University of Bristol

6-10 September 2010

Dynamics Days Europe 2010

Compiled and Edited by

Yuliya Kyrychko

Dynamics Days Europe 2010

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Local organising committee:

Yuliya Kyrychko (co-chair)
John Hogan (co-chair)
Mike Jeffrey
Alan Champneys
Kerrie Walker

Conference webpage: http://www.enm.bris.ac.uk/anm/ddays10/index.html

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Welcome

The Organising Committee would like to welcome you to the 30th Dynamics Days Europe which is a major annual international conference aimed at bringing together theoretical and experimental scientists from a variety of disciplines, including nonlinear dynamics, theoretical physics, fluid dynamics and mathematics.

Dynamics Days meetings are also held anually in the US, the Asia-Pacific region and, starting in 2010, in South America. See `www.dynamicsdays.info` for a complete history of the meetings.

The meeting is being hosted by the Applied Nonlinear Mathematics Group `www.enm.bris.ac.uk/anm` in the Department of Engineering Mathematics at the University of Bristol.

Bristol is an historic city with a strong maritime tradition. The historic port is unusual because the tidal range of River Avon that flows through the city is the largest in Europe. The modern port is now situated where the Avon meets the Severn estuary 4 miles to the northwest at Avonmouth. These days Bristol is known as a European centre for the aviation industry (with the original Bristol Aeroplane Company celebrating its centenary this year), the creative industries (e.g. Wallace and Gromit and just about any wildlife documentary you have ever seen are produced in Bristol), micro-electronics, insurance and renewable energy. There is also a strong cultural tradition in Bristol with the Bristol 'Old Vic' being the oldest continually operating theatre in England.

The University of Bristol `www.bris.ac.uk`, which celebrated its centenary in 2009, is known internationally for its excellence in both teaching and research. The University was the first in the UK to admit women students and was the founding partner of the Worldwide University Network consortium of Universities.

We hope that all participants find the conference stimulating and fruitful and enjoy their stay in Bristol.

Conference Location

All the conference activities will take place in the Victoria Rooms, University of Bristol, Queens Road, BS8 1SA UK.

In case of emergencies, during office hours the Victoria Rooms can be contacted through the University switchboard on +44 (0)117 928000. The University's Security Services' 24-hour emergency number is +44 (0)117 33 112233.

The registration desk is located in the lobby of Victoria Rooms and will be open from Sunday 5pm and Monday-Thursday from 8.30am until 11am. The registration fee includes a book of abstracts, tea/coffee breaks, welcome reception on Monday evening, conference excursion and a conference dinner. Other than on the Wednesday evening, participants are free to have lunch and dinner on their own and may wish to choose from the wide variety of restaurants and sandwich shops in the immediate vicinity of the Victoria Rooms. A map indicating many suggestions is included in the pack.

Scientific Programme

The meeting will begin at 10.15am on Monday 6th September 2010 and will finish at noon on Friday 10th September.

The programme will consist mostly of invited presentations, and invited mini-symposia. In keeping with the traditions of Dynamics Days to keep the number of parallel sessions to a minimum, there is space in the programme only for a few contributed talks. Most participants from students to full professors will present their work in one of two dedicated poster sessions.

In keeping with the informal spirit of previous Dynamics Days meetings, there will be no published proceedings. We hope though that participants make maximum use of the tea, coffee and lunch breaks and the dedicated breakout rooms for informal interactions and scientific discussions.

Social Programme

On Wednesday, 8th September 2010 there will be a choice of excursions to famous Bristol landmarks including Bristol Zoo, Bristol Botanical Gardens and the SS Great Britain. Coaches for the excursions will leave the Victoria Rooms at 2pm. Final numbers for each excursion will be required by lunchtime of Monday 6th September. Please finalise your choice at the reception desk.

The excursion will be followed by the conference dinner at Bristol Marriott City Centre starting at 7.30pm. Participants are encouraged to make their own way to the Marriott, directions to which are contained on a map enclosed in your pack.

Presentations

Plenary talks will take place in the main room of the Victoria Rooms, the Auditorium. The talks are 45 minutes plus 10 minutes for questions. Mini-symposium talks will be run in three parallel sessions, and will consist of four 25-minute presentations, with 5 minutes for questions. Contributed talks last 15 minutes plus five minutes for questions. If you have a talk scheduled, please arrive at least 15 minutes before the start of your session to set up your presentation. Everyone is asked to adhere to the timetable to ensure that all parallel session are run in synchrony.

Poster Sessions

There will be two dedicated poster sessions, on Tuesday 7th September 2010 and Thursday, 9th September both starting at 4pm.

Posters for the first session can be set up any time starting from 9am on Monday 6th September, but must be removed by the end of Tuesday. Posters for the second session can be set up any time starting from 9am on Wednesday 8th September, and must be removed by noon on Thursday. Any posters left after either of these times are liable to be destroyed.

If you are due to present a poster please note which of the two sessions you have been allocated, and make sure that you are present at your poster during the appropriate dedicated poster session.

Internet connection

Wireless Internet connection will be available throughout the conference in Auditorium, Victoria's Room, Theatre Bar and Lecture Room G12. The university is a member of the *eduroam* federation. Visitors from any organisation within the federation can access wireless at Bristol using their regular username and password from their home organisation. If you do not have *eduroam* account, please enquire at reception about a temporary account.

<table>
<tr><th colspan="4">Schedule</th></tr>
<tr><td></td><td>Monday,
6th September</td><td>Tuesday,
7th September</td><td>Wednesday
8th September</td></tr>
<tr><td>09:00
09:30</td><td>Registration Opens</td><td>I4 (A)
Jane Wang</td><td>I6 (A)
Reinhard Richter</td></tr>
<tr><td>10:00</td><td>Welcome and Opening Remarks (A)</td><td>Tea/coffee break (RR)</td><td>Tea/coffee break (RR)</td></tr>
<tr><td>10:30
11:00</td><td>I1 (A)
Ingo Fischer</td><td rowspan="2">M4 (LR) Cell Dynamics
M5 (VR) Molecular Dynamics
M6 (A) Nonlinear Optical Systems</td><td rowspan="2">M7 (LR) Traffic Dynamics
M8 (VR) Dynamics of Hearing
M9 (A) Open Quantum Systems</td></tr>
<tr><td>11:30
12:00</td><td>I2 (A)
Angela McLean</td></tr>
<tr><td>12:30
13:30</td><td>Lunch</td><td>Lunch</td><td>Lunch</td></tr>
<tr><td>14:00
14:30
15:00
15:30</td><td>M1 (A) Networks & Synchronization
M2 (VR) Sustainable Energy
M3 (LR) Stochastic Transport in Periodic Potentials</td><td>C5 (LR) Complex Networks
C6 (AB) Complex Systems in Biology
C7 (VR) Coupled Oscillators I
C8 (A) Dynamical Systems</td><td>Excursion</td></tr>
<tr><td>16:00</td><td>Tea/coffee break (RR)</td><td>Tea/coffee break (RR)</td><td></td></tr>
<tr><td>16:30
17:00</td><td>C1 (LR) Localised Patterns
C2 (AB) Quantum Chaos
C3 (VR) Time Series and Control
C4 (A) Chemistry</td><td>Poster Session I (RR)</td><td></td></tr>
<tr><td>17:30
18:00</td><td>I3 (A)
Michael Berry</td><td>I5 (A)
Christof Schütte</td><td></td></tr>
<tr><td>18:30</td><td rowspan="2">Welcome Reception</td><td rowspan="2"></td><td></td></tr>
<tr><td>19:00</td><td>Dinner (Marriott)</td></tr>
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A - Auditorium
LR - Lecture Room G12
AB - Albert's Bar
VR - Victoria's Room
RR - Recital Room

<table>
<tr><th colspan="6">Schedule</th></tr>
<tr><td></td><td colspan="4">Thursday,
9th September</td><td>Friday,
10th September</td></tr>
<tr><td>09:00</td><td colspan="4" rowspan="2">I7 (A)
Emily Shuckburgh</td><td rowspan="2">I9 (A)
Ute Ebert</td></tr>
<tr><td>09:30</td></tr>
<tr><td>10:00</td><td colspan="4">Tea/coffee break</td><td>Tea/coffee break (RR)</td></tr>
<tr><td>10:30</td><td rowspan="4">M10 (VR)
Wave
Chaos and
Chaotic
Transport</td><td rowspan="4">M11 (A)
Multiple
Time
Scales</td><td colspan="2" rowspan="4">M12 (LR)
Mathematical
Modelling
of Engineering
Applications</td><td rowspan="2">I10 (A)
Michelle Girvan</td></tr>
<tr><td>11:00</td></tr>
<tr><td>11:30</td><td rowspan="2">Award of prizes (A)
Closing Remarks (A)</td></tr>
<tr><td>12:00</td></tr>
<tr><td>12:30</td><td colspan="4" rowspan="2">Lunch</td><td rowspan="2">Lunch</td></tr>
<tr><td>13:30</td></tr>
<tr><td>14:00</td><td rowspan="4">C9 (LR)
Fluid
Dynamics</td><td rowspan="4">C10 (AB)
Coupled
Oscillators II</td><td rowspan="4">C11 (VR)
Micro-
scale
Systems</td><td rowspan="4">C12 (A)
Traffic
and
Granular
Flow</td><td rowspan="4"></td></tr>
<tr><td>14:30</td></tr>
<tr><td>15:00</td></tr>
<tr><td>15:30</td></tr>
<tr><td>16:00</td><td colspan="4">Tea/coffee break (RR)</td><td></td></tr>
<tr><td>16:30</td><td colspan="4" rowspan="3">Poster Session II (RR)</td><td rowspan="3"></td></tr>
<tr><td>17:00</td></tr>
<tr><td>17:30</td></tr>
<tr><td>18:00</td><td colspan="4" rowspan="2">I8 (A)
Panayotis Kevrekidis</td><td rowspan="2"></td></tr>
<tr><td>18:30</td></tr>
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Conference Programme

MONDAY, 6th September 2010

9:00-10:30	Registration and coffee

Plenary Talks (Auditorium)

10:30-11:30	*Dynamics and applications of delay-coupled systems*
I1	Ingo Fischer (Palma de Mallorca, Spain)
11:30-12:30	*Modelling the evolution and spread of immune escape in HIV*
I2	Angela McLean (Oxford, UK)
12:30-14:00	Lunch

M1 Mini-symposium: Networks and Synchronisation (Auditorium)
Organiser: Eckehard Schöll (TU Berlin, Germany)

14:00-14:30	*Dynamical properties of chimera states*
M1.1	Oleh Omel'chenko (WIAS, Berlin, Germany & Kiev, Ukraine)
14:30-15:00	*Chaos synchronization of networks with time-delayed couplings*
M1.2	Wolfgang Kinzel (Würzburg, Germany)
15:00-15:30	*Generalized stability criterion for synchronous states of nonlocally coupled phase oscillators with propagation delays*
M1.3	Gautam Sethia (Bhat, India)
15:30-16:00	*Clustering in network dynamics*
M1.4	Thomas Dahms (TU Berlin, Germany)

M2 Mini-symposium: Sustainable Energy (Victoria's Room)
Organiser: Peter Berg (University of Ontario, Canada)

14:00-14:30	*Endogenous economic growth and patenting*
M2.1	Andrew Pickering (Bristol, UK)
14:30-15:00	*Modeling and simulation of solar updraft and energy towers*
M2.2	I. Gasser (Hamburg, Germany)
15:00-15:30	*Dynamics of a wave energy converter and its affect to reliability and cost*
M2.3	L. Johanning (Exeter, UK)
15:30-16:00	*Nano-scale fluid mechanics of polymer electrolyte membranes in PEM fuel cells*
M2.4	Peter Berg (Oshawa, Canada)

M3 Mini-symposium: Stochastic Transport in Periodic Potentials (Lecture Room G12)
Organisers: Lutz Schimansky-Geier (HU Berlin, Germany) & Dirk Hennig (Portsmouth, UK)

10:30-11:00	*Avalanches in a nonlinear oscillator chain in a periodic potential*
M3.1	Dirk Hennig (Portsmouth, UK)
11:00-11:30	*Transport in energetic and entropic periodic potentials*
M3.2	S. Martens (HU Berlin, Germany)
11:30-12:00	*Diffusive transport in confined geometries*
M3.3	Gerhard Schmid (Augsburg, Germany)
12:00-12:30	*Dimer currents on one dimensional asymmetric substrates*
M3.4	S.E. Savel'ev (Loughborough, UK)

C1 Contributed Talks: Localised Patterns (Lecture Room G12)

16:30-16:50	*Single localized structures in a three-component reaction-diffsion system*
C1.1	S. Gurevich (Münster, Germany)
16:50-17:10	*Convection rolls in a at rotating box filled with a granular mixture*
C1.2	F. Rietz (Magdeburg, Germany)
17:10-17:30	*Solvable model of spiral wave chimeras*
C1.3	Erik A. Martens (Göttingen, Germany)

C2 Contributed Talks: Quantum Chaos (Albert's Bar)

16:30-16:50	*Ray model and ray-wave correpondence in coupled optical microdisks*
C2.1	Jung-Wan Ryu (Dresden, Germany)
16:50-17:10	*Chaos and the quantum: how nonlinear effects can explain quantum paradoxes*
C2.2	Wm. C. McHarris (Michigan, USA)
17:10-17:30	*Wave chaos in oceanic acoustic waveguides*
C2.3	D.V. Makarov (Vladivostok, Russia)

C3 Contributed Talks: Time Series and Control (Victoria's Room)

16:30-16:50	*Accelerating chaos control*
C3.1	Christian Bick (Göttingen, Germany)
16:50-17:10	*Superstatistical fluctuations in time series*
C3.2	Erik Van der Straeten (QMUL, London, UK)
17:10-17:30	*The n-level spectral correlations for chaotic systems*
C3.3	Sebastian Müller (Bristol, UK)

C4 Contributed Talks: Chemistry (Auditorium)

16:30-16:50	*A geometric model for mixed-mode oscillations in a chemical system with multiple time-scales*
C4.1	Chris Scheper (Cornell, USA)
16:50-17:10	*Progress of an auto catalytic reactions inside slugs in microchannels*
C4.2	S. Pushpavanam (Madras, India)
17:10-17:30	*Endex thermoreactive systems and application to carbon separation from fuels and flue gases*
C4.3	Rowena Ball (Canberra, Australia)

Plenary Talk (Auditorium)

17:30-18:30	*Tsunami asymptotics*
I3	Michael Berry (Bristol, UK)
18:30-20:00	**Welcome Reception**

TUESDAY, 7th September 2010

Plenary Talk (Auditorium)

9:00-10:00	*How Insects Fly and Turn*
I4	Jane Wang (Cornell, USA)
10:00-10:30	Tea/coffee break

M4 Mini-symposium: Cell Dynamics (Lecture Room G12)
Organisers: Diego di Bernardo (TIGEM, Italy) & Mario di Bernardo (Bristol, UK)

10:30-11:00	*Modelling gene networks in systems and synthetic biology*
M4.1	Diego di Bernardo (Naples, Italy)
11:00-11:30	*Oscillations in CDC14 release and sequestration reveal the circuit underlying mitotic exit*
M4.2	Andrea Ciliberto (Milan, Italy)
11:30-12:00	*Modelling and analysis tools for biochemical networks*
M4.3	A. Papachristodoulou (Oxford, UK)
12:00-12:30	*Development and implementation of a modular strategy for gene regulatory network control*
M4.4	Filippo Menolascina (Naples, Italy)

M5 Mini-symposium: Molecular Dynamics (Victoria's Room)
Organisers: Ben Leimkuhler (Edinburgh, UK) & Frédéric Legoll (ENPC, France)

10:30-11:00	*Effective dynamics for the overdamped Langevin equation*
M5.1	Frédéric Legoll (ENPC, France)
11:00-11:30	*On the efficiency of temperature controls for molecular dynamics*
M5.2	B. Leimkuhler (Edinburgh, UK)
11:30-12:00	*Covariant tangent-space dynamics for simple particle systems*
M5.3	Harald A. Posch (Vienna, Austria)
12:00-12:30	*Shadowing the trajectories of molecular dynamics*
M5.4	P.F. Tupper (Simon Fraser, Canada)

M6 Mini-symposium: Nonlinear Optical Systems (Auditorium)
Organiser: Sebastian Wieczorek (Exeter, UK)

10:30-11:00 *Complex dynamics in two-mode semiconductor lasers*
M6.1 A. Amann (Cork, Ireland)

11:00-11:30 *Nonlinear dynamics of semiconductor microring lasers*
M6.2 G. Van der Sande (Brussels, Belgium)

11:30-12:00 *Frequency and phase locking in a cavity soliton laser*
M6.3 T. Ackemann (Glasgow, UK)

12:00-12:30 *Noise synchronisation and stochastic bifurcations in lasers*
M6.4 Sebastian Wieczorek (Exeter, UK)

12:30-14:00 Lunch

C5 Contributed Talks: Complex Networks (Lecture Room G12)

14:00-14:20 *Experimental analysis and synchronization of networks of Chua circuits*
C5.1 Marco Colandrea (Naples, Italy)

14:20-14:40 *Determination of the scale of coarse graining in complex network of earthquake*
C5.2 Norikazu Suzuki (Chiba, Japan)

14:40-15:00 *Systematic network analysis*
C5.3 Sumeet Agarwal (Oxford, UK)

15:00-15:20 *The price of fair flows in complex networks*
C5.4 Rui Carvalho (QMUL, London, UK)

15:20-15:40 *Information transport on complex networks*
C5.5 A. Igarashi (Kyoto, Japan)

15:40-16:00 *Localization of Laplacian eigenvectors on complex networks*
C5.6 Hiroya Nakao (Kyoto, Japan)

C6 Contributed Talks: Complex Systems in Biology (Albert's Bar)

14:00-14:20 *Analyzing the influence of the neural network topology on pattern formation*
C6.1 J. Starke (Lyngby, Denmark)

14:20-14:40 *Motion control of groups of Artemias*
C6.2 L. Fortuna (Catania, Italy)

14:40-15:00 *Stochastic effects in a two-component signalling system*
C6.3 R.B. Hoyle (Surrey, UK)

15:00-15:20 *Insights into the evolution and emergence of a novel infectious disease*
C6.4 Ruben J Kubiak (Oxford, UK)

15:20-15:40 *A simple model for spike-burst dynamics of neurons*
C6.5 D.Roy (Marseille, France)

15:40-16:00 *Modeling molecular motors: Myosin-V processivity and the framework for comparison of stepping models*
C6.6 N. J. Boon (Surrey, UK)

C7 Contributed Talks: Coupled Oscillators (Victoria's Room)

14:00-14:20 *Routes to complex dynamics in a ring of unidirectionally coupled systems*
C7.1 M. Wolfrum (WIAS, Berlin, Germany)

14:20-14:40 *Modelling and global bifurcations of coupled laser arrays*
C7.2 H. Erzgräber (Exeter, UK)

14:40-15:00 *Dynamical response and phase modes of two ratchets coupled through a dynamic environment*
C7.3 U. E. Vincent (Lancaster, UK)

15:00-15:20 *Ratcheting effect for coupled phase oscillators*
C7.4 Özkan Karabacak (Exeter, UK)

15:20-15:40 *Tailoring current reversal within ratchet systems driven by chaotic noise*
C7.5 L.Y. Chew (Singapore)

15:40-16:00 *Semiconductor laser subject to feedback from two external filters*
C7.6 P. Słowiński (Bristol, UK)

C8 Contributed Talks: Dynamical Systems (Auditorium)

14:00-14:20 *Paradoxes of dissipation-induced destabilization or who opened Whitney's umbrella?*
C8.1 O.N. Kirillov (Darmstadt, Germany)

14:20-14:40 *Multistability and multimode dynamics in lasers*
C8.2 A. Pérez-Serrano (Palma de Mallorca, Spain)

14:40-15:00 *Estimation of the Kolmogorov entropy in the generalized number system*
C8.3 A. Fülöp (Budapest, Hungary)

15:00-15:20 *Interference-induced tunneling emission in deformed micro-cavities*
C8.4 S.-Y. Lee (Seoul, Korea)

15:20-15:40 *Towards stabilization of odd-number orbits in experiments*
C8.5 V. Flunkert (Berlin, Germany)

15:40-16:00 *The dynamics of a simplified pin-ball machine*
C8.6 Karin Mora (Bath, UK)

16:00-16:30 Tea/coffee break

16:30-17:30 Poster Session

Plenary Talk (Auditorium)
17:30-18:30 *Markov state models for complex systems*
I5 Christof Schütte (FU Berlin, Germany)

WEDNESDAY, 8th September 2010

Plenary Talk (Auditorium)

9:00-10:00	*On growth and form of ferrofluids*
I6	Reinhard Richter (Bayreuth, Germany)
10:00-10:30	Tea/coffee break

M7 Mini-symposium: Traffic Dynamics (Lecture Room G12)
Organiser: Gábor Orosz (UCSB, USA)

10:30-11:00	*Dynamical phenomena induced by bottleneck*
M7.1	I. Gasser (Hamburg, Germany)
11:00-11:30	*A mechanism to describe the formation and propagation of stop-and-go waves in congested freeway traffic*
M7.2	Jorge A. Laval (Georgia Tech, USA)
11:30-12:00	*Traffic jams - dynamics and control*
M7.3	Gábor Orosz (California, USA)
12:00-12:30	*On the nature of traffic instabilities - what traffic data and theory can tell us*
M7.4	M. Treiber (Dresden, Germany)

M8 Mini-symposium: Dynamics of Hearing (Victoria's Room)
Organiser: Daniele Avitabile (Bristol, UK)

10:30-11:00	*Transduction, Adaptation and Amplification in the Drosophila ear: Theoretical and experimental approaches to the operation of an active sense organ*
M8.1	Joerg T. Albert (UCL, UK)
11:00-11:30	*The neural basis of active hearing in the mosquito* Aedes aegypti*: experiments and modelling*
M8.2	Katie Lucas (Bristol, UK)
11:30-12:00	*Mechanical preprocessing of sound in the mammalian cochlea*
M8.3	Nigel P. Cooper (Keele, UK)
12:00-12:30	*Spatial and temporal delays in the inner ear*
M8.4	Robert Szalai (Bristol, UK)
16:00-16:30	**Tea/coffee break**

M9 Mini-symposium: Open Quantum Systems (Auditorium)
Organiser: Henning Schomerus (Lancaster, UK)

10:30-11:00	*Quantum Resonant States in Open Chaotic Systems*
M9.1	J.P. Keating (Bristol, UK)
11:00-11:30	*Resonances states in open chaotic cavities*
M9.2	F. Mortessagne (Nice, France)
11:30-12:00	*Wave intensity distributions in complex structures*
M9.3	Gregor Tanner (Nottingham, UK)
12:00-12:30	*Classical trajectory correlations and their semiclassical consequences*
M9.4	Jack Kuipers (Regensburg, Germany)
12:30-14:00	Lunch
14:00-18:30	Conference Excursions
19:30-	Conference Dinner (Bristol Marriott City Centre)

THURSDAY, 9th September 2010

Plenary Talk (Auditorium)

9:00-10:00	*Transport, mixing and dynamics of the atmosphere and oceans*
I7	Emily Shuckburgh (Cambridge, UK)
10:00-10:30	Tea/coffee break

M10 Mini-symposium: Wave Chaos and Chaotic Transport (Victoria's Room)

Organiser: Klaus Richter (Regensburg, Germany)

10:30-11:00	*Rogue waves: refraction of Gaussian seas and rare event statistics*
M10.1	L. Kaplan (New Orleans, USA)
11:00-11:30	*Dynamical tunneling in systems with a mixed phase space*
M10.2	A.Bäcker (TU Dresden, Germany)
11:30-12:00	*Distinguishing quantum and classical transport through nanostructures*
M10.3	C. Emary (TU Berlin, Germany)
12:00-12:30	*Unidirectional light emission along unstable manifolds in deformed microdisks*
M10.4	J. Wiersig (Magdeburg, Germany)

M11 Mini-symposium: Multiple Time-Scales Behaviour (Auditorium)

Organisers: Mike Jeffrey & Mathieu Desroches (Bristol, UK)

10:30-11:00	*Singular perturbations of piecewise smooth systems and a simple ocean circulation model*
M11.1	Paul Glendinning (Manchester, UK)
11:00-11:30	*Flow Curvature Method for Canard Computation*
M11.2	J.M. Ginoux (France)
11:30-12:00	*Canards and bifurcation delays in reaction-diffusion equations*
M11.3	Peter De Maesschalck (Hasselt, Belgium)
12:00-12:30	*Canard a l'orange: a new recipe for multiple time scales*
M11.4	Mike Jeffrey & Mathieu Desroches (Bristol, UK)

M12 Mathematical Modelling of Engineering Applications (Lecture Room G12)
Organisers: Yuliya Kyrychko & John Hogan (Bristol, UK)

10:30-11:00	*Stabilization with time-periodic delayed feedback of higher derivatives*
M12.1	Gabor Stepan (Budapest University of Technology and Economics, Hungary)
11:00-11:30	*Emergent scaling laws in dielectric materials*
M12.2	Chris Budd (University of Bath, UK)
11:30-12:00	*The Effects of Bathymetry Variations on Two Dimensional Tidal Flow*
M12.3	Steven Parkinson (University of Bristol, UK)
12:00-12:30	*Application of non-smooth dynamics to mechanical engineering*
M12.4	Marian Wiercigroch (University of Aberdeen, UK)
12:30-14:00	Lunch

C9 Contributed Talks: Fluid Dynamics (Lecture Room G12)

14:00-14:20	*Non equilibrium phase transitions and invariant measures in the 2D Navier-Stokes equations and in other systems with long range interactions*
C9.1	F. Bouchet (Los Alamos, USA) & (Lyon, France)
14:20-14:40	*Hydrodynamics of anisotropic fluids in 2D: Experiments and modeling of flow in thin freely suspended films*
C9.2	R. Stannarius (Magdeburg, Germany)
14:40-15:00	*Oscillations of soap bubbles: experiments and numerical analysis*
C9.3	K. Harth (Magdeburg, Germany)
15:00-15:20	*Dissolution dynamics of slowly miscible liquids*
C9.4	A. Vorobev (Southampton, UK)
15:20-15:40	*Particle self-ordering in periodic flows*
C9.5	D.O. Pushkin (Brussels, Belgium)
15:40-16:00	*Transition to turbulence for flows without linear instability*
C9.6	M. Nagata (Kyoto, Japan)

C10 Contributed Talks: Coupled Oscillators II (Albert's Bar)

14:00-14:20	*Localised Patterns in Continuum Limits of Coupled-Cell Systems*
C10.1	J.H.P. Dawes (Bath, UK)
14:20-14:40	*Chaotic behaviour of identical phase oscillators with full permutation symmetry*
C10.2	P. Ashwin (Exeter, UK)
14:40-15:00	*Coherence and reliability of noisy oscillators with delayed feedback*
C10.3	D.S. Goldobin (Leicester, UK)
15:00-15:20	*Collective phase diffusion and temporal precision in networks of noisy oscillators*
C10.4	H. Kori (Tokyo, Japan)
15:20-15:40	*On synchronization analysis in networks with delay*
C10.5	P. Hövel (TU Berlin, Germany)
15:40-16:00	*Delay and symmetry in coupled oscillator networks*
C10.6	O. D'Huys (Brussels, Belgium)

C11 Contributed Talks: Micro-scale Systems (Victoria's Room)

14:00-14:20	*Nonlinear dynamics of the internal degrees of freedom and transport of benzene on graphite*
C11.1	Astrid S. de Wijn (Nijmegen, the Netherlands)
14:20-14:40	*Chaotic scattering in astrodynamics*
C11.2	Tamás Kovács (Dresden, Germany)
14:40-15:00	*The dynamics of quantum equilibration*
C11.3	Anthony J. Short (Cambridge, UK)
15:00-15:20	*Phase transitions induced by microscopic disorder*
C11.4	Niko Komin (Palma de Mallorca, Spain)
15:20-15:40	*Variational Formulation for the KPZ Equation: Consistency, Galilean-invariance violation, and fluctuation-dissipation issues in real-space discretization*
C11.5	H.S. Wio (Cantabria, Spain)
15:40-16:00	*Asymptotics of work distributions in driven non-equilibrium systems*
C11.6	D. Nickelsen (Oldenburg, Germany)

C12 Contributed Talks: Traffic and Granular Flow (Auditorium)

14:00-14:20	*Mixing by cutting and shuffling in 3D tumbled granular flows*
C12.1	R. Sturman (Leeds, UK)
14:20-14:40	*3D aspects of mixing and transport in tumbled granular flow*
C12.2	I.C. Christov (Northwestern, USA)
14:40-15:00	*Chaotic advection of inertial particles in gravitational field*
C12.3	Gábor Drótos (Budapest, Hungary)
15:00-15:20	*Air traffic Complexity: a dynamical system approach*
C12.4	S. Puechmorel (ENAC, France)
15:20-15:40	*Few new models of crowd dynamics*
C12.5	M.G. Sadovsky (Krasnoyarsk, Russia)
15:40-16:00	*Characterization of the nontrivial and chaotic behavior that occurs in a simple bus traffic model*
C12.6	J. Villalobos (Bogotá, Colombia)
16:00-16:30	Tea/coffee break
16:30-17:30	Poster Session II (Recital Room)
Plenary Talk (Auditorium)	
17:30-18:30	*Dark solitons and vortices in Bose-Einstein condensates: oscillations and precessions, dynamics and interactions in an ultracold world*
I8	Panayotis Kevrekidis (Massachusetts, USA)

FRIDAY, 10th September 2010

Plenary Talk (Auditorium)	
9:00-10:00	*The multiscale dynamics of sprites and lightning*
I9	Ute Ebert (Amsterdam, the Netherlands)
10:00-10:30	Tea/coffee break
Plenary Talk (Auditorium)	
10:30-11:30	*Effects of network structure in simple models of gene regulation*
I10	Michelle Girvan (Maryland, USA)
11:30-12:00	Award of Prizes & Closing Remarks (Auditorium)

Monday, 6th September 2010

Plenary Talk I1 (Auditorium)

Dynamics and Applications of Delay-Coupled Systems

I1
10:30

Ingo Fischer*

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Oscillators being coupled with finite coupling delays often exhibit dynamical instabilities. This has for the first time been identified about 50 years ago. Since then delay-instabilities have been recognized to play an important role in as diverse systems as semiconductor lasers, brain activity, traffic systems and more. In addition, delayed coupling gives rise to particular synchronization properties. In delay-coupled laser systems, generalized leader-laggard type synchronization and zero-lag (identical) synchronization have been demonstrated in experiments and modeling.

In this talk we review the particular dynamical and synchronization properties of delay-coupled systems and show how they can be utilized for functional purposes. We demonstrate how synchronization without correlation can be achieved and how this can be utilized for key exchange. We discuss how the delay dynamics of semiconductor lasers can be employed to generate random bit sequences at high bit rates. Finally, we show, how delay dynamics can even be utilized to perform information processing, by realizing the reservoir computing concept.

Plenary Talk I2 (Auditorium)

Modelling the evolution and spread of immune escape in HIV

I2
11:30

Angela McLean[1]*, Helen Fryer[1], John Frate[1], Rodney Phillips[1]

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HIV evolves so quickly that it can be seen to adapt to its environment within one infected person. Evolutionary escape from immunity is particularly well-described. Immune escape variants can be transmitted to new hosts, where, if costly, they may revert. Here we present a mathematical model of the three processes of within-host evolution of escape mutants, transmission of those variants between hosts and subsequent reversion in new hosts of costly mutations. With this model we reconcile a number of diverse data-sets on HIV immune escape, highlighting where multiple data sources agree on the underlying rate processes at play, and where they disagree. The several-dozen immune epitopes we survey reveal a relatively sedate rate of evolution with average rates of escape measured in years and reversion in decades.

We explore the implications of these findings for the durability of a putative anti-HIV vaccine.

Mini-symposium M1: Networks and Synchronization (Auditorium)

Organiser: Eckehard Schöll (TU Berlin, Germany)

Dynamical properties of chimera states

M1.1
14:00

O.E. Omel'chenko[1]*, M. Wolfrum[1], Y.L. Maistrenko[2]

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Chimera states are a recently new discovered dynamical phenomenon displaying a self-organized spatially intermittent pattern of regions with coherent and incoherent motions [1, 2, 3]. They are generally observed in systems of non-locally coupled oscillators, also in the case when the system possesses a high degree of symmetry, i.e. all oscillators are identical, organized on a circle or on a torus, and the coupling is identical for all of them. Such type of behavior constitutes a new paradigm of dynamical theory as it shows that structured dynamical patterns can emerge from otherwise structureless networks.

