Direct and Indirect Causal Effects: a helpful distinction?

Current real examples:

Anthrax vaccine:	immunogenicity; survival
Anti-epileptic drug:	PK (concentration); PD (response)
School choice:	attend private school; achievement
Job training:	get job; wages

"Effect" – often not causal, but sometimes is – be careful.

"Direct/Indirect" usually makes it even less clear.

If “effect” of treatment $W = 0$ vs. 1 on Y intended to be causal,

Must describe (hypothetical or actual) intervention with $W = 0$

vs. $W = 1$, and thereby define <u>Potential Outcomes</u> $Y(0)$ & $Y(1)$.

In my examples, W = getting vaccine, drug, scholarship, trained,

and Y = survival, PD, achievement, wages,

and X = covariates.

If you can do this, you can contemplate an <u>Assignment Mechanism</u>

$\Pr(W \mid X, Y(0), Y(1))$, a generally stochastic function.

If there are both "Direct" and "Indirect" causal effects,
Then must have two interventions, *W* as before,
And *V* = immunogenicity, PK, attend PS, get job.
These are "On the causal pathway" ... ??

Thus, must have, potential outcomes *Y(W,V)*, and compound assignment mechanism $\Pr(W \mid X, Y(-,-))\Pr(V \mid X, Y(-,-), W)$.

Can avoid compound assignment mechanism only by treating *V*, like *Y*, as an outcome and using Principal Stratification (Frangakis & Rubin, 2002).

Exclusion restrictions can play roles in both perspectives.

But, these are two different templates for how data are generated.

VERY easy to get confused when using direct/indirect jargon –
In my view, unhelpful for precise scientific inquiry.

Either: principal stratification – be explicit

or

compound assignment mechanism – be explicit

Even one of the “greats” (Fisher) was confused by this.

DOE 1st Ed. (1935, p. 169, Ch. IX §55)

8th Ed. (1966, p. 164-5)

In agricultural experiments involving the yield following different kinds of treatments, it may be apparent that the yields of the different plots have been much disturbed by variations in the number of plants which have established themselves. If we are satisfied that this variation in plant number is not itself an effect of the treatments being investigated, or if we are willing to confine our investigation to the effects on yield, excluding such as flow directly or indirectly from effects brought about by variations in plant number, then it will appear desirable to introduce into our comparisons a correction which makes allowance, at least approximately, for the variations in yield directly due to variation in plant number itself.

SM for RW 14th Ed. (1970, §49.1, pp. 283-4)

Thus, if we were concerned to study the effects of agricultural treatments upon the purity index of the sugar extracted from sugar-beet, a variate which might be much affected by concomitant variations in (a) sugar-percentage, and (b) root weight, an analysis of covariance applied to the three variates, purity, sugar percentage, and root weight, for the different plots of the experiment, would enable us to make a study of the effects of experimental treatments on purity alone; i.e., after allowance for any effect they may have on root weight or concentration, without our needing to have observed in fact any two plots agreeing exactly in both root weight and sugar percentage.

Figure 1. An Example With A Treatment Effect on the Concomitant, C, But No Treatment Effect on the Primary Outcome, Y.

Fraction of Population	Potential Outcomes				Observed Data		
	$C(1)$	$C(0)$	$Y(1)$	$Y(0)$	W	C_{obs}	Y_{obs}
1/4	3	2	10	10	0	2	10
1/4	3	2	10	10	1	3	10
1/4	4	3	12	12	0	3	12
1/4	4	3	12	12	1	4	12

Figure 2. An Example With A Constant Treatment Effect on the Outcome, Y, Even For Units With No Treatment Effect on the Concomitant, C.

Fraction of Population	Potential Outcomes				Observed Data		
	$C(1)$	$C(0)$	$Y(1)$	$Y(0)$	W	C_{obs}	Y_{obs}
1/6	3	2	11	10	0	2	10
1/6	3	2	11	10	1	3	11
1/6	3	3	13	12	0	3	12
1/6	3	3	13	12	1	3	13
1/6	4	3	15	14	0	3	14
1/6	4	3	15	14	1	4	15

Figure 1A. When $W = 0$, Randomize to $C_{obs} = 2, 3$ with Prob 1/2, 1/2

Fraction of Population	Potential Outcomes $Y(0,2)$	$Y(0,3)$	Observed Data W	C_{obs}	Y_{obs}
1/4	10	12	0	2	10
1/4	10	12	0	3	12

Figure 1B. When $W = 1$, Randomize to $C_{obs} = 3, 4$ with Prob 1/2, 1/2

Fraction of Population	Potential Outcomes $Y(1,3)$	$Y(1,4)$	Observed Data W	C_{obs}	Y_{obs}
1/4	10	12	1	3	10
1/4	10	12	1	4	12

Figure 1C. All Potential Outcomes – Averages

$Y(0,2)$	$Y(0,3)$	$Y(0,4)$	$Y(1,2)$	$Y(1,3)$	$Y(1,4)$
10	12	?14?	?8?	10	12

Figure 2A. When $W = 0$, Randomize to $C_{obs} = 2, 3$ with Prob 1/3, 2/3

Fraction of Population	Potential Outcomes $Y(0,2)$	$Y(0,3)$	Observed Data W	C_{obs}	Y_{obs}
1/6	10	13	0	2	10
1/6	10	12	0	3	12
1/6	10	14	0	3	14

Figure 2B. When $W = 1$, Randomize to $C_{obs} = 3, 4$ with Prob 2/3, 1/3

Fraction of Population	Potential Outcomes $Y(1,3)$	$Y(1,4)$	Observed Data W	C_{obs}	Y_{obs}
1/6	11	15	1	3	11
1/6	13	15	1	3	13
1/6	12	15	1	4	15

Figure 2C. All Potential Outcomes – Averages

$Y(0,2)$	$Y(0,3)$	$Y(0,4)$	$Y(1,2)$	$Y(1,3)$	$Y(1,4)$
10	13	?16?	?9?	12	15