text{school}_{56}_i + \beta_{57} \text{school}_{57}_i + \beta_{58} \text{school}_{58}_i + \beta_{59} \text{school}_{59}_i + \beta_{60} \text{school}_{60}_i + \\ & \beta_{61} \text{school}_{61}_i + \beta_{62} \text{school}_{62}_i + \beta_{63} \text{school}_{63}_i + \beta_{64} \text{school}_{64}_i + \beta_{65} \text{school}_{65}_i \end{aligned}$$

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## Can ordinary linear regression method estimate the variation between schools?

However, it is inefficient and inadequate “ because it involves estimating many times coefficients...because it does not treat schools as a random sample...”

(Rasbash, et al., 2005)

Think about a national data set with hundreds of schools.....

## Multilevel modelling

A statistical technique that allows an analysis to take account of the levels of hierarchical structure in the population so that we can

- treat sample as random
- specify and fit a wide range of multilevel models
- understand where and how effects are occurring

(Rasbash, et al., 2005)

# Statistical software packages

There are some statistical packages have the function

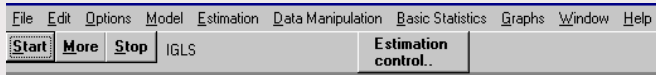
- *MLwiN* is one of them



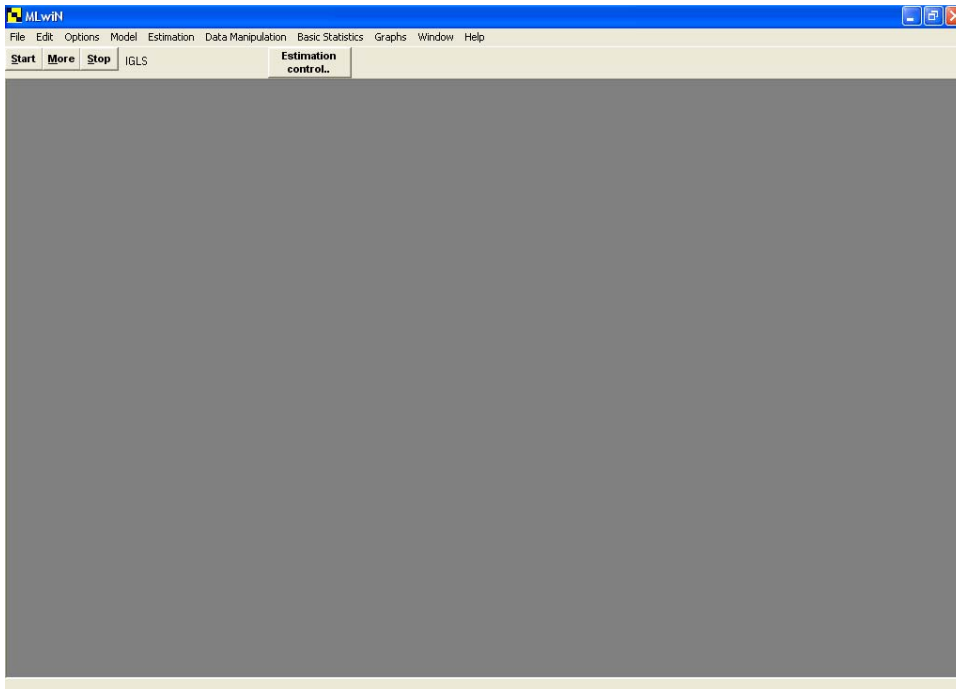
MLwin.Ink



Menu bar



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## Get started with creating a *MLwiN* worksheet

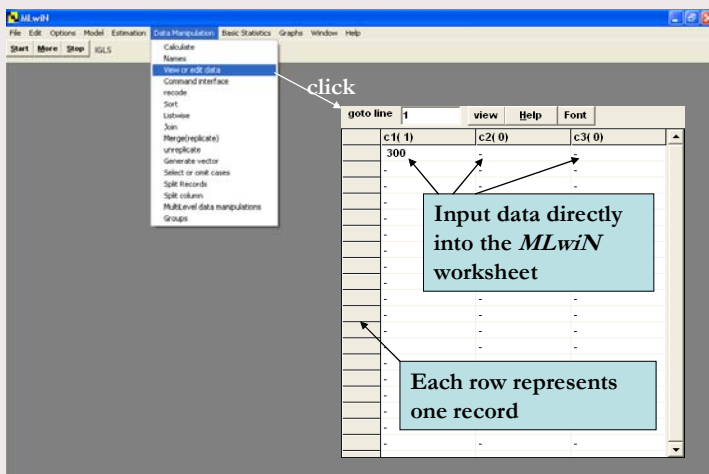
*MLwiN* can only input and output **numerical** data

- code data numerically
- assign an identical numerical code to all missing data
- three ways of creating a *MLwiN* worksheet:
  - input data into a *MLwiN* worksheet
  - copy and paste data into a *MLwiN* worksheet
  - import ASCII data from a text file

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## Input data into a *MLwiN* worksheet

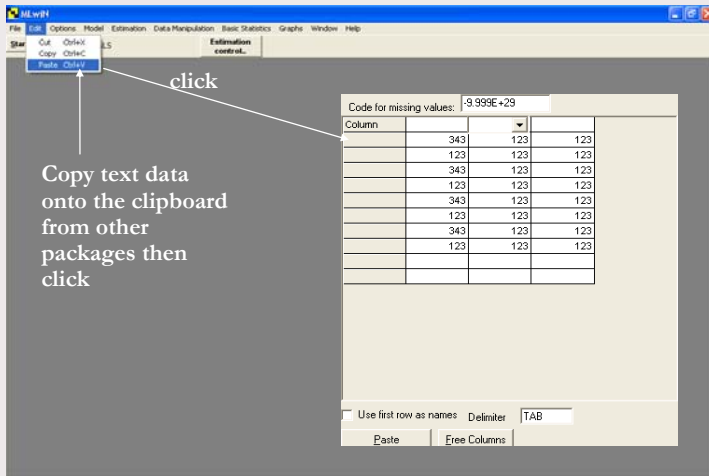


Data Manipulation/View or edit data

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## Paste data into a *MLwiN* worksheet

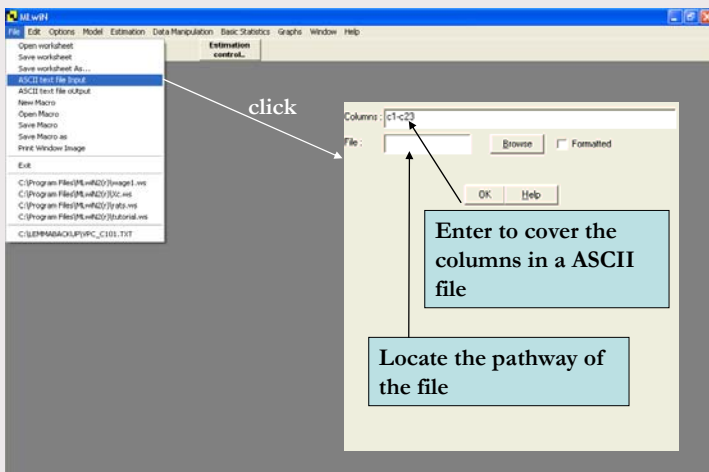


Edit/Paste

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## Import ASCII data from a text file