Most of previous studies considered chimera states in the thermodynamic limit (when the number of coupled oscillators is infinite) only and used as the main tool a refined version of Kuramoto's self-consistency approach [1, 4]. However, it turns out that many important dynamical features of chimera states remain uncovered in this way. They originate from the discrete nature of the medium where chimera states "live", and can be revealed on the level of finite size effects only. In this presentation we are going to describe such new dynamical features of chimera states and explain their scaling properties and dependence on the coupling range and other system parameters.

We base our analysis on a detailed numerical study of the chimera-like dynamics in a ring of N identical non-locally coupled phase oscillators similar to the model used in [1, 2]. In particular, our numerical observations certainly indicate that chimera states are not chaotic attractors but chaotic transients only. However, the lifetime of such transients increases exponentially as the system size N grows leading to the neutral stability of chimera states in the thermodynamic limit ($N \to \infty$).

[1] Y. Kuramoto and D. Battogtokh, *Nonlinear Phenom. Complex Syst.* **5**, 380 (2002).

[2] D.M. Abrams and S.H. Strogatz, *Phys. Rev. Lett.* **93**, 174102 (2004).

[3] A.E. Motter, *Nature Physics* **6**, 164 (2010).

[4] E. Ott and T.M. Antonsen, *Chaos* **18**, 037113 (2008).

Chaos synchronization of networks with time-delayed couplings

M1.2
14:30

Wolfgang Kinzel[1]*, Ido Kanter[2]

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Several chaotic units interacting with time-delayed bi-directional couplings can synchronize completely. Although the signals are transmitted with arbitrary long delay time, chaos synchronization occurs without any time shift (zero lag synchronization, ZLS).

In this presentation we review our work on chaos synchronization. First we point out interesting applications of ZLS on fast random bit generators [1] with interacting chaotic semiconductor lasers, and on novel methods of secure communication on public channels [2]. Then we introduce analytic methods which allow to calculate phase diagrams of simple chaotic networks exactly [3], using the master stability function and the theory of polynomials. Predictions on ZLS of two units with multiple time-delayed couplings are supported by experiments on chaotic semiconductor lasers [1].

Finally we present analytic results for large networks. Two interacting chaotic networks of N units with all-to-all time-delayed interactions can synchronize only if they are connected by N couplings. This holds for $N \to \infty$, for finite N it is possible to synchronize two networks with a single coupling, only, if the chaos is sufficiently weak [5]. We show that it is possible to synchronize chaotic networks with uni-directional couplings, without feedback and with a single delay time. The configurations with optimal synchronization are found using methods of combinatorial optimization [6].

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[1] I. Kanter et al, An optical ultrafast random bit generator, *Nature Photonics* **4**, 58 (2010)

[2] I. Kanter, E. Kopelowitz, W. Kinzel, Public Channel Cryptography: Chaos Synchronization and Hilbert's Tenth Problem, *Phys. Rev. Lett.* **101**, 084102 (2008)

[3] J. Kestler, E. Kopelowitz, I. Kanter and W. Kinzel, Patterns of chaos synchronization, *Phys. Rev. E* **77**, 046209 (2008).

[4] A, Englert et al, Zero Lag Synchronization of Chaotic Systems with Time Delayed Couplings, *Phys. Rev. Lett.* **104**, 139902 (2010)

[5] M. Paulig, unpublished

[6] F. Geissler, unpublished

Generalized stability criterion for synchronous states of nonlocally coupled phase oscillators with propagation delays

M1.3
15:00

Gautam C Sethia[1]*, Abhijit Sen[1], Fatihcan M. Atay[2]

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Networks of oscillators with time delayed coupling have been found to be of great interest in recent times [1-9]. In the simplest models, the oscillators are described by their phases only, with amplitude variations neglected. We consider a continuum of identical phase oscillators, arranged on a circular ring, labeled by $x \in [-1, 1]$ with periodic boundary conditions, whose dynamics is governed by :

$$\frac{\partial}{\partial t}\phi(x,t) = \omega - \int_{-1}^{1} G(|z|) f\left(\phi(x,t) - \phi\left(x - z, t - |z|/v\right)\right) dz. \tag{1}$$

where $\phi(x, t) \in [0, 2\pi)$ is the phase of the oscillator at location x and time t, whose intrinsic oscillation frequency is $\omega > 0$, and $G : [-1, 1] \to \mathbb{R}$ is a normalized even function describing the coupling kernel. We also choose the interaction function f to be an odd one. The quantity v denotes the signal propagation speed which gives rise to a time delay of $|z|/v$ for distance $|z|$ from the locations x. Note that space and time are dimensionless in the above equation.

Our numerical results suggest a heuristic *necessary* (but not *sufficient*) condition for stability of the synchronous states to be :

$$\int_{-1}^{1} |z| G(|z|) f'\left(\Omega|z|/v\right) dz \ > \ 0 \tag{2}$$

and further a *sufficient* condition for the stability is given by :

$$\int_{-1}^{1} G(|z|) f'\left(\Omega|z|/v\right) \left[1 - \cos\left(\pi z\right)\right] dz > 0 \tag{3}$$

where $\Omega = \omega - \int_{-1}^{1} G(|z|) f\left(\Omega|z|/v\right)\, dz$ denotes the frequency of the synchronous states.

Our results show that the synchronous states become unstable via a saddle-node bifurcation process and the most unstable perturbation correspond to the lowest mode of the spatial perturbation on the ring of oscillators. The synchronization condition (Eq.3) turns out to be an intricate balance between the temporal and the spatial scales in the system.

[1] H. G. Schuster and P. Wagner, Prog. Theor. Phys., **81**, 929 (1989).

[2] E. Niebur, H. G. Schuster, and D. M. Kammen, Phys. Rev. Lett., **67**, 2753 (1991).

[3] S. Kim, S. H. Park and C. S. Ryu, Phys. Rev. Lett. 79, 2911 (1997).

[4] S. M. Crook, G. B. Ermentrout, M. C. Vanier, and J. M. Bower, J. Comput. Neurosci., **4**, 161 (1997).

[5] M. K. S. Yeung and S. H. Strogatz, Phys. Rev. Lett. 82, 648 (1999).

[6] D. H. Zanette, Phys. Rev. E, **62**, 3167 (2000).

[7] M. G. Earl and S.H. Strogatz, Phys. Rev. E, **67**,036204 (2003).

[8] T. K. Ko and G. B. Ermentrout , Phys. Rev. E, **76**, 056206 (2007).

[9] G. C. Sethia, A. Sen and F. M. Atay, Phys. Rev. Lett., **100**, 144103 (2008).

Clustering in network dynamics

M1.4
15:30

T. Dahms[1*], C.-U. Choe[2], J. Lehnert[3], P. Hövel[3], E. Schöll[3]

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[2] Department of Physics, University of Science, Unjong-District, Pyongyang, DPR Korea
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Recent work on synchronization in networks – including small motifs as well as complex topologies – has focused on isochronous or in-phase synchronization, where all nodes act identically at a given time. However, in a wide range of applications, nonidentical synchronization arises. We focus on cluster synchronization, where several subsets of a network are completely synchronized, but not the network as a whole. There may be a certain phase lag between these clusters, introducing an ordering of the nodes. We develop a generalization of the well-established master stability approach for nonidentical synchronization, which is partly based upon work by Sorrentino and Ott [1]. We extend this approach by introducing time delays into the coupling and show that this is crucial for the emergence of nontrivial dynamical scenarios.

Motivated by the range of applicability, we study several classes of dynamics occurring on networks: oscillatory, excitable, and chaotic, using Stuart-Landau oscillators, FitzHugh-Nagumo neurons, and Lang-Kobayashi semiconductor lasers, respectively.

The Stuart-Landau system is a paradigmatic oscillator model as it resembles any oscillatory system near a Hopf bifurcation. We show that for every delay-coupled network featuring a circulant adjacency matrix, cluster and splay states emerge, which can be controlled by a complex coupling parameter. By tuning its phase, we can deliberately switch between any of these states [2].

We observe similar cluster states in neural networks, whose nodes are modeled by the FitzHugh-Nagumo system. The parameters are chosen such that this neural network operates in the excitable regime, i.e., it exhibits no self-sustained, but coupling-induced oscillations. As it turns out, the dynamics on regular networks is highly multistable exhibiting isochronous synchronization as well as cluster dynamics. Among the cluster states, we observe multistability with respect to both the number of clusters and the phase lags between nodes in otherwise identical cluster states.

Finally, there is much interest in chaotic synchronization recently, especially in coupled semiconductor lasers. We show, on one hand, the emergence of isochronous and cluster synchronization, where several subgroups in the networks act isochronously. On the other hand, introducing a few different delays into the couplings enables two distant nodes to synchronize, while all other nodes are seemingly uncorrelated and do not lock into this dynamics.

[1] F. Sorrentino and E. Ott, *Network synchronization of groups*, Phys. Rev. E **76**, 056114 (2007).

[2] C.-U. Choe, T. Dahms, P. Hövel, and E. Schöll, *Controlling synchrony by delay coupling in networks: from in-phase to splay and cluster states*, Phys. Rev. E **81**, 025205(R) (2010).

Mini-symposium M2: Sustainable Energy (Victoria's Room)

Organiser: Peter Berg (University of Ontario, Canada)

Endogenous Economic Growth and Patenting

M2.1
14:00

Andrew Pickering[1] *

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This paper examines optimal patent length in a dynamic economic model of endogenous growth. Models of endogenous growth typically exhibit economies of scale at the aggregate level, enabled both directly through research and development (R&D) and indirectly through productive externalities. Through replication, innovation and specialisation these mechanisms can explain the sustained continuous growth of global GDP observed in the past two centuries.

Patent length has two opposing effects on the drivers of endogenous growth. As patent lengths are increased the benefits to innovation are internalised, allowing the monopoly rents to accrue to the innovator. The incentives to innovate are sharpened and R&D is increased to the limit at which the marginal returns are equalised with those of the production economy. Furthermore, as the stock of knowledge increases the capacity to bring forward future discovery is enhanced.

On the other hand rigorously enforced patenting entails monopoly, and its attendent welfare loss. Additionally and especially important in a model of economic growth it that inhibits replication. Other firms, or indeed low income countries, cannot replicate the new technology and hence the capacity to develop through 'learning by doing', or other mechanisms through which positive productive externalities are generated, are retarded. Given a plausibile parameterisation of the benchmark model it is found that for an objective of maximising economic growth, then the optimal balancing of these opposing forces implies patent lengths of 10-15 years. These findings are in the 'ball park' of actual practice, though on the whole argue for a slight weakening of the current practice of 20 years.

However, when the objective instead is to maximise global economic welfare, as captured in a conventional utility maximisation problem, then for an income distribution as observed in the world today these numbers are approximately halved. When financial transfers are politically or technically infeasible (these in theory would be the simplest way of maximising welfare), as often argued in the literature on aid effectiveness, then allowing replication can be an effective means of improving living standards in low income countries. The model thus presents a welfare case for a weakening in patenting law.

Modeling and simulation of solar updraft and energy towers

M2.2
14:30

I. Gasser[1*], M. Bauer[2]

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We present a new modeling approach for the description of the gasdynamics in both solar updraft and energy towers. The models are derived from the full Euler equations of gas dynamics. After scaling we perform a small Mach number asymptotics. We simulate the final models for various situations and compare them to measured data. The results underline the validity of this modeling approach.

[1] I. Gasser, Modelling and Simulation of a Solar Updraft Tower, *Kinetic and Related Models KRM*, **2**, 191-204 (2009).

[2] M. Bauer and I. Gasser, Modeling and simulation of an energy tower, *submitted* (2010).

Dynamics of a Wave Energy Converter and its affect to reliability and cost

M2.3
15:00

Dr. L. Johanning[1*]

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Resonance modes of floating wave energy converters (WECs) can be assumed to be close to excitation modes as a result of the body size and its application to produce power from first order waves. In the case for moored WEC devices additional complexity is given through the analysis requirement of the coupled system. The implications this can have to the design of WEC devices are manifold and hence a good understanding of the dynamic response of the floating system is required. This is because the reliability, design performance, expected lifetime and subsequently (cost-) optimisation of WEC devices are directly linked to the prediction of the load and response characteristics.

Often for the response and load analysis of typical oil and gas offshore structures a 'quasi'-static approach can be applied, since such floating devices are designed well off the resonance mode. Whilst numerical simulation models are improving steadily, allowing for a dynamic analysis of floating structures, these models often rely on input parameters such as damping characteristics. An adequate prediction of the external loads and the resulting response and tension characteristics of a floating system, moored in a realistic environment, is dependent on how accurately the coupled interrelation between these can be calculated. For a fully dynamic analysis variation in stiffness and damping characteristics needs to be identified to inform the numerical models. To identify the correct response of a coupled moored WEC device, the equation of motion has not only to consider the load and response behaviour of the structure, but in addition the influence of the mooring system and

the power-take-off (PTO) system.

For mooring applications where the station keeping of a structure cannot be considered to be in a 'static' mooring regime, non-linear mooring behaviour contributes importantly to the tension characteristics of the moor. As discussed by Mavrakos et.al. (1996) this became a great concern for deep sea mooring installations, where the elastic stiffness of the cable becomes the principal parameter affecting the mooring line's dynamic response and, hence, it's non-linear behaviour. Non-linear mooring behaviour has been addressed in several publications (Papazoglou et.al. 1990, Webster 1995, Raajimakers and Battjes 1997, Aranha and Pinto 2001, Johanning et.al. 2007). One of the most important factors governing the non-linear response is the role of viscous energy dissipation, when either the amplitude of displacement or the frequency of the top-end motion (induced by the attached device) increases. Considering a coupled moored system the contribution of non-linear response not only influences importantly the tension characteristics of the lines, but also the mooring reactance and resistance in form of stiffness and damping respectively and, hence, the motion characteristics of the floating structure and component reliability.

Within this presentation the need for further investigation of parameters related to the coupled dynamic behaviour of moored WECs devices is discussed. Work already undertaken to inform numerical models and future research will be discussed. It will be argued how this information can be used to enhance the designs of WECs to support reliable and cost effective installations. Finally, the research of the Exeter-PRIMaRE- research group will be presented, identifying their research methodology to inform the marine renewable energy.

Nano-scale Fluid Mechanics of Polymer Electrolyte Membranes in PEM Fuel Cells

M2.4
15:30

P. Berg[1*]

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Fuel cells are generally expected to provide a cornerstone of future energy conversion technologies. However, many scientific and engineering challenges remain, both at the applied and fundamental level. Regarding the mathematical modelling of these devices, the vast range of time and length scales within the system pose perhaps the largest obstacle to a comprehensive description of the physical phenomena, typically resulting in stiff equations.

As an example for the small scale, in this talk we will explore a continuum modelling approach to investigate proton and water transport in nano-pores of polymer electrolyte membranes (PEM), including limits to its applicability. Our focus will be on Poisson-Nernst-Planck (PNP) equations, coupled to Stokes flow, which entail system parameters and boundary conditions that need to be chosen carefully (e.g. slip vs. no-slip conditions).

We will discuss analytical solutions of simple geometries [1] as well as numerical solutions pertaining to general geometries. Protonic conductivity and water drag coefficients are the key experimental quantities that need to be matched by such micro-simulations.

[1] P. Berg and K. Ladipo, *Proc. Roy. Soc. A* **465**, 2663-2679 (2009).

Mini-symposium M3: Stochastic Transport in Periodic Potentials (Lecture Room G12)

Organisers: Lutz Schimansky-Geier (HU Berlin, Germany) & Dirk Hennig (University of Portsmouth, UK)

Avalanches in a nonlinear oscillator chain in a periodic potential

M3.1
14:00

Dirk Hennig[1]*

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We consider the dynamics of a chain of coupled units evolving in a periodic substrate potential. The chain is initially in a flat state and situated in a potential well. A bias force, acting as a weak driving mechanism, is applied at a single unit of the chain. We study the instigation of directed transport in two types of system: (i) a microcanonical situation associated with deterministic and conservative dynamics and (ii) the Langevin dynamics when the system is in contact with a heat bath. Interestingly, for the deterministic and conservative dynamics the directed transport is drastically enhanced compared with its Langevin counterpart. In particular, in the deterministic and conservative regime a self-organised redistribution of energy triggers huge-sized avalanches yielding ultimately accelerated transport of the chain. In contrast, in the thermally-assisted process between avalanches the chain settles always into a pinned metastable state impeding continual accelerated chain motion.

Transport in energetic and entropic periodic potentials

M3.2
14:30

S. Martens[1]*, L. Schimansky-Geier[1], D. Hennig[2], G. Schmid[3], P. Hänggi[3]

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Transport phenomena play a fundamental role in many physical systems. For example, the dynamics of Josephson tunneling junctions, diffusion of atoms and molecules on crystal surfaces, biophysical processes such as neural activity and intracellular transport [1], etc., can be described by spatially periodic potential [2, 3]. Analytic expressions for the mean particle velocity and the diffusion coefficient[4] for non-interacting particles in 1D are known. We study the one-dimensional as well as the two-dimensional overdamped Langevin dynamics of the simplest cooperative

system - the dimer - evolving in a spatially periodic substrate potential. In contrast to previous works [5], the impact of an external localized point force applied at only one of the two components on the dynamic of the dimer system is considered. For the one-dimensional case, two accurate approximations for the center of mass mobility and its diffusion coefficient are obtained. We found a set of optimal parameter values maximizing both quantities. Interestingly, while in the 1D case the mobility as a function of the thermal energy is a monotonic function of the latter in 2D we found the effect of negative resistance, i.e., the mobility possesses a minimum at a finite value of thermal energy for a given overcritical external force magnitude.

The dynamics of Brownian motion in two-dimensional channels, exhibiting periodically varying cross-sections, is ubiquitous in biological cells, ion channels, zeolites, and micro?uidic devices. We consider the motion of Brownian particles in presence of an external force acting along the direction of the channel axis. Both, the cases of single point-like particles and the dimer system evolving in a sinusoidal channel [6] are studied. For the single particle case expressions for the mobility and the effective diffusion coefficient within Fick-Jacobs (FJ) approximation [7] are derived. Applying the method of standard long-wave analysis higher corrections can be calculated which secure an agreement beyond the validity of FJ [8]. Studying interacting particles an additional entropic effect comes into play, viz., a temperature dependent mean elongation of the dimer. Thus the mobility is not a monotonically increasing function of the thermal energy, as it is in the single particle case. To be more precisely, the mobility possesses a minimum at a given temperature. The value of the latter depends on the applied force and the coupling constant.

[1] M.A. Zaks et. al., *Phys. Rev. E* **68**, 066206 (2003).

[2] D. Hennig et. al., *Phys. Rev. E* **78**, 011104 (2008).

[3] S. Martens et. al., *Phys. Rev. E* **78**, 041121 (2008).

[4] P. Reimann et. al., *Phys. Rev. Lett.* **87**, 010602 (2001).

[5] E. Heinsula et. al., *Phys. Rev. E* **77**, 021129 (2008).

[6] P.S. Burada et. al., *BioSystems* **93**, p. 16 - 22 (2008).

[7] D. Reguera et. al., *Phys. Rev. Lett.* **96**, 130603 (2006).

[8] S. Martens et. al., *submitted*.

Diffusive Transport in confined geometries

M3.3
15:00

Gerhard Schmid[1]*, [1]

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Transport of biased Brownian particles in symmetric tubes in two and three dimensions with periodically varying cross-section is studied. Bottlenecks which produce entropic barriers hinder the motion of the particles and exhibit peculiar characteristics in the transport behavior which are different from that taking place in systems with energy barriers [1, 2]. The constrained dynamics is responsible for an existence of a scaling regime for the particle current and the diffusion in terms of ratio between the work done to the particles and thermal energy. Our findings, genuine of the entropic nature of the periodic barriers, can be used in the control of transport through quasi-one dimensional structures, such as pores, ion channels or zeolites, in which irregularities of the boundaries may induce entropic effects.

In the talk, both theoretical and numerical concepts for describing forced Brownian motion in one-dimensional periodic potentials or in two- and three-dimensional tubes are introduced and transport characteristics are discussed.

[1] D. Reguera, G. Schmid, P. S. Burada, J. M. Rubi, P. Reimann, and P. Hänggi, *Phys. Rev. Lett.* **96**, 130603 (2006).

[2] P. S. Burada, P. Hanggi, F. Marchesoni, G. Schmid, and P. Talkner, Diffusion in confined geometries, *ChemPhysChem* **10**, 45 (2009).

Dimer currents on one dimensional asymmetric substrates

M3.4
15:30

S.E. Savel'ev[1*], A. Pototsky[2], N.B. Janson[2], F. Marchesoni[2]

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Transport on a one dimensional asymmetric substrate is studied for the case of short, soft charged dimers, formed by two equal masses carrying charges of opposite sign. The stationary dimer currents are computed as functions of the drive orientation and the dimer parameters. Optimal transport conditions are predicted to depend on the dimer elastic constant, in close agreement with the simulation data. Correspondingly, the rectification dimer currents sustained by zero frequency ac drives, reveal a rich phenomenology, also investigated in detail.

Contributed Talks C1: Localised Patterns (Lecture Room G12)

Single localized structures in a three-component reaction-diffusion system

C1.1
16:30

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We are interested in the stability of the nontrivial stationary solution (so-called dissipative soliton) of a three-component reaction-diffusion system with one activator and two inhibitors. In the simplest case it is a stationary localized structure with rotational symmetry, which is stable in a certain parameter region. In addition, in a parameter range near to the Turing bifurcation, stationary solitons possess spatially oscillating tails, which enable the formation of stationary bound states. By changing system parameters single solitons can undergo a drift-bifurcation, losing their rotational symmetry. Another example is breathing bifurcation, leading to the oscillating in space localized solution. Moreover, an interaction between these two unstable modes is possible, giving rise for a co-dimensional two bifurcation. All these situation are analyzed performing a multiple scale perturbation expansion in the vicinity of the bifurcation points and the corresponding order parameter equations are obtained. Also numerical simulations are carried out showing good agreement with the analytical predictions.

Convection rolls in a flat rotating box filled with a granular mixture

C1.2
16:50

Frank Rietz[1]*, Ralf Stannarius[1]

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We fill a flat container with a mixture of spherical beads of different sizes and rotate it slowly about a horizontal axis. When the filling level is below a certain value, one can observe the formation of regular segregation bands known e.g. from cylindrical mixers. Then we add some more beads to make the box almost completely filled and the free motion of the individual beads is now rather limited. However, we observe that the segregation pattern as well as its dynamics are qualitatively changed and the beads are now moving in convection rolls [1]. At first sight, they have characteristics similar to other rolls in liquids or shaken granulates. By making slight modifications of the container geometry we can deliberately choose the wave lengths of the rolls and determine the dispersion relation for the evolution of rolls of different wave lengths. In small containers we observe unusual dynamics of single rolls. Moreover, the experiment was done in vacuum for the purpose of finding out the influence of the interstitial air on the process of granular dynamics.

Despite the fact that the experiment seems rather simple, it could not be explained by applying any known mechanism for granular convection. The authors focus on the quantitative description of the structures and compare the characteristic features of the rolls to the other known structures in shaken and rotated systems.

[1] F. Rietz and R. Stannarius. On the brink of jamming: Granular convection in densely filled containers. *Phys. Rev. Lett.* **100**, 078002 (2008).

Solvable Model of Spiral Wave Chimeras

C1.3
17:10

Erik A. Martens[1]*, Carlo R. Laing[1], Steven H. Strogatz[1]

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We study networks of phase-coupled oscillators, based on the Kuramoto model, with nonlocal coupling

$$\frac{\partial}{\partial t}\phi(\mathbf{x},t) \quad = \quad \omega + \int_D G(\mathbf{x}-\mathbf{x}')\sin[\phi(\mathbf{x}',t)-\phi(\mathbf{x},t)+\alpha]\,d^2\mathbf{x},$$

where ϕ are the phases of individual oscillators with natural frequency ω located at $\mathbf{x}$ in the domain D. The nonlocal coupling is given by a kernel G such that the coupling strength decays with distance on the domain, e.g. $G \sim \exp(-\kappa|x|)$. Kuramoto and Battogtokh discovered that such systems exhibit intriguing states where regions of synchronized and desynchronized oscillators coexist and form a stable pattern [1].

This phenomena has since then gained strong interest in the community and been investigated in various settings to better understand its specific nature [4, 6, 5, 3]. Chimera states have been observed on different spatial structures such as rings and simple networks of oscillator populations [2, 8, 9]. On 2D lattices of oscillators they emerge in the shape of spirals: in this case, the spiral arms form the synchronized region, whereas the typically observed topological singularity in the center is replaced with a zone of desynchronized oscillators. We demonstrate how we obtain analytical solutions of this system and present a method to calculate the radius of the desynchronized core analytically [7]. We finish with a brief overview of results and open questions.

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[3] G. Sethia and A. Sen. Clustered Chimera States in Delay-Coupled Oscillator Systems. *Phys. Rev. Lett.*, 100, 144102 (2008).

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[7] E. A. Martens, C. R. Laing and S. H. Strogatz. Solvable Model of Spiral Wave Chimeras. *Phys. Rev. Lett.*, 104:044101 (2010).

[8] E. A. Martens, Bistable chimera attractors on a triangular network of oscillator populations. *(to be published in Phys. Rev. E), arXiv:1003.2827* (2010).

[9] E. A. Martens, Chimeras in a network of three oscillator populations with varying network topology. *(submitted to Chaos), arXiv:1003.2916.*

Contributed Talks C2: Quantum Chaos (Albert's Bar)

Ray model and ray-wave correpondence in coupled optical microdisks

C2.1
16:30

Jung-Wan Ryu[1]*, Martina Hentschel[1]

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Coupled optical microdisks have recently attracted much interest in the context of device applications such as photonic molecules and coupled-resonator optical waveguide according to the advances in material science and nano-fabrication techniques. In addition, the coupled optical microdisks have been also studied on theoretical prospects and experimental realizations such as optical mode coupling and producing directional light emission. However, in coupled optical microcavities, the ray dynamical model and ray-wave correspondence are not simple because of the intrinsic ray splitting.

In this work, we introduce the ray model for coupled optical microdisks, where we select coupling-efficient rays among the splitting rays by deterministic selection rule. Using the ray model, we report regular island structures arising from coupling between two microdisks and calculate the stabilities and decay rates of the islands. We also find many resonances relating to the islands in coupled optical microdisks and study how the island structures have an effect on the spectral properties of resonance, especially, such as distribution of loss which is represented as imaginary part of complex resonance.

Chaos and the Quantum: How Nonlinear Effects Can Explain Quantum Paradoxes

C2.2
16:50

Wm. C. McHarris[1]*

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In recent years we have suggested that many of the so-called paradoxes resulting from the Copenhagen interpretation of quantum mechanics have more logical parallels based in nonlinear dynamics and chaos theory [WCM, *Complexity* **12**(4), 12 (2007), and references therein]. From this one can infer that quantum mechanics might not be strictly linear as is commonly postulated. Indeed, during this past year experimentalists have reported observing definite signatures of chaos in a quantum system [S. Chadhury et al., *Nature* **461**, 768 (8 Oct. 2009)]. As an illustration of what can go wrong when quantum effects are forced into a linear interpretation, I examine Bell-type inequalities. In conventional derivations of such inequalities, a classical system is found to impose upper limits on the statistical correlations between, say, the properties of a pair of (entangled) particles measured at an effectively infinite separation. Quantum mechanics, on the other hand, allows greater correlations, and numerous experiments have found the quantum mechanical predictions to be correct. The implications are that a measurement made on one particle *instantaneously* collapses the wave function of the other particle, i.e., superluminal signals are transmitted between the particles—Einstein's "spooky action-at-a-distance" is at work, implying that "local reality" does not exist in the quantum realm! I argue that there is nothing wrong with the quantum derivations (the usual point of attack by those attempting to debunk Bell-type arguments), but that implicit in the classical derivations (such as for the CHSH inequality) are independent, uncorrelated particles. Thus, one is comparing uncorrelated probabilities with conditional probabilities rather than comparing classical mechanics with quantum mechanics—making moot the conclusions drawn from all those elaborate experiments. Classical nonlinear systems are known to exhibit correlations that can easily be as great as those found in quantum systems—nonextensive thermodynamics [*Nonextensive Entropy: Interdisciplinary Applications*, M. Gell-Mann and C. Tsallis, Eds., Oxford: Oxford Univ. Press, 2004] has verified this with numerous examples. As an example of the nonintuitive nature of conditional probabilities, I examine the so-called Monty Hall paradox, and I also demonstrate how non-ergodic behavior can easily ape action-at-a-distance. Perhaps quantum mechanics after all does contain basic nonlinear elements: Nonlinear dynamics and chaos could provide a bridge between the determinism so dear to Einstein, yet provide the statistical interpretation essential to Bohr and his Copenhagen cohorts. Ultimately, both Einstein and Bohr could have been right in their debates over the fundamentals of quantum mechanics.

Wave chaos in oceanic acoustic waveguides

C2.3
17:10

D.V. Makarov[1*], L.E. Kon'kov[1], M.Yu. Uleysky[1]

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It is well known that sound waves can propagate in an ocean over distances of thousands kilometers due to the presence of an underwater sound channel. Underwater sound channel is a refractive acoustic waveguide conditioned by a nonmonotonic dependence of the sound speed on the ocean depth. It prevents sound waves from a contact with a lossy bottom, maintaining them within a water column. About two decades ago it was realized that a weak horizontal inhomogeneity of an underwater sound channel, mainly caused by sound-speed variations due to oceanic internal waves, induces rays chaos. This circumstance plays an important role in different applications, the main of them is long-range acoustic tomography. Since ray trajectories obey coupled Hamiltonian equations, and the parabolic equation describing an acoustic wavefield formally coincides with the time-dependent Shrödinger equation, with range coordinate playing the role of time, the phenomenon of ray/wave chaos can be studied using methods and approaches borrowed from the theory of quantum chaos [1].

Analysis of an idealized range-periodic model of an underwater sound channel in terms of the Floquet theory shows that ray chaos can be significantly suppressed by wave-based corrections. In particular, the Floquet modes belonging to the chaotic sea can reveal surprisingly regular patterns, despite of the absence of any islands of stability [2]. It is found that these patterns can be associated with the clusters of periodic orbits with relatively small stability exponents.

In order to generalize the methods of the Floquet theory to the case of a realistic noisy-like inhomogeneity, we utilize the specific Poincaré map [3, 4] which is designed to visualize domains of finite-time stability in phase space of noisy-driven classical systems, We consider the quantum counterpart of the specific Poincaré map. It is an evolution operator connecting two states separated by a range interval which can be specified arbitrarily. Calculating quasienergy spectra of this operator, we show that statistics of level spacings corresponds to the Berry-Robnik regime even for range intervals of hundreds kilometers, indicating on the presence of domains of integrability. It means that wave dynamics can be very regular despite of the randomness of environment.

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Contributed Talks C3: Time Series and Control (Victoria's Room)

Accelerating Chaos Control

C3.1
16:30

Christian Bick[1]*, Christoph Kolodziejski[1], Marc Timme[1]

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Some chaotic attractors contain an infinite amount of repelling periodic orbits. The goal of chaos control is to stabilize these periodic orbits and turn them into attracting ones. This idea has applications in many fields [1]. For example, it has been demonstrated in neural circuits [2] but has recently also been used actively in order to control robot behavior [3].

Chaos control can be achieved by perturbing parameter values of the corresponding dynamical system [4] or by adding small control perturbations that render unstable fixed points stable [5]. A recent work by Schmelcher and Diakonos on the latter has the advantage that it is of global nature, i.e., the target periodic orbit does not need to be known. Due to the nature of their transformation, however, convergence can become very slow as you stabilize more and more periodic points.

In applications the speed of stabilization, i.e., the speed of convergence to one of the newly attracting periodic points, can be of crucial importance. Take for example a robot with a chaos control mechanism where a specific movement is linked to the period of some periodic orbit. For the robot to react to changing environments, new periodic orbits with different periods have to be stabilized as fast as possible resulting in the corresponding reactive movements. Hence, reaction time is linked to the convergence time of the stabilization mechanism. If we hypothesize similar mechanisms to be present in biological neural dynamical systems, the same argumentation would apply.

Our work takes a step towards an adaptive method accelerating the convergence of standard chaos control algorithms [5]. Based on recent results [3, 5] we present an algorithm that adapts the control parameter in order to minimize the spectral radius of the derivative of the transformed function at the target periodic orbit. If we assume that the function (whose iteration leads to the chaotic attractor) has certain properties, the algorithm estimates the spectral radius at the limit point and will try to adapt the parameter to optimize convergence speed without compromising convergence. An invariant measure can give lower bounds on the probability the algorithm will converge given a random initial condition. Furthermore, we study the dependence of the parameters of the algorithm on the target period, a crucial relationship in systems targeting multiple periods.

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[2] M. I. Rabinovich and H. D. I. Abarbanel, *Neuroscience* **87** (1998), no. 1, p. 5–14.

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[5] P. Schmelcher and F. K. Diakonos, *Physical Review E* **57** (1998), no. 3, p. 2739–2746.

Superstatistical fluctuations in time series

C3.2
16:50

Erik Van der Straeten[1]*, Christian Beck[1]

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Time series are omnipresent in daily life. Examples are the closing prices of shares at the stock market and the daily amount of rainfall at one particular geographical place. From a physical point of view, these time series are generated by an unknown complex system. The aim of superstatistics [1] is to build accurate models for this kind of systems that reproduce the most important features of the given time series. The crucial assumption of the this approach is that the dynamics of the system under study is a superposition of two dynamics with well separated time scales. Then, one also assumes that locally the system is described by a simple Gaussian stochastic process, e.g. described by the Langevin equation, and only on a large scale the mixture of various Gaussian processes leads to more complex (non-Gaussian) behaviour. Applications of superstatistical methods include hydrodynamic turbulence [3], wind velocity fluctuations [4], the statistics of train departure delays [5], ... Recent progress in the superstatistical modeling of share price dynamics was made in [6].

I will illustrate the basic theoretical considerations of superstatistics for the example of the share prices. During calm periods, the fluctuations of the share price around its mean will be reasonably small, while during active periods with a lot a trading transactions, the fluctuations will be more violent. An important assumption of superstatistics is that the stock market can always be described by basically the same model. However, the values of the model parameters will depend on the state of the stock market. As such, apart from the proposed basic model, the statistical properties of the values of the model parameters are also very important in the methodology. Together, they determine the exact outcome of the theory that has to be compared with the real data.

This talk consists of two parts. In the first part, I will outline the general ideas of superstatistics [1, 6]. In the second part, I will discuss some applications of the superstatistical approach [6, 6].

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The n-level spectral correlations for chaotic systems

C3.3
17:10

Taro Nagao[1], Sebastian Müller[2*]

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We study the n-level spectral correlation functions of classically chaotic quantum systems without time-reversal symmetry. According to Bohigas, Giannoni and Schmit's universality conjecture, it is expected that the correlation functions are in agreement with the prediction of the Circular Unitary Ensemble (CUE) of random matrices. A semiclassical resummation formalism allows us to express the correlation functions as sums over pseudo-orbits. Using an extended version of the diagonal approximation on the pseudo-orbit sums, we derive the n-level correlation functions identical to the $n \times n$ determinantal correlation functions of the CUE.

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Contributed Talks C4: Chemistry (Auditorium)

A Geometric Model for Mixed-Mode Oscillations in a Chemical System with Multiple Time-Scales

C4.1
16:30

Chris Scheper[1]*, John Guckenheimer[1]

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Our work presents a detailed analysis of mixed-mode oscillations (MMOs) in the autocatalator, a three-dimesional chemical model with multiple time-scales, given by

$$\begin{aligned} \dot{a} &= \mu(\kappa - c) - ab^2 - b \\ \epsilon\dot{b} &= ab^2 + a - b \\ \dot{c} &= b - c. \end{aligned} \tag{1}$$

MMOs in this system were studied by both Petrov, et. al in [2] and Milik and Szmolyan [1], who treated it as a slow-fast system. A return map to an appropriately chosen cross-section near the fold in the critical manifold at $b = 1$ exhibits both full rank and rank deficient behavior. The two regions are separated by boundary layer consisting of canards, which follow a two-dimensional repelling slow manifold. The intersection of the cross section with the family of attracting slow manifolds is exponentially close to a one-dimensional manifold Γ_a. A family of one-dimensional induced return maps are defined on Γ_a. The induced map approximates the returns of the two-dimensional return map.

The bifurcations of the induced maps are used to characterize the bifurcations of the MMOs exhibited by the system. We analyze the induced maps using kneading theory. Specifically, we show that the kneading theory allows us to analyze and predict the sequences of bifurcations that occur in the transition between stable bands of MMOs. The kneading theory describes a variety of behavior, including bistability, saddle-node bifurcations leading to chaotic behavior, and period-doubling sequences. We use continuation methods to compare the bifurcations of the MMOs in the full system to those predicted by the kneading theory. It is also shown that the MMOs are generated by a singular Hopf bifurcation that occurs in the system.

[1] A. Milik and P. Szmolyan. *Multiple time scales and canards in a chemical oscillator*, volume 122 of *The IMA Volumes in Mathematics and its Applications*, pages 117–140. Springer, 2001.

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Progress of an auto catalytic reactions inside slugs in microchannels

C4.2
16:50

S. Pushpavanam[1*], Siva Kumar Reddy Cherlo[1]

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In heterogeneous liquid-liquid flows slug flow is commonly observed regime. Each slug can be viewed as a reactor which is not perfectly mixed. Mass transfer across the slug surface can be induced by concentration gradients. The progress of a reaction is affected by both mixing in the slug and kinetics. The evolution of the system to different possible steady states depends not only on the initial concentration but also on mixing. The behavior of ideal reactors is based on assumptions on mixing. In real reactors mixing may not be perfect and this determines the progress of reactions.

The present work analyzes the effect of mixing on a representative auto catalytic reaction inside a slug in microchannel. The kinetics is chosen so that the system can exhibit multiple steady states. Different models with varying degree of complexity i.e lumped parameter models, 1D model, 2D CFD model are used to analyze mixing and study their effect on progress of an auto catalytic reaction. Existence of travelling waves propagating in either direction and the effect of initial arrangement of reagents inside the slug on progress of reaction is analyzed. The kinetics chosen has implications on blood clotting. However the study helps determine the effect of various parameters on system behavior and hence can be used to design systems with specific applications.

Endex thermoreactive systems and application to carbon separation from fuels and flue gases

C4.3
17:10

Rowena Ball[1*]

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An Endex thermoreactive system consists of one or more exothermic reactions that are directly thermally coupled and kinetically matched with endothermic reactions to achieve thermal stability, autothermal operation, full recovery of chemical energy, and co-production of valuable products. Endex theory and principles were co-invented by the author [1] and published in 1999 [2, 3], but the concepts were virtually ignored at the time. In the mid to late 90s climate change mitigation was not a political priority, fossil fuels were still relatively cheap, and carbon capture and storage (CCS) had barely been heard of.

Times have changed dramatically. Now, some 15 years later, Endex principles are underpinning a new CO_2 separation technology of unprecedented economic viability [4]. From one perspective the Endex story is a morality tale about the enormous value to industry and society of supporting high quality fundamental research. It illustrates how such research can be of great value down the track to new problems and in different contexts from those in which it was originally carried out.

Why CO_2 separation? Even with the strongest possible investment in renewables fossil fuels will continue to be used for power generation plants for decades to come. The current economics of CCS is such that the cost of emitting CO_2 to the atmosphere is still less than the cost of implementing CCS. The question is simple, blunt and understandable: "How much does the price of electricity go up if we adopt your CCS technology?" The answer is just as simple: most people and most industries are unable to pay very much more for electricity than they pay currently. So the problem is to reduce markedly the cost of CCS technologies.

Endex-based technologies have the potential to achieve that critical reduction in cost for carbon separation, and give high efficiency to very large plants. It is important to appreciate the potential for adiabatic, or thermally insulated, operation of an Endex-configured reactor. A conventional chemical reactor is almost never run adiabatically (unless it is a bomb calorimeter) because for most common industrial reactions the adiabatic temperature rise for full conversion is dangerously high. Purpose-built cooling systems are usually necessary, which can become technically very elaborate and expensive when the reactor is large. The *ideal adiabatic* Endex reactor, where the additional conservation condition of enthalpy flux conservation holds to a good approximation, is scale-invariant. Quite simply, there are no surface-to-volume ratios, explicit or hidden, in the equations.

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Plenary Talk I3 (Auditorium)

Tsunami asymptotics

I3
17:30

Michael Berry[1]*

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For most of their propagation, tsunamis are linear dispersive waves whose speed is limited by the depth of the ocean and which can be regarded as diffraction-decorated caustics in spacetime. For constant depth, uniform asymptotics gives a very accurate compact description of the tsunami profile generated by an arbitrary initial disturbance. Variations in depth can focus tsunamis onto cusped caustics, and this 'singularity on a singularity' constitutes an unusual diffraction problem, whose solution indicates that focusing can amplify the tsunami energy by an order of magnitude. In the opposite water-wave regime, 'Antitsunamis', dominated by surface tension, possess interesting complementary asymptotics.

Tuesday, 7th September 2010

Plenary Talk I4 (Auditorium)

How Insects Fly and Turn

I4
9:00

Jane Wang[1*]

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Insects' aerial acrobatics result from the concerted efforts of their brains, flight muscles, and flapping wings. To understand insect flight, we started from the outer scale, analyzing the unsteady aerodynamics of flapping flight, and are gradually working toward the inner scale, deducing control algorithms. In this approach, the dynamics of flight informs us about the internal control scheme for a specific behavior.

I will first describe the aerodynamic tricks that dragonflies employ to hover and fly efficiently. I will then discuss how fruit flies recover from aerial stumbles, and how they make subtle wing movements to induce sharp turns in tens of wing beats, or 40-80ms. The observed yaw maneuver can be explained by a quantitative mechanical model that connects a single control variable to the body dynamics.

Mini-symposium M4: Cell Dynamics (Lecture Room G12)

Organisers: Diego di Bernardo (TIGEM, Italy) & Mario di Bernardo (University of Bristol, UK)

Modelling gene networks in systems and synthetic biology

M4.1
10:30

Lucia Marucci[a,b1*], Irene Cantone[a1], Francesco Iorio[a1], Vincenzo Belcastro[a1], Maria Aurelia Ricci[a1], Stefania Santini[b1], MariaPia Cosma[b1], Mario di Bernardo[b1], Diego di Bernardo[a,b,c1]

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[b]Department of Engineering Mathematics, University of Bristol, U.K.
[c]Department of Computer and Systems Engineering, Federico II University, Naples 80125, Italy.
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Systems Biology approaches are extensively used to model and reverse-engineer gene regulatory networks from experimental data. Conversely, Synthetic Biology allows constructing de novo a regulatory network to seed new functions in the cell. At present, usefulness and predictive ability of modelling and reverse-engineering cannot be assessed and compared rigorously. We built in the yeast *Saccharomyces cerevisiae* a synthetic network, IRMA, for In-vivo benchmarking of Reverse-engineering and Modelling Approaches. The network is composed of five genes regulating each other through a variety of regulatory interactions; it is negligibly affected by endogenous genes and is responsive to small molecules. We measured promoter strengths, as well as, steady-state and time-series expression data following multiple perturbations. These data were used to assess state-of-the-art modelling and reverse-engineering techniques. A dynamical model of IRMA is able to capture and predict its behaviour; reverse-engineering based on differential equations correctly infers regulatory interactions for informative datasets. An in-depth bifurcation analysis of the model is used to uncover possible regions in parameter space where the network can behave as a switch or an oscillator.

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Oscillations in Cdc14 release and sequestration reveal the circuit underlying mitotic exit

M4.2
11:00

Romilde Manzoni[a,d1], Francesca Montani[b,d1], Clara Visintin[b1], Fabrice Caudron[c1], Andrea Ciliberto[a1*], Rosella Visintin[b1]

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Mitotic exit requires the reversal of the effect of the Clb-dependent CDKs. The Cdc14 phosphatase plays a key role in the process, both acting on many of Clb-CDK substrates, and also indirectly inhibiting Clb-CDKs themselves. During most of the cell cycle, Cdc14 is sequestered into the nucleolus and inactive. When cells transit into anaphase the phosphatase is released and can actively counteract the Clb-CDKs, after which it is inactivated by re-sequestration into the nucleolus before cells exit from mitosis. Here, we investigate the molecular circuit underlying Cdc14 re-sequestration at the end of mitosis. We show that in cells blocked in mitosis by high levels of non-degradable Clb2, Cdc14 undergoes periodical cycles of sequestration and release. Oscillations are based upon a negative feedback loop that includes Cdc14 itself, the APC/C cofactor Cdh1 and the polo-kinase Cdc5.

Analogous oscillations have also been described for yeast bud formation and centrosome duplication. A common theme emerges where events that must happen only once per cycle, although intrinsically capable of oscillations, are limited to one occurrence by the cyclin-CDK cell cycle engine. With the help of mathematical analysis, we suggest that oscillatory dynamics might have been selected because they guarantee robust and complete execution of events activated by the cell cycle engine.

[d] These authors contributed equally to the work.

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Modelling and Analysis Tools for Biochemical Networks

M4.3
11:30

A. Papachristodoulou[1*], James Anderson[1], Yo-Cheng Chang[1]

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Mathematical modeling is key for understanding the properties of dynamic processes inside cells, which usually consist of complicated networks of interacting genes/proteins. However, modeling and analyzing such pathways presents a number of mathematical and computational challenges, as the models that are considered are usually complicated nonlinear differential equations with several time-scales and unknown parameters. In this talk, we present a number of issues related to biological network modeling and analysis and discuss ways to address them.

In particular, we present new mathematical and algorithmic tools to understand the properties of biological networks - e.g., equilibrium stability and input-output performance in the presence of parametric uncertainties [1]. We consider Ordinary Differential Equation models for these systems, for which the vector fields are polynomial or rational (resulting from Mass Action or Michaelis-Menten kinetics). We then discuss how Lyapunov functions can be algorithmically constructed for these systems to verify, e.g., robust stability. The techniques we use are rooted in robust control and dynamical systems theory, but use recent developments in the theory of positive polynomials and Sum of Squares [2]. Computation is done using Semidefinite Programming and SOSTOOLS [3].

Finally, we discuss the limitations that this approach has and consider how dynamic model decomposition and reduction can enable the algorithmic analysis of larger system models [4]. A series of biologically meaningful examples will be given to illustrate the methodology.

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[1] H. El Samad, A. Papachristodoulou, S. Prajna, J. C. Doyle, and M. Khammash. Advanced methods and algorithms for biological networks analysis. *Proceedings of the IEEE*, 94(4), 2006.

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Development and implementation of a modular strategy for gene regulatory network control

M4.4
12:00

Filippo Menolascina[a,b1]*, Mario di Bernardo[b1], Diego di Bernardo[a,b1]

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Disruption of the endogenous control of gene regulatory networks has been demonstrated to be the origin of many incurable diseases. For this reason a huge effort has been spent in the last decades in studying and modelling gene networks from a quantitative point of view. In this context, the control of biological networks is a primary objective of current biomedical sciences: starting from a rough model of the biological process we want to be able to drive gene expression at will by applying principles from control systems theory. Therefore, by applying such a strategy we aim at forcing gene networks to follow a pre-determined behavior, possibly bringing unhealthy cells to a healthy state again. Even though no theoretical hindrance should hold, to the best of our knowledge, no experiment has been carried out so far to validate this hypothesis. The reasons for this mainly lie in the lack of suitable technological platforms to carry out the experiments (which require a fine control over concentration of inducer molecules, thus preventing their accomplishment in macro-scale devices like flasks). On the other hand, microfluidics proved to be a valuable aid in overcoming such limitations. For these reasons we decided to apply engineering principles to a synthetic network constructed in the yeast S.

cerevisiae, in order to develop and test a control strategy in vivo to be carried out in a microfluidic device. Here we derive a new hybrid model of the synthetic network implemented in S. Cerevisiae and we propose a control strategy based on hybrid systems theory and developed to achieve both regulation and tracking control of a biological networks. The strategy proposed herein has been designed to tackle some of the most common issues found in biological networks namely: nonlinearities, transcriptional delays and switch-like transcription activation functions (leading to discontinuities). In-silico control design has been validated by computer simulations and preliminary experiments have been carried out to test the functionalities and constraints of the platform proposed herein: a full overview of both theoretical and practical aspects of this approach, therefore, will be provided in this contribution.

Mini-symposium M5: Molecular Dynamics (Victoria's Room)

Organisers: Ben Leimkuhler (University of Edinburgh, UK) & Frédéric Legoll (ENPC, France)

Effective dynamics for the overdamped Langevin equation

M5.1
10:30

Frédéric Legoll[1*], Tony Lelièvre[1]

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The question of coarse-graining is ubiquitous in molecular dynamics. In this work, we are interested in deriving effective properties for the dynamics of a coarse-grained variable $\xi(X)$, where X describes the configuration of the system in a high-dimensional space, and ξ is a smooth scalar function (typically a reaction coordinate). We assume that the configuration X_t of the complete system evolves according to the overdamped Langevin stochastic differential equation, and we propose an effective closed dynamics that approximates the evolution of $\xi(X_t)$, see [1]. Such an effective dynamics may be useful to compute more efficiently e.g. transition rates from one configuration of the system to another one.

The accuracy of the effective dynamics compared to the reference evolution $\xi(X_t)$ is analyzed through rigorous error estimates between the laws of the two stochastic processes, at any given time. Numerical simulations illustrate the efficiency of the approach as well as the accuracy of the proposed dynamics according to various criteria, including residence times in potential energy wells.

[1] Frédéric Legoll and Tony Lelièvre, "Effective dynamics using conditional expectations", arxiv preprint `http://arxiv.org/abs/0906.4865`

On the Efficiency of Temperature Controls for Molecular Dynamics

M5.2
11:00

B. Leimkuhler[1]*, E. Noorizadeh[1], O. Penrose[2]

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When molecular dynamics is used to find time-dependent properties, such as correlation functions, at a prescribed temperature, there is a conflict between the need for the thermostat to perturb the time evolution of the observables as little as possible and the need to establish equilibrium. Some alternatives to traditional methods such as Langevin dynamics have recently appeared that introduce stochastic perturbation in novel ways, offering, potentially, greater flexibility in the way by which equilibrium is achieved, including the stochastic scaling method [1] and the Nose-Hoover-Langevin method [2]. In this article we suggest a mathematical framework for comparing thermostats based on the concept of *efficiency*, defined as the ratio of the rate of thermal equilibration to the rate of build-up of observable perturbations. We show how to estimate this quantity analytically, and proceed to carry out the analysis for several thermostats. We observe that the convergence to equilibrium for Nosé-Hoover-Langevin and stochastic scaling can be modelled by one or two dimensional relaxation equations. Using these reductions, we find strong efficiency improvements for each of the new schemes [1,2] compared to Langevin Dynamics. Numerical experiments are presented which confirm the theoretical results.

[1] Bussi, G. Donadio, D. and Parinello, M., Canonical sampling through velocity-rescaling, *J.Chem.Phys* **126**, 014101, 2007.

[2] Samoletov, A., Dettmann, C. and Chaplain, M.,Thermostats for "slow" configurational modes, *J.Stat.Phys* **128**, 1321-1336 , 2007.

Covariant tangent-space dynamics for simple particle systems

M5.3
11:30

Harald A. Posch[1]*, Hadrien Bosetti[1]

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Recently, a new algorithm for the computation of covariant Lyapunov vectors and of corresponding local Lyapunov exponents has become available [1]. Here we study the properties of these still unfamiliar quantities for a number of simple models, including an harmonic oscillator coupled to a thermal gradient with a two-stage thermostat, which leaves the system ergodic and fully time reversible [2]. We explicitly demonstrate how time-reversal invariance affects the perturbation vectors in tangent space and the associated local Lyapunov exponents. We clarify, in particular, the relation between the covariant vectors and the more familiar Gram-Schmidt vectors. If the system is phase-volume conserving, the Gram-Schmidt

vectors faithfully reflect that symmetry, but the covariant vectors do not. We demonstrate that the local covariant exponents may vary discontinuously along directions transverse to the phase flow.

Next, we carry out extensive computer simulations to study the covariant vectors of a two-dimensional hard disk system in a rectangular box with periodic boundary conditions [3]. The system is large enough to allow the formation of Lyapunov modes parallel to the x axis of the box. The Oseledec splitting into covariant subspaces of the tangent space is considered by computing the full set of covariant perturbation vectors co-moving with the flow in tangent-space. These vectors are shown to be transversal, but generally not orthogonal to each other. Only the angle between covariant vectors associated with immediate adjacent Lyapunov exponents in the Lyapunov spectrum may become small, but the probability of this angle to vanish approaches zero. The stable and unstable manifolds are transverse to each other and the system is hyperbolic. The mode-supporting degenerate subspaces are dynamically isolated, and the Oseledec splitting is dominated.

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[1] F. Ginelli, P. Poggi, A. Turchi, H. Chaté, R. Livi, and A. Politi, "Characterizing dynamics with covariant Lyapunov vectors", Phys, Rev. Lett. **99**, 130601 (2007).

[2] H. Bosetti, H.A. Posch, C. Dellago and Wm.G. Hoover, "Time-reversal symmetry and covariant Lyapunov vectors for simple particle models in and out of thermal equilibrium", *Phys. Rev. E*, submitted (2010); arXive: 1004.4473

[3] H. Bosetti and H.A. Posch, "Covariant Lyapunov vectors for rigid disk systems", *Chemical Physics*, submitted (2010).

Shadowing the Trajectories of Molecular Dynamics

M5.4
12:00

P.F. Tupper[1]*

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An open theoretical problem is to explain the apparent reliabilty of molecular dynamics simulations. The difficulty is that individual trajectories computed in molecular dynamics are accurate for only short time intervals, whereas useful information can be extracted from very long simulations [1]. One conjecture is that numerical trajectories are *shadowed* by exact trajectories: that is, for every numerical trajectory there is an exact trajectory with different initial conditions that remains close to the numerical trajectory over long time intervals [4].

I will demonstrate for a simple yet representative molecular dynamics system that long numerical trajectories are not shadowable. The system I will consider is a version of the Lorentz gas: a single moving particle interacting via the Lennard-Jones potential with an infinite array of fixed particles [3]. I will explicitly construct a numerical trajectory that is not shadowable for this system. Then I will demonstrate through numerical experiments and the use of a software package by Wayne Hayes [2] that almost all sufficiently long trajectories are not shadowable for this system.

These results notwithstanding, numerical simulations of long trajectories still predict macroscopic features of the Lorentz gas system accurately. I will illustrate this phenomenon with the example of the diffusion coefficient of the moving particle. I will discuss why this is despite the inaccuracy of microscopic simulations [5].

[1] D. Frenkel and B. Smit, "Understaning Molecular Simulation", Academic Press (2002).

[2] Wayne Hayes and Ken Jackson, *SIAM J. Sci. Comp.* **29:4**, 1738–1758 (2007).

[3] R. Klages and Christoph Dellago, *J. Stat. Phys.* **101**, 145–159 (2000).

[4] S. Reich, *SIAM J. Num. Anal.* **36:5**, 1549–1570 (1999).

[5] P. Tupper, chapter in "Mathematics and Computation", 95–108, Spinger (2008).

Mini-symposium M6: Nonlinear Optical Systems (Auditorium)

Organiser: Sebastian Wieczorek (University of Exeter, UK)

Complex dynamics in two-mode semiconductor lasers

M6.1
10:30

A. Amann[1*], B. Wetzel[1], S. Osborne[1], S. O'Brien[1]

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Optically injected semiconductor lasers are paradicmatic non-linear systems which exhibit a number of dynamical phenomena of wider interest, including locking, multistability, and chaotic dynamics. For a single mode laser, the bifurcations between different regimes under optical injection have been successfully explained on the basis of relatively simple rate equation models. Much less is known for the dynamics of multimode lasers, which have recently received increased interest due to their potential in providing sophisticated optical functionality.

In this presentation we consider the injection dynamics of a two-mode laser diode with a mode spacing of the primary modes in the THz regime. Injection in one of the primary modes of the device leads to a rich variety of single and two-mode dynamical scenarios, including antiphase dynamics and bistability between two mode and single mode equilibria. These phenomena are reproduced with remarkable accuracy by a four dimensional rate equation model, which contains the established single mode model in an invariant submanifold.

We have found regimes of large amplitude bursting of the uninjected mode, which are organised by codimension two points at which saddle node of limit cycle and transcritical bifurcation lines tangentially intersect. At the saddle node of limit cycle bifurcation lines the time interval between bursts and the geometric length of the orbit diverge, similarly to the Blue Sky Catastrophe in generic systems. We discuss the associated phase space structure, and compare with other infinite period bifurcations described in the literature.

Nonlinear dynamics of semiconductor microring lasers

M6.2
11:00

G. Van der Sande[1]*, L. Gelens[1], L. Mashal[1], W. Coomans[1], S. Beri[1], G.Verschaffelt[1], J. Danckaert[1], G. Mezosi[2], M. Sorel[2]

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Semiconductor ring lasers (SRLs) are a particular type of semiconductor lasers where the laser cavity consists of a ring-shaped waveguide. Due to their symmetry, SRLs can emit in one of two counter-propagating modes and are therefore recognized to be promising sources in photonic integrated circuits. In particular, this possibility of bistable directional operation has paved the way for encoding digital information in the emission direction of SRLs with record low values for switching times and energies [1].

We have applied a singular perturbation technique to reduce a set of rate equations for a SRL to two equations for the relative modal intensity and the phase difference between the two counter-propagating modes [2]. Not only do these reduced equations simplify the bifurcation analysis of the possible steady-state solutions considerably, they also allow for a two-dimensional phase-space description of the laser. In particular, the shape of the invariant manifolds of the saddle point in the system is studied and a full bifurcation analysis of this two-dimensional laser system is carried out. This approach also reveals that semiconductor ring lasers are an optical prototype of nonlinear systems with Z_2 symmetry.

We investigate both theoretically and experimentally the stochastic switching between two counter-propagating lasing modes of a SRL. Experimentally, the residence time distribution cannot be described by a simple one parameter Arrhenius exponential law and reveals the presence of two different mode-hop scenarios with distinct time scales. The topological phase-space picture of the two-dimensional dynamical system gives insight into the observed features [3]. Expanding on the mode-hopping study in the bistable region, we also show how the operation of the device can be steered to multistable dynamical regimes, predicted by the two-dimensional model. By analyzing the phase-space in this model, we predict how the stochastic transitions between multiple stable states take place and confirm it experimentally [4].

Finally, we show theoretically and experimentally that by perturbing the Z_2 symmetry, SRLs can be driven into a regime of excitability. The global shape of the invariant manifolds of the saddle in the vicinity of a homoclinic loop are shown to determine the origin of excitability and the features of the excitable pulses [5].

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Frequency and phase locking in a cavity soliton laser

M6.3
11:30

T. Ackemann[1*], N. Radwell[1], C. McIntyre[1], G.-L. Oppo[1], W. J. Firth[1]

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Synchronization of nonlinear self-sustained oscillators via frequency and phase locking is a basic paradigm of Nonlinear Science as well as self-localization via nonlinearities in solitons. Quite recently a new type of laser, the cavity soliton laser (CSL) was realized based on semiconductor microcavities (see [1, 2, 3]). In a CSL, a broad transverse area (transverse indicates the plane orthogonal to the cavity axis) is pumped in a marginally stable plano-planar cavity. Emission does not take place over the whole aperture but on filaments which are much smaller than the pumped aperture and which are stabilized by nonlinearities, in the simplest case by self-lensing. Hence they are referred to as laser cavity solitons (LCS). They represent small coherent emitters, i.e. microlasers, which typically have different individual frequencies due to disorder. It seems to be a very interesting question to ask whether these self-localized states show frequency and phase locking behavior as conventional lasers. We are going to present a first demonstration in this paper.

The experimental system is based on a vertical-cavity surface-emitting laser (VCSEL) emitting at 980 nm with a diameter of 200 μm. It is coupled to a volume Bragg grating (VBG) providing narrow-band frequency-selective feedback via a self-imaging cavity [4].

Tuning current and alignment of the VBG, we can observe fringes in the far field plane of the VCSEL, the spacing of which corresponds to the separation of the LCS in near field. The fringes can have essentially perfect as well as incomplete visibility, which is interpreted as complete and partial locking (corroborated by the behavior of the optical spectra). Indications for hysteresis is found. The (minute) tilting of the VBG controls the detuning conditions. It is a global control parameter and hence cannot be adapted to the local disorder responsible for the frequency spread of the LCS but locking of three LCS can be also obtained in some situations.

The origin of locking is not well established although coupling via evanescent fields like in coupled solid-state lasers [5] is thought to contribute. In addition, some coupling might arise from the external cavity which is currently under consideration.

The interaction of propagating spatial solitons in conservative systems is known to be phase-sensitive and at the origin of a wealth of intriguing phenomena of spatial dynamics. However, the phases are fixed by the launching conditions. The long term vision of the research on CSL is the realization and the study of the interaction of a complex network of self-sustained oscillators which have freedom of choosing frequency, phase (possibly polarization) and location simultaneously.

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[3] T. Ackemann, G.-L. Oppo, and W. J. Firth, Adv. Atom. Mol. Opt. Phys. **57**, 323 (2009).

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Noise Synchronisation and Stochastic Bifurcations in Lasers

M6.4
12:00

Sebastian Wieczorek[1]*

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Synchronisation of nonlinear oscillators to external noise is an interesting mathematical problem of importance in biology, applied science, and engineering [1]. The key difference to synchronisation to an external periodic signal is the lack of a simple functional relationship between the noisy input and the synchronised output, making the phenomenon much less evident. Rather, synchronisation to noise is defined as identical response of two or more identical but uncoupled oscillators to the same external noise. This definition is equivalent to obtaining reproducible output from a single oscillator driven repeatedly by the same external noise, each time starting at a different initial state. Hence, noise synchronisation is also known as reliability [2] or consistency [3], and represents the ability to encode irregular external signals in a reproducible manner.

Typically, a small amount of external noise causes synchronisation. However, as the external-noise strength increases, one can observe loss of synchrony in oscillators with amplitude-phase coupling (also called nonisochronicity, chirp or shear) [1]. Mathematically, loss of synchrony, consistency or reliability is a manifestation of the same phenomenon, namely stochastic bifurcation to a *random strange attractor*.

Motivated by fundamental interest and applications alike, this paper studies general properties of such stochastic bifurcation using noise-driven laser model and Hopf normal form [4]. We demonstrate onset of noise synchronisation followed by one transition to asynchronous behaviour and another transition back to synchrony in a single laser with external white noise. We show that both transitions are due to stochastic bifurcation to a random strange attractor and study this bifurcation with dependence on the three parameters: the laser pump which controls the onset of Hopf bifurcation (laser threshold), the linewidth enhancement factor which quantifies the amount of amplitude-phase coupling, and the external-noise strength. The analysis uncovers an experimentally accessible region of chaos that is purely noise-induced; the noise-free laser does not posses any type of a chaotic set in its phase space. For small external-noise strength, the numerically obtained stochastic bifurcation in the laser system follows a square-root law in agreement with the normal form of Hopf bifurcation. However, discrepancies arise with increasing external-noise strength. In particular, the region of noise-induced chaos in the laser system is significantly enhanced owing to one additional degree of freedom and damped relaxation oscillation.

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Contributed Talks C5: Complex Networks (Lecture Room G12)

Experimental analysis and synchronization of networks of Chua circuits

C5.1
14:00

Marco Colandrea[1], Massimiliano de Magistris[1], Mario di Bernardo[2], Sabato Manfredi[2*]

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The paper describes experimental results on the analysis of synchronization [1] for a networks of Chua's circuits. Over the past few years a large numbers of papers have appeared in the literature reporting a wide range of different strategies to achieve synchronization. Most papers report theoretical results and use the MSF approach to support the theoretical derivation which is often validated via numerical simulations on networks of a small set of paradigmatic nonlinear examples such as Lorenz or Roessler systems or Chua circuits. The experimental validation of the theoretical results can be found only in a relatively small number of papers. Typically, experiments are limited to small networks of few nodes. For instance, in [4], networks of 3 Chua circuits are studied. Despite being an acceptable example, when only three nodes are used it is difficult to properly assess experimentally one of the most striking theoretical findings on synchronization: the effect of the network topology on its dynamics (see for instance [1]). The aim of this paper is to overcome this limitation and discuss the effects of topological variations and changes in the strength of the mutual coupling between neighboring nodes by means of experiments carried out on networks of 4 nodes. In so doing, we shall seek to compare the observed dynamics with the predictions obtained by using the MSF approach [2, 3]. Also, we will investigate the robustness properties of synchronization by considering networks of non-identical Chua circuits. We will show that, in order to understand and capture the key features of the experimental results, it is possible to use a new extendend MSF (EMSF) which was recently proposed in [5] to investigate the synchronization of networks of non-identical nodes. To our knowledge, the effectiveness of the EMSF as an analytical tools to obtain reliable predictions is tested here experimentally for the first time in the existing literature. As a representative example, we consider the experimental investigation of networks of four or more Chua circuits coupled via a diffusive term proportional to the mismatch between neighboring circuits. The experimental set-up will allow the variation of the network topology, changes in the parameters of the Chua at the nodes and a modulation of the coupling gain being used to couple them. In so doing, we will show to what extent the theoretical predictions of both the MSF and the EMSF match the experimental observations and discuss the level and quality of synchronization detected in different conditions. We wish to emphasize that Chua circuits are good paradigmatic examples of nonlinear systems exhibiting complex dynamics as often noted in the literature (e.g. [2]). Hence, we expect the results reported in this paper to be of interest, beyond nonlinear circuit theory, to the broad scientific community studying synchronization of complex networks.