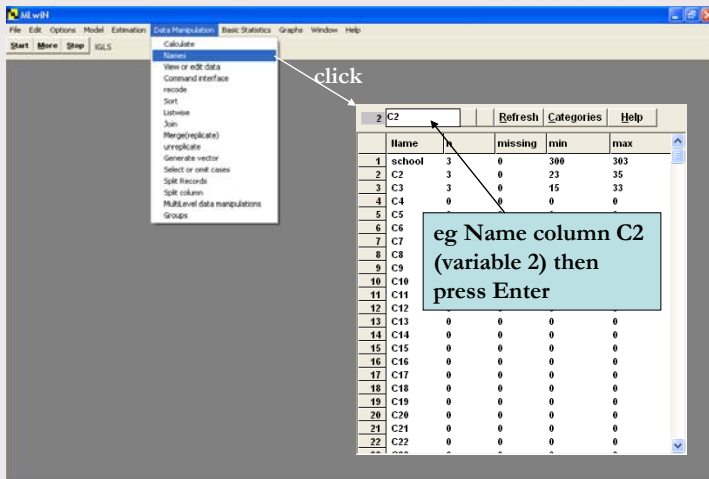


File/ASCII text file Input

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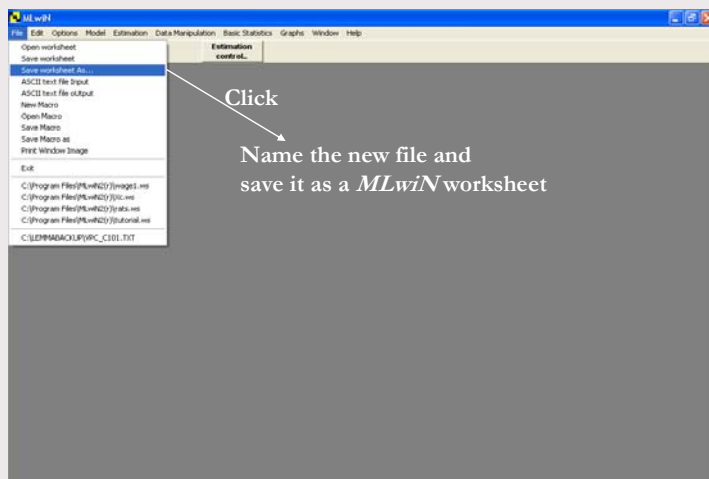


## Name columns - variables



Data Manipulation/Names

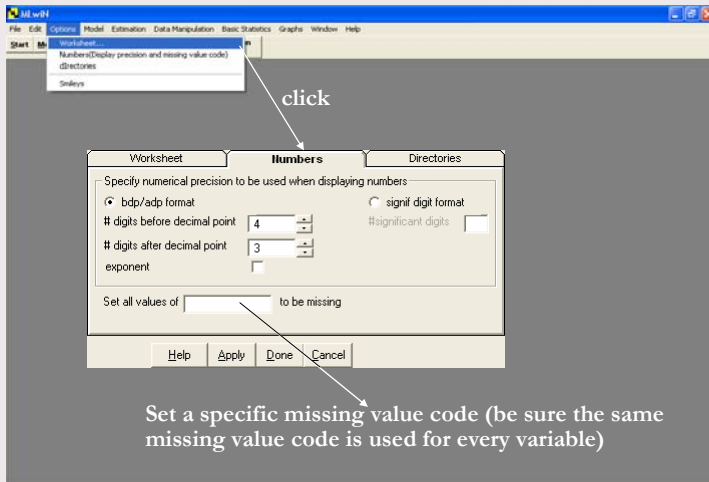
## Save the file as a *MLwiN* worksheet



File/Save worksheet As...



## Declare the missing data



Options/Worksheet/Numbers

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## Sort a dataset to reflect its hierarchical structure

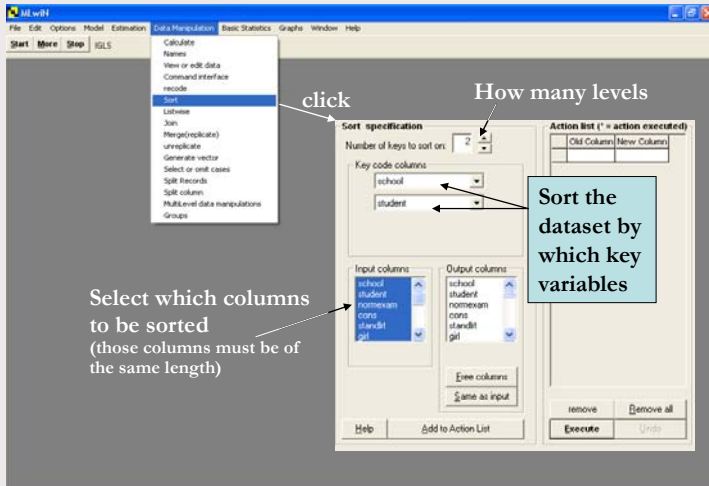
“before trying to fit a multilevel model to a dataset.....the dataset must be sorted so that all records for the same highest-level unit are grouped together and within this group, all records for a particular lower level unit are contiguous”

(Rasbash, et al., 2005)

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## Sort the dataset

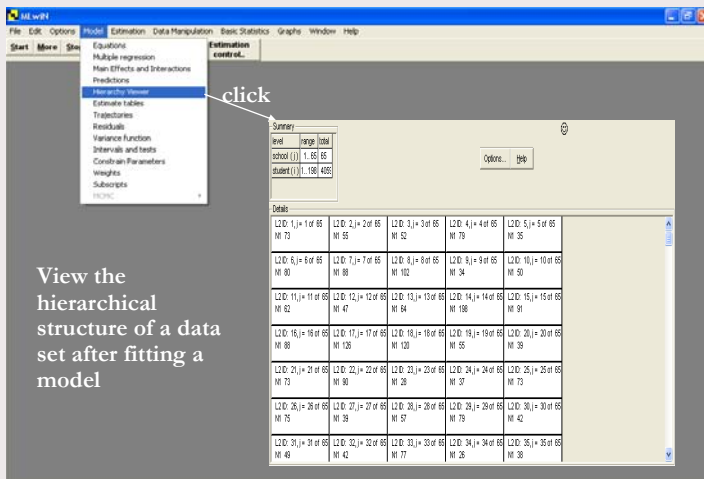


Data Manipulation/Sort

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## View hierarchical structure



Models/Hierarchy Viewer

(Note: to View after fitting a model)

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## Checklist

- All value codes are numerical?  $\checkmark$
- An identical missing value code?  $\checkmark$
- The dataset has been sorted?  $\checkmark$
- The dataset is a MLwin worksheet?  $\checkmark$

## Understand the notation used in *MLwiN*

An example – linear regression with continuous variables  $x$  and  $y$  for one school with  $i$  number of pupils

$$\hat{y}_i = a + bx_i \quad i = 1, 2, 3 \dots \text{the number of pupils}$$

$$y_i = \hat{y}_i + e_i \quad e_i = \text{residual (or error) ie, the difference}$$
$$= a + bx_i + e_i \quad \text{between } y_i \text{ and } \hat{y}_i - \text{pupil level}$$

$a$  = intercept (average across all pupils)

$b$  = slope (coefficient – the effect of  $x$ )

$a$  (intercept) and  $b$  (slope of  $x$ ) define the average line across all pupils in the school.