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Determination of the scale of coarse graining in complex network of earthquake

C5.2
14:20

Norikazu Suzuki[1]*

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The seismic data are mapped to growing random networks [1, 2]. Vertices and edges of such networks correspond to coarse-grained events and event-event correlations, respectively. Yet unknown microscopic dynamics governing event-event correlations and fault-fault interactions is replaced by these edges. Global physical properties of seismicity can then be explored by examining its geometric (e.g., topological etc.), statistical and dynamical properties. Firstly, we show that the earthquake network is scale free, being characterized by the power-law connectivity distribution [3, 4]. We give a physical interpretation to this result based on network growth with the preferential attachment rule together with the Gutenberg-Richter law. Secondly, we study the small-world structure of the earthquake network reduced to an undirected simple network [4]. The value of the clustering coefficient is found to be much larger than that of the classical random network. In addition, the average path length is very small. Thirdly, we show that the earthquake network possesses hierarchical organization [5]. We interpret this fact in terms of vertex fitness and vertex deactivation by the process of stress release at faults. Fourthly, We find that the earthquake network has the property of assortative mixing. This point is an essential difference of the earthquake network from the Internet that has disassortative mixing. Combined with other dynamical properties [6], the present results imply that yet unknown mechanism governing seismicity may be so-called glassy dynamics on a growing complex network. These observations have obvious importance for constructing and improving physical models of seismicity such as the ones exhibiting self-organized criticality.

In the discussions above, the cell size, which is the scale of coarse graining needed for constructing an earthquake network, has remained as a free parameter. We finally report a method for determining it based on the scaling behavior of the network [7]. Quite remarkably, both the exponent of the power-law connectivity distribution and the clustering coefficient are found to approach the respective universal values and remain invariant as the cell size becomes larger than a certain value, l_*, which depends on the number of events contained in the analysis, in general. This l_* fixes the scale of coarse graining. Universality of the result is demonstrated for all of the networks constructed from the data independently taken from California, Japan and Iran.

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Systematic Network Analysis

C5.3
14:40

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Many real-world systems are most naturally represented as networks, and a variety of measures exist for their analysis. However, studies of networks typically employ only a small, largely arbitrary subset of these, and the lack of a systematic comparison makes it unclear which metrics are redundant or complementary. We present a framework for systematic analysis of networks and network metrics, and use it to analyse a large variety of real networks, several synthetic network models, and hundreds of metrics or summary statistics thereof. We demonstrate the utility of the framework for finding redundant metrics, fitting models to real networks, classification of networks, studying evolving networks, and determining the robustness of metrics to network damage and sampling effects.

The price of fair flows in complex networks

C5.4
15:00

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Although there have been considerable advances over the last decade in understanding dynamical processes in complex networks, little is known on how the structure of the network will impact an allocation of flows which will not unnecessarily "starve" any commodity. An elegant approach is the notion of *max-min fairness*. Loosely, a vector of flows is max-min fair if no flow may be increased without simultaneously decreasing another flow which is already smaller. Formally, $\{f_{m_q}\}$ is max-min fair if it is feasible and if, for any other feasible vector f', $\exists m_p : f'_{m_p} > f_{m_p} \implies \exists m_q : f'_{m_q} < f_{m_q} \leq f_{m_p}$.

The majority of work on max-min fairness has been motivated by communication networks. Researchers have focused mainly on the setting in which connections are specified by a fixed set of paths, and one wants to allocate rates to these paths [**?**, **?**]. However, comparatively little effort has been devoted to understanding the relationship between the way in which one selects paths for routing, and the amount of throughput one obtains from the resulting fair allocation on these paths. One notable exception is [**?**, **?**], where Megiddo showed that in the setting of single-source fractional flow (in which flow must be sent from a single source to a set of sinks, without pre-allocation of paths) the max-min fair allocation is a maximum flow [**?**, **?**]— one does not sacrifice throughput by imposing fairness. Moreover, Megiddo showed that the max-min fair flow can be computed in polynomial time [**?**].

Here we measure the loss of flow from max-flow, which takes place due to a fair allocation of flows on complex networks. In particular, we derive analytical results for Watts-Strogatz networks with $k = 4$ and $p = 0$ and show by numerical simulations how network fairness implies a loss of flow from max-flow for $p = 0$ as a function of k. We also show that for $p \neq 0$ fair flows have a higher throughput than for $p = 0$, indicating that even weak rewiring of the network can increase throughput considerably under the constraint that flows should be fair.

Information transport on complex networks

C5.5
15:20

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Three subjects on information transport on complex networks are investigated.

First, we investigate routing strategies on complex networks. Using neural networks, we introduce a routing strategy where path lengths and queue lenghts are taken into account within a framework of statistical physics. The performance in this strategy becomes more efficient from improvement of the distance term. At the same time, we analyze how the properties of networks influence the performance of this strategy. Moreover, we propose a routing strategy where connection weights in neural networks are adjusted by local information. We also confirm how the distance term and the properties of networks influence the performance of this adjustive strategy.[1]

Secondary, we investigate new packet routing strategies which mitigate traffic congestion on complex networks. Instead of using shortest paths, we propose efficient paths which avoid hubs on scale-free networks with a weight of each node. Firstly, we compare the routing strategy using degree-based weights with that using betweenness-based weights on two types of scale-free networks. The strategy using degree-based weights is more efficient than that using betweenness-based weights on scale-free networks generated by a preferential attachment. On the other hand, the ascendancy of the strategy using degree-based weights over that using betweenness-based weights is reversed on scale-free networks composed by taking into account the distance between nodes. Next, we consider the heuristic algorithm which improves step by step routing properties on congestion by using the information of betweenness of each node in every step. We propose new heuristic algorithm which balances traffic on networks by achieving minimization of the maximum betweenness in the much smaller number of iteration steps.

Finally, we investigate peer-to-peer (p2p) networks. Recently p2p file-sharing systems have become a new communication paradigm. We model virus spreading on p2p networks. P2p networks have two notable aspects. One is that a peer searches for a file and downloads it from a 'host peer', which has the requested file. To determine whether a peer has the requested file or not in modeling of search and download process, we introduce a parameter, which expresses the normalized amount of files stored in a peer. The other aspect is that peers leave and join the network repeatedly. The topology of the network, therefore, changes gradually with time and the behaviour of the density of infected peers becomes complicated. We appropriately simulate virus spreading on the networks under the change of its topology by the effect of leave and join. Using mean-field approximation, we obtain an analytical formulation and emulate virus spreading on the network and compare the results with those of simulation. We attain the fact that viruses spread on the network independently of the infection rate.

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Localization of Laplacian Eigenvectors on Complex Networks

C5.6
15:40

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The Laplacian matrix describes diffusion processes on networks and appears in various mathematical models of network-organized dynamical systems. Consider a network consisting of N nodes specified by a symmetric adjacency matrix, whose element A_{ij} takes 1 if nodes i and j are connected and 0 otherwise. The Laplacian matrix L_{ij} is defined as $L_{ij} = A_{ij} - k_i \delta_{ij}$, where $k_i = \sum_{j=1}^{N} A_{ij}$ is the degree of node i. Diffusively coupled dynamical systems on networks are described by

$$\frac{d}{dt}\mathbf{X}_i(t) = \mathbf{F}(\mathbf{X}_i) + \mathbf{D}\sum_{j=1}^{N} L_{ij}\mathbf{X}_j \ (i = 1, \cdots, N),$$

where $\mathbf{X}_i(t)$ is the dynamical state of node i at time t, $\mathbf{F}(\mathbf{X})$ represents intrinsic node dynamics, and $\mathbf{D}$ is a matrix consisting of diffusion constants. This class of network-organized systems includes coupled phase oscillators [2], coupled limit cycles [3], and activator-inhibitor systems [4], each of which exhibits interesting network dynamics like synchronization, chaos, and Turing pattern formation.

In analyzing network-organized systems, e.g., in the linear stability analysis of uniform states, Laplacian eigenvalues $\Lambda^{(\alpha)}$ ($\alpha = 1, \cdots, N$) and corresponding Laplacian eigenvectors $\mathbf{v}^{(\alpha)} = (v_1^{(\alpha)}, \cdots, v_N^{(\alpha)})$ satisfying $\sum_{j=1}^{N} L_{ij} v_j^{(\alpha)} = \Lambda^{(\alpha)} v_i^{(\alpha)}$ naturally appear. Their properties are thus essentially important in understanding dynamical processes on networks. Now, Laplacian eigenvectors of the well-known random scale-free networks generated by the standard Barabási-Albert algorithm [1] are especially known to localize on subsets of nodes with close degrees [2, 4]. Namely, active components $\{v_\ell^{(\alpha)}\}$ of each Laplacian eigenvector mostly concentrate on a subset of nodes $\{\ell\}$ whose degrees $\{k_\ell\}$ are close to a characteristic value $k^{(\alpha)}$. Moreover, the Laplacian eigenvalue $\Lambda^{(\alpha)}$ is approximately proportional to the characteristic degree as $\Lambda^{(\alpha)} \simeq -k^{(\alpha)}$.

It is naturally expected that strong degree heterogeneity of the scale-free networks are responsible for these interesting properties. Based on numerical calculations using several types of random networks and perturbation analysis of the eigenvalue problem with mean-field approximation of the networks, it is argued that the localization of Laplacian eigenvectors generally takes place in a class of unstructured random networks with broad degree distributions. Some applications of the results to network dynamics will also be mentioned.

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Contributed Talks C6: Complex Systems in Biology (Albert's Bar)

Analyzing the influence of the neural network topology on pattern formation

C6.1
14:00

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Equation-free techniques allow for systematic investigations of macroscopic neural network activities and their dependence on biological parameters such as kinetic parameters or the network topology. As example, the olfactory bulb network, consisting of mitral cells which are coupled in an inhibitory way via granular cells, is considered and described as spike response model. The input of this network is well defined by odor evoked stimulus specific spatio-temporal patterns on a so-called glomeruli level. This is taken from in vivo experimental data obtained by optical imaging techniques. A spatial independent component analysis of this high-resolution imaging data was used to identify and separate different neuronal populations based on their stimulus specific spatio-temporal activation. The investigated topology dependent phenomena of spatio-temporal pattern formation include first contrast enhancement between several spatially close activations, second hysteresis effects in odorant discrimination between similar odorants depending on the concentration ratios of odorant mixtures and third traveling waves which were analyzed with equation-free continuation methods and bifurcation analysis.

This is in parts joint work with C. Ellsaesser, A. Grinvald, J. Midtgaard, D. Omer, J. Reidl, H. Spors.

Motion control of groups of Artemias

C6.2
14:20

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There are many biological organisms that live in an aggregation [1]. Some of them form various dynamics and patterns when they act. For example, migratory fish such as bonito and herring form large groups. Small birds such as sparrows and swallows form disordered aggregate and fly in wandering. Large birds such as cranes and geese migrate in well-ordered formations with constant cluster velocity. All previous studies are based on the study, analysis and modeling of the group behavior [2]. In this study, our study aims to control the group behavior (i.e., formation, direction or speed), by exploiting some natural properties of the group itself.

The Artemia Salina belongs to a genus of very primordial crustacean. The Artemia larvae are called nauplii. A nauplius has only one eye containing a photo receptor that is light sensitive. In this paper, we studied the Artemias behavior as a group, and the effects of light intensity on the direction of their motions, and the response of a population of Artemias to light wavelength (red, green, blue and white). We achieved several experimental setups in order to design a strategy to control motion of Artemias, by exploiting the effect of different patterns of light with different intensities and colors. Our results proved that Artemias are attracted by high intensity light with short wavelength. We also discuss some results about the effect of magnetic, electromagnetic field and acoustic signal on the behavior of Artemia obtaining, in this way, several possibilities to perform wireless control of their motion.

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Stochastic effects in a two-component signalling system

C6.3
14:40

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Two-component signal transduction is a mechanism frequently used by bacteria to adapt to changing environmental signals. We analyse a detailed stochastic kinetic model of a two-component signalling system, motivated by the desire to understand switching behaviour, such as that involved in the transition between rapid growth and dormancy in *Mycobacterium tuberculosis*. Kierzek, Zhou and Wanner [1] recently discovered in simulations a novel mixed mode stochastic switching response, where the level of reporter gene expression in a proportion of cells varies in a graded manner as the external signal strength increases, while the remaining, genetically identical, cells continue to express the gene at a basal level. We seek to understand the relationship between positive autoregulation of the response regulator gene, and ‘all-or-none’, graded and mixed mode stochastic switching responses.

We compare deterministic and stochastic approaches - including equation-free methods - to the analysis of this system, and show that the mixed mode response is an intrinsically stochastic phenomenon. We further show that stochasticity results in an ‘all-or-none’ bistable response being observed over a much wider range of external signals than would be expected on deterministic grounds.

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Insights into the Evolution and Emergence of a Novel Infectious Disease

C6.4
15:00

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Estimates show that three-quarters of all emerging human pathogens are of zoonotic origin, revealing the importance of a better understanding of zoonotic emergences for public health. Most zoonotic, novel infectious disease in humans appears as sporadic infections with spatially and temporally restricted outbreaks, as seen with influenza A(H5N1). Adaptation is often a key factor for successfully establishing sustained human-to-human transmission. Though some diseases are able to cross the inter-species barrier and cause infections in humans, every zoonotic pathogen will ultimately fail in emergence and go extinct in the human reservoir without sufficient adaptation to the human physiology. Previous work has studied models of within-host evolution and between-host transmission in which an initially poorly transmitting pathogen acquires adaptations to human hosts, following repeated zoonotic introductions until it achieves pandemic potential. These make the natural, simplifying assumption that the host population is homogeneous, so that changes in infection parameters entirely reflect adaptations in the biology of host infection. In reality, however, factors such as human contact patterns and other host heterogeneity may also shape the risk and tempo of emergence events.

We concentrate here on the first of these factors, an area which has hitherto received little attention in the context of adapting pathogens. We give an overview of our modelling approach that consists of simple mathematical models to describe different adaptation scenarios with particular reference to spatial host contact heterogeneity within the human population. We present analytical results for the probability of emergence per introduction, as well as the waiting time to a successful emergence event. Furthermore, we show general analytical results for the statistical properties of emergence events, including the probability distribution of outbreak sizes. We first ask: how large does a single, finite host population have to be, for population size to have a negligible effect? We next present a model in which a small village is connected, by human travel, with a large city. We use this model to ask: how strong do human travel connections have to be for us to safely ignore the separation of a population between cities and villages; and how do these thresholds compare with typical human mobility patterns? We compare our analytical results with a stochastic model, which has previously been studied computationally.

Our results suggest that, for typical connection strengths between small communities and larger populations, spatial heterogeneity has only a weak effect on outbreak size distributions, and on the risk of emergence per introduction. For example, if the average reproductive number R_0 is 1.4 or larger, any village connected to a large city by just ten commuters a day is effectively just a part of the city when considering the chances of emergence and the outbreak size distribution. We present empirical data on commuting patterns and show that the vast majority of communities for which such data is available are at least this well interconnected. For plausible parameter ranges the effects of spatial heterogeneity are likely to be dominated by the evolutionary biology of host adaptation.

A simple model for spike-burst dynamics of neurons

C6.5
15:20

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Neuronal spike-burst activity is characterized by recurrent transition between rest state and firing state with a burst of multiple spikes. Certain cells in the mammal brain, for eample, neurons in the thalamus during periods of drowsiness, in attentiveness, and sleep are known to exhibit this type of spike-burst behavior [1]. Nerve cells can exhibit autonomous or induced bursting by firing discrete groups of action potentials in time. Autonomously bursting neurons are found in a variety of neural systems, from the mammalian cortex [2] and brain stem to identified invertebrate neurons. The nervous system is made up of large networks of coupled neurons that are important for many different activities. The configurations of these networks produce different modes of behavior, including synchrony and phase-locking. In the brain alone, synchrony and phase-locking has been implicated as a mechanism for memory, cognition, sensory processing, motor planning, and execution. Many neurological diseases, including Parkinson??s, schizophrenia, and epilepsy, are the result of abnormal synchronization. A clear understanding of the basic mechanisms that produce synchrony and phase locking will ultimately allow us to repair or replace functionality lost through illness and accident. We introduce a simple one dimensional model which requires only one phase variable to describe the phenomenon of parabolic bursting [3, 4]. The analysis in the continum limit reveals that for any unimodal distribution of frequencies the qualitative properties of the full and the reduced model are identical. Further, we derive analytically based on [5] an exact low dimensional description of a globally coupled network of bursting oscillators for our model. Study of the stability for this low dimensional model reveals different dynamical signatures in the parameter space. We demonstrate that the structure of the parameter space remains independent of the number of spikes per burst. The model is simple to integrate and can serve as a simple bursting unit in a network of oscillators. We study synchronization behavior of a network of bursters under different operating conditions.

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Modeling Molecular Motors: Myosin-V Processivity and the Framework for Comparison of Stepping Models

C6.6
15:40

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A molecular motor is a nano-scale protein which converts chemical energy, typically obtained from the hydrolysis of adenosine triphosphate (ATP), into mechanical work. Myosin-V is a double headed processive molecular motor which transports a variety of cargos within cells. It achieves this by walking head-over-head along an actin filament, passing through a sequence of biochemical reactions and mechanical motions, taking several successive steps before detaching.

The exact nature of the stepping mechanism of myosin-V remains unresolved due to the noise to which nano-scale measurements are subject. However, some experimental quantities can be observed. Average velocities and run lengths of the molecules have been measured against varying cellular conditions and single molecule experiments have shown that in one stepping cycle, myosin-V can take at least one large step and perhaps also a smaller step. Our work focuses on theoretical methods to extract more information - such as the stepping mechanism - from these experimental results.

A particular stepping mechanism can be encoded into a discrete stochastic model describing the processes a molecule undergoes in taking one step. Several possible different versions have previously been proposed, each encoding a different sequence of biochemical and mechanical processes. Run lengths, velocities and dwell times can be calculated for different chemical concentrations and compared against experiment.

Mathematical methods which use experimental evidence to determine the parameters of such models are discussed. Probable energetic relationships are postulated through agreement across different models. Methods for assessing the degree to which each model can reproduce the experimental evidence are presented. A quantitative analysis of competing stepping cycles and their agreement with experimental results is also discussed.

Contributed Talks C7: Coupled Oscillators (Victoria's Room)

Routes to complex dynamics in a ring of unidirectionally coupled systems

C7.1
14:00

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The emergence of complex dynamics in networks of coupled oscillators has been studied extensively in the recent years. The investigation of such systems can contribute to the understanding of nonlinear phenomena in various fields, ranging from physics and engineering to biology and neuroscience. An important question is, how the specific properties of the individual oscillator, and the coupling architecture can give rise to different types of collective behavior

We present here a system that shows a transition from stationary to periodic, chaotic, and hyperchaotic behavior only by increasing the coupling strength. Our example system is a ring of unidirectionally coupled Duffing oscillators. We show the emergence of rotating wave solutions and their transition to chaotic rotating waves. For a large number of oscillators, we observe the coexistence of several periodic solution branches with a stability that can be interpreted in terms of the classical Eckhaus scenario as a sideband instability within a family of solutions with different periods in space and time. We calculate also the unstable periodic orbits, identify their role in the skeletons of chaotic and hyperchaotic attractors and point out their importance for the development of spatio-temporal structures.

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Modelling and global bifurcations of coupled laser arrays

C7.2
14:20

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Laser arrays are technologically important examples of coupled nonlinear oscillators. They typically consist of semiconductor lasers, which exhibit a strong coupling between the amplitude and the phase of the optical field. Understanding the complex dynamics of such systems has attracted considerable interest over recent years. Mathematically, the amplitude-phase coupling causes a strong expansion of the phase-space in certain directions, and it is one of the reasons for the observed complex and chaotic dynamics [1].

We use bifurcation analysis to quantify the instabilities that arise in coupled laser arrays as a result of the amplitude-phase coupling. To this end, we develop low-dimensional coupled ordinary differential equation models that describe the spatio-temporal dynamics in these laser arrays. An important question in this context is the accurate treatment of the optical coupling that arises in such a spatially extended laser array. We use two different modelling approaches. In the weakly coupled-laser model we quantify the coupling between individual lasers by the optical fields injected from neighbouring lasers [2]. In the composite-cavity model we quantify the coupling between spatially extended modes of the entire laser array by their overlap with the laser active medium [3].

We concentrate here on the case of three coupled lasers in a linear configuration, which constitutes a first step toward understanding the stability properties of larger arrays [4]. This system exhibits a reflectional $\mathbb{Z}_2$ symmetry when the two outer lasers are identical. We present a comprehensive bifurcation analysis, where we concentrate on the transitions from stable locking to chaotic dynamics. These transitions are organised by several global bifurcations, including non-central homoclinic saddle-node bifurcations and Belyakov points. Moreover, we calculate the bifurcations that influence the synchronisation properties of the system. These are, first, bifurcations within the fixed point subspace of the $\mathbb{Z}_2$ symmetry, which maintain synchronisation between the outer lasers, and, second, bifurcations that break this symmetry and hence lead to a loss of synchronisation.

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Dynamical response and phase modes of two ratchets coupled through a dynamic environment

C7.3
14:40

B. R. Nana Nbendjo[1], U. E. Vincent[2*], P.V.E. McClintock[2]

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The lack of reflection symmetry of the ratchet potential poses challenges with regard to analytical treatment of the dynamical response. In this paper, we propose an analytical approach that allows us to obtain the dynamical response of ratchet systems by developing and transforming the asymmetric nonlinear characteristics of an asymmetric periodic ratchet potential given in variable s by [1]

$$v(s) = C - \frac{1}{4\pi^2\delta}[\sin 2\pi(s - x_0) + 0.25 \sin 4\pi(s - x_0)]$$

to be symmetric in a new variable x, where $x = s + \frac{\beta}{3\gamma}$; and β and γ are constants. We employ the harmonic balance technique to obtain the dynamical response of the deterministic ratchet system. By coupling two identical ratchets to an environment, we further examine the existence of in-phase and out-of-phase modes; and then determine the effect of the environment on the dynamical response. The quenching of chaotic regime on the two systems simultaneously are obtained along with the domain of efficient control gain parameters. The effect of the environment on transport properties is explored by computing the current, the result of which demonstrates efficient control of transport in the presence of the environment.

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Ratcheting Effect for Coupled Phase Oscillators

C7.4
15:00

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Ratcheting is a novel phenomenon for coupled oscillator systems that gives rise to asymmetric desynchronization of oscillators [1]. A particular type of robust heteroclinic network that we call heteroclinic ratchet induces the ratcheting effect. Namely, the oscillators loses frequency synchronization only when a positive detuning (mismatch in natural frequencies) is applied, whereas for small negative detuning they maintain frequency synchronized. Similarly, arbitrary small noise results in asymmetric desynchronization of certain pairs of oscillators, where particular oscillators have always larger frequency after the loss of synchronization.

In this presentation, the ratcheting effect for coupled oscillators will be explained with an example of 4-cell coupled system of phase oscillators. We show that the existence of a robust heteroclinic ratchet for this example does not depend on symmetry but depends on the specific connection structure of the coupled system. Some other examples which also show the ratcheting effect will be discussed and the consequences in synchronization of the oscillators will be shown using simulation results. Finally, the definition of heteroclinic ratchet will be given for more general dynamical systems on N-dimensional torus and the relation between heteroclinic ratchets and different synchronization types of oscillators will be analysed.

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Tailoring current reversal within ratchet systems driven by chaotic noise

C7.5
15:20

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The presence of directed transport in a static symmetric or asymmetric periodic potential driven by the class of zero mean chaotic noise from Tchebyscheff maps has been addressed in [1]. This class of chaotic noise has also been shown to modify the escape rate of an overdamped particle over a potential well [2] and also lead to the phenomenon of chaotic resonance [3]. In this talk, I will present my recent results on how current reversals can be tailored for ratchet systems driven by this class of chaotic noise and explain why it differs from those generated by random Gaussian noise. In particular, on-off ratchets with constant flashing and dichotomous flashing will be discussed. In addition, I will show how and why current reversals can result when the intensity of the chaotic noise is spatially dependent.

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Semiconductor laser subject to feedback from two external filters

C7.6
15:40

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We study the solution structure and dynamics of a semiconductor laser receiving delayed filtered optical feedback from two filter loops, also referred to as the 2FOF laser. The motivation for this study comes from applications where two filters are used to control and stabilize the laser output. We present an analysis of the continuous-wave solutions of the 2FOF laser, as described by a mathematical model with delay due to the travel time in the two filter loops. These basic solutions are known as external filtered modes (EFMs). Compared to a laser with a single filtered optical feedback loop, the introduction of the second filter significantly influences the structure of the EFMs and, therefore, the laser's operation.

Specifically, we compute and represent the EFMs as surfaces in the space of frequency ω_s, inversion level N_s of the laser, and filter phase difference dC_p. The parameter dC_p is associated with interference between the two filter fields, and it is identified as a key of the EFM structure. We make extensive use of numerical continuation techniques for DDEs and associated transcendental equations to compute the EFM surface in (ω_s, dC_p, N_s)-space and how it changes with other parameters, including the two filter detunings and the two delays. Furthermore, we use singularity theory to identify and classify the generic changes in the EFM-surface.

Overall, our geometric approach allows us to perform a comprehensive multi-parameter analysis of the EFM structure of the 2FOF laser model. To investigate the associated dynamics of the system we identify stability regions of EFMs and regions of bifurcating stable frequency and relaxation oscillations. We also show how these regions depend on feedback strength, delay times and the filter's detunings.

Contributed Talks C8: Dynamical Systems (Auditorium)

Paradoxes of dissipation-induced destabilization or who opened Whitney's umbrella?

C8.1
14:00

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There is a fascinating category of mechanical and physical systems which exhibit the following paradoxical behavior: when modeled as systems without damping they possess stable equilibria or stable steady motions, but when small damping is introduced, some of these equilibria or steady motions become unstable. Moreover, the threshold of instability in the case of small damping may be considerably smaller than that for the same system without damping [1, 4, 6]. In other words, there is a wide range of parameters for which the undamped system is stable, but which produce instability as soon as a tiny bit of damping is added to the system. This paradox of destabilization of a conservative or non-conservative system by small dissipation, or Ziegler's paradox (1952), has stimulated an ever growing interest in the sensitivity of reversible and Hamiltonian systems with respect to dissipative perturbations [5, 9, 10]. Since the last decade it has been widely accepted that dissipation-induced instabilities are closely related to singularities arising on the stability boundary[3, 7, 8]. What is less known is that the first complete explanation of Ziegler's paradox by means of the Whitney umbrella singularity dates back to 1956. We revisit this undeservedly forgotten pioneering result by Oene Bottema [2] that outstripped later findings for about half a century. We discuss subsequent developments of the perturbation analysis of dissipation-induced instabilities and applications over this period, involving structural stability of matrices, Krein collision, Hamilton-Hopf bifurcation, and related bifurcations.

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Multistability and multimode dynamics in lasers

C8.2
14:20

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Ring Lasers (RLs) present a large variety of dynamical regimes arising from the nonlinear interaction of the counter-propagating waves mediated by the active medium, e.g. bidirectional continuous wave emission, oscillation regimes [1], mode-locking [2] and chaos. In addition, RLs with low degrees of backscattering present regimes of bistable unidirectional emission [3] which can be exploited for all-optical signal processing and storage [4]. Moreover, they can also present states of stable bidirectional emission [3] that have immediate potential applications for the development of laser gyroscopes [5].

In bidirectional RLs, the dynamics can be described by means of a travelling wave model (TWM) with the appropriate boundary conditions that allow us to follow the metamorphosis from a RL to a Fabry-Pérot (FP) configuration as the reflectivity increases [6]. We study the different dynamical regimes of the system and the bifurcations connecting them for a homogeneously broadened two-level medium. We compute the steady state solutions of the system via a shooting method and we numerically perform their linear stability analysis by diagonalising the jacobian obtained from the evolution operator of the discretized system.

Close to threshold, RLs emit bidirectionally in a single-longitudinal mode due to the residual facet reflectivity, but they easily reach a regime of bistable, almost unidirectional emission via a pitchfork bifurcation. Often, this pitchfork bifurcation is preceded by a Hopf bifurcation that leads to antiphase oscillations of the power emitted in each direction [3]. The regime of unidirectional emission can become unstable via a reentrant Risken-Nummedal-Graham-Haken instability [7] depending on the parameter values. In the regime of stable unidirectional emission, RLs exhibit multistability among several longitudinal modes even when one mode is resonant with the atomic gain line. In this regime the emission wavelength can be selected among that of several cavity modes by injection of external optical pulses, in agreement with experimental reports [8]. We also find a novel dynamical regime where the emission in each direction is almost single-mode, but each direction lases at a different wavelength. This emission state may coexist with states of mode-locked unidirectional emission for large gain bandwidth and relatively small detuning [6]. Finally we compare the physical mechanisms that lead to different dynamics for RLs and FPs.

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Estimation of the Kolmogorov entropy in the generalized number system

C8.3
14:40

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The Kolmogorov entropy K_q [1, 2] that considers given time series of measurement to play important role of statistical physics[3] and number theory [4] also.

$$K_q = -\lim_{\tau\to 0}\lim_{\epsilon\to 0}\lim_{d\to\infty}\frac{1}{d\tau}\frac{1}{q-1}\ln\sum_{i_1,\dots i_d} p^q(i_1,\dots i_d) \tag{1}$$

We numerically determined the spectrum of the order-q quantities K_q in a subset of complex plain which is generated by a coefficient system in the theory of generalized number systems[5, 6].

This set is defined as follows.[7] We say that (A, M) is a number system, where M is an $n \times n$ type regular matrix with integer entries and A is digit set. Let

$$H = \{z|z = \sum_{i=1}^{\infty} M^{-i} f_i, f_i \in A\}. \tag{2}$$

If (A, M) is a number system, then $\cup_{\gamma\in\Gamma}(H + \gamma) = \mathbb{C}$, where Γ_l denotes the set of $\gamma = \varepsilon_0 + \varepsilon_1\Theta + \cdots + \varepsilon_l\Theta^l$, ε_i run over A, then $\Gamma = \cup_{l=0}^{\infty}\Gamma_l$. Futhermore $\lambda(H+\gamma_1 \cap H+\gamma_2) = 0$ holds for every $\gamma_1, \gamma_2 \in \Gamma$, $\gamma_1 \neq \gamma_2$. Let H is the fundamental region with respect to (A, M). Let $H \cap H + \gamma \neq 0$, where S be the set of those $\gamma \in \Gamma$, $\gamma \neq 0$ for which

$$B_\gamma = H \cap H + \gamma. \tag{3}$$

We study the entropy of order q in the finite subset approximation of B_γ. A map is introduced on this set by the analogy of a classical dynamical deterministic systems. We gain information from the series of measurements making a partition on the subset of complex plain. The entropy of order-q is determined by Grassberger-Procaccia algorithm [1] and it is extrapoleted to large size limit.

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Interference-induced tunneling emission in deformed microcavities

C8.4
15:00

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Deformed microcavity has attracted much attention as a realistic analog of open quantum billiards and an essential ingredient in applications like microlaser and optoelectronics. Wave mechanical combination of nonlinear dynamics and openness gives rise to many intriguing phenomena such as qasiscarred modes and nontrivial mode-energy surfaces etc.[1, 2] In this context, one of key issues is the emission directionality of high-Q modes. In a strong deformation regime exhibiting chaotic ray dynamics, the emission directionality has been well understood by unstable-manifold structure near the critical line in phase space[3, 4, 5]. However, for a weak deformation regime showing almost regular Poincaré surface of section above the critical line, only a few works have treated the emission property of high-Q modes[6, 7]. In fact, the emission in this regime is closely associated with the nontrivial multi-dimensional tunneling.

We study tunneling emissions of high-Q modes in weakly deformed microcavities. We experimentally observed that the tunneling emission is *asymmetric*, in contrast with the symmetric feature in the previous experiment[6], and the degree of the asymmetry become severe as the cavity deformation increases, resulting one main tunneling output within a $\pi/2$-angle image scanning. Also, the distance δ between cavity surface and the main tunneling-output image decreases with the deformation parameter. Through a numerical analysis, we found that the dynamical-tunneling concept is appropriate to understand the experimental observations. The asymmetric feature can be explained by the effect of an asymmetric short-time ray dynamics during the dynamical tunneling process, and the variation of δ can be understood by interference of inhomogeneous tangential waves, refractively coming out after the tunneling process.

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Towards stabilization of odd-number orbits in experiments

C8.5
15:20

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Recently, it has been shown [1], using the normal form of a subcritical Hopf bifurcation, that contrary to accepted opinion [2] time-delayed feedback control can stabilize odd-number orbits, i.e., periodic orbits with an odd number of unstable Floquet multipliers. However, the control matrix used in this counterexample is rather specific and the control is thus only applicable to experiments in special situations [3].

In this work we consider feedback matrices suited for direct experimental realization, and derive conditions for stabilization of unstable periodic orbits in Hopf bifurcations. We also introduce an additional latency time, which allows stabilization in even more general situations.

Finally, we show, using normal form analysis and numerical simulations, how our method can be apply to stabilize a subcritical orbit in a laser [4] by applying optoelectronic feedback.

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The Dynamics of a Simplified Pin-Ball Machine

C8.6
15:40

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This talk focuses on the dynamics of a combination of a map with a gap and a square root map which results from impact oscillators, e.g. simplified pin-ball machines (with one bumper).

Passive impacts which have been thoroughly studied in the past are modelled by a reset law that is governed by Newton's Law of Restitution. In contrast, the reset law of active impacts contains a further term V, which is the additional velocity at which the bumper in a pinball machine repels the ball. The dynamics of these two maps differ greatly, for example for the same parameters intervals of chaotic behaviour can be observed when $V = 0$ but we see only periodic orbits when $V > 0$. I will demonstrate in this talk that the dynamics of the active impact problem can be described in terms of the dynamics of a one-dimensional map with a gap, which is a linear/ square-root map with a discontinuity of size proportional to V. This talk will present results on the periodic orbits of this map as well as a further analysis of the associated bifurcations.

Poster Session I (Recital Room)

About the sprite influence on the chemical composition of the mesosphere

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Nowadays the researchers pay much attention to the influence of high-altitude discharges on the chemical composition of the atmosphere. During the outbreak of the sprite a sharp increase in the electron temperature and electric field appears, which continues from tens of microseconds to milliseconds, and results in greatly resent the concentration of electrons, ions, excited atoms and molecules. In this paper shown the plasma-chemical model which describes the dynamics of perturbations of chemical composition of the mesosphere at altitudes of 75 and 85 km in the generation of sprites and elves. As a result of calculations we can see that:

1. We selected the most strongly perturbed components, examined the dynamics of relaxation, color reactions, responsible for the perturbation of the chemical components. The analysis was carried out for the night and daytime conditions at altitudes of 75 and 85 km.

2. We analyzed the nonlinear correspondence between the perturbations of the chemical components of the duration of the sprite and the electric field.

3. We analyzed the sequence of the two sprites. The conclusions about the possibility of accumulation of chemical perturbations were made.

4. The necessity of taking into account the excited atoms and molecules to quantitatively describe the influence of the sprite on the composition of the mesosphere was shown.

This work was supported by the Programme PSD RAS Electrodynamics of the atmosphere, radiophysical methods of atmospheric processes.

Master Stability Function for time-delayed chaotic networks

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Chaos synchronization in a network with time delayed couplings and self-feedbacks plays an important role in secure communication, neural models or chaotic laser systems. The Master Stability Function (MSF) is a well known method to analyse the stability of the synchronization of such a network. We analyse the MSF for Bernoulli networks by using the Schur-Cohn theorem and derive symmetry arguments for the possible appearance of zero lag synchronization. We apply these results to chaotic semiconductor laser systems in simulations using the Lang-Kobayashi equations and present experimental results.

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Revolving ferrofluid flow due to rotating flat plate in porous medium

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The problem of revolving flow of ferrofluid over a moving plate in a porous medium is theoretically investigated by solving boundary layer equations for the Neuringer Rosenweig model. Non linear differential equations are solved by power series method. The effect of porosity on the velocity components, pressure profile and displacement thickness of the boundary layer are observed. The inclusion of three effects, the porosity, rotation and ferrofluid characteristics have shown some interesting results.

Hierarchically consistent geometric optimal control of nonlinear dynamical systems

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Most of the real modern systems are complex, nonlinear, and large-scale. Large-scale systems are typically imposed a hierarchical structure in order to manage complexity. In hierarchical control models, the notion of consistency is much important, as it ensures the implementation of high-level objectives by the lower level systems. In this work, I present a dynamic model for synthesis of hierarchical control systems for complex nonlinear multidimensional and multiconnected dynamical systems. A hierarchical system is understood as a set of interacting subsystems that organize separate hierarchical levels. On each level the system has its local goals, *invariants*. The goals are defined in the state space of the system. The set of goals G has a hierarchical structure; it is decomposed into subsets $G_i, i = \overline{1, \mu+1}$, consisting of the goals of each of the subsystems, $\cup_{i=1}^{\mu+1} G_i = G$. On each level the subsystems are described in the corresponding space of parameters and variables, some of which are *polymorphic*. Each of the subsystems or a class of subsystems is immersed onto the intersection of the corresponding *local attractors*, invariant manifolds, related to the concrete subset of goals $G_j, j = \overline{1, \nu}, \nu \leq \mu$, but the whole system is immersed into the *global attractor*, called *hierarchical state manifold*, which corresponds to the set of system goals G; this approach is applicable at any level of abstraction. The dynamics of high-level subsystem is described by the equation

$$S_{\mu+1} \quad : \quad \begin{aligned} \dot{\mathbf{x}}(t) &= f\left[\mathbf{x}(t), \mathbf{u}(t), \mathbf{q}(t)\right], \\ \mathbf{y}(t) &= h\left[\mathbf{x}(t), \mathbf{y}(t)\right], \end{aligned} \tag{1}$$

where $\mathbf{x} \in \mathcal{X} \subset \mathbb{R}^n, \mathbf{u} \in \mathcal{U} \subset \mathbb{R}^m, \mathbf{y} \in \mathcal{Y} \subset \mathbb{R}^r, \mathbf{q} \in \mathcal{Q} \subset \mathbb{R}^d$; $\mathbf{x}(t), \mathbf{u}(t), \mathbf{y}(t)$, and $\mathbf{q}(t)$ represent, respectively, the state vector, the control vector, the output vector, and the disturbance vector. The dynamics of each of the subsystems is described by the equations

$$S_j \quad : \quad \begin{aligned} \dot{\mathbf{z}}(t) &= g^{(j)}\left[\mathbf{z}^{(j)}(t), \mathbf{w}^{(j)}(t), \mathbf{v}^{(j)}(t)\right], \\ p_j &= s^{(j)}\left[\mathbf{z}^{(j)}(t), \mathbf{w}^{(j)}(t)\right], \end{aligned} \tag{2}$$

where $\mathbf{z}^{(j)} \in \mathcal{Z}^{(j)} \subset \mathbb{R}^{k_j}, \mathbf{w}^{(j)} \in \mathcal{W}^{(j)} \subset \mathbb{R}^{\ell_j}, \mathbf{v}^{(j)} \in \mathcal{V}^{(j)} \subset \mathbb{R}^{\lambda_j}$; $\mathbf{z}^{(j)}(t), \mathbf{w}^{(j)}(t)$, $\mathbf{v}^{(j)}(t)$, and p_j represent, respectively, the state vector, the control vector, the disturbance vector, and the output variable of j-th subsystem; and $\dim G_j \leq \dim \mathcal{W}^{(j)}$. The neighbor levels are connected by the equation $\mathbf{x} = \mathbf{F}(\mathbf{p})$, where $\mathbf{p} = [p_1, p_2, ..., p_n]^T$. Based on the decomposition G_i, I define the notion of *consistent hierarchical dynamics* by introducing the *sequential-parallel* invariant manifolds that satisfy a vector functional equation, and the *aggregated* invariant manifolds.

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Estimation of local scour downstream of positive step stilling basins by support vector machines

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The estimation of local scour below low head spillways with reasonable accuracy is important for technical, safe and economic design of the spillways. Scouring is a complex and dynamic phenomenon that is governed by many parameters. Accurate prediction of local scour formed downstream of positive step stilling basin has been mainly based on the experimental studies using physical modeling and results obtained are inconclusive. It has been found that the traditional as well as new regression based approaches are not successful in predicting the local scour depth satisfactorily downstream of positive step stilling basins of spillways. Because of non linearity and complexity of flow, bed material and geometric parameters of spillways, more robust tools are required to model scour process downstream of the spillways. In this paper, prediction of local scour downstream of spillways (based on 101 data sets of laboratory measurements) using linear regression and SVM (rbf and polynomial kernels) techniques have been attempted. The performance of modeling techniques is validated in training and testing data sets in terms of common statistical measures such as correlation coefficient and root mean square error. The findings of this study, encourages the application of SVM modeling approach in predicting the local scour downstream of positive step stilling basin of spillways as an alternative technique to the empirical relations and the linear regression.

The mechanism of vorticity depletion and large time asymptotics of the 2D Euler equations

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The 2D Euler equations is a time reversible dynamical system. Still the large scales of the flow show a irreversible relaxation towards steady solutions of the 2D Euler equations [4]. This very important reversibility/irreversibility paradox can be understood for the first time, directly from a detailed dynamical study, thanks to new results for the linearized dynamics on one hand and a controlled multiple time scales expansion for the non-linear dynamics on the other hand. The results also have implication for real turbulent flow subject to stochastic forces.

We study the large time asymptotic behavior and the asymptotic stability of the 2D Euler equations and of the 2D linearized Euler equations close to parallel flows [2]. We focus on flows with spectrally stable profiles $U(y)$ and with stationary streamlines $y = y_0$ (such that $U'(y_0) = 0$), a case that has not been studied previously. We describe a new dynamical phenomenon: the depletion of the vorticity at the stationary streamlines. An unexpected consequence is that the velocity decays for large times with power laws, similarly to what happens in the case of the Orr mechanism for base flows without stationary streamlines. The asymptotic behaviors of velocity and the asymptotic profiles of vorticity are theoretically predicted and compared with direct numerical simulations. We argue on the asymptotic stability of this ensemble of flow profiles even in the absence of any dissipative mechanisms.

In order to make prediction for the fluctuations of real turbulent flows, we present theoretical results for the prediction of the velocity and of the vorticity fluctuations, in the context of the Navier Stokes stochastic (NSS) equation, with weak stochastic forcing and dissipation [3]. Theoretical arguments and numerical evidences show that flows are then close to equilibria of the Euler equation. At leading order, fluctuations around such equilibria are then described by the linearized NSS equation.

We thus study theoretically the linearized NS equation with random forces. In the limit of zero dissipation, as expected no stationary distribution exist for the Gaussian vorticity field. By contrast, the Gaussian stream function or velocity fields strikingly converge toward a stationary Gaussian process. The velocity field thus acts similarly to a dissipative system, when dissipation is no more present. An explanation of this seemingly anomalous behavior and its relation to the deterministic Landau damping of plasma physics and Orr mechanism for 2D vortices will be given.

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The influence of discrete symmetries on universal level repulsion

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In recent years periodic-orbit theory has indicated why quantum chaotic systems should, in the semiclassical limit, have energy level distributions predicted by random matrix theory (RMT) [1, 2], where the correct choice of RMT ensemble depends on the time-reversal invariance properties of the system. Interestingly, by incorporating extra discrete spatial symmetries these RMT predictions have to be amended and the choice of ensembles becomes somewhat counterintuitive. However through incorporating additional methods [3, 4] we can use the periodic-orbit theory approach to explain these predictions.

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Different feedback-induced bifurcation scenarios of quantum-dot and quantum-well lasers

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We compare the complex dynamics of semiconductor quantum-dot (QD) and quantum-well (QW) lasers subjected to weak optical feedback from a distant mirror. For the QW laser we use a standard Lang-Kobayashi model consisting of one equation of motion for the complex amplitude of the electrical field and one equation of motion for the carrier density [1]. The QD laser system is modeled by a modified Lang-Kobayashi equation for the electric field combined with microscopically based rate equations for the carriers in the QDs and the surrounding wetting layer. In contrast to the simpler Lang-Kobayashi model our model separately treats electrons and holes in the QDs and the wetting layer allowing for insights into the internal nonlinear dynamics of the laser [2].

In dependence of the feedback strength and the phase-amplitude coupling (linewidth-enhancement factor α) we study the bifurcation scenarios of the optical output of the lasers. They show periodic and chaotic intensity pulsations alternating with windows of stable continuous wave (cw) operations.

The simplest solutions of both systems with feedback are the external cavity modes (ECMs). These are circular periodic orbits (rotating waves) having constant photon-, and carrier densities and a phase varying linearly in time. At the beginning of each instability region one of the ECMs loses stability in a supercritical Hopf-bifurcation, leading to more complex solutions with an intensity that is periodically modulated with the relaxation oscillation frequency. With increasing feedback strength this solution undergoes a period-doubling or quasi-periodic route to chaos. By performing a linear stability analysis, we can analyze the stability of the modes in dependence of the feedback strength.

For large $\alpha > 3$ we find several bifurcation cascades interrupted by short regions of cw operation. Furthermore the systems exhibit multi-stability in the instability regions. However, for small $\alpha < 1$ the models display enhanced feedback tolerance and thus perform stable cw operation over a wide range of increasing feedback strength [3].

Comparing the bifurcation diagrams of QD and QW lasers, we are able to attribute the reduced feedback sensitivity of QD lasers to their small phase-amplitude coupling (small α) as well as to their strongly damped relaxation oscillations. The nonlinear microscopic scattering rates that determine the damping rate and relaxation oscillation frequency of the solitary QD laser are shown to determine the quantitative position, i.e. the critical feedback, at which the lasers become unstable in a supercritical Hopf-bifurcation.

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Implications of Low and High-frequency Periodic Perturbations in Maps

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Real dynamical systems are always subject to perturbations, some of which are intrinsically periodic and linked to its origin. One example is the day-night cycle, in which life itself has evolved since the beginnings. In simple models of populational reproduction this factor can be introduced into the model simply as a periodic disturbance of some parameter (a food-linked one, for example) in the system. This perturbation will have its own frequency, which can be locked or not with the main frequency of the system. The spam of frequency values is quite large in systems, ranging for instance from a day-night cycle for a bacterial culture, with many generations being affected by a single oscillation, to changes in biochemical interplay in the brain, where periodic neuronal shots of just order of milliseconds in some specific neuronal population can lead to oscillations of the order of minutes in other sets of neurons.

In this paper we start studing a periodic disturbance in the logistic map aiming to extend to maps of neuronal models. This problem is classical in some sence [1], but here we are interested in how extreme changes in frequency (far from the main oscillation frequencies of the map) affect bifurcation diagram and nature of the system itself. This concept of high and low frequency may seem strange at first sight, because when the model begins the period doubling cascade we quickly see that it tends to an infinite period. Here the concept of high frequency is related to the very slow changes in the control parameter, far away from the main periodicities.

In order to get started, consider a periodic perturbation to the Logistic equation in the form

$$x_{t+1} = \mu[1 + \epsilon \sin(\omega t)]x_t(1 - x_t), \tag{1}$$

where $\epsilon \ll 1$, ω is a real frequency and $0 < x_t < 1$. This type of system leads to problems of multistability (which are difficult to treat) within the limits of frequencies that are near the main system frequencies.

We can divide the results into two types: for high values of ω ($\omega \gg 1$) we note that the effect of introducing the perturbation is the same as increasing a stochastic perturbation on the system. This is easily explainable because, in each generation, the system would be forced in a completely different way. In fact we split this systema in a higher number of unlocked-phase systems with similar behaviour. In the case of periods with $\omega \ll 1$, we noticed one of the richest structures, where the periodic orbits now tend to locate themselves on continuum regions and chaos emerges more sharply as we increase the control parameter.

In the sequence of the work we will explore the dependencies of the routes to chaos with μ, ϵ and ω.

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Molecular dynamics of multiple relaxation processes in copolyesteramide (Vectra B 950) polymer liquid crystal

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Molecular dynamics associated with the various relaxation processes in copolyesteramide (Vectra B950) polymer liquid crystal has been investigated using FITSC technique. The current maxima appearing in the poling/ depoling current characteristics were analyzed under different field and temperature conditions. The four major relaxation processes have been recognized namely β', β, α and δ with their respective location around 30,110, 160 and 220C. The β- relaxation arises from dipolar nature of carbonyl group present in HNA and TA non LC phase of vectra-B whereas β' - and α - relaxations are due to the dipolar nature of ester groups and liquid crystalline phase in vectra B. The high temperature δ -relaxation has been associated to onset of melting of LC phase. Cole- Cole distribution has also been obtained for these relaxation processes.

Anomalous diffusion and formation of barriers in nontwist area-preserving maps

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Nontwist maps appear in many physical problems, in particular, when modeling continuous systems that describe features of fluids and plasmas. Much previous research indicates that transport properties of such systems can be well described by area preserving maps [1]. Many examples exist, e.g., in the fields of fluid mechanics and plasma physics; in particular, in models that describe zonal flows that occur in geophysics, atmospheric science, and fusion plasma physics . The standard nontwist map (SNM) is a well-known paradigm for investigating transport in reverse shear systems [2]. The nonmonotonicity property of the SNM gives rise to transport barriers due to robust tori (invariant curves) that occur in zonal flows. These tori separate regions of the two-dimensional phase space. Moreover, the influence of these barriers on the transport remains even after the breakup of the tori. This phenomenon, i.e., the difficulty encountered in crossing broken barriers, is explained by examining the stickiness of orbits that occur in some regions of the map phase space. For a certain range of control parameters, these regions emerge near resonances. In order to investigate the transport properties of nontwist systems, we analyze the barrier escape time and barrier transmissivity for the standard nontwist map [3]. We interpret the sensitive dependence of these quantities upon map parameters by investigating chaotic orbit stickiness and the associated role played by the dominant crossing of stable and unstable manifolds. We show that the values of transmissivity for different values of parameters has strong dependency of the manifolds crossings configuration.

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Toy model for tidal synchronization and orbit circularization

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We study the simplest possible system that displays tidal synchronization and orbit circularization, published in [1], with a minimal model that takes into account only the essential ingredients of tidal deformation and dissipation in the secondary body. We model an extended secondary body of mass m by two point masses of mass $m/2$ connected with a damped spring. This composite body moves in the gravitational field of a primary of mass $M \gg m$ located at the origin. In this simplest case oscillation and rotation of the secondary are assumed to take place in the plane of the Keplerian orbit. The gravitational interactions of both point masses with the primary are taken into account, but that between the point masses is neglected.

Despite its simplicity, our model can account for both synchronization into the $1:1$ spin-orbit resonance and the circularization of the orbit as the only true asymptotic attractor together with the existence of relatively long-lived metastable orbits with the secondary in $p:q$ (coprime integers) synchronous rotation.

In order to understand the long-term evolution of the orbit and the stability of the resonances it is necessary to study the details of the energy and the angular momentum transfer between the orbital and roto-vibrational degrees of freedom.

We perform a Taylor expansion on the exact equations of motion to isolate and identify the different effects of tidal interactions. We compare both sets of equations and study the applicability of the approximations, in the presence of chaos. We introduce a resonance function as a resource to identify resonant states.

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Scaling properties for a set two dimensional Hamiltonian maps

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We study the transition from integrability to non-integrability for a set of two dimensional Hamiltonian mappings exhibiting mixed phase space. The phase space of such mappings show a large chaotic sea surrounding KAM islands and limited by a set of invariant tori. The description of the phase transition is made by the use of scaling functions for average quantities of the mapping averaged along the chaotic sea. The critical exponents are obtained via extensive numerical simulations. Given the mappings considered are parametrized by an exponent γ in one of the dynamical variables, the critical exponents that characterize the scaling functions are obtained for many different values of γ. Therefore classes of universality are defined.

We consider the following expression for the two dimensional mapping:

$$T : \begin{cases} x_{n+1} = \left[x_n + \frac{a}{y_{n+1}^{\gamma}}\right] \mod 1 \\ y_{n+1} = |y_n - b\sin(2\pi x_n)| \end{cases} , \tag{1}$$

where a, b and γ are the control parameters. The determinant of the Jacobian matrix is Det J $= \text{sign}(y_n - b\sin(2\pi x_n))$ where $\text{sign}(u) = 1$ if $u > 0$ and $\text{sign}(u) = -1$ if $u < 0$. Note that if $\gamma < 0$, depending on the initial conditions and control parameters, one can observe unlimited growth for the variable $\bar{y}$. Such growth is observed since large values of y implies in a large number of oscillations for the sin function. Then, in the regime of very large oscillations, the sin function works more likely a random function yielding in an unlimited growth for $\bar{y}$. Since we want avoid such a condition in order to use the scaling approach, in this paper we consider only values for $0 < \gamma \leq 1$. Note however if we choose $\gamma = 1$ and change the dynamical variables to $x \to v$ and $y \to \phi$, the mapping T recovers the one-dimensional Fermi-Ulam accelerator model where v represents the velocity of the particle and ϕ denotes the phase of the moving wall. For this case the critical exponents are well defined [1]. It is important to emphasize that there are two control parameters in mapping T that control the transition from integrability to no integrability, namely $a = 0$ or $b = 0$.

The critical exponents were obtained via extensive simulations and scaling hypotheses were all supported by a perfect collapse of all the curves of the deviation around the average quantities for the chaotic sea [2]. Lyapunov exponents have also been used to quantify the intensity of chaos of the mappings.

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Phase reduction of stochastic limit cycle oscillators

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Many physical systems are mathematically modeled by limit cycle oscillators. A fundamental theoretical technique for studying oscillator dynamics is the phase reduction method [1]. Dynamics of oscillators subject to noise has attracted much interest. It is important to develop the phase reduction theory for noisy oscillators, in order to clarify the roles of noise in oscillatory physical systems. The conventional phase equation is not a proper approximation for an oscillator subject to noise even when the noise is weak. We present the phase equation valid for noisy oscillators.

Consider the stochastic differential equation

$$\dot{\boldsymbol{x}} = \boldsymbol{F}(\boldsymbol{x}) + \boldsymbol{G}(\boldsymbol{x})\eta(t), \tag{1}$$

where $\boldsymbol{x} \in \mathbb{R}^N$, $\boldsymbol{F}$ and $\boldsymbol{G}$ are smooth vector functions, and $\eta(t)$ represents a noise. The unperturbed system $\dot{\boldsymbol{x}} = \boldsymbol{F}(\boldsymbol{x})$ is assumed to have a limit cycle $\boldsymbol{x}_0(t)$ with the frequency ω. For simplicity, we assume that the Floquet matrix for $\boldsymbol{x}_0$ is diagonalizable. Let ϕ be a phase coordinate defined in a neighbourhood of $\boldsymbol{x}_0$ in phase space. If we appropriately define the other $N-1$ coordinates $\boldsymbol{r} = (r_1, \ldots, r_{N-1})$

such that $\boldsymbol{r} = \boldsymbol{0}$ on $\boldsymbol{x}_0$, equation (1) is rewritten as

$$\dot{\phi} = \omega + h(\phi, \boldsymbol{r})\eta(t), \tag{2}$$

$$\dot{r}_i = -\lambda_i r_i + f_i(\phi, \boldsymbol{r}) + g_i(\phi, \boldsymbol{r})\eta(t), \tag{3}$$

where $i = 1, \ldots, N-1$ and λ_i is a positive constant.

Let $\xi(t)$ be the white Gaussian noise such that $\langle \xi(t) \rangle = 0$ and $\langle \xi(t)\xi(s) \rangle = 2D\,\delta(t-s)$. In the case that $\eta(t) = \xi(t)$ and the Stratonovich interpretation is employed for Eq. (1), the phase equation is given by the Stratonovich-type stochastic differential equation

$$\dot{\phi} = \omega + DY(\phi) + Z(\phi)\xi(t), \tag{4}$$

where $Z(\phi) = h(\phi, \boldsymbol{0})$ and $Y(\phi) = \sum_{i=1}^{N-1} \{\partial h/\partial r_i(\phi, \boldsymbol{0})\}\, g_i(\phi, \boldsymbol{0})$ [2]. Existence of the Y-term in Eq. (4) is the difference from the conventional phase equation. This term is important because it describes a shift of the oscillator frequency induced by the noise. The noise-induced frequency shift significantly influences entrainment properties of the oscillator. Since time correlation of noise is ubiquitous in the real world, it is also important to generalize the phase equation to the case of colored noise. We consider a class of colored noise which are generated from white Gaussian noise via differential equations. A typical and simple example is the Ornstein-Uhlenbeck noise, which is generated via the equation $\dot{\eta} = -\gamma\eta + \gamma\xi(t)$, where $\gamma > 0$ is a constant. We show that the phase equation is obtained in a form similar to Eq. (4) for this class of colored noise [3].

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Stability and bifurcations in an epidemic model with temporary immunity

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In this talk I will present the derivation of an epidemic model with distributed time delay that in order to describe the dynamics of infectious diseases with varying immunity. I will show that solutions of the model are always positive, and the model has at most two steady states: disease-free and endemic. It is proved that the disease-free equilibrium is locally and globally asymptotically stable. When an endemic equilibrium exists, it is possible to analytically prove its local and global stability using Lyapunov functionals. Bifurcation analysis is performed using DDE-BIFTOOL and traceDDE to investigate different dynamical regimes in the model using numerical continuation for different values of system parameters and different integral kernels.

Noise-induced synchronization in the Sakaguchi-Kuramoto model

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Because real systems are inevitably subject to noise, it is important to understand the effect of noise on the synchronization of periodic oscillators. There are many situations where a single noise process, such as that originating from environmental fluctuations, acts on an entire system. Whether such common noise enhances or inhibits synchronization is actually unclear. This issue is thought to be relevant to biological pacemaker tissues in that external noise could have a positive effect on synchronization.

The effect of common noise on uncoupled oscillators or coupled-oscillator networks with small sizes has been extensively studied for periodic oscillators, and rigorous theoretical frameworks have been proposed [1, 2]. In contrast, for a large population of coupled oscillators, theoretical treatment is still an open and challenging problem.

We study a large population of globally coupled phase oscillators, known as the Sakaguchi-Kuramoto model [3], subject to common white Gaussian noise

$$\frac{\mathrm{d}\theta_i}{\mathrm{d}t} = \omega_i + \frac{K}{N}\sum_{j=1}^{N}\sin(\theta_j - \theta_i + \beta) + p(t)\sin\theta_i, \tag{1}$$

$$\langle p(t)\rangle = 0, \tag{2}$$

$$\langle p(t)p(s)\rangle = 2D\delta(t-s), \tag{3}$$

where θ_i and ω_i are the phase and the natural frequency, respectively, of the i-th oscillator, $K > 0$ is the coupling strength, β is a parameter of the coupling function ($-\pi/2 < \beta < \pi/2$), and $p(t)$ is a common external random force. Utilizing the anzatz recently proposed by Ott and Antonsen [4], we analytically show that the addition of common noise leads to a decrease in the critical coupling strength for synchronization transition by D as compared to that in the original Kuramoto transition. Our prediction is corroborated by direct numerical simulations of the model. We also numerically confirm that globally coupled limit-cycle oscillators show the same dependence on common noise.

The employed phase model approximates many realistic systems with weak coupling and weak forcing. Thus, our results suggest that weak common noise generally promotes synchronization of oscillators with week and global coupling. It would be interesting to experimentally investigate the effect of common noise on coupled biological and chemical oscillators.

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Enlargement of a stochastic web and enhancement of chaotic transport through it

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Low-dimensional stochastic webs represent a remarkable chaotic phenomenon. They occur in 1D Hamiltonian systems subjected to a perturbation that is periodic in time and space while the period of small eigenoscillations is equal, or nearly equal, to a multiple of the perturbation period. The system may diffuse along such a web for a significant distance in any given dynamical variable, a feature that may be important in a variety of applications [1, 2]. However, in the archetypal web arising in the oscillator perturbed by the resonant traveling wave [1, 3], chaotic transport from the centre of the web is limited to rather short distances because the width of the chaotic layer decreases exponentially fast with distance from the centre.

We now propose a method of greatly increasing the rate of chaotic transport and the distance over which it occurs. This may be either a slow modulation of the wave angle, or by the addition of the auxiliary wave with a slightly shifted frequency. The former option may e.g. greatly enhance the control of quantum transport properties of electrons in nanometre-scale semiconductor superlattices with an applied voltage and magnetic field (compare [2]).

Consider an example of an equation of motion of the relevant type:

$$\ddot{q} + q = 0.1 \sin[15q - 4t - h \sin(0.02t)] \tag{1}$$

The efficiency of the method is demonstrated by its numerical integration, first for $h = 0$ (i.e. for the conventional case with parameters as in [1, 3]), and secondly for $h = 0.1$. Although the modulation in the latter case is **weak** (i.e. of amplitude small compared to the 2π period of the wave angle), the resultant increase in the size of the web is **large**: by factors of approximately $n = 6\times$ in coordinate and momentum and by $n^2 = 36\times$ in energy.

To account for these results we develop the analytic theory. The theory is generalized for the off-resonant case [1, 4] too, using the general method developed recently in [5]. We also discuss applications of the results.

It is anticipated that the method can also be generalized for uniform webs [1, 6], leading to an exponentially strong enhancement of chaotic transport through them.

Some of the results have been published in our review on stochastic webs [7].

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A family of stadium-like billiards with parabolic boundaries under scaling analysis.

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A two dimensional billiard consists of a domain Q with a piecewise-smooth boundary ∂Q inside which a point-mass particle (billiard ball) moves freely along straight lines with constant velocity. Reaching the boundary, the particle is reflected from it elastically such that the normal component changes its sign while the tangent one is preserved. This condition leads that the incidente angle is equal to the angle of reflection.

Often, the dynamics of the system depends on the control parameters, and they can control the intesity of the nonlinearity of the system. As they change, the dynamics can presents subtle modifications, in particular phase transitions. Many of these transitions can be described using scaling arguments [1]. In this work we consider a family of stadium-like billiards with fixed parabolic boundaries described by a two-dimensional nonlinear area preserving map [2], that is given by

$$T : \begin{cases} \xi_{n+1} = \xi_n + \dfrac{l}{a} \tan \psi_n \quad \mod 1 \\ \psi_{n+1} = \psi_n - \dfrac{8b}{a} (2\xi_{n+1} - 1) \ , \end{cases} \tag{1}$$

where x_n is the collision point inside the billiard and ψ_n is the angle between the trajctory and the vertical line and it is measured counter-clockwise.

We have shown that geometrical parameters of this family can be rescaled in the sense that the maximal Lyapunov exponent of chaotic components of the billiard map is scaling invariant under the billiard geometry [3].

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Anomalous transport in open horizon billiards with oscillating boundaries

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One of the principal points of contemporary non-equilibrium statistical physics is to achieve a deep understanding of the macroscopic transport properties by parameters which appear in deterministic systems of equations. A considerable progress has been made here for some low-dimensional models. Very effective constructions among them turned out to be spatially periodic systems (open horizon billiards or Lorentz gas) in which moving non-interacting particles collide with fixed scatterers.

Apart from specific interests, open horizon billiards present the simplest physical models of classical statistical mechanics which provide clear mathematical examples of classical chaos. However, physically more realistic models than periodic billiards with fixed obstacles are systems in which scatterers are swaying nearby their equilibrium state. Indeed, because the Lorentz gas symbolizes the flow of light electrons through an array of hard ions, then one should imply that ions must oscillate with a small enough amplitude. This model naturally leads to the introduction of stochastic oscillations (i.e. time dependency) of the billiard boundary [1]. Recently, using such an approach a thermodynamic billiard approach [2] has been developed.

Billiards with oscillating boundaries are also very useful models for the explanation of some important physical phenomena. So, the observed diffusion acceleration of adatoms during the crystal growth, when adatom interacts with the diffusing island regions of the growing layer, can be described in terms of time-dependent billiards [3]. Also, considerations of billiard systems turn out to be very important in solving the problem related to irreversibility, correlation of random and dynamical factors in the diffusion motion, etc.

In this context, one of the essential problems of time-dependent billiards is to describe several consequences of dynamical chaos such as transport properties. It is obvious that, owing to scatterer oscillations these properties can be qualitatively changed. In particular, Fermi acceleration inherent in such billiards may crucially influence on the velocity of diffusion.

The main purpose of the present paper is to describe Fermi acceleration in open horizon billiards with stochastically oscillating boundaries and, on the basis of the obtained results, investigate their transport properties. We focus our attention on two-dimensional periodic billiards, where the scatterers form a periodic array and stochastically oscillate nearby their equilibrium state.

To describe the transport properties of such billiards, the motion of the billiard particle is considered on the basis of Machta-Zwanzig approximation, i.e. via a set of successive passages (jumps) from one cell of the billiard configuration space to an adjacent cell. Therewith, the probability of such passages in a unit time is determined via the particle velocity, the size of channels which connect these cells, and the area of cells. This approach clearly reveals that Fermi acceleration plays a vital part in the dynamics of open horizon billiards and leads to anomalous transport.