## Understand the notation used in *MLwiN*

For one school

$$y_i = a + bx_i + e_i$$

For a number of schools

$$y_{i1} = a_1 + bx_{i1} + e_{i1}$$

$$y_{i2} = a_2 + bx_{i2} + e_{i2}$$

.....

$$y_{ij} = a_j + bx_{ij} + e_{ij}$$

There is

$$a_j = a + u_j - \text{school level}$$

Thus

$$y_{ij} = a + bx_{ij} + u_j + e_{ij}$$

## Understand the notation used in *MLwiN*

For a number of schools

$$y_{ij} = a + bx_{ij} + u_j + e_{ij}$$

Introduce  $x_0 (=1)$  and symbols

$\beta_0$  and  $\beta_1$  to denote  $a, b$

$$y_{ij} = \beta_0 x_0 + \beta_1 x_{ij} + u_{0j} x_0 + e_{0ij} x_0$$

$x_0$  called cons in *MLwiN*

$$y_{ij} = \beta_{0ij} x_0 + \beta_1 x_{ij}$$

$$\beta_{0ij} = \beta_0 + u_{0j} + e_{0ij}$$

general notation

$i = \text{pupil level}, j = \text{school level}$

$\beta_0$  and  $\beta_1$  define the average line across all pupils in all schools.

## The fixed and random parts in *MLwiN*

The fixed part of the model:

$\beta_0, \beta_1$  – multilevel modelling regression coefficients  
– explained in the model

The random part of the model:

$\sigma^2_{u0j}$  – the variance of the school level random effects  $u_{0j}$   
 $\sigma^2_{e0ij}$  – the variance of the pupil level random effects  $e_{0ij}$   
– unexplained in the model

(<http://tramss.data-archive.ac.uk/documentation/MLwiN/chapter1.pdf>)

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## Fit a multilevel model in *MLwiN* - start with simple models -

“Multilevel modelling is like any other type of statistical modelling and a useful strategy is to start by fitting simple models and slowly increase the complexity.”

“It is important...to know as much as possible about your data and to establish what questions you are trying to answer.”

(Rasbash, et al., 2005)

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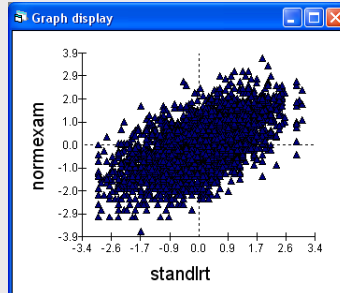


## Research questions

We are interested in exploring – via data modelling – the size, nature and extent of the school effect on progress in normexam.

**Q1** – What the relationship between the outcome attainment measure normexam and the intake ability measure standlrt would be?

**Q2** – How this relationship varies across schools (what the proportions of the overall variability shown in the plot attributable to schools and to student)?



(<http://tramss.data-archive.ac.uk/documentation/MLwiN/chapter1.pdf>)

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## Considering the following 3 models

**Cons Model** – a random intercepts Null model with ‘normexam’ as the response variable, no predictor/explanatory variables apart from the Constant (ie representing the intercept) which is allowed to vary randomly across schools and with the levels defined as pupils (level 1) in schools (level 2)

**Model A** – a random intercepts/variance components model  
– Cons Model with also an explanatory variable (standlrt)

**Model B** – a random intercepts/slopes model  
– Model A with also the parameter associated with standlrt being allowed to vary randomly across schools (ie random slopes as well as intercepts)

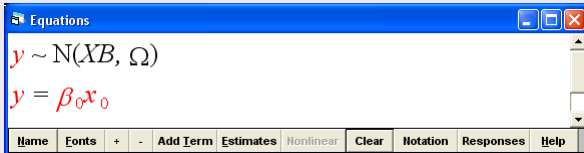
(Thomas, 2007, <http://www.cmm.bristol.ac.uk/research/Lemma/two-level.pdf>)

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# Fitting Cons Model

Select Model/Equations from Menu bar



Notice the red colour parts? – indicating that the variable and the parameter associated with it has not yet been specified

$y \sim N(XB, \Omega)$  – the default distributional assumption:

“The response vector has a mean specified in matrix notation by the *fixed part*  $XB$ , and a random part consisting of a set of random variables described by the *covariance matrix*  $\Omega$ .”

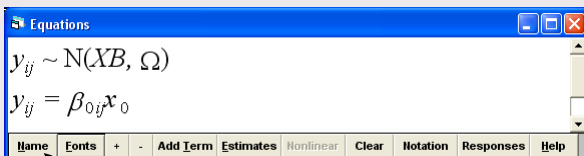
(<http://tramss.data-archive.ac.uk/documentation/MLwiN/chapter1.pdf>)

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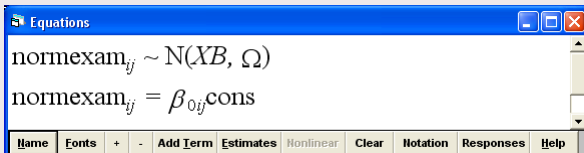


Click either of the  $y$ 's to specify the response variable – normexam, assign  $i$  and  $j$  at pupil and school levels respectively.

Click either  $x_0$  or  $\beta_0$  to specify the explanatory variable – cons, assign  $i$  and  $j$  at pupil and school levels respectively to model the intercept.



Click Name to show the names of the variables.



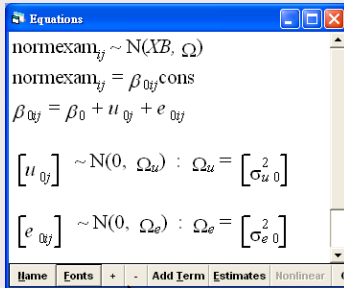
Notice the variables and parameters have changed from red to black? – indicating that specification is completed.

(<http://www.cmm.bristol.ac.uk/research/Lemma/two-level.pdf>)

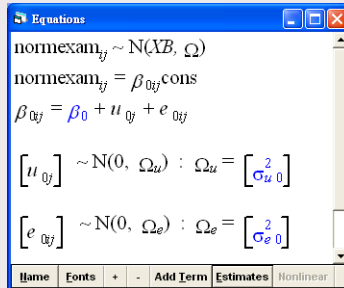
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## Cons Model has now been specified

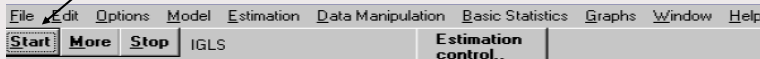


Click the + or - buttons to see the composition of  $\beta_{0ij}$ .



Click Estimates to see the parameters highlighted in blue that are to be estimated.

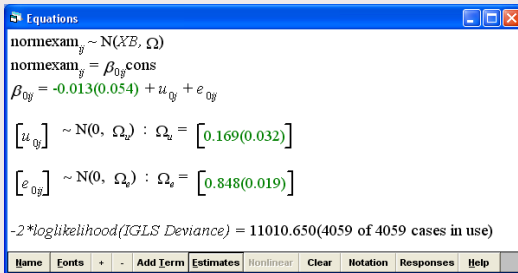
Click Start on Menu bar to start estimation.



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## Completion of the parameters estimation



Note that the default method of estimation is iterative generalised least squares (IGLS).

The blue highlighted parameters in the Equations window change to green to indicate convergence.

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## What the parameter estimates tell us?

```

Equations
normexamy ~ N( $\bar{X}E$ ,  $\Omega$ )
normexamy =  $\beta_{0y}$ cons
 $\beta_{0y}$  = -0.013(0.054) +  $u_{0y}$  +  $\varepsilon_{0y}$ 

[ $u_{0y}$ ] ~ N(0,  $\Omega_u$ ) :  $\Omega_u$  = [0.169(0.032)]
[ $\varepsilon_{0y}$ ] ~ N(0,  $\Omega_e$ ) :  $\Omega_e$  = [0.848(0.019)]

-2*loglikelihood(IGLS Deviance) = 11010.650(4059 of 4059 cases in use)
    
```

Note that normexam scores were normalised to have a proximately standard normal distribution

- Overall mean -0.013 (approach to zero)
- Total variance  $0.169 + 0.848 = 1.017$  (approach to 1) (if children were taken from the whole population at random the variance would be)
- Intra-school correlation  $0.169 / (0.169 + 0.848) = 16.6\%$  (the proportion of the total variance attributable to the school)

## Graphing prediction

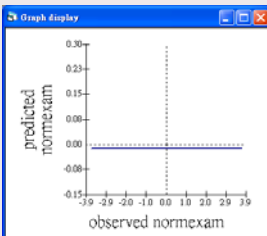
```

Models/Prediction
normexamy =  $\hat{\beta}_0$ cons
variable cons
fixed  $\beta_0$  ←-----
level 2  $u_{0y}$ 
level 1  $\varepsilon_{0y}$ 
    
```

```

Graphs/Customised Graph(s)
Customised graph : display 2, data set 1
ds# Y X
1 c11 normexam
2
3
4
5
6
7
8
9
10

Details for for data set number (ds#) 1
plot what? plot style position error bars other
y c11 x normexam
filter [none] group [none]
plot type line
    
```

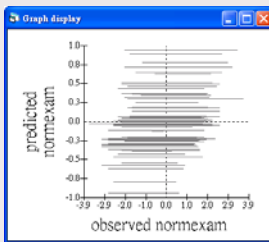


In Prediction window, click  $\beta_0$  to calculate the average predicted line produced from the intercept coefficient  $\beta_0$  - this is the predicted overall mean normexam (= -0.013) line for all pupils in all schools.

## Graphing prediction

### Models/Prediction

$\hat{\text{normexam}}_y = \beta_{0j} \text{cons}$   
 variable cons  
 fixed  $\beta_0$  ←-----  
 level 2  $u_{0j}$  ←-----  
 level 1  $e_{0j}$  ←-----



### Graphs/Customised Graph(s)

Customised graph: display 3, data set 1  
 ds # Y X  
 1 c12 normexam  
 Details for for data set number (ds#) 1  
 plot what? plot style position error bars other  
 y c12 x normexam  
 filter [none] group school  
 plot type line

Click also  $u_{0j}$  to include the estimated school level intercept residuals in the prediction function and produce the predicted lines for all 65 schools. The line for school  $j$  departs from the average prediction line by an amount  $u_{0j}$ .

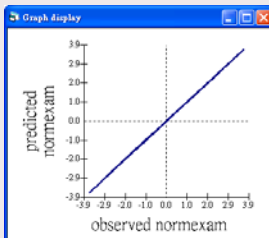
WJ Peng



## Graphing prediction

### Models/Prediction

$\hat{\text{normexam}}_y = \beta_{0ij} \text{cons}$   
 variable cons  
 fixed  $\beta_0$  ←-----  
 level 2  $u_{0j}$  ←-----  
 level 1  $e_{0ij}$  ←-----



### Graphs/Customised Graph(s)

Customised graph: display 4, data set 1  
 ds # Y X  
 1 c13 normexam  
 Details for for data set number (ds#) 1  
 plot what? plot style position error bars other  
 y c13 x normexam  
 filter [none] group school  
 plot type line

Click  $e_{0ij}$  to include the estimated pupil level intercept residuals in the prediction function too. Plot shows identical predicted and observed normexam ( $r = 1$ ). Pupil  $i$  in school  $j$  departs from the predicted line for school  $j$  by an amount  $e_{0ij}$ .

WJ Peng



## Fitting Model A - an random intercepts model -

The screenshot shows the MLwiN Equations window with the following content:

$$y_{ij} \sim N(\beta_{0j}, \Omega)$$

$$y_{ij} = \beta_{0j}x_{i0} + \beta_{1j}x_{i1}$$

$$\beta_{0j} = \beta_0 + u_{0j} + e_{0ij}$$

$$\begin{bmatrix} u_{0j} \end{bmatrix} \sim N(0, \Omega_u) : \Omega_u = \begin{bmatrix} \sigma_u^2 \end{bmatrix}$$

$$\begin{bmatrix} e_{0ij} \end{bmatrix} \sim N(0, \Omega_e) : \Omega_e = \begin{bmatrix} \sigma_e^2 \end{bmatrix}$$

The loglikelihood (IGLS Deviance) is 11010.650 (4059 of 4059 cases in use).

“Note that  $x_0$  has no other subscript but that  $x_1$  has collected subscripts  $ij$ .

*MLwiN* detects that  $\text{cons}$  is constant over the whole data set, whereas the values of  $\text{standlrt}$  change at both level 1 and level 2.”

Click Add Term to add an explanatory variable – `standlrt`.

(<http://tramss.data-archive.ac.uk/documentation/MLwiN/chapter1.pdf>)

Click the +, -, and Name buttons to see how much the detail of the model is displayed.

The screenshot shows the MLwiN Equations window with the following content:

$$\text{normexam}_{ij} \sim N(\beta_{0j}, \Omega)$$

$$\text{normexam}_{ij} = \beta_{0j}\text{cons} + \beta_{1j}\text{standlrt}_{ij}$$

$$\beta_{0j} = \beta_0 + u_{0j} + e_{0ij}$$

$$\begin{bmatrix} u_{0j} \end{bmatrix} \sim N(0, \Omega_u) : \Omega_u = \begin{bmatrix} \sigma_u^2 \end{bmatrix}$$

$$\begin{bmatrix} e_{0ij} \end{bmatrix} \sim N(0, \Omega_e) : \Omega_e = \begin{bmatrix} \sigma_e^2 \end{bmatrix}$$

The loglikelihood (IGLS Deviance) is 11010.650 (4059 of 4059 cases in use).

$\beta_0$  (the intercept) and  $\beta_1$  (the slope of `standlrt`) define the average line across all pupils in all schools.

“The model is made multilevel by allowing each school’s summary line to depart (be raised or lowered) from the average line by an amount  $u_{0j}$ .” Pupil  $i$  in the school  $j$  departs from its school’s summary line by an amount  $e_{0ij}$ .

(<http://tramss.data-archive.ac.uk/documentation/MLwiN/chapter1.pdf>)

In other words.....

```
normexamij ~ N(XB, Ω)
normexamij = β0jcons + β1standlij
β0j = β0 + u0j + e0ij

[ u0j ] ~ N(0, Ωu) : Ωu = [ σu02 0 ]
[ e0ij ] ~ N(0, Ωe) : Ωe = [ σe02 0 ]

-2*loglikelihood(IGLS Deviance) = 11010.650(4059 of 4059 cases in use)
```

$u_{0j}$  – the level 2 or school level *residuals* (one for each school);  
distributed Normally with mean 0 and variance  $\sigma^2_{u0}$

$e_{0ij}$  – the level 1 or pupil level *residuals* (one for each pupil);  
distributed Normally with mean 0 and variance  $\sigma^2_{e0ij}$

(<http://tramss.data-archive.ac.uk/documentation/MLwiN/chapter1.pdf>)

## Estimate the parameters of the specified model

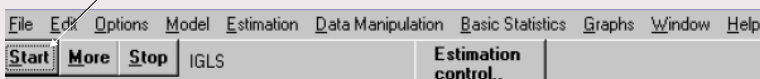
```
normexamij ~ N(XB, Ω)
normexamij = β0jcons + β1standlij
β0j = β0 + u0j + e0ij

[ u0j ] ~ N(0, Ωu) : Ωu = [ σu02 0 ]
[ e0ij ] ~ N(0, Ωe) : Ωe = [ σe02 0 ]

-2*loglikelihood(IGLS Deviance) = 11010.650(4059 of 4059 cases in use)
```

The parameters highlighted in **blue** are to be estimated.

Click Start on Menu bar to start estimation.



(<http://tramss.data-archive.ac.uk/documentation/MLwiN/chapter1.pdf>)

## Completion of the parameters estimation

```
Equations
normexam_y ~ N(XB, Omega)
normexam_y = beta_0j*cons + 0.563(0.012)*standlrt_y
beta_0j = 0.002(0.040) + u_0j + e_0j
[u_0j] ~ N(0, Omega_u) : Omega_u = [0.092(0.018)]
[e_0j] ~ N(0, Omega_e) : Omega_e = [0.566(0.013)]
-2*loglikelihood(IGLS Deviance) = 9357.242(4059 of 4059 cases in use)
```

**Slope** – the slopes of the lines across schools are all the same, of which the common slope is 0.563 with SE = 0.012

**Intercept** – the intercepts of the lines vary across schools. Their mean is 0.002 with SE = 0.040. The intercept of school  $j$  is  $0.002 + u_{0j}$  with a variance of 0.092 and SE = 0.018.

(<http://tramss.data-archive.ac.uk/documentation/MLwiN/chapter1.pdf>)

WJ Peng



## What the parameter estimates tell us?

```
Equations
normexam_y ~ N(XB, Omega)
normexam_y = beta_0j*cons + 0.563(0.012)*standlrt_y
beta_0j = 0.002(0.040) + u_0j + e_0j
[u_0j] ~ N(0, Omega_u) : Omega_u = [0.092(0.018)]
[e_0j] ~ N(0, Omega_e) : Omega_e = [0.566(0.013)]
-2*loglikelihood(IGLS Deviance) = 9357.242(4059 of 4059 cases in use)
```

**Total variance** ( $0.092 + 0.566 = 0.658$ ) – the sum of the level 2 and level 1 variances

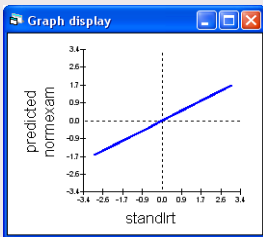
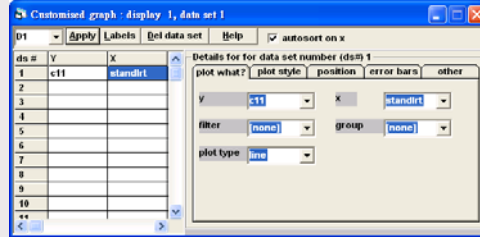
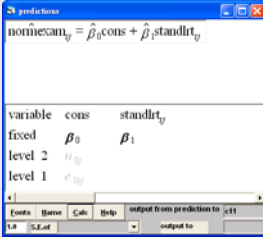
**Intra-school correlation** ( $0.092/0.658 = 0.140$ ) – measuring the extent to which pupils' scores in the same school are more alike as compared with those from pupils at different schools

(<http://tramss.data-archive.ac.uk/documentation/MLwiN/chapter1.pdf>)

WJ Peng



## Graphing predication



$$\hat{y} = \beta_0 \text{cons} + \beta_1 x$$

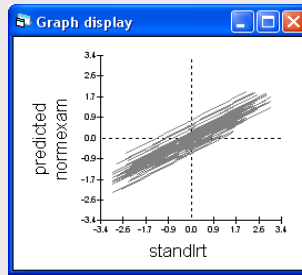
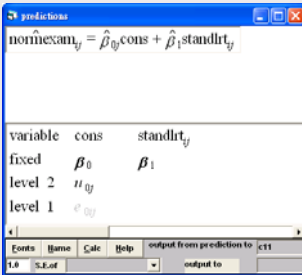
$$\hat{y} = 0.002 + 0.563 \text{standlrt}$$

The average line across all pupils in all schools

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## Graphing predication



$$\hat{y} = \beta_{0j} \text{cons} + \beta_1 x$$

$$\beta_{0j} = \beta_0 + u_{0j}$$

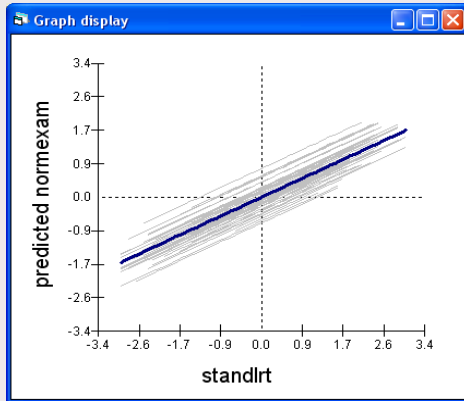
$$= 0.002 + u_{0j}$$

One line for each school

WJ Peng



## Graphing predication

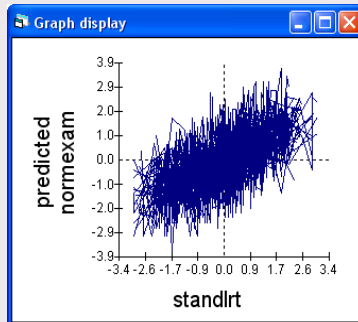
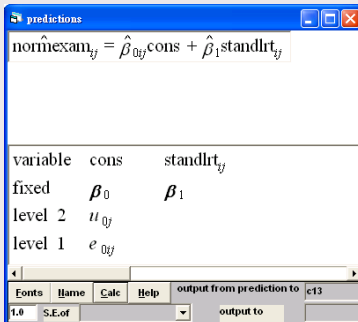


The line for school  $j$  departs from the average prediction blue line by an amount  $u_{0j}$ .

WJ Peng



## Graphing predication



$$\hat{y} = \beta_{0ij} \text{cons} + \beta_1 x$$

$$\beta_{0ij} = \beta_0 + u_{0j} + e_{0ij}$$

$$\beta_{0ij} = 0.002 + u_{0j} + e_{0ij}$$

Pupil  $i$  in school  $j$  departs from the school  $j$  summary line by an amount  $e_{0ij}$ .

WJ Peng

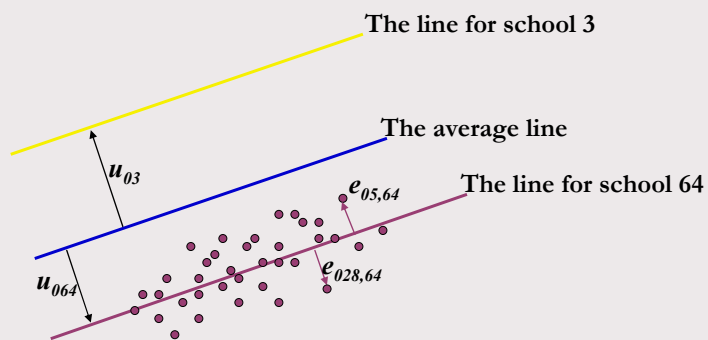


## What all these about – Model A?

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In brief, by employing multilevel modelling approach...



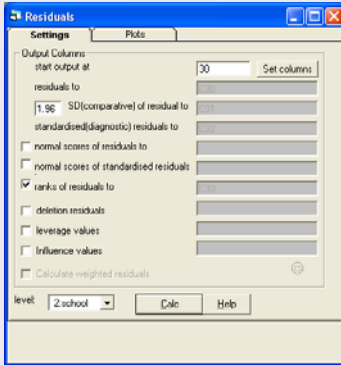
$u_{03}$  – residual for School 3  
 $u_{064}$  – residual for School 64  
 $e_{05,64}$  – residual for pupil 5 in school 64  
 $e_{028,64}$  – residual for pupil 28 in school 64

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## Graphing residuals

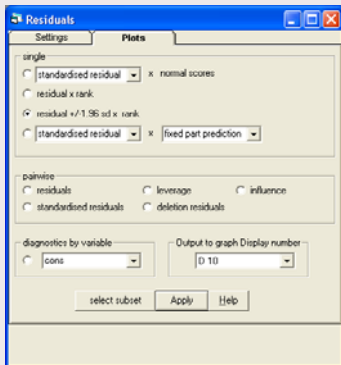


Model/Residuals/Settings

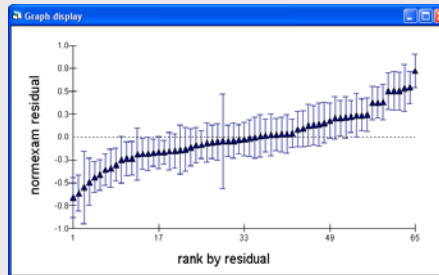
	Name	n	missing	min	max
23	c23	0	0	0	0
24	c24	0	0	0	0
25	c25	0	0	0	0
26	c26	0	0	0	0
27	c27	0	0	0	0
28	c28	0	0	0	0
29	c29	0	0	0	0
30	schres	65	0	-0.6583684	0.7233134
31	sch2sd	65	0	0.1284661	0.5170535
32	c32	65	0	-2.4094	2.5058
33	schresrank	65	0	1	65
34	c34	0	0	0	0

Residuals for individual schools, of which their mean is 0 and their estimated variance of 0.092

## Graphing residuals

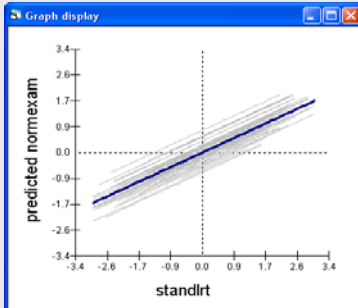


Model/Residuals/Plots

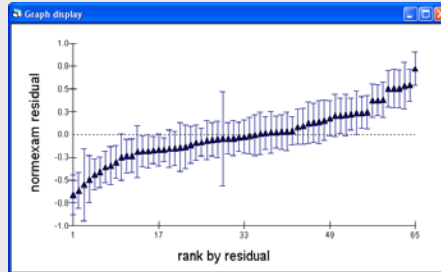


Each vertical line represents a residual with 95% confidence interval estimated for each school.

## What is meant by residual?



**School residual**  
– the departure of a school (grey) line from the average (blue) line



These school residuals might be regarded as school effect – expressed by the term ‘value added’ in school effectiveness and improvement research.

## What is meant by value added?

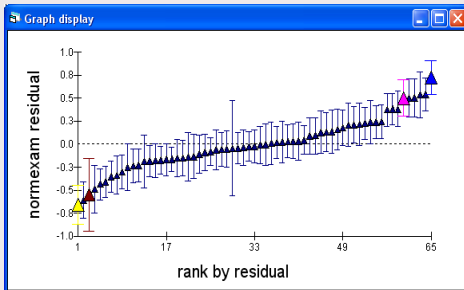
In this case, value added (or residual) for each school represents the differences between the *observed* level of school performance (pupil normexam scores taken at age 16) and what would be *expected* on the basis of pupils’ prior attainment (pupul standlt scores taken at age 11).

In other words “value added is a measure of the relative progress made by pupil in a school over a particular period of time (usually from entry to the school until public examinations in the case of secondary schools, or over particular years in primary schools – in this case, between age 11 and 16) in comparison to pupils in others schools in the same sample.”

(Thomas, 2005)

(See Thomas (2005) Using indicators of value added to evaluation school performance in UK. Educational Research Journal. 2005 September 2005. China National Institute of Educational Research: Beijing – in Chinese)

## Were some schools doing better than others?



- a positive value added score (i.e. residual) indicating a school may be performing above expectation.

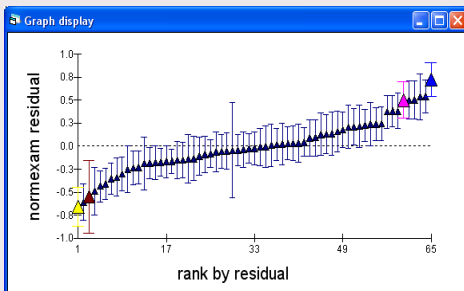
- a negative value added score indicating a school may be performing below expectation

“However, information about the 95% confidence interval (CI) is required to evaluate whether an individual school’s value added performance is likely to have occurred by chance.

In other words, the confidence interval is vital to judge whether a school’s performance above or below expectation is *statistically significant* .”

(Thomas, 2005)

## Were some schools doing better than others?



Information is also needed about the statistical uncertainty of performance measures when different schools are compared.

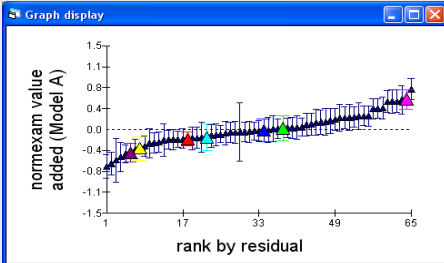
So...

Was School▲ doing better than school▲ ?

Was School▲ doing worse than school▲ ?

How about school▲ and school▲ ?

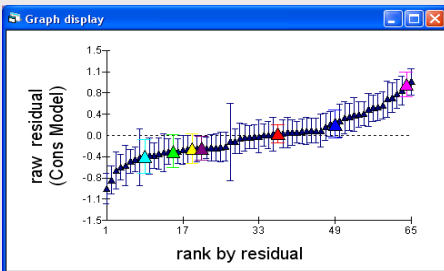
## Compare raw (Cons Model) and value added residuals



On average, how were these schools performing in their raw normexam and value added (VA) scores as compared to other schools?

As compared to other schools:

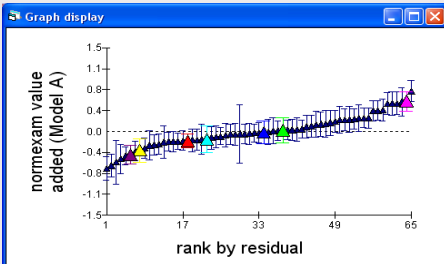
- ▲ performed higher than expected in both scores
- ▲ performed as expected in both scores
- ▲ performed lower than expected in both scores
- ▲ performed as expected in raw but lower than expected in VA
- how about other schools?



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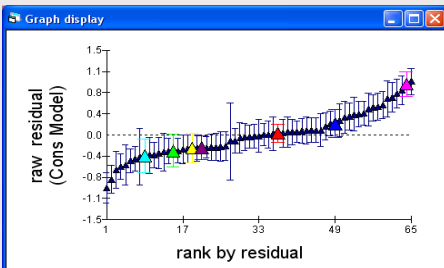


## Compare level 2 (and level 1) variance between two models



Variance between schools  
- which residual curve ~ is steeper  
- what the implication of it

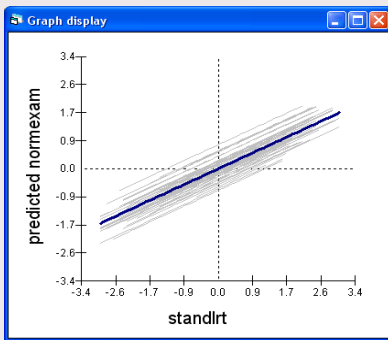
Variance within schools  
- the likely bounds (95%CI) of variation on schools for raw residuals wider than the ones for value added  
- What the implication of it



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## Fitting Model B - random intercepts/slopes model -



Graph of Model A - variance components model

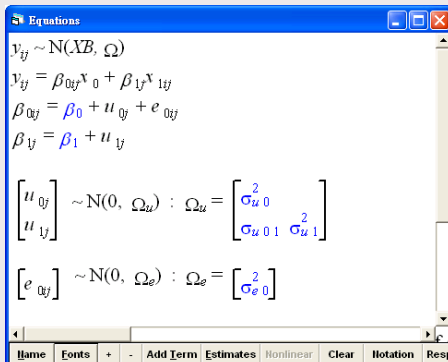
(<http://tramss.data-archive.ac.uk/documentation/MLwiN/chapter1.pdf>)

Model A which we have just specified and estimated assumes that the only variation between schools is in their intercepts. “However, there is a possibility that the school lines have different slopes. This implies that the coefficient of standlrt will vary from school to school.”

WJ Peng



## Specifying Model B



Click  $\beta_1$  to specify the coefficient of standlrt which is random at level 2.

“The terms  $u_{0j}$  and  $u_{1j}$  are random departures or ‘residuals’ at the school level from  $\beta_0$  and  $\beta_1$ . They allow the  $j$ ’th school’s summary line to differ from the average line in both its slope and its intercept.”

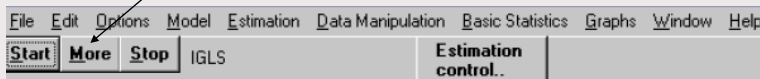
(<http://tramss.data-archive.ac.uk/documentation/MLwiN/chapter1.pdf>)

WJ Peng



## Specifying Model B

To fit this new model we could click Start as before, but it will probably be quicker to use the estimates already obtained from the 1<sup>st</sup> model as initial values for the iterative calculations. Click More.



(<http://tramss.data-archive.ac.uk/documentation/MLwiN/chapter1.pdf>)

WJ Peng



## Completion of the parameters estimation

	mean (SE)	variance (SE)
individual school slopes vary	0.557 (0.020)	0.015 (0.004)
school line intercepts vary	-0.012 (0.040)	0.090 (0.018)

(<http://tramss.data-archive.ac.uk/documentation/MLwiN/chapter1.pdf>)

WJ Peng



## Completion of the parameters estimation

```

Equations
normexamij ~ N(XB, Ω)
normexamij = β0jcons + β1jstandlrtij
β0j = -0.012(0.040) + u0j + e0j
β1j = 0.557(0.020) + u1j

[ u0j ] ~ N(0, Ωu) : Ωu = [ 0.090(0.018)
[ u1j ] ~ N(0, Ωu) : Ωu = [ 0.018(0.007) 0.015(0.004) ]

[ e0j ] ~ N(0, Ωe) : Ωe = [ 0.554(0.012) ]

-2*loglikelihood(IGLS Deviance) = 9316.870(4059 of 4059 cases in use)
    
```

“The positive covariance between intercepts and slopes estimated as +0.018 (SE = 0.007) suggests that schools with higher intercepts tend to some extent to have steeper slopes and this corresponds to a correlation between the intercept and slope (across schools) of 0.49.”

(<http://tramss.data-archive.ac.uk/documentation/MLwiN/chapter1.pdf>)

WJ Peng



## Completion of the parameters estimation

```

Equations
normexamij ~ N(XB, Ω)
normexamij = β0jcons + β1jstandlrtij
β0j = -0.012(0.040) + u0j + e0j
β1j = 0.557(0.020) + u1j

[ u0j ] ~ N(0, Ωu) : Ωu = [ 0.090(0.018)
[ u1j ] ~ N(0, Ωu) : Ωu = [ 0.018(0.007) 0.015(0.004) ]

[ e0j ] ~ N(0, Ωe) : Ωe = [ 0.554(0.012) ]

-2*loglikelihood(IGLS Deviance) = 9316.870(4059 of 4059 cases in use)
    
```

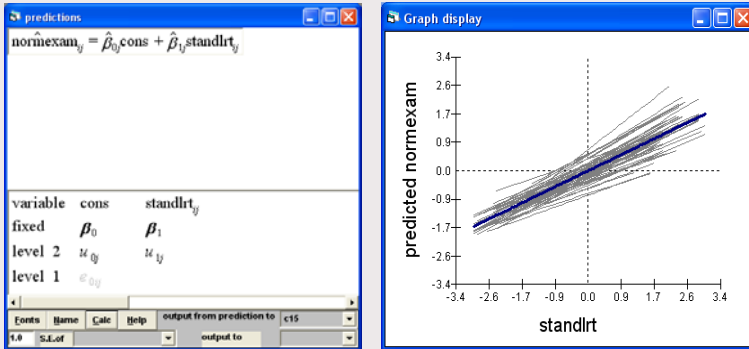
“The pupils' individual scores vary around their schools' lines by quantities  $e_{0ij}$ , the level 1 residuals, whose variance is estimated as 0.554 (SE = 0.012).”

(<http://tramss.data-archive.ac.uk/documentation/MLwiN/chapter1.pdf>)

WJ Peng

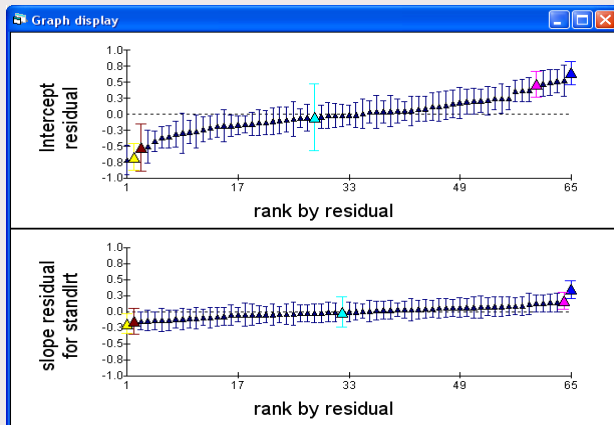


## Graphing prediction



The positive covariance between slopes and intercepts leading to a fanning out pattern when plotting the schools predicted lines (the average line = blue line)

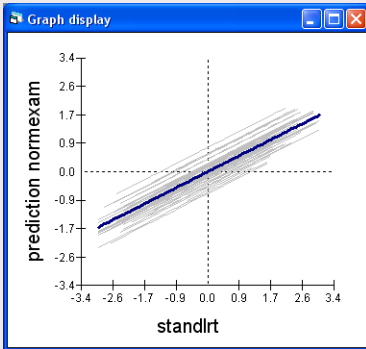
## Graphing residual



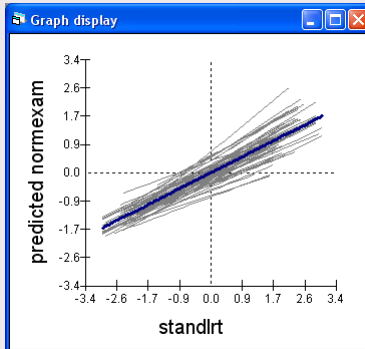
One residual plot for the intercepts of individual school lines  
One residual plot for the individual line slopes



## Compare the two models



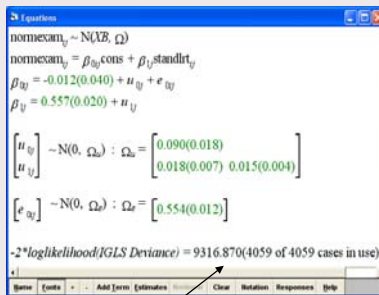
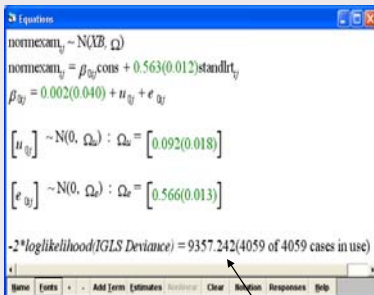
Model A



Model B

Which model is better fit?

## Which model was better fit?

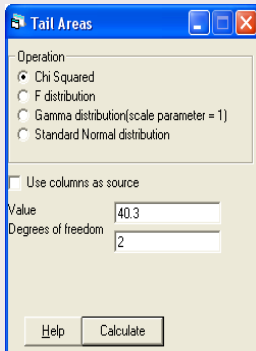


“A  $-2 \log$ -likelihood value is the probability of obtaining the observed data if the model were true and can be used in the comparison of two different models.”

(Rasbash, et al., 2005)

## Which model is better fit – the likelihood ratio test

Basic Statistics/Tail Areas



- the change of the two  $-2 \cdot \log$ -likelihood values  
 $9357.2 - 9316.9 = 40.3$ .
- the change in the  $-2 \log$ -likelihood value (which is also the change in deviance) has a chi-squared distribution on 2 degrees of freedom under the null hypothesis that the extra parameters have population values of zero.
- two extra parameters involved in the 2<sup>nd</sup> model
  - (1) the variance of the slope residuals  $u_{1j}$
  - (2) their covariance with the intercept residuals  $u_{0j}$ .

(Rasbash, et al., 2005)

The change is very highly significant, confirming the better fit of the 2<sup>nd</sup> model, a more elaborate model to the data.

WJ Peng



## Examples of other modelling

### Gender effects

- Do girls make more progress than boys? (F)
- Are boys more or less variable in their progress than girls? (R)

### Contextual effects

- Are pupils in key schools less variable in their progress? (R)
- Do pupils do better in urban schools (or key schools)? (F)
- Does gender gap vary across schools? (R)

### Cross-level interaction

- Do boys learn more effectively in a boys' or mixed sex school? (F)
- Do low ability pupils fare better when educated alongside higher ability pupils? (F)

(Jones, 2007; Rasbash, et al., 2005)

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## Examples of other hierarchical structures in education settings

Level 1	level 2	level 3	level 4
Pupils	classes	schools	
Pupils	schools	regions	
Pupils	schools	regions	countries
Pupils	neighbourhoods	schools	regions

**NB:**  
What happens if we have other types of data (eg ordered/unordered categorical data, binomial/multinomial data, repeated data) or non-hierarchical structure (eg pupils changing schools)?

## Useful references/links

**Getting start with the concept of value added in school effectiveness and improvement research:**

Thomas (2005) Using indicators of value added to evaluation school performance in UK. Educational Research Journal, September 2005, CNIER: Beijing. (translated into Chinese)

**Getting start with how to fit a model in MLwiN:**

Centre for Multilevel Modelling, Graduate School of Education,  
University of Bristol  
<http://www.cmm.bristol.ac.uk/MLwiN/index.shtml>

Teaching Resources and Materials for Social Scientists, ESRC  
<http://tramss.data-archive.ac.uk/documentation/MLwiN/what-is.asp>

MLwiN - Tutorials - Mozilla Firefox


File Edit View History Bookmarks Tools Help

http://tramss.data-archive.ac.uk/documentation/MLwiN/tutorials.asp

Windows Marketplace

Bristol University homepage... PLASC - Google Search MLwiN chapter1.pdf (application/p... MLwiN - Tutorials

## MLwiN - Tutorials



This page contains the detailed tutorials. These can be opened directly or downloaded.

**Educational example:**

- [Chapter 1](#) : Random intercept and random slope models
- [Chapter 2](#) : Residuals
- [Chapter 3](#) : Graphical procedures for exploring the model
- [Chapter 4](#) : Contextual effects
- [Chapter 5](#) : Variance Functions

**Mortality example:**

[View/download tutorial](#)

The tutorial files are in Acrobat \*.pdf format. You can read Acrobat files either after copying or downloading them, or directly within a suitable web browser. If you wish to view acrobat files from within a web browser then you will need Internet Explorer 3 or later or Netscape 3.0 or later. Please consult your browser documentation for configuration information. In either case you will need to install the free Reader (version 3.0 or later) on your computer.

- If you wish to have more information about Acrobat go to Adobe's web site <http://www.adobe.com/acrobat/> or go directly to <http://www.adobe.com/prodindex/acrobat/readstep.html> from where you will be able to download the reader.

If you wish to work through the tutorials on the example datasets with MLwiN , go to the [software download page](#).

**Next Section:** [Software](#) ▶

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## a software package for fitting multilevel models

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MLwiN was created by the Centre for Multilevel Modelling [team](#) with colleagues in other centres. MLwiN has benefited enormously from the input of numerous people. Michael Healy wrote the original version of NANOSTAT which formed the basis for MLn, a predecessor of MLwiN. Professor Bill Browne provided the foundations and coding of the MCMC features of the software. The (ESRC) have provided essential support over the years.

If you are considering purchasing MLwiN and would like to find out more about how the program works, why not download the [free training version](#) (Teaching Materials and Resources for Social Sciences) website. You can try out MLwiN on example data sets supplied with the program. (Note you will not be able to load your own data sets into this version).

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