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A Discrete Time Dissipative Forced Oscillator

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It is know that a very interesting case of maps is that of a conservative harmonic oscillator perturbed by sequence of periodically applied δ-like pulses (kicks) at times $t = T, 2T, 3T, \ldots$. This system has been proposed as a model for studying quantum chaos in trapped ions [1, 2], ionization of atoms by a train of pulses [3] or electronic transport in semiconductor super-lattices [4]. Usually, the system is resolved by integrating the equations of motion over the period T [5, 6]. When the perturbation is sinusoidal and the dissipation is zero a well-know discrete map is obtained, called *web-map* [7]. We follows a different approach [8]. We start from the equation, which describes the motion of a linear oscillator subjected to an external position-dependent force $\ddot{x} + \omega_0^2 x + 2\gamma\dot{x} = F(x)$. By discretizing the time $t_n = n\Delta t$, and introducing suitable variables and parameters, we obtains the map

$$\begin{aligned} z_{n+1} &= [bw_n + k\varphi(z_n)]\sin(\alpha) + z_n\cos(\alpha) \\ w_{n+1} &= [bw_n + k\varphi(z_n)]\cos(\alpha) - z_n\sin(\alpha), \end{aligned} \tag{1}$$

where $\varphi(z)$ models the perturbation, b the dissipation, and $\alpha = 2\pi/q$ (q = integer) sets the resonance condition. In fact, discretization introduces an additional half degree of freedom and produces an effect equivalent to the action of a periodic sequence of δ-pulses. Besides, differently from what happens for the standard treatment, the map (1) can be continued to $b = 0$, where becomes one-dimensional.

For weak k, dissipation destroys the stochastic net of phase plane ($b = 1$) and periodic attractors P_N of period $N = nq$, ($n = 1, 2, \ldots$) appear and disappear by saddle node bifurcation and period doubling.

Increasing k toward the instability region of origin, chaotic bands and strange attractors appear. We studied (i) sinusoidal $\varphi(z) = \sin(z)$ and (ii) Gaussian $\varphi(z) = ze^{-z^2}$ pulses, which give complementary properties to the system: forcing symmetry and resonance symmetry dominance, respectively. With (i), coexistence between chaotic and periodic attractors (P_1 attractors, together their fast period doubling cascade) is evidenced in particular intervals of the forcing parameter k. With (ii), q-symmetrical periodic points are found also for very large k, and chaotic states bifurcate by crisis in periodic windows.

Therefore, the system shows various and interesting properties, which can be investigated in the parameters space.

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Fuzzy clustering of biologically active compounds using impedance spectroscopy

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The problem of forecasting the relation between the structure of various medicines and their biological activity is an important issue in medical chemistry and clinical pharmacology. Development of new tools for studying this relation provides a significant advance in reducing financial investments required for the design of new biologically active compounds. One of the existing experimental techniques used for efficient identification of chemical properties is impedance spectroscopy. This method provides a number of spectral characteristics by means of finding a dependence of a complex impedance on the frequency of applied voltage. These characteristics and other structural parameters are used to perform clustering of biologically active compounds, i.e. relating them to clusters of medicines with known pharmacological properties.

In this work we propose a new method of fuzzy clustering that augments the classical fuzzy c-means by an additional criterion based on information entropy. Such two-criterion clustering procedure provides a much sharper and efficient fuzzy clustering of multi-parameter objects. The method we propose can be applied to perform fuzzy clustering of biologically active compounds in a wide range of practical contexts and experimental setups.

Axi-symmetric ferrofluid flow with magnetic field-dependent viscosity

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In present case, we have investigated the effect of magnetic-field dependent viscosity on revolving flow of ferrofluid. Neuringer Rosenweig model has been used in this problem. Non linear differential equations with boundary conditions are solved by power series approximation. Radial, tangential, axial velocities and pressure have been calculated numerically and their profiles are expressed graphically. Displacement thickness is also obtained. The flow of ferrofluid is taken steady and Axi symmetric. Further, numerical results of velocities, pressure profile and boundary layer displacement thickness are compared with the results of ordinary viscous fluid.

Synchronization Transition in The Kuramoto Model with Colored Noise

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The last decade has seen a renewed interest in the synchronization transition in systems of coupled oscillators. Even though the phase reduction method of Kuramoto and his seminal work on the synchronization of globally coupled oscillators [1] have been around for over 30 years, the discovery of chaotic phase synchronization and the shift of interest in the physical community to dynamic processes on networks have kept phase models popular. They show a rich dynamical behavior that often captures the mechanism of physical effects, despite their simplicity.

I derive the critical condition for the onset of synchronization in the Kuramoto model subject to colored noise

$$\dot{\vartheta}_n = \frac{1}{N}\sum_m g(\vartheta_m - \vartheta_n) \; + \; \sigma\eta_n(t) \tag{1}$$

at the examples of a dichotomous Markov-process $\eta(t) \in \{-1, 1\}$ with switching rate γ and an Ornstein-Uhlenbeck process $\eta(t) \sim N(0,1)$ with unit variance and relaxation rate γ. At very large or very small values of the ratio $x = \gamma/\sigma$ the timescales of the noise and the oscillator dynamics are separated as the known limits for white noise ($x \to \infty$, $\sigma x^{-1} = D = const$) and quenched noise ($x \to 0$, $\sigma = const$) are approached [1]. However, natural systems are often subject to perturbation from a wide range of timescales. Moreover, if the noise is generated dynamically by the system itself, as for instance in chaotic oscillators or in sparse networks, where $\sigma\eta(t)$ is a local deviation from the global mean field, the time scale of the noise and the oscillators is usually not separated. The presented method may be applicable to also estimate the critical coupling strength in these systems.

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Random sequential packings of arbitrary ellipsoids

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We introduce a procedure for packing ellipsoids sequentially taken from prescribed distributions for sizes and aspect rations. The procedure uses molecular dynamics algorithms and an analytical formulation for detecting collisions between ellipsoids. The notion of separation plane of particle pairs is used for optimizing the collision detection. The algorithm is devised for simulating sedimentation of particles with the aim of modeling the stages of sandstone formation, namely, sedimentation, compression and crystallization.

Separatrix chaos: a new theoretical approach, with applications

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Separatrix chaos plays a fundamental role in Hamiltonian chaos and may be important in a variety of contexts [1]. The basic class of systems where it arises is a time-periodically perturbed 1D Hamiltonian system possessing a separatrix. For high or adiabatically low frequencies of the perturbation, separatrix chaotic layers have been thoroughly studied and are well understood [1, 2, 3, 4]. However the range of **moderate and moderately low frequencies** has barely been studied even though it is the **most interesting range** in the sense that the layer **width in energy** (which is one of the most important quantities characterizing the layer) most often takes its **maximum** in this range. Moreover, the width as a function of the frequency possesses multiple peaks in this range.

We have developed [5, 6] a rigorous method allowing us to describe the layer in this frequency range and we applied it both to the single separatrix layer [6] and to the onset of global chaos between neighbouring separatrices [5]. The results are remarkable. In particular, we have obtained the first ever theoretical description of the layer boundaries in the **Poincaré sections**, and it closely agrees with computer simulations; in accordance with our theory, all single-separatrix systems are divided in two classes in terms of the dependence of the width on the perturbation amplitude and, for one of the classes, the maximum of the largest peak of the width exceeds the formerly assumed maximum [1] by a **logarithmically large** factor, in an excellent agreement with computer simulations; for double-separatrix systems, we have rigorously described the drastic facilitation of the onset of global chaos (at certain frequencies) discovered by us earlier in computer simulations [7].

We suggest **several applications** of our results, in particular methods for: a substantial facilitation of the onset of global chaos in time-periodically perturbed systems (even if unperturbed systems do not possess any separatrix) due to the enhanced overlap of nonlinear resonances; a strong enhancement of the noise-induced escape over double-barrier structures; a substantial enlargement of stochastic webs.

The approach is anticipated to allow us also to explicitly describe the effect of **stickiness** of chaotic trajectories to stability islands [1].

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Multiple resonance effects in grazing incident angle diatom - surface collisions

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The response of a system to a weak perturbation is strongly conditioned by its internal dynamics and the strength, form, and duration of the external interaction. In the case of a integrable system with $N \geq 2$ degrees of freedom with initial frequencies in the proximity of $R \leq (N-1)$ resonance conditions, the response to a transient perturbation can be characterized by the adiabatic invariance of $N-R$ actions [1, 2]. These adiabatic invariants determine a well defined correlation between the initial and final states of the system.

This work presents a classical analysis of the effects associated with the response of nearly resonant systems to a transient perturbative interaction, arising in the grazing incident angle collision of a diatom with a periodic surface. In these processes the initial internal molecular state and the incident direction along the surface can define different multiple resonance conditions, whose effects could be reflected in the diffraction spectrum.

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Quasi stationary states in long-range interacting many particle systems

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In a free-electron laser (FEL) electrons interact by coherent radiation, which they emit as they are accelerated by a magnetic field in an undulator. Since the interaction can be described by a long-range potential, the saturation is achieved via a combination of kinetic mixing and collisionless relaxation. In the saturation regime, the electron distribution displays non-equilibrium quasi-stationarity and non-Maxwellian tails. In particular, since the single-particle distribution function obeys a Hamiltonian functional equation $\partial_t f = \{f, \mathcal{H}[f]\} + C[f]$, where $\mathcal{H}[f]$ is the Hamiltonian functional, $C[f]$ is the collision operator and $\{,\}$ is the symplectic bracket, it has well-known stationary solutions $f = \Phi(E)$, parametrized by the mean-field energy E. In many interesting cases $C \approx 0$ in the sense that collective behaviour rather than collisions shape the evolution. While low temperatures are characterized by strong instability and efficient kinetic mixing, non-equilibrium Φ are observed when the instability is weaker. In this case the intermediate, quasi-stationary states are observed. Interestingly, weak instability can result in collective oscillations in f which violate $f = \Phi(E)$. The latter effect appears to be dynamical. It has been conjectured that averaging over the mixing "degrees of freedom", and taking into account Casimir invariants as constraints of the theory (which are imposed by the structure of the symplectic bracket), could explain why and how do quasi-stationary distributions depend on the initial conditions. I will present our work in this direction.

Evolutionary approach to model the cell cycle dynamics

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Cell cycle is a series of events taking place from one cell division to another. A state of a cell is described by n extensive variables (substance abundances). According to a general growth theory, they are supposed to be interdependent due to allometry $x_i = \alpha_i x_1^{\gamma_i}$, $i = 2, \ldots, n$. Evolution optimality prohibits a maintenance of α_i and γ_i permanent during the entire cycle, since no stable regime may take place. Hence, a cell cycle is divided into stages, where α_i and γ_i are constant, during a stage. A change of a stage for another yields a change in α_i and γ_i [1].

Suppose, a change between two stages is an instant process. Then, a system could be considered as a hybrid automate with a set of discrete states Q; these latter are the stages of a cell cycle. And a continuous dynamics F (that is a cell growth at each stage) is determined by a system of differential equations:

$$\dot{x_i} = \exp(k \cdot \beta_i^j) \cdot x_i\,, \tag{1}$$

where k is a growth factor, while the parameters vector $\vec{\beta}^j$ is specific, for each discrete state of the automate.

For each growth stage, a growth trajectory is a straight line, in the space space of $\ln x_i$. The set I_0 of all initial values of extensive variables and the set I_d of all terminal values of extensive variables are some areas on two parallel planes located in the space of extensive variables; the distance between them is equal to $\ln 2$, since a cell is supposed to divide into two equal "daughter" cells. Predicates P determine the switching planes L. Let p_m is a switching predicate in the state q_m, then an intersection of the switching plane and the "growth" plane determines a set of initial values of the logarithms of extensive variables corresponding to the state q_m.

Hence, a feasibility of a path consisting from a series of the states $q_0, q_{k_1}, \ldots, q_{k_m}$, q_d requires that $U(0, k_1, \ldots, d) = I'_0 \cap I'_{k_1} \cap \cdots \cap I'_d \neq \emptyset$, where I'_{k_j} is the image of I_{k_j} for a composition of two mappings $T_{k_j,k_{j+1}} \circ \cdots \circ T_{k_m,d}$, where

$$T_{k_i,k_{i+1}}(\ln \vec{X}) = \ln \vec{X} - \frac{l_{k_{i+1}}(\vec{X})}{\vec{n}_{k_{i+1}}\vec{\beta}^{k_i}} \cdot \vec{\beta}^{k_i} \tag{2}$$

is the mapping of the plane l_{k_i} on the plane $l_{k_{i-1}}$. If here $U'(0, k_1, \ldots, d) \subseteq D(0, k_1, \ldots, d)$, then $U'(0, k_1, \ldots, d)$ contains a steady state. Here $D(0, k_1, \ldots, d) = I_0 \cap I''_{k_1} \cap \cdots \cap I''_d$; I''_{k_j} is the image of I_{k_j}, for the composition of two mappings $T_{k_j,k_{j-1}} \circ \cdots \circ T_{k_1,0})$ while $U'(0, k_1, \ldots, d)$ is a projection of $U(0, k_1, \ldots, d)$ on the "birth" plane.

The system described above exhibits a great variety of limit (stable) regimes ranging from a steady state, to the most complicated chaos-like dynamics, through a series of limit cycles of various length and nature.

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Change of tunneling mechanism for multi-dimensional barrier systems with control parameters and initial condition

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In multi-dimensional barrier tunneling, there exist different (semiclassical) mechanisms, i.e., the well-established instanton mechanism and the recently discovered mechanism called SUMGT(stable-unstable manifold guided tunneling)[1, 2]. SUMGT is a kind of dynamical tunneling in which tunneling trajectories are guided by complexified stable and unstable manifolds of the saddle orbit(NHIM) above the top of a potential barrier. It is considered as a simplified situation reflecting the inherent multidimensionality and it may provide a unified theoretical understanding for various novel tunneling phenomena peculiar to multi-dimensional systems, including chaos-assisted and resonance-assisted tunneling.

The difference of the tunneling mechanisms is reflected in tunneling spectrum: a well localized spectrum formed by instanton and a widely spread plateau by

SUMGT, whose width corresponds to the oscillating range of the unstable manifold[1, 2, 3]. The judgement which mechanism, SUMGT or instanton, dominates the tunneling process for given parameters and initial condition is done with comparing the initial imaginary depths of characteristic trajectories between SUMGT and instanton. The tunneling rate of SUMGT is well approximated by a simplified method based on the Melnikov method without going into the full-semiclassical calculation[3, 4].

Taking the Eckart potential perturbed by a periodic force (1.5D system) or a harmonic channel(2D system) as a model system, we first investigate in what regime of the frequency SUMGT works as the leading mechanism of multi-dimensional barrier tunnelling[4]. In the large and small limits of the frequency, the tunneling rate is well evaluated based on the instanton picture. It is in the intermediate frequency range that SUMGT dominates the tunneling process. The tunneling rate takes the maximum value in this intermediate range.

Next, with taking the perturbation frequency in the intermediate range, we observe the change of tunneling mechanism with change of the input energy along the reaction coordinate. It is surprised that the range of perturbation strength in which the SUMGT dominates increases with decrease of the input reaction energy. This is because the imaginary time path of instanton becomes extremely longer in the low energy range and it means that SUMGT is the dominate mechanism for a thick barrier, if it is periodically perturbed.

We will also mention the case of a periodically perturbed rounded-off-step potential for which SUMGT is only the dominate tunneling mechanism for the whole frequency range[5].

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NetEvo: Computational Evolution of Dynamical Networks

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Networks in some form underpin virtually all complex systems making their study a fundamental tool for understanding system level behaviours. Much existing work considers networks in a static context, neglecting the fact that in many real-world systems structure changes over time, evolving due to new requirements. In some cases rules exist governing this process and can be used to efficiently evolve complex networks for our own purpose, or to reliably control existing systems.

This work introduces methods for incorporating dynamic attributes into complex networks, with the aim of providing a coherent framework to investigate and model evolving structures. This will be based on a computation-based optimisation approach where systems are described as networks of coupled ordinary differential equations with node and coupling dynamics that can vary. We introduce the idea of a network "supervisor" whose task it is to re-wire the network such that a chosen cost function is minimised. To illustrate some of these methods a new simulation tool called NetEvo will be presented to analyse how differing constraints and performance measures can effect the types of network produced. Finally, we outline the future directions of this work and some possible applications.

Controllability of quantum states by a weak external field

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Controllability of quantum states is one of the current problems in theoretical and experimental fields of physics and chemistry [1]. Quantum manipulation by a weak external field, in particular, is interesting theoretically and important in various application areas such as quantum computing, chemical reactions, etc [2].

A coarse-grained approach to steering chaotic quantum states was proposed in our previous works [3, 4]. This leads to an expression of the control field,

$$\varepsilon(t) = \frac{\pi\hbar}{\bar{V}^2 T}\mathrm{Re}\left[e^{-i\theta}\langle\phi_0(t)|\hat{V}|\chi_0(t)\rangle\right], \tag{1}$$

represented by two unassisted states, $|\phi_0(t)\rangle$ and $|\chi_0(t)\rangle$, which are identical to the initial and the target states, respectively. For sufficiently chaotic cases, the control field Eq.(1) was proved to be valid when the target time is long and the field strength is weak enough [5]. It was also shown that the control-time depends on the universality class of the system, i.e., distribution of nearly zero elements, and that the field is applicable to banded-random cases after some modification [6].

In this contribution, we present numerical results of controlled dynamics on a quantum kicked-rotor system,

$$H(q,p;t) = \frac{p^2}{2} + \frac{K}{\tau}\cos(q)\sum_{n=-\infty}^{\infty}\delta(t-n\tau) + \varepsilon(t)V(q), \tag{2}$$

where the external field $\varepsilon(t)$ is represented as a function of a continuous variable t [4]. Since the interaction operator $V(q)$ has banded-random elements in p-representation, this can also be a model of realistic systems. Based on these numerical studies, we discuss relation between controllability of quantum states and stochastic structures of the phase space from semiclassical points of view.

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Hamiltonian ratchets with autoresonance

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Motion of an ensemble of non-interacting particles in a space-periodic potential subjected to a AC perturbation is considered. The perturbation is taken in the form of a plane wave with adiabatic phase modulation, and its amplitude is small compared with a height of the potential. It is shown that there can occur directed ballistic transport even if the particle ensemble is initially cold, i. e. particle energies are close to the minimal value. Since dissipation is neglected, this system is an example of a Hamiltonian ratchet. We consider three cases: (i) wavenumber of the perturbation is large and the phase modulation is space-dependent, (ii) wavenumber is large and the phase modulation is space-independent, and (iii) wavenumber of the perturbation is equal to the wavenumber of the unperturbed periodic potential and the phase modulation is space-independent. In the case (i), there appears a regime of explosion-like acceleration along resonant channels in phase space [1, 2]. In the case (ii), particles become trapped into a chaotic layer induced by scattering on resonance with rapid oscillations of the perturbation. This chaotic layer moves upwards or downwards in the energy space and, thus, can transfer particles from the lowest energy range to the ballistic regime, and vice versa [3]. A similar situation occurs in the case (iii), where the role of the transporting chaotic layer is played by the island of nonlinear resonance 1 : 1. All these three cases exemplify the phenomenon of autoresonance when a particle is retained in resonance due to temporal or spatial variations of parameters. It should be emphasized that the same mechanism can be used for particle cooling as well.

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Research on WSN Routing based on Clustering Algorithm

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Recently, researchers have been increasingly interested in wireless sensor networks (WSN). This rising interest is in large part due to many useful and varied applications of wireless sensor networks when once they are deployed. In a wireless sensor network, hundreds to thousands of small, sensor nodes are scattered over some environment for the purpose of gathering data. These sensor nodes collaborate among themselves to establish a sensing network. Because of the remote nature of these networks and the size of the individual nodes, however, nodes do not have access to unlimited energy. Thus, in order to prolong system lifetime, energy efficient algorithms and protocols should be used. In this paper, we consider the design of routing scheme that harnesses certain desirable properties such as connectivity, energy efficiency, and throughput. One way to meet these requirements is to partition the WSN into clusters, where each cluster is managed by a cluster head (CH) covered by a number of cluster members. We formulate the energy consumption and study their estimated lifetime based on a clustering mechanism. Following the analysis, we propose a novel clustering protocol which minimizes the energy consumption by selecting optimal number of CHs. The proposed protocol prolongs the lifetime of the network and eases the maintenance of wireless sensor network.

The role of invariant manifolds in the dynamics of an optically injected laser

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We consider the following model for a laser with optical injection:

$$\begin{cases} \dot{E} &= K + \left(\frac{1}{2}(1+i\alpha)n - i\omega\right) E, \\ \dot{n} &= -2Gn - (1+2Bn)(|E|^2 - 1). \end{cases} \tag{1}$$

Here $E = E_x + iE_y$ is the complex electric field, and n is the population inversion (number of electron-hole pairs in case of a semiconductor laser). This model was derived in [6] and describes a single-mode laser that receives optical injection at amplitude K and detuning ω; furthermore, B, G and α describe material properties of the laser.

An extensive bifurcation analysis for this model can be found in [5] where, among many other dynamical features, Shilnikov homoclinic bifurcations [2, 3, 4] (homoclinic orbits to a saddle-focus p) are presented and include both the simple and the chaotic case. This means that, under certain conditions, the two-dimensional stable manifold $W^s(p)$ of the saddle-focus p is an excitability threshold for the laser.

Moreover, the dynamics in our region of interest is organized by $W^s(p)$ and by the stable manifolds of periodic orbits created from the homoclinic bifurcation.

However $W^s(p)$ might, and indeed does, change through the bifurcation [1]. Therefor, we study the topological and geometrical changes $W^s(p)$ undergoes and its interaction with other invariant objects of the dynamical system (1). In this way, we are able to understand the complicated organization of solutions in phase space.

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Periodic Quantum Walks

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In the last few years the Quantum Random Walk (QRW) model has been attracting attentions of many mathematicians and physicists for the potential applications in Quantum Information and Solid State Physics [1]. The QRW can be understood as an electron in a one dimensional crystal where each atom flips the electronic spin when the electron passes near the atom. The case in which all the atoms flips the spin in the same way is straightforward and the electron travels with constant speed, i.e. the model is ballistic, and the electron position distribution for large times can be calculated [2,3,4]. In this poster it will be presented the periodic case, where the electronic spin flips periodically or in other words different atoms are organized periodically in a crystal. An extension of the theorem by Grimmett-Janson-Scudo [4] to periodic QRW will be presented and as a corollary is obtained a method to calculate the limit distribution and that periodic QRW are ballistic (as appeared in [5]). The period two lattice will be used to illustrate the theorem since the position distribution for large times can be explicitly calculated.

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Discrete breathers in a pin jointed mechanical lattice

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Discrete Breathers have been found in many different physical systems such as DNA, micro-mechanical systems and Josephson junctions, however, there are very few studies of discrete breathers in fully macroscopic engineering systems. In this talk I will describe how we have located discrete breathers in a numerical model of a pin jointed mechanical structure. The stability and localisation properties of these discrete breathers will be presented. We will also see how this example increases the size of the class of ordinary differential equations in which we know discrete breathers can be found.

Characteristic Lyapunov vectors of time-delayed systems (and its relation with space-time chaos)

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In dynamical systems, characteristic (or covariant) Lyapunov vectors (CLVs) indicate the intrinsic direction associated with each Lyapunov exponent (in contrast with backward or Gram-Schmidt vectors, which depend on the definition of the scalar product). CLVs serve to address fundamental properties [1] such as hyperbolicity or Oseledec splitting, but could also be interesting from an applied point of view (e.g. weather forecasting) due to their inherent stability [2].

Systems with delay appear naturally in optics and other fields, and exhibit a particular type of high-dimensional chaos that is believed to be a form of space-time chaos. Nevertheless the relation between delayed systems and space-time chaos in extended systems is not completely understood, and further progress should be welcomed.

In this communication we show how to compute CLVs in time-delayed systems like the Mackey-Glass model:

$$\frac{dy(t)}{dt} = -ay(t) + b\frac{y(t-\tau)}{1+y(t-\tau)^{10}} \tag{1}$$

We find that the structure of CLVs is in quantitative terms like that of CLVs of dissipative systems with space-time chaos [3, 4].

We also give theoretical and numerical support to the relation between the main Lyapunov vector and the Kardar-Parisi-Zhang equation. This was pointed out years ago [5] (and later questioned in [6]) but intensive simulations allow us to conclude that in (scalar) delayed systems like (1) the main Lyapunov vector belongs to the universal class of KPZ for sufficiently large τ.

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Brain Computer Interface for Control of Dynamics and Robot Systems

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The Brain Computer Interface (BCI) system allows the direct connection between Brain Signals and Real World by using Electroencephalography (EEG). The task of this work concerns with the possibility of controlling an artificial hand (KTHANS) and a rover robot moving in a complex scenario, by using detected Brain Signals and their nonlinear processing. In particular, two different techniques have been used. One is based on the use of sensory-motor rhythms, while the second one on evoked potential. The two techniques have been applied to the control of the two systems described and then compared. One of the experiments performed is available online (http://www.youtube.com/watch?v=AmfAynGhVaE&feature=fvsr).

The final project consists in building cheap robot-based systems that can be controlled by the brain. The useful achievements of this research must lead to use robots to create desired artistic representations.

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Extended and Multiple Duffing Oscillation Model in Multi-folding System with multiple bifurcation points

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In fundamental research of nonlinear dynamics, it is important to understand the essential property for nonlinear mechanics such as bifurcation and chaotic behaviour, as these are physical factors in the loss of stability in structures. It is very useful to exactly demonstrate the nonlinear behaviour of structures by numerical analysis because such behaviour is so complex. There are also a number of factors that can disrupt the stability of a structure. The problems related to the elastic stability of a structure have been researched by many scientists in this field, notably the work of Thompson and Hunt [1] has been identified with the general theory of elastic stability using catastrophe theory in engineering, and Thompson and Stewart [2], who introduced nonlinear dynamics and chaos based upon the instability of a structural model.

This paper reviews the theoretical basis for nonlinear dynamic behaviour to examine the structural instability of the folding truss layered with hilltop-type bifurcations. The dynamic analysis allows for geometrical non-linearity based upon static bifurcation theory. We have found that a simple folding structure based on Multi-Folding-Microstructures theory is more interesting in a strange trajectory in multiple heteroclinic orbits than a well-known standard homoclinic orbit, as a model of an extended Duffing oscillation. It is found that there is global and local dynamic behaviour for a folding multi-layered truss which corresponds to the structure of the multiple heteroclinic orbits. This means the numerical solution depends on the dynamic behaviour of the system subject to the forced cyclic loading such as folding or expanding action. The authors suggest that understanding global and local dynamic behaviour, which depends on the static bifurcation problem, is very useful for forecasting simulations of the extended Duffing oscillation model as essential and invariant nonlinear phenomena.

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An Implementation of FPGA-Based Delay Circuit

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A delay circuit is one block which could intentionally delay the input signal with a given time duration. A large amount of applications of delay circuits have been developed in different fields. For example, the highly integrated time-multiplexing device [1] and the particle detector [2] or time analog-to-digital converter [3] were used in measurement. In addition, delay circuit plays an important role in Delay-locked loop (DLL) [4] which enhances the quality of the integrated circuits.

On the other hand, delays are fundamental in many mathematical models of dynamical systems and, in particular, in neuron models, where synapses are characterized by the presence of delays. Delays are also important in coupled circuits and in networks where their presence can introduce significant changes in the dynamics or may constitute a fundamental element of the model required to account for the dynamics really observed in experiments. From these considerations and in view of the circuit implementation of mathematical models of networks, neuron models, and coupled dynamical systems, the need of strategies for the implementation of delays arises.

In previous researches, some solutions for delay circuit realized with Field Programmable Gate Array (FPGA) have been presented, such as using logic gates [1, 2] or delay units [4]. The delay step was constant; both the input and output signal were digital. Hence, these features reduce the flexibility and limit functions of delay circuit. The valuable applications of delay circuit motivate us to develop a novel FPGA-based delay circuit which can make a controllable range of delay with high precision. The FPGA approach will be used to realize new nonlinear circuits with time-delays able to show the emergence of complex behavior, like chaos.

Shift register configuration will be utilized to optimize our design. One of the most important advantage is that it is easy to change the delay time by varying the duration of the clock or the number of shifted bits. Our programmable delay circuit is comprised of a variety of delay blocks connected in cascade. The minimum delay can be changed conveniently according to the user's demand. Moreover, there are ADC and DAC parts to support the interfacing with analog signals. In the first stage, the implemented FPGA-based delay circuit has three control inputs which allows us to choose 8 taps of delays from 0.1ms to 0.8ms. Despite its flexibility and a considerable amount of delay time steps make it the suitable candidate for applying in applications such as implementations of delayed chaotic dynamics and networks.

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Solving of the Navier–Stokes equations by Monte Carlo methods

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Let Ω is bounded domain in $\mathbb{R}^n$ and $T > 0$ is fixed value, $Q \equiv \Omega \times \big(0,\ T\big)$. We consider the following initial boundary value problem

$$\frac{\partial \vec{u}}{\partial t} - \nu \Delta \vec{u} + \sum_{i=1}^{n} u_i D_i \vec{u} + \nabla p = \vec{f} \ \text{ in } \ Q, \tag{1}$$

$$\sum_{i=1}^{n} \frac{\partial u_i}{\partial x_i} = 0 \ \text{ in } \ Q, \tag{2}$$

$$\vec{u} = 0 \ \text{ on } \ \partial Q \times \big(0,\ T\big), \tag{3}$$

$$\vec{u}(x,0) = \vec{u}_0(x) \ \text{ in } \ \Omega. \tag{4}$$

Here $\vec{u}:\ \Omega \times \big[0,\ T\big] \to \mathbb{R}^n$ is unknown vector function (velocity of the fluid). $p:\ \Omega \times \big[0,\ T\big] \to \mathbb{R}$ is unknown scalar function of fluid pressure, $D_i \equiv \frac{\partial}{\partial x_i}$ $\big(1 \leq i \leq n,\ n = 3\big)$. $\vec{f}$ and $\vec{u}_0$ are given functions defined on $\Omega \times \big[0,\ T\big]$ and Ω respectively.

We obtain discrete equation (1) with respect to time variable using four two-ply schemes: 1. Completely implicit scheme; 2. Crank-Nicholson scheme; 3. Implicit scheme in linear part and explicit scheme in nonlinear part of equation; 4. Explicit scheme, [1]. After that we apply Uzawa and Arrow-Hurwicz algorithms. As a result we obtain four Dirichlet problems for elliptic equations. Solutions of the last problems are estimated by two Monte Carlo algorithms: "random walks on spheres", "random walks on lattices" and probability difference method. [2] [3], [4].

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Modeling of Artemia Flock

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Multi-robot systems are becoming more and more significant in industrial, commercial and scientific applications. Distributed control has advantages over hierarchical control because the robots can be autonomous. In nature, flocking is a kind of ubiquitous self-organized phenomenon, such as flock of birds, school of fish, and crowds of people. And it is a form of collective behavior of a large number of interacting agents with a common group objective. The study of flocks, schools and swarms has attracted a lot of researchers in different fields such as biology, physics, robotics and control engineering.

Mathematical modeling is the primary tool for exploring the connection between individual properties and group properties. Using models, researchers are able to test sets of interaction mechanisms and visualize the resulting group behavior.

The Artemia Salina belongs to a genus of very primordial crustaceans. Their collective behaviors are good model for the development of useful distributed control systems (DCS) for robot applications. In this paper we discuss a mathematical model for Artemia behavior as group. This mathematical model describes the effects of light intensity on direction of Artemia trajectories, and the response of a population of Artemias to light wavelength (red, green, blue and white).

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Well-posedness for an abstract second order fractional differential equation

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Of concern is an abstract general second order differential equation involving different derivatives of non-integer order in the nonlinearity. This problem covers many of the well-known problems involving integer order derivatives. Some of the problems which arise in applications will be discussed. We establish some existence and uniqueness theorems for different types of nonlinearities. Moreover, we introduce nonlocal conditions of fractional type. The key tools are the cosine families and some fixed point theorems. Cosine families are the equivalent of semigroups for the case of a differential equation of first order.

A Hybrid Method to Predict the Distribution of Vibroacoustic Energy in Complex Built-up Structures

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Finding the distribution of vibro-acoustic energy in complex built-up structures in the mid-to-high frequency regime often is a challenging task. This is especially the case for mid-frequency problems where exact numerical methods such as the boundary element method (BEM) or the finite element method (FEM) are beginning to overstretch the capacity of even the most advanced computer architectures while high frequency techniques such as statistical energy analysis (SEA) are not yet available. One of the possible ways to circumvent this difficulty is to develop hybrid methods which incorporate both FEM/BEM and SEA.

Mid-frequency problems usually occur in structures with large variation of local wavelengths and/or characteristic scales. The method presented here is based on splitting the whole system into a number of subsystems which can be treated with either FEM or an SEA approach depending on the local wavelength. The subsystems where the wavelength is of the same order as the characteristic scale are labelled "deterministic" and are treated with FEM. The other subsystems are labelled "stochastic" and their averaged response can be reduced to a weighted random diffuse field correlation function. Finally, it is demonstrated that our approach results in an SEA-like set of linear equations which can be solved for the energies in the stochastic subsystems.

Flexible Parallel Logic Gates by Synthetic Genetic Network

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Logic gates, such as AND, OR, NOR, NAND and XOR form the basis of universal general-purpose computation. So different physical principles that can yield logic outputs are of paramount interest. A new direction in this endevour uses the interplay between noise and nonlinearity to enhance the reliability of logic gates [1].

In this work we investigate the possibility of obtaining robust logic outputs from a noisy biological system of considerable interest, that is to say, a synthetic single gene network. We demonstrate the crucial role of noise in the enhancement of the logic performance in the system. We also show that the power of varying noise levels can flexibly yield different kinds of logic : namely the genetic network can effectively behave as a *reconfigurable biological logic gate* with noise acting as a logic pattern selector. Furthermore, we show the capacity of a higher dimensional system for *parallel processing logic functions*, such as simultaneous OR and XOR operations. The latter observation indicates that more complex systems may have inherently higer computational capability arising from greater parallel processing capacity.

The obtained results in this work imply possibilities for significantly increasing computational power not only in synthetic genetic networks, but also in many other natural and engineered systems. So the observations here may have potential applications in the design of biological gates with added capacity of reconfigurability of logic operations and parallel processing.

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Transmission and Reflection Probabilities for the Stadium Billiard: Time reversal symmetry breaking

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Transmission and reflection probabilities are important in many applications including ballistic transport through semiconductor micro-structures. We investigate the respective survival probabilities for the chaotic stadium billiard with two holes (one on a straight segment and one on a curved) and argue that, classically, these distributions differ depending on the choice of hole through which particles are injected into the system. When injected through the hole on the curved segment the transmission and reflection survival probabilities decay exponentially ($\sim e^{-\gamma t}$), while in the case where particles originate from the hole on the straight segment, an algebraic tail $\mathcal{O}(t^{-2})$ follows the exponentially decaying reflection distribution and therefore, contrary to what would be expected, offers an example where time-reversal symmetry is broken. This distinction comes in effect due to the bouncing ball orbits present, creating a sticky region in phase space, which is identified and separated by the presence of the hole on the straight segment, and is inaccessible to particles originating from the hole on the curved. Exact expressions are found to leading order and are confirmed numerically, therefore suggesting new ways of calibrating and controlling experimental devices such as quantum dots.

Fermi acceleration and suppression of Fermi acceleration in a time-dependent Lorentz Gas

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In our work, we revisit the problem of a Lorentz Gas [1] with triangular configuration considering both the static as well as time-dependent boundary. For the static case, we derived a two dimensional nonlinear map that describe the dynamics of the model. We obtain the phase space and we show that it has a chaotic component with positive Lyapunov Exponent. After that, for the first time, we introduce a different kind of time dependent perturbation. We assume that the radius of the scatters are fixed and the center of mass changes according to an harmonic function. Our main goal is, for the conservative case, to verify the validity of the Loskutov-Ryabov-Akinshin (LRA) conjecture [2] which states that: Chaotic dynamics of a billiard with a fixed boundary is a sufficient condition for the Fermi acceleration in the system when a boundary perturbation is introduced. It was confirmed when we studied the behaviour of the average velocity for an ensemble of particles. Since the phenomenon of Fermi Acceleration is present in this model our next step is to introduce dissipation into the model via damping coefficients. For dissipative billiards, Leonel proposed the following conjecture [3]: For one-dimensional billiard problems that show unlimited energy growth for both their deterministic and stochastic dynamics, the introduction of inelastic collision in the boundaries is a sufficient condition to break down the phenomenon of Fermi acceleration. Such conjecture was verified recently for a two dimensional time dependent oval billiard [4]. In our approach, we are studying a two-dimensional system close to the transition conservative to dissipative. Our results allow us to confirm Leonel's conjecture for this new kind of time-dependent perturbation when dissipation is introduced. In both, conservative as well as dissipative case, we describe the behaviour of average velocity using scaling formalism [5, 6].

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Coupled phenomena for particles evolving in a periodic and symmetric washboard potential

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I explore the scattering of particles evolving in a two degree of freedom system, in which both degrees of freedom are open. Particles, initially having all kinetic energy, will be sent into a so called interaction region where there will be an exchange of energy with particles that are initially at rest. The open nature of both components of this system eliminates any restrictions on which particles can escape. Notably, it will be shown that two particles can co-operate in a mutual exchange of energy allowing both particles to escape and travel large distances before settling into their long term behaviour. It will also be shown that this level of cooperation is highly sensitive to the coupling strength between both components of the system. Indeed, large uctuations of the magnitude and direction of the current have been observed for small changes of this coupling parameter. Another interesting observation has been that even with the presence of chaotic scattering, it is possible that the system, for certain parameter regimes, will express a vanishing current. This is in contrast to many other systems, especially externally modulated non-autonomous systems, where it is chaos that promotes transport.

The aim of this study has been to examine the Hamiltonian dynamics of the system and to give insight into the structures of phase that allow for such complex behaviour in a seemingly simple system.

Omnivory and critical transitions in lake ecosystems

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Trophic levels theory is a very important ecological theory and has been successfully applied in recovery of many eutrophic lakes and reservoirs in temperate zones. Classical food chain theory has assumed that omnivory is rare in natural systems; however, this is clearly not the case for food chains of tropical lakes and reservoirs. In these systems, omnivorous fish can play a central role in food chain dynamics and might lead to patterns not predicted by classical food chain theory. Omnivorous filter-feeding Nile tilapia, *Oreochromis niloticus*, is one of the most abundant and economically important fish of tropical regions worldwide. It might affect plankton dynamics and water quality in a manner not predicted by models which ignore omnivory. Our aim in the present work was to theoretically evaluate consequences of tilapia omnivory in planktonic populations and water quality. We further developed our former model of two coupled differential equations proposed to describe the dynamics of the populations of algae and planktonic animals [1], adding an extra equation in order to discriminate between two different kinds of algae and take into account cyanobacteria, potentially toxic to humans and/or animals. Parameter p describes omnivory of tilapia. In the limit $p \to 1$, tilapia feed only on zooplankton, whereas $p \to 0$ corresponds to a restricted phytoplankton consumption. In this dynamical system a critical value of parameter p or of fish density F gives rise to a Hopf bifurcation, where original predator-prey oscillation between the two kinds of phytoplankton and the zooplankton is broken, originating a global attractor in the system. This state, exhibiting high density of phytoplankton and low concentration of zooplankton, is identified as the eutrophication of the reservoir.

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Plenary Talk I5 (Auditorium)

Markov State Models for Complex Systems

I5
17:30

Ch. Schuette[1]*, M. Sarich[1], N. Djurdjevac[1]

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We consider the dynamical behavior of complex systems on high-dimensional state spaces and want to find low-dimensional structure-preserving approximations of the process in the sense that the longest timescales of the dynamics of the original process are reproduced well. Recent years have seen the advance of so-called Markov state models (MSM) for Markov processes on very large state spaces exhibiting metastable dynamics. It has been demonstrated that MSMs are especially useful for modelling the interesting slow dynamics of biomolecules [3] and materials. From the mathematical perspective, MSMs result from Galerkin projection of the transfer operator underlying the original process onto some low-dimensional subspace which leads to an approximation of the dominant eigenvalues of the transfer operators and thus of the longest timescales of the original dynamics [2]. Until now, most articles on MSMs have been based on full subdivisions of state space, i.e., Galerkin projections onto subspaces spanned by indicator functions. We show how to generalize MSMs to alternative low-dimensional subspaces with superior approximation properties, and how to analyse the approximation quality (dominant eigenvalues, propagation of functions) of the resulting MSMs [2, 1]. To this end, we give an overview of the construction of MSMs, the associated stochastics and functional-analysis background, and its algorithmic consequences. Furthermore, we illustrate the mathematical construction with numerical examples, and discuss the application of MSMs to protein folding.

[1] M. Sarich, F. Noé, and Ch. Schuette. On the approximation quality of Markov state models. *to appear in Multiscale Modeling and Simulation*, 2010.

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Wednesday, 8th September 2010

Plenary Talk I6 (Auditorium)

On Growth and Form of Ferrofluids

I6
9:00

Reinhard Richter[1*], Christian Gollwitzer[1], Thomas Friedrich[1], Tobias Lang[1]

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Pattern formation has mostly been investigated in systems driven *far from equilibrium* [1]. This lopsided orientation is due to the belief that mainly the analysis of those systems can bring us a step forward to unravel fundamental riddles like the self organisation of structures, including life on earth [2]. However, pattern emerge as well in systems without a permanent energy throughput.

A beautiful example is a layer of magnetic liquid subjected to a magnetic field oriented normally to the surface. Upon increase of the field strength various pattern of liquid crests evolve. This normal field, or Rosensweig instability [3] will be tackled by a linear and a nonlinear description. In the *linear regime* of small amplitudes we measure the wave number of maximal growth, its corresponding growth rate and the oscillatory decay of metastable pattern, and compare our results with predictions of the linear stability analysis. In the *nonlinear regime* predictions of the fully developed peak pattern can be obtained by a minimization of the free energy [4], by amplitude equations [5], and by numerics employing the finite element method. We record the three-dimensional surface profiles via absorption of X-rays and compare them with the predictions. Moreover, in the bistable regime of the flat and hexagonal state we generate localized states which are recovered in analytical and numerical model descriptions. For higher fields an inverse hysteretic transition from hexagonal to square planforms is measured. Adding a horizontal field component the symmetry can be broken in the experiment, resulting in liquid ridges and distorted hexagons, as predicted by theory. Replacing ferrofluid by ferrogel also an elastic energy contribution has to be taken into account, yielding a linear shift of the threshold, which is tested in experiments [6]. *Parametric excitation* in combination with magnetic fields is widening the horizon of pattern formation even further. As a recent example we study ferrouidic transport and surface pattern generated by magnetic traveling-stripe forcing [7]. Due to limited space for references we refer the gentle reader for most of our experimental findings to a recent review [8].

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[8] R. Richter and A. Lange, *Surface Instabilities of Ferrofluids in Colloidal Magnetic Fluids: Basics, Development and Applications of Ferrofluids*, Vol. 763 of *Lect. Notes Phys.*, edited by S. Odenbach (Springer, Berlin, Heidelberg, New York, 2009), Chap. 3, pp. 157-243.

Mini-symposium M7: Traffic Dynamics (Lecture Room G12)

Organiser: Gabor Orosz (University of California, USA)

Dynamical Phenomena induced by Bottleneck

M7.1
10:30

I. Gasser[1]*, B. Werner[1]

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We study a microscopic follow-the-leader model on a circle with a bottleneck. Allowing large bottleneck strengths we encounter very interesting traffic dynamics. Different types of waves - traveling and standing waves and combinations of both wave types - are observed. The way to find these phenomena requires a good understanding of the complex dynamics of the underlying (nonlinear) equations. Some of the phenomena, like the Ponies-on-a-Merry-Go-Round-solutions (POMs) are mathematically well known from completely different applications. Mathematically speaking we use Poincare maps, bifurcation analysis and continuation methods beside numerical simulations.

[1] I. Gasser and B. Werner, Dynamical Phenomena induced by Bottleneck, *Phil. Trans. R. Soc. Lond. A*, to appear (2010).

[2] T. Seidel, I. Gasser and B. Werner: Microscopic car-following Models revisited: from road works to fundamental diagrams, *SIAM Journal of Applied Dynamical Systems SIADS*, **8(5)**, 1305-1323 (2009).

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A mechanism to describe the formation and propagation of stop-and-go waves in congested freeway traffic

M7.2
11:00

Jorge A. Laval[1*], Ludovic Leclercq[2]

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This presentation introduces a parsimonious theory for congested freeway traffic that describes the spontaneous appearance of oscillations and their ensuing transformation into stop-and-go waves. Based upon the analysis of detailed vehicle trajectory data, we conclude that timid and aggressive driver behaviors are the cause for this transformation. We find that stop-and-go waves arise independently of the details of these behaviors. Analytical and simulation results are presented.

Traffic jams – dynamics and control

M7.3
11:30

Gábor Orosz[1*], Francesco Bullo[1], Jeff Moehlis[1], R. Eddie Wilson[2], Gábor Stépán[3]

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[2] Department of Engineering Mathematics, University of Bristol, Bristol, BS8 1TR, UK
[3] Department of Applied Mechanics, Budapest University of Technology, Budapest, 1521, Hungary
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The introduction of the assembly line in the automotive industry about a century ago allowed the mass production of automobiles, which, in turn, revolutionized land transportation. At the same time a problem was also generated that has not yet been resolved: traffic congestion. In this talk we review the most common modelling approaches used in the vehicular traffic community and present the state-of-the-art methods that may be applied to classify the dynamical behavior of these models. We will show that using sophisticated techniques form dynamical systems theory may allow one to characterize the dynamical phenomena behind traffic jam formation. Stable and unstable motions will be described that may give the skeleton of traffic dynamics and the effects of driver behavior will be explained in determining what state is approached by a vehicular system. Such dynamical principles need to be taken into account when one wishes to eliminate congestion by controlling traffic. The material presented has been published in the papers [1, 2, 3, 4].

[1] G. Orosz and G. Stépán, Subcritical Hopf bifurcations in a car-following model with reaction-time delay. *Proceedings of the Royal Society A*, **462**(2073), 2643–2670, (2006).

[2] G. Orosz, R. E. Wilson, R. Szalai, and G. Stépán, Exciting traffic jams: Nonlinear phenomena behind traffic jam formation on highways. *Physical Review E*, **80**(4), 046205, (2009).

[3] G. Orosz, J. Moehlis, and F. Bullo, Robotic reactions: Delay-induced patterns in autonomous vehicle systems. *Physical Review E*, **81**(2), 025204(R), (2010).

[4] G. Orosz, R. E. Wilson, and G. Stépán, Traffic jams – dynamics and control. *Philosophical Transactions of the Royal Society A*, in press, (2010).

On the nature of traffic instabilities – what traffic data and theory can tell us

M7.4
12:00

M. Treiber[1]*, A. Kesting[1]

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We analyze detector data from several hundred traffic jams on freeways in Germany, Holland, England [1], and the USA [2] with respect to the nature of traffic instabilities. By applying dedicated analyzing tools [3] to a large database of congestions on a German freeway (see `traffic-states.com`), we summarize the qualitative aspects in terms of the *stylized facts* of jam propagation, such as the following:

- Jammed regions can be extended or localized. The localized jams are either standing waves, or travel in the upstream direction at a fixed velocity c.
- The downstream front of an extended jam is either pinned at a bottleneck, or propagates upstream with the velocity c.
- Within extended jams, emerging oscillations grow and propagate upstream (all at the same velocity c). Consequently, congested traffic near the bottleneck is essentially stationary while stop-and-go waves evolve further upstream.

In the overwhelming majority of all cases, the data are compatible with linear string instabilities of the *convective* type, i.e., perturbations grow but eventually propagate out of the congested region. We quantify the instability in terms of linear growth rate, wavelength of the oscillations, and propagation velocity as a function of the empirically determined bottleneck strength.

On the theoretical side, we derive analytical criteria for linear and convective stability applicable to a wide range of microscopic and macroscopic models and divide these models into five stability classes which uniquely determine the set of observable spatiotemporal patterns [4, 5]. It turns out that one of the five classes is consistent with most of the observed findings. It is characterized by metastability at capacity and a density range of convective instability in the congested regime.

Finally, we show, by means of approximate analytic solutions to systems with temporary or sustained localized noise, that the observations mentioned above can be quantitatively explained by the phenomenon of convective instability. We conclude that the concept of convective instability, widely ignored in the traffic community, plays a crucial role in uncovering the nature of traffic jams.

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[3] M. Treiber, A. Kesting, and R. Eddie Wilson, Computer-Aided Civil and Infrastructure Engineering, in press (2010).

[4] D. Helbing, M. Treiber, A. Kesting, and M. Schnhof, The European Physical Journal B **69,** 583 (2009).

[5] M. Treiber, A. Kesting, and D. Helbing, Transportation Research Part B, in press (2010), DOI:10.1016/j.trb.2010.03.004.

Mini-symposium M8: Dynamics of Hearing (Victoria's Room)

Organiser: Daniele Avitabile (University of Bristol, UK)

Transduction, Adaptation and Amplification in the Drosophila ear: Theoretical and experimental approaches to the operation of an active sense organ

M8.1
10:30

Joerg T. Albert[1*]

[1] UCL, Ear Institute, London, UK
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Ears are remarkable sensory systems that attract attention from a wide variety of scientists. Neurobiologists are stunned by the sensitivity and temporal precision of their operation, engineers marvel at the complexity and sophistication of their design, and frustrated molecular biologists, finally, at least honour the persistency with which they hide the details of their molecular implementation. In fact, our remarkably deep insights into the mechanistic principles of hearing stand in stark contrast to our (likewise remarkable) lack of knowledge about their molecular construction. Till this day, we have not identified the auditory transducer channels, i.e. those ion channels that are responsible for the elementary conversion of mechanical force into a bio-electrical signal, for any known hearing organ. Recent computational and experimental advances have recommended the ear of the ubiquitous fruit fly Drosophila melanogaster as a scientific model organ that allows for ascribing distinct functions to identified molecules and thus may eventually help closing this gap of knowledge. The presentation will give a brief design review of the computational and experimental dissection of the fly's ear and shortly introduce some of the candidate molecules along with their supposed roles for hearing.

The neural basis of active hearing in the mosquito Aedes aegypti: experiments and modelling

M8.2
11:00

Katie Lucas[1*], Daniele Avitabile[2], Daniel Robert[3]

[1] School of Biological Sciences, University of Bristol, Bristol, UK
[2] Department of Engineering Mathematics, University of Bristol, Bristol, UK
[3] School of Biological Sciences, University of Bristol, Bristol, UK
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Male mosquitoes are faced with the difficult task of finding females in order to mate on the wing, and do so by detecting the low-intensity, low-frequency sounds produced by her wing beat. Hearing in mosquitoes is mediated through their antenna that respond to nanoscale mechanical displacements, which in turn initiates an electrical response in Johnston's organ (JO), the auditory sensory organ housed at the base of the antenna. Johnston's organ is arguably one of the most complex hearing organs found in insects, with almost 16 000 sensory cells in male mosquitoes, and 7500 in females. These sensory cells, organized into scolopidial units arranged radially throughout JO, have a dual sensory and motor function to both detect and contribute to the mechanical vibration of the antenna. This active sensing is mediated by synchronization between sensory cells, which reveals itself through the twice-frequency forcing of sets of neurons. We recorded this twice-frequency forcing extracellularly in local subsets of scolopidia in the mosquito species Aedes aegypti. Using a Hilbert transform to calculate the instantaneous frequency and phase of the neural response in the time domain, it was revealed that neurons located in the distal region of JO exhibited greater firing at twice the sound stimulus frequency than those located proximally. As the sound stimulus intensity was increased, so too did the proportion of cells firing at twice the frequency, suggesting entrainment to generate coherent oscillations. This is in accordance with a recent model of the active hearing process in the mosquito JO that found that scolopidia located distally provide the largest contribution to the mechanical amplification of the antenna through the entrainment of the neural response to twice the stimulus frequency. Our findings provide further evidence that this active amplification is a peripheral phenomenon mediated by the neurons in JO.

Mechanical preprocessing of sound in the mammalian cochlea

M8.3
11:30

Nigel P. Cooper[1*]

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A brief review of the mammalian cochlea's mechanical responses to sound will be given, focussing on the nonlinear dynamics and the active amplification revealed at the level of the basilar membrane. The data will be interpreted in terms of a highly simplified numerical model that simulates the actions of an instantaneous, saturating nonlinearity embedded within a positive feedback loop

Spatial and temporal delays in the inner ear

M8.4
12:00

R. Szalai[1*], A.R. Champneys[1], M. Homer[1]

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It is now widely accepted that in order to model the response of the mammalian inner ear accurately one has to include a delay in the model. The source of this delay is still unclear, however there are many candidates. The most promising and obvious underlying mechanism is due to the longitudinal tilting of hair cells and the Deiter's cells phalangeal processes that couple the vibrations forward with respect to the direction of wave propagation. Another viable mechanism is a secondary traveling wave either on the tectorial membrane or in the tunnel of Corti. Solving these models is not obvious and requires careful numerical considerations. In particular, spatially delayed models are neutral delay equations that are known to be difficult to solve, hence most authors use various approximations to eliminate the spatial delay. In this talk we compare solution techniques and show the inaccuracies introduced by the commonly used methods.

Generally a feedback loop can induce instability, hence the need of stability calculations. This is rarely carried out in the literature and even then it is based on rough estimates. We introduce methods to compute the stability of both the temporally and the spatially delayed system. In the first case we reduce the stability calculation to computing the roots of a so-called characteristic function while in the second case we discretize the infinitesimal generator. The results show substantial differences between these two types of models and suggest a direction for further bifurcation analysis.

Mini-symposium M9: Open Quantum Systems (Auditorium)

Organiser: Henning Schomerus (Lancaster University, UK)

Quantum Resonant States in Open Chaotic Systems

M9.1
10:30

J.P. Keating[1*]

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I shall describe some recent ideas, conjectures and results relating to the distribution of quantum resonances in open chaotic systems and to the morphology of the associated eigenfunctions. The conjectures in question connect the semiclassical limits of these quantum measures to properties of the corresponding classical strange repellor, that is to the fractal set of classical trajectories trapped for all forward and backward times. They appear to be deep and difficult. Rigorous results exist for only an extremely limited class of toy models.

Resonances states in open chaotic cavities

M9.2
11:00

F. Mortessagne[1*], C. Poli[1], O. Xeridat[1], L. Labonté[1], P. Sebbah[1], O. Legrand[1], D. V. Savin[2]

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In the domain of wave or quantum chaos, open systems are currently actively investigated both from experimental and theoretical points of view. The most salient feature of open systems is the set of resonances which are quasibound states embedded in the continuum. A natural way to address them analytically is via the energy-dependent scattering matrix, $S(E)$. Following the Heidelberg approach, the poles of the S-matrix turn out to be the complex eigenvalues of an effective non-Hermitian Hamiltonian. Its Hermitian part corresponds to the Hamiltonian of the closed system and its anti-Hermitian part models the coupling to the environment. More precisely, the openness is introduced by means of the $N \times M$ coupling matrix V whose elements V_n^j connect the $n = 1, \ldots, N$ states to the $j = 1, \ldots, M$ scattering channels.

Universal properties of resonance scattering in the chaotic regime can then be analyzed by applying random matrix theory (RMT) that amounts to replacing the actual non-Hermitian Hamiltonian with an RMT ensemble of the appropriate symmetry class. By now, complex eigenvalues of such non-Hermitian random matrices have been studied quite systematically. However, the statistical properties of the corresponding (left and right) eigenvectors are less understood. Quite a substantial progress in this direction has been achieved by Schomerus *et al.* [1], who studied mainly systems with broken time-reversal symmetry. Here we present results obtained for a wave system whose closed limit displays time reversal symmetry. To quantify the complexness of the field associated to the nth resonance state, it is convenient to introduce the ratio of the variances of the imaginary and real parts of the field as a single parameter: the complexness parameter q_n^2.

A few years ago, by analyzing hundreds of resonance states of a 2D chaotic microwave cavity at room temperature, we showed evidence of a linear relationship between q_n and Γ_n [2]. This result was then confirmed using the effective Hamiltonian formalism in the limit $M \gg 1$, relevant in the experiment [3]. More recently, a linear relationship between q_n and Γ_n was also verified in an elastodynamics experiment for a given resonance when the uniform coupling is varied uniformly [4]. We theoretically investigated the complexness parameter for arbitrary values of M by means of its probability distribution in the regime of weak coupling [5]. Very recently, we successfully confronted our theoretical predictions to numerical solutions of the Maxwell equations in a lossy microwave chaotic cavity[6].

[1] H. Schomerus et. al., *Physica A* **278**, 469 (2000).

[2] J. Barthélemy et. al., *Europhys. Lett.* **70**, 162 (2005).

[3] D. Savin et. al., *Europhys. Lett.* **76**, 774 (2006).

[4] O. Xeridat et. al., *Phys. Rev. E* **80**, 035201(R) (2009).

[5] C. Poli et. al., *Phys. Rev. E* **80**, 046203 (2009).

[6] C. Poli, L. Labonté, O. Legrand, and F. Mortessagne, in preparation.

Wave intensity distributions in complex structures

M9.3
11:30

Gregor Tanner[1]*, David Chappell[1], Stefano Giani[1], Hanya Ben Hamdin[1], Dmitrii Maksimov[1]

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The vibro-acoustic response of mechanical structures can in general be well approximated in terms of linear wave equations. Standard numerical solution methods comprise *finite* or *boundary element method* (FEM, BEM) in the low frequency regime and *Statistical Energy Analysis* (SEA) in the high-frequency limit. Major computational challenges are posed by so-called *mid-frequency problems* - that is, composite structures where the local wave length may vary by orders of magnitude across the components.

Recently, a new approach towards determining the distribution of mechanical and acoustic wave energy in complex built-up structures improving on standard SEA has been proposed in [1]. The technique interpolates between SEA and *ray tracing* containing both these methods as limiting cases. The method has its origin in studying solutions of wave equation with an underlying chaotic ray-dynamics - often referred to as *wave chaos*. Within the new theory - *Dynamical Energy Analysis* (DEA) - SEA is identified as a low resolution ray tracing algorithm and typical SEA assumptions can be quantified in terms of the properties of the ray dynamics.

We have furthermore developed a hybrid SEA/FEM method based on random wave model assumptions for the short-wavelength components. This makes it possible to tackle mid-frequency problems under certain constraints on the geometry of the structure. Extensions of the technique towards a DEA/FEM hybrid method will be discussed.

DEA and SEA/FEM calculations for a range of multi-component model systems will be presented. The results are compared with both SEA results and FEM as well as BEM calculations. DEA emerges as a numerically efficient method for calculating mean wave intensities with a high degree of spatial resolution and capturing long range correlations in the ray dynamics.

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[1] *Dynamical energy analysis - Determining wave energy distributions in vibro-acoustical structures in the high-frequency regime* Tanner G 2009 *Journal of Sound and Vibration* **320** 1023.

Classical trajectory correlations and their semiclassical consequences

M9.4
12:00

Jack Kuipers[1]*

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Quantum systems have notably different properties depending on whether their classical counterparts are chaotic or not. A powerful way to understand these properties is to use semiclassical methods, valid in the limit $\hbar \to 0$, where the density of states can be expressed through Gutzwiller's trace formula [1] as a sum over the classical periodic orbits of the system. We focus on systems with chaotic dynamics and an often considered statistic is the spectral form factor $K(\tau)$ which is the Fourier transform of the two-point correlation function of the density of states. This can then be expressed semiclassically in terms of a double sum over periodic orbits and importantly includes a complex exponential $\mathrm{e}^{\frac{\mathrm{i}}{\hbar}(\mathrm{S}_{\gamma'}-\mathrm{S}_{\gamma})}$ with the action difference $S_{\gamma'} - S_\gamma$ of the two periodic orbits γ, γ' in the sum.

To understand the typical statistics of quantum chaotic systems, the task was to find classically correlated pairs of periodic orbits whose action difference is small on the scale of $\hbar$. Moving beyond the diagonal approximation of comparing periodic orbits with themselves or their time reversals [2] the first such pair was found in [3] and consists of an orbit with a small angle self crossing and a partner that follows almost the same trajectory, but which however avoids crossing.

These ideas and calculations were further extended in [4] to cover orbits with an arbitrary number of such encounters each involving an arbitrary number of stretches. By using the hyperbolicity and long time ergodicity of the chaotic dynamics, as well as considering the number of different possible configurations of orbit pairs, they were able to generate all terms of the small τ expansion of the form factor.

We will review these ideas and how they carry over to the large field of quantum transport [5, 6, 7], where again quantum quantities can be expressed in terms of classical scattering trajectories. In particular we will focus on our recent results which show how these subtle correlations between ever larger sets of classical trajectories lead not only to the bounding of the distribution of the delay times of a scattering system [8], but also to a hard gap in the density of states of Andreev billiards where the scattering lead is connected to a superconductor [9].

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Thursday, 9th September 2010

Plenary Talk I7 (Auditorium)

Transport, mixing and dynamics of the atmosphere and oceans

I7
9:00

Emily Shuckburgh[1]*

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The urgent need to provide reliable predictions of future climate has meant that it is particularly important that we have a comprehensive understanding of the dynamical processes in the atmopshere and oceans and an ability to quantify and model them accurately. The atmosphere and oceans can be thought of as thin layers of fluid on a rotating sphere, and consequently many of the same dynamical processes are relevant to both. However, the Coriolis effect endows rotating fluids with unusual properties [1]. The dynamics of both the atmosphere and the oceans are dominated by jets and by instabilities that result in "eddies" - vortical structures with scales of ~1000 km in the atmosphere and ~10-100 km in the ocean. In the lower atmosphere, the eddies form the storm systems that are familiar from weather charts and satellite pictures. In this talk I will focus on the transport and mixing processes associated with jets and eddies and how these influence the distribution of important chemical species such as ozone in the atmosphere [2] and carbon dioxide in the ocean [3], as well as dynamically active tracers such as heat and salinity. I will describe how we have used concepts from turbulence and dynamical systems theory to quantify the mixing and transport from observations and numerical model output [4]. Finally, I will explain how these transport and mixing processes can form critical feedbacks for climate change (e.g. [5]) and describe the challenge of properly representing these feedbacks in climate models.

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Mini-symposium M10: Wave Chaos and Chaotic Transport (Victoria's Room)

Organiser: Klaus Richter (University of Regensburg, Germany)

Rogue Waves: Refraction of Gaussian Seas and Rare Event Statistics

M10.1
10:30

L. Kaplan[1*], L. H. Ying[1], E. J. Heller[2], R. Höhmann[3], U. Kuhl[3], H.-J. Stöckmann[3]

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We study weak scattering of Gaussian random ocean waves by Gaussian random currents. Scattering creates persistent, branch-like, local energy density variations, which survive dispersion over wavelength and incoming wave propagation direction. These variations lead to a long, non-Rayleigh tail in the wave height distribution. Using the linearized equations for deep ocean surface gravity waves, and the ray limit of these equations, we obtain quantitative predictions for the increased probability of rogue wave formation when the refractive effects are taken into account. The resulting distribution depends primarily on the "freak index", which measures the strength of refraction relative to the angular spread of the incoming sea. For parameter values typically occurring in nature, the tail of the probability distribution can be easily enhanced by a factor of 10 to 100 as compared with the random wave model. Interesting analogies exist with electron transport in a two-dimensional electron gas, and with recent experiments in disordered microwave cavities. Strong additional enhancement in the rogue wave formation probability is obtained when scattering by currents is combined with nonlinear wave instability.

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Dynamical tunneling in systems with a mixed phase space

M10.2
11:00

A. Bäcker[1*]

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Typical dynamical systems have a mixed phase space, in which regions of chaotic motion coexist with regions of regular motion. While classically these regions are dynamically separated, quantum mechanically, these regions are connected by the fundamental process of tunneling. In one-dimensional integrable systems this is well understood, but the quantitative prediction of so-called dynamical tunneling rates in systems with a mixed phase space is a long-standing open challenge.

In this talk we give an introduction to the dynamics in systems with a mixed phase space, their quantum properties and the prediction of dynamical tunneling rates. These tunneling rates describe the decay of states localized inside the regular region towards the so-called chaotic sea. For their determination we introduce the fictitious integrable system approach [1]. Agreement with numerical data will be presented for kicked systems, two-dimensional billiards [2] and optical microcavities [3]. If prominent nonlinear resonances are present, they can have a strong influence on the behaviour of tunneling rates. We combine the direct regular-to-chaotic tunneling mechanism with the resonance assisted mechanism, which leads to a quantitative description of tunneling rates, even for generic systems such as the standard map [4].

[1] A. Bäcker, R. Ketzmerick, S. Löck, and L. Schilling, *Regular-to-Chaotic Tunneling Rates Using a Fictitious Integrable System*, Phys. Rev. Lett., **100** (2008) 104101.

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[3] A. Bäcker, R. Ketzmerick, S. Löck, J. Wiersig, and M. Hentschel, *Quality factors and dynamical tunneling in annular microcavities*, Phys. Rev. A **79** (2009) 063804.

[4] S. Löck, A. Bäcker, R. Ketzmerick, P. Schlagheck, *Regular-to-Chaotic Tunneling Rates: From the Quantum to the Semiclassical Regime*, Phys. Rev. Lett., **104** (2010) 114101.

Distinguishing quantum and classical transport through nanostructures

M10.3
11:30

C. Emary[1*], Neill Lambert[2], Franco Nori[2], Yueh-Nan Chen[3]

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[2] Advanced Science Institute, The Institute of Physical and Chemical Research (RIKEN), Saitama 351-0198, Japan
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Quantum coherence of electrons is the essential ingredient behind many interesting phenomena in transport through nanostructures, and considerable progress has recently been made in the experimental investigation of such coherent effects with both charge and current measurements. Typically, Rabi oscillations in the current are taken as a distinctive signature of quantum coherence. However, since even classical autonomous rate equations can admit oscillatory solutions, oscillations by themselves cannot be considered as a definitive proof of the existence of quantum coherent dynamics. In this talk we present a set of inequalities that would allow an experimentalist to exclude the possibility of a classical description of transport through a nanostructure. The inspiration for this comes from the Leggett-Garg inequality [1], which has been described as a single-system temporal version of the famous Bell inequality.

In the context of nanostructures weakly coupled to contacts, such that a generalized master equation description is appropriate, we derive and study two inequalities [2]. The first concerns correlations between local charge measurements performed, e.g., by a quantum point contact (QPC). We formulate this inequality in quite general terms, applicable to a range of nanostructures and related systems. For charge operator Q with maximum value $Q_{\max}$ the inequality reads

$$|2\langle Q(t)Q\rangle - \langle Q(2t)Q\rangle| \leq Q_{\max}\langle Q\rangle. \tag{1}$$

Violation of this inequality precludes a description of the dynamics in terms of a classical Markovian theory; and in the stationary limit, any classical theory obeying macroscopic realism.

Our second inequality explicitly focuses on an open two-level system (double quantum dot or 'transport qubit'), and provides a bound for the correlation functions of the current flowing through the structure. Whilst similar in form to Eq. (1), the derivation and significance are rather different, due to the quantum jump description of current flow in the master equation.

[1] A. Leggett and A. Garg, Phys. Rev. Lett **54**, 857 (1985); A. Leggett, J. Phys.: Condens. Matter **14**, R415 (2002).

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Unidirectional light emission along unstable manifolds in deformed microdisks

M10.4
12:00

J. Wiersig[1]*

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Dielectric optical microcavities are important for a wide range of research areas and applications [1], such as ultra-low threshold lasers and single-photon sources. In the nonlinear dynamics and quantum chaos community microdisk cavities with deformed cross-sectional shape attracted considerable attention since they can be used to study the ray-wave correspondence in open systems in direct comparison to experiments [2, 3].

Deforming the boundary of a microdisk cavity is also very interesting from a practical point of view as it allows for improved directionality of light emission and therefore for more efficient extraction and collection of light. We discuss an approach that results in unidirectional light emission without spoiling the quality of the modes (photon lifetime in units of the light period) [4]. This approach uses a combination of wave phenomena (wave localization along unstable periodic ray trajectories) and chaotic ray dynamics in open systems (escape along unstable manifolds). It is demonstrated that high-quality modes in the so-called limaçon cavity exhibit a universal far-field emission pattern as long as the ratio between wavelength and cavity diameter is large enough. For smaller ratios, this universal behavior breaks down which is accompanied by a transition from an individual to a collective ray-wave correspondence [5].

Financial support by the DFG research group “Scattering Systems with Complex Dynamics” is acknowledged.

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Mini-symposium M11: Multiple Time-Scales Behaviour (Auditorium)

Organisers: Mike Jeffrey & Mathieu Desroches (University of Bristol, UK)

Singular perturbations of piecewise smooth systems and a simple ocean circulation model

M11.1
10.30

Paul Glendinning[1]*

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We consider the simplest class of singular perturbation (slow-fast) analysis of piecewise continuous systems and apply the results to the bifurcations in a box model of the Atlantic Ocean thermohaline currents.

This is joint work with Piotr Kowalczyk.

Flow Curvature Method for Canard Computation

M11.2
11:00

J.M. Ginoux[1]*

[1] Laboratoire PROTEE, I.U.T. de Toulon — Universit du Sud, BP 20132, F-83957 La Garde Cedex, France
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The aim of this talk is to present a new method entitled: Flow Curvature Method [1] which is based on the use of Differential Geometry local metrics properties of curvatures. Starting from the velocity vector field of any n-dimensional dynamical system it is generally possible to compute its successive time derivatives, i.e. acceleration, over-acceleration etc., ... Then, since curvature definition only involves this time derivatives, curvatures of trajectory curves integral of any n-dimensional dynamical system may be analytically computed. Thus, it has been stated that the Flow Curvature Manifold defined as the location of the points where the highest curvature vanishes directly provides the Slow Invariant Manifold of any n-dimensional dynamical system (singularly perturbed or non-singularly perturbed) up to suitable order of approximation. Such approximation may be simply improved by taking the time derivatives of the Flow Curvature Manifold. In this work Flow Curvature Method will be applied for the computation of canards solutions into the famous Van der Pol 2D-model [2] and into a 3D-model studied by Wechselberger [3] in the case of a folded node.

[1] J. M. Ginoux, *Differential Geometry Applied to Dynamical Systems*, World Scientific Series on Nonlinear Science, Series A, Vol. 66, Singapore: World Scientific, 2009.

[2] E. Benoît, J. F. Callot, F. Diener, and M. Diener, Chasse au canard, *Collect. Math.*, **3132**, pp. 37119 (1981).

[3] M. Wechselberger Existence and bifurcations of canards in R3 in the case of a folded node, *SIAM J. Appl. Dyn. Syst.* **4(1)**, p.101139 (2005).

Canards and bifurcation delays in reaction-diffusion equations

M11.3
11:30

Peter De Maesschalck[1*], Nikola Popović[2], Tasso J. Kaper[3]

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[2] University of Edinburgh
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In ordinary differential equations of singular perturbation type, the dynamics of solutions near saddle-node bifurcations of equilibria are rich. Canard solutions can arise, which, after spending time near an attracting equilibrium, stay near a repelling branch of equilibria for long intervals of time before finally returning to a neighborhood of the attracting equilibrium (or of another attracting state). As a result, canard solutions exhibit bifurcation delay. In this talk, we analyze some linear and nonlinear reaction-diffusion equations of singular perturbation type, showing that solutions of these systems also exhibit bifurcation delay and are, hence, canards. Moreover, it is shown for both the linear and the nonlinear equations that the exit time may be either spatially homogeneous or spatially inhomogeneous, depending on the magnitude of the diffusivity. The presented results are published in [1].

[1] P. De Maesschalck, N. Popović, T. J. Kaper, Canards and bifurcation delays of spatially homogeneous and inhomogeneous types in reaction-diffusion equations, *Adv. Differential Equations* **14** (2009), no. 9–10, 943–962.

Canard a l'orange: a new recipe for multiple time scales

M11.4
12.00

Mike Jeffrey[1*], Mathieu Desroches[1*]

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[*] Electronic Address: M.Desroches@bristol.ac.uk

We present a fresh look at phenomena associated with multiple time scale dynamics. In particular, we look at the extreme changes of curvature that give rise to canards and mixed-mode oscillations. The 'fresh look' involves viewing singularly perturbations from the perspective of discontinuities. We pinch away regions of fast dynamics, leaving a system that switches between slow dynamics and slides in the pinched region between. This reveals a link between singular perturbation theory, Filippov's differential inclusions, and nonstandard analysis. We also present a criterion to establish whether the conditions for canards exist; namely, based on inflection curves, we derive an upper bound in the time scale ratio for the occurrence of canard explosions.

At the end we will invite discussion on the theme of the minisymposium: the different methods used to study multiple time scales, their relative merits, similarities, and compatibilities, and we invite your thoughts on the state of the art in the mathematical understanding of multiple time scales.

Mini-symposium M12: Mathematical Modelling of Engineering Applications (Lecture Room G12)

Organisers: Yuliya Kyrychko & John Hogan (University of Bristol, UK)

Stabilization with time-periodic delayed feedback of higher derivatives

M12.1
10:30

G. Stepan[1*], T. Insperger[1]

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Acceleration signal is rarely used in feedback loops to control mechanical systems since the stability of trivial solutions of the corresponding neutral delay-differential equation is sensitive for parameter variations. If the jerk is used with delay, the correspondng linear system is usually unstable. Those systems are investigated where the higher derivatives are used in sampled feedback loops and the results are compared to those known for advanced differential equations. As a case study, the balancing of a single degree-of-freedom inverted pendulum is discussed having the governing equation in the form

$$\ddot{\phi}(t) - a^2\phi(t) = -d\dot{\phi}(t-\tau(t)) + \epsilon d\dddot{\phi}(t-\tau(t)) - p\phi(t-\tau(t)) + \epsilon p\ddot{\phi}(t-\tau(t))$$

where a is the parameter of the unstable uncontrolled system, while p and d are constant parameters of the PD control. If the time delay does not depend on the time t and it is a constant τ, the system can be transformed into an advanced differential equation of the form

$$\dddot{\phi}(t) + \frac{p}{d}\ddot{\phi}(t) - \frac{1}{\epsilon}\dot{\phi}(t) - \frac{p}{\epsilon d}\phi(t) = \frac{1}{\epsilon d}\ddot{\phi}(t+\tau) - \frac{a^2}{\epsilon d}\phi(t+\tau)\,.$$

Although this system is always unstable with constant delay, time-periodic delays like the ones arising in case of sampling can stabilize it. Conclusions are also given for the dynamics of the Segway.

Emergent scaling laws in dielectric materials

M12.2
11:00

Chris Budd[1*], N. McCullen[1], D. Almond[1]

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Dielectric materials are widely used in many applications in engineering particularly those related to materials with novel electrical properties.

When an alternating potential is placed across such dielectric material the current through the material usually depends upon the frequency of the potential. Understanding the relation between the resulting material admiitance and the frequency is very important in the design and use of such materials.

For low and high frequencies this dependence is dominated by percolation paths through the material and is very ssensitive to the material composition. However for intermediate frequencies with more regular conduction paths, it is often possible to observe emergent behaviour which is much less dependent upon the precise form of the material composition. In this behaviour we often see anomolous power laws in which the material admmitance depends on a fractional power of the potential frequency.

In this talk we will aim to explain this behaviour by modelling the dielectric as a random binary network. The electrical properties can then be described by certain types of random operator. We will show that the power law and percolation behaviour is then a consequence of certain prperties of the spectrum of this operator. We will then show how these results on the spectrum can be generalised to exaplain the electrical properties of many other materials.

The Effects of Bathymetry Variations on Two Dimensional Tidal Flow

M12.3
11:30

S. Parkinson[1*], A. R. Champneys[1], S. J. Hogan[1], A. R. Hogg[2]

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Currently, there is a lot of interest in the use of marine current turbines to produce energy. These devices generate power in a similar manner to wind turbines, [2]. This would help enhance, in some small measure, the renewable energy sector in terms of power output and reliability. The main advantage with generating energy from tidal forces is the consistent nature of this resource. In addition, it has been suggested that tidal turbines would not raise as many objections as other renewables such as wind turbines which are deemed to be unsightly.

In this talk a robust, two-dimensional, quasi-analytic description of the flow speed variation in a tidal channel for some arbitrary bathymetry $z_b(x)$ is presented. Starting from the primitive equations used in ocean-scale flow modelling; assumptions are made to produce a reduced order model which yields results with a similar accuracy to numerical oceanographic models using a less numerically intensive approach. The resulting equations of motion are solved using a Galerkin spectral method with an appropriate set of basis fuctions in the height variable z for either a slip or no-slip seabed boundary condition, [1]. The reduced equations are thus formulated as a set of ODEs in the flow direction x, which can be solved for an arbitrary bathymetry function $z_b(x)$. Simulations have been performed for several different bathymetries specified as piecewise continuous functions, $z_b(x)$. The system is solved using `MatLab` and the results are compared to the Regional Ocean Modelling System (ROMS) where excellent agreement has been found when using a no-slip boundary condition.

One of the primary applications for such a model would be to help design hydraulic structures which would operate in tidal channels. This approach could lead to a tool that may be used to optimise the placement of an array of marine current turbines. Similar design tools are already used in the renewable wind energy industry. The inspiration for this type of qualitative analysis came from the well cited paper [3] by Jackson & Hunt, which models the speed up of turbulent wind flow due to surface topography. This kind of analysis has the advantage that the solution method is often more straightforward, the set of parameters which characterise the physical system are more intuitive and the analysis can be extended to model other phenomenon.

[1] Davies A. M. *Application of the Galerkin Method to the Formulation of a Three-Dimensional Nonlinear Hydrodynamic Numerical Sea Model*, Applied Mathematical Modelling (Dec 1980) **Vol. 4**, pg 245-255.

[2] Fraenkel, P. L. *Power From Marine Turbines*, Journal of Power and Energy, Proc. Instn Mech. Engrs. **Vol. 16**.

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Applications of non-smooth dynamics to mechanical engineering

M12.4
12:00

Marian Wiercigroch[1*]

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In this lecture I will paint a broad picture of the non-smooth dynamical systems and examine implications of this view on mechanical engineering. Through carefully chosen examples I will try to show just how non-smooth and dynamic our world really is.

In the first part, I will define non-smooth dynamical systems in simple terms. Then through a biased view I will introduce the major developments and models used to explain the physics behind the typical non-smooth phenomena such as impacts and dry friction.

The second part will be devoted to what we might call Nonlinear Dynamics for Engineering Design where I will present results from my recent projects, where non-smooth dynamic interactions have been used to enhance the performance of real systems and structures. I will put a special emphasis on one large project from oil and gas industry, where we have developed a revolutionary downhole drilling technology. I will argue that this would not be possible in a smooth and static world!

Contributed Talks C9: Fluid Dynamics (Lecture Room G12)

Non equilibrium phase transitions and invariant measures in the 2D Navier-Stokes equations and in other systems with long range interactions

C9.1
14:00

F. Bouchet[1]*, M. Corvellec[1], E. Simonnet[2]

[1] Center for Non Linear Studies (CNLS, Los Alamos-USA) and Ecole Normale Supérieure de Lyon (ENSL, CNRS-France)
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We consider systems with long range interactions driven out of equilibrium by stochastic forces. When these are balanced with linear dissipation they reach non equilibrium steady states (NESS), without detailed balance [1, 4]. We study dynamics both of particle systems (Hamiltonian Mean Field model) or fields (2D Navier Stokes equations with stochastic forces). We discuss the existence of non-equilibrium phase transitions and describe explicitly sets of invariant measures.

For instance, for the two-dimensional Navier-Stokes equations with weak stochastic forces and dissipation, the existence of an invariant measure has been mathematically proved recently, together with mixing and ergodic properties. This problem has however never been considered from a more physical point of view. We thus address the following issues: when is the measure concentrated on an inertial equilibrium, how are the large scales selected by the forcing, what is the level of the fluctuations?

The most striking result is the existence of non-equilibrium phase transitions. One observe transitions from one type of flow (unidirectional) to an other one (dipole), at random time [2, 3, 4]. This is similar to the classical two wheel potential with noise. By contrast, in our case, no such potential exists and the turbulent nature of the flow (infinite number of degrees of freedom) renders the phenomena much richer. Analogies with the Earth magnetic field reversal, and with similar phenomena in experiment of two dimensional and geophysical flows will be discussed.

We also discuss explicit invariant measures of partial differential equations such as the 2D Euler, Vlasov equations [5]. We describe sets of invariant Young measures and discuss briefly their stability and the consequence for the ergodicity of the dynamics. These results are foreseen to be very important as they give access to explicit solution to the hierarchy of equations that describe the moment of the field, for instance in turbulence problems.

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[1] Freddy Bouchet, Shamik Gupta, David Mukamel, Thermodynamics and dynamics of systems with long-range interactions, in Press in Physica A, doi:10.1016/j.physa.2010.02.024, arXiv:1001.1479,

[2] F. Bouchet and E. Simonnet, Random Changes of Flow Topology in Two-Dimensional and Geophysical Turbulence, Physical Review Letters 102 (2009), no. 9, 094504.

[3] A. Venaille and F. Bouchet, Statistical Ensemble Inequivalence and Bicritical Points for Two-Dimensional Flows and Geophysical Flows, Physical Review Letters 102 (2009), no. 10, 104501.

[4] F. Bouchet and A. Venaille, Equilibrium and Non Equilibrium Statistical Mechanics of Two-Dimensional and Geophysical Flows, submitted to Physics Report, available on http://perso.ens-lyon.fr/freddy.bouchet/ .

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Hydrodynamics of anisotropic fluids in 2D: Experiments and modeling of flow in thin freely suspended films

C9.2
14:20

R. Stannarius[1*], A. Eremin[1], K. Harth[1], S. Baumgarten[1]

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Free-standing films of thermotropic smectic phases represent the simplest model for a quasi-two-dimensional liquid. Their layered structure stabilizes films with areas up to several square centimeters and thicknesses of a few molecular layers. Films with tilted layer structure (smectic C or C*) represent quasi-2D anisotropic fluids. Distortions of the local tilt azimuth (c-director) mediate elastic torques. Viscous coupling of orientational and flow fields can lead to complex pattern formation. Among the interesting aspects of these systems from the hydrodynamic point of view are the description of simple relaxation problems [1], the motion of particles on inclined film planes and interactions of inclusions with each other via the c-director field.

We have performed experiments by means of polarising light microscopy. The films were drawn across a metal or glass frame. Simple non-equilibrium states like spiral or target patterns were prepared by means of external electric fields. During the relaxation of these regular structures in the field off state, the distorted director field drives a macroscopic flow in the film plane. The dynamic equations - the incompressible quasi Navier-Stokes equation and the torque balance equation for the director - were solved with standard finite elements software (COMSOL) in order to describe the flow and director dynamics in 2D.

Inclusions can be generated either randomly on the film e.g. by melting some of the material in the vicinity of the clearing point, but one can also deposited liquid droplets in a controlled way by picoliter dispensers. Such droplets interact with each other and with the film via the c-director field. An interesting aspect is the observation of droplets moving on oblique or vertical films under the action of gravity. The falling particles create a Schlieren type director pattern along their path. The experiment is very simple, but its description involves five viscosities and two elastic constants of the liquid crystal. We present experimental observations and compare them with calculated scenarios.

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Oscillations of soap bubbles: experiments and numerical analysis

C9.3
14:40

K. Harth[1*], U. Kornek[1], R. Stannarius[1], A. Hahn[2], S. Ganesan[2], L. Tobiska[2]

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There are many situations where the dynamics of a fluid confined in another fluid attracts interest from physical as well as technological points of view. Typical problems are the surface dynamics of a liquid droplet in a gaseous environment or, vice versa, the behavior of gas bubbles in fluids. Soap bubbles represent a particularly simple system to study the dynamics of a volume of gas enclosed by the same of a different gas, separated only by a micrometer-thick soap film. While in the cases mentioned before, the density of the gas is much lower than the fluid density and thus often neglected, in soap bubbles the inner and the outer fluids must be considered. The dynamics are governed by the viscosities of the inner as well as the outer gas and the surface tension of the membrane. Under simplifying assumptions, the governing equations had been derived by the end of the 19th century.

In our experiments [1], we prepare special axisymmetric nonequilibrium initial states where two soap bubbles approach each other and coalesce. The evolution of the fused bubbles towards a final spherical equilibrium state is observed with a high-speed camera. Numerical calculations are performed with a finite elements method for axisymmetric problems on a moveable mesh [2], solving the Navier-Stokes-equations for both fluids and considering the surface tension of the soap film. As initial states for the calculations we chose bubble shapes that have been extracted from the experimental data, and we compare their evolution with the experimentally observed oscillations. In addition, a variety of simple initial states were specifically chosen for an analysis of the influence of bubble shape parameters on the oscillations. We vary parameters such as the bubble size, viscosities of the gases, and the surface tension, analyse eigenmodes of the oscillations, the nonlinear coupling behaviour between them and the flow and pressure fields themselves. The goal of this study is the comparison of analytic models, numerical simulations and experiment, and general insight into the characteristics of the flow field inside an oscillating fluid volume.

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Dissolution dynamics of slowly miscible liquids

C9.4
15:00

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Fluid mixtures are divided into immiscible (e.g. oil/water mixture) and miscible (e.g. honey/water mixture or mixture of two gases). For immiscible systems, an interface is introduced; there is no mass transfer through this interfacial boundary; and the interfacial boundary is endowed with the surface tension. The mixture of two gases is an opposite case: the concept of interface is not used as molecules can freely co-diffuse through the boundary of two initially separated gases.

In the current work we examine the thermo- and hydro-dynamic behaviour of the mixtures exhibiting an intermediate behaviour, called the mixtures of slowly miscible liquids. All completely or partially miscible liquids, for which the intermolecular forces cannot be neglected, may be referred to this case. The behaviour of a honey droplet in water is one of the examples: a strict interface can be visible for a long period of time after immersion of a honey droplet into water, but after a slow dissolution process honey/water mixture becomes homogeneous and the interface disappears. The concept of interface is required to describe the behaviour of a slowly miscible system. Such an interface is characterised by the surface tension coefficient which varies over dissolution process [1, 2].

It is important to notice that the surface tension determines both the morphology of the interfacial boundary and the rate of mass transfer through it. For immiscible systems, we may argue that the surface tension is sufficiently high to exclude co-diffusion of molecules between adjoining liquids; while, in the case of slowly miscible systems, the mixing occurs but its intensity is restricted by the value of the surface tension.

In the current work, we study the interactions between the thermodynamic transition and hydrodynamic flows which would characterise a thermo- and hydrodynamic evolution of a binary mixture in a dissolution/nucleation process. The Cahn-Hilliard approach is used to model the behaviour of evolving and diffusing interfaces. An important peculiarity of the full Cahn-Hilliard-Navier-Stokes equations [3] is the use of the full continuity equation even for a binary mixture of incompressible liquids, firstly, due to dependence of mixture density on concentration and, secondly, due to strong concentration gradients at liquids' interfaces.

By using the multiple-scale method and the averaging procedure we separate the physical processes occurring on different time scales and, ultimately, provide a strict derivation of the Boussinesq approximation for the Cahn-Hilliard-Navier-Stokes equations. This approximation forms a universal theoretical model that can be further employed for a thermo/hydro-dynamic analysis of the multiphase systems with the phase change, i.e. for the processes involving dissolution/nucleation, evaporation/condensation, solidification/melting, polymerisation, etc.

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Particle self-ordering in periodic flows

C9.5
15:20

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We show that small heavier than fluid particles may spontaneously organize in ordered structures in time-periodic incompressible flows [1]. The ordering mechanism hinges on the nonlinear effect known as *phase-locking* or *frequency-locking.* This novel mechanism of particle ordering explains the surprising assembly of particles into rotating spirals that was discovered experimentally in thermocapillary flows more than a decade ago[2]. Although the phenomenon has been extensively studied on the ground and on board of the International Space Station, its comprehensive explanation has been lacking. In our exposition we introduce a simple model of fluid-particle interaction, which can be reduced to the circle map. This theoretical approach allows to establish a correspondence between the geometrical shapes of the ordered particulate spirals, which have been observed in experiments and our numerical simulations, and the phase-locked regions of the circle map known as *Arnold tongues*[3]. We discuss further parallels between dynamical behavior of the circle map and features of particle ordering observed in experiments and numerical simulations, such as intermittent rise and fall of the coherent particulate structures. We remark that similar to chaotic advection and intermittency, here the dynamical systems perspective has proven illuminating in explaining a phenomenon that appeared puzzling from the viewpoint of traditional fluid mechanics.

We anticipate that this novel mechanism may cause localization and ordering of particles in large-scale periodic flows that abound in nature. Their examples include flows in laminar stirred tanks, pyroclastic surges, microfluidic, convective, atmospheric, oceanic, planetary and astrophysical flows.

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Transition to turbulence for flows without linear instability

C9.6
15:40

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Instability of fluid flows and their transition to turbulence are widespread phenomena in engineering and natural environment. Yet, the transition to turbulence in canonical unidirectional shear flows is still an unsolved problem in fluid mechanics. The difficulty consists in extracting relevant information from the Navier-Stokes equations to understand what is observed in real life turbulence. It is well known that plane Couette flow (PCF), pipe flow (PF) and square duct flow (SDF) are linearly stable against arbitrary three-dimensional perturbations at any finite Reynolds number, so that transitions from the basic laminar states, if they exist, must be abrupt. Due to this lack of linear instability, weakly nonlinear analysis does not work in general and only numerical approaches must be resorted.

The absence of a linear instability mechanism has previously prevented the discovery of nonlinear solutions to these linearly stable canonical flows. However, Nagata (1990)[1] found, for the first time, time-independent three-dimensional solutions to PCF by first focusing on Taylor-Couette flow between co-rotating cylinders. The solutions to PCF were obtained by bringing the rotation rate to zero. Later, several nonlinear solutions have been obtained for PF, first by Faisst & Eckhardt (2003)[2] using the idea of a self-sustaining process (SSP) of turbulence proposed by Waleffe (1998)[3], followed by Wedin & Kerswell (2004)[4]. It is only recently that nontrivial nonlinear states for SDF are discovered numerically at finite Reynolds number [5, 6]. The onset of turbulence for these flows is believed to be related to the appearance of steady states and travelling-waves, called *exact coherent structures*, caused by disturbances of finite amplitude which take the flows out of the basin of attraction of the laminar state in the phase space.

In the present paper we report our recent discovery of new travelling-waves in PCF[7] and SDF[8]. Furthermore, we introduce another flow which also exhibits no linear instability, namely sliding Couette flow in an annulus, and show our discovery of nonlinear travelling-waves for this flow [9]. We believe that these travelling-waves constitute the skeleton, around which a time-dependent trajectory in the phase space is organized, and contribute toward understanding non-equilibrium turbulent processes.

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[3] F. Waleffe, *Phys. Rev. Lett.* **81**, 4140 (1998).

[4] H. Wedin & R. R. Kerswell, *J. Fluid Mech.*, **508**, 333 (2004).

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[7] K. Deguchi & M. Nagata, *Phys. Rev. Lett.*, Submitted (2010).

[8] S. Okino et al., *J. Fluid Mech.*, Under consideration (2010).
[9] K. Deguchi & M. Nagata, *J. Fluid Mech.*, Submitted (2010).

Contributed Talks C10: Coupled Oscillators II (Albert's Bar)

Localised Patterns in Continuum Limits of Coupled-Cell Systems

C10.1
14:00

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We consider the discrete coupled cell system

$$\dot{u}_n = h^{-2}(u_{n+1} + u_{n-1} - 2u_n) + \mu u_n + s u_n^3 - u_n^5, \tag{1}$$

where the $u_n(t)$, for $n \in \mathbb{Z}$, are real variables on an infinite 1D lattice and $h, s > 0$ and $\mu < 0$ are real parameters. This is perhaps the simplest possible toy model for a spatially discrete system with a bistable nonlinearity, steady states of which are relevant to many fields including nonlinear optics (where steady states of (1) are also time-periodic solutions of a discrete collection of nonlinear Schrödinger equation) and biological pattern formation modelled at a cellular scale. Through the interaction between the bistability and the discreteness, localised states are stabilised over open intervals of the bifurcation parameter μ and are organised into the well-known 1D snaking bifurcation diagram.

As the lattice spacing parameter h decreases, the interval in μ over which the localised states persist shrinks. In the natural continuum limit in which $h \to 0$, the coupling term is replaced by u_{xx}: there is no pinning and hence no steady localised states exist.

In this presentation we will discuss higher-order PDE approximations to (1), the simplest of which is the sixth-order PDE

$$u_t = \left[1 + \left(\frac{h}{2\pi}\right)^2 \frac{\partial^2}{\partial x^2}\right]^2 u_{xx} + \mu u + s u^3 - u^5,$$

which explicitly contain the coupling parameter, and discuss to what extent they are able to preserve the snaking behaviour.

Chaotic behaviour of identical phase oscillators with full permutation symmetry

C10.2
14:20

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The Kuramoto model (and modifications thereof) has been used for many years as an archetypal model for the collective dynamics of identical coupled oscillators. However for the conventional coupling function, the system is known to typically have only periodic or quasiperiodic dynamics, unless the symmetries are broken by, for example, including non-identical oscillators. In this contribution we will explore the appearance of chaotic attractors in systems of the form

$$\dot{\varphi}_i = \omega + \sum_{j=1}^{N} g(\varphi_i - \varphi_j)$$

where $g(\varphi)$ is a periodic function and $(\varphi_1, \cdots \varphi_N)$ periodic variables, even when the oscillators remain identical. We will investigate the appearance of chaotic attractors of various types on variation of N and parameters in the function g.

Coherence and reliability of noisy oscillators with delayed feedback

C10.3
14:40

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For noisy self-sustained oscillators, both reliability, stability of a response to a noisy driving, and coherence understood in the sense of constancy of oscillation frequency belong to the main characteristics. Though the both characteristics and techniques for controlling them received great attention of researchers, owing to their importance for neurons, lasers, clocks, electric generators, *etc.*, these characteristics were previously considered separately.

Quantitatively, stability of response, reliability, is characterized by the largest Lyapunov exponent λ (LE). For smooth limit cycle oscillators LE is negative meaning that the system is reliable. However, a large noise may lead to a positive LE (antireliability for neuronlike systems in a "classic" experimental setup has been previously forecast as well). Another quantifier, coherence, is measured by the diffusion constant D of the oscillation phase $\varphi(t)$; $\langle(\varphi(t) - \langle\varphi(t)\rangle)^2\rangle \propto D\,t$.

In Ref. [1] an extremely efficient technique for controlling coherence by a weak delayed feedback has been proposed and theoretically analyzed. Presumably, this delayed feedback may considerably affect the response stability as well.

We present results on the reliability of noisy oscillators subject to delayed feedback control [2], suggesting an effective mean for controlling the reliability. Analysis of these results in the context of controlling coherence reveals strong quantitative relations between the reliability and the coherence for a weak noise;

$$-\lambda/D \approx const\,.$$

(The constant depends on the parameters of the control-free noiseless system.) Thus, a high reliability occurs for a weak coherence, and *vice versa* the weaker reliability the higher coherence.

In particular, the disclosed relation imposes strong limitations on the implementation of the technique of coherence improvement by virtue of a linear delayed feedback. For instance, in an ensemble of uncoupled identical self-sustained oscillators synchronized by a common external noisy driving, small intrinsic noise is always present and leads to spreading of oscillator phases: $\Delta\varphi \propto \varepsilon_{\mathrm{in}}/\sqrt{-\lambda}$ ($\varepsilon_{\mathrm{in}}$ is the amplitude of intrinsic noise). In such an ensemble the delayed feedback improvement of the coherence results in a mutual spreading of oscillator phases which may sometimes be undesirable.

Detailed calculation of the Lyapunov exponent [2] discloses the essentially different nature of various contributions to the Lyapunov exponent and the diffusion constant. However, the final quantitative effect of delayed feedback on these dissimilar independent properties of oscillatory systems somewhat surprisingly turns out to be identical (note, while for one quantifies this effect means enhances regularity, for another the regularity is impaired). The reported phenomenon being valid for a general class of limit cycle oscillators is neither intuitively expected nor trivial.

The work was supported by CRDF (Y5-P-09-01) and MESRF (2.2.2.3/8038).

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[2] D.S. Goldobin, Coherence versus reliability of stochastic oscillators with delayed feedback, Phys. Rev. E **78**, 060104(R) (2008).

Collective phase diffusion and temporal precision in networks of noisy oscillators

C10.4
15:00

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In nature, there are many situations in which a population of oscillators forms a collective oscillation. Understanding the dynamical properties of synchronous oscillators is an important issue because of their broad applications in disciplines ranging from biology to engineering [1, 2, 3].

Natural oscillators are inevitably subjected to external or intrinsic noise. Therefore, their phase dynamics, and thus their cycle-to-cycle periods, fluctuate with time. However, precision in oscillations is an important requirement, particularly for biological pacemakers, such as the circadian pacemaker and the pacemaker organ in weakly electric fish. One may expect that a larger population of oscillators would tend to suppress fluctuations, resulting in more precise oscillations. Indeed, there is experimental and numerical evidence supporting this idea. However, as A. Winfree pointed out [2], the mathematical essence behind such an improvement remains unrevealed.

In this presentation, we propose a general theoretical framework with which to describe fluctuations in phase dynamics (referred to phase diffusion [3]) and the precision of cycle-to-cycle periods in networks of noisy oscillators. Our framework is based on phase models, which are known to approximate networks of real oscillators [1]. By confining ourselves to long time behavior or a strong coupling case, we obtain a concise relationship between the inherent phase diffusion constant D_i of an isolated oscillator and the collective phase diffusion constant σ^2 of a synchronous network. We then relate σ to the precision of collective oscillations. It turns out that σ is strongly dependent on network structure; *e.g.*, the number of oscillators N and the detailed connectivity. In particular, we find (i) in undirected networks, σ scales as $1/\sqrt{N}$, which is in accordance with the central limit theorem, and (ii) in directed networks, σ shows slower or even nonvanishing decay with N. Examples including a directed scale-free network are provided to demonstrate such network effects. Furthermore, we demonstrate our results by experiments using chemical oscillators. Our approach provides a clear answer, in certain cases, to the classical problem of collective enhancement of precision [2].

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[1] Y. Kuramoto, *Chemical Oscillations, Waves, and Turbulence*, Springer, New York, 1984.

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[3] A. Pikovsky, M. Rosenblum, J. Kurths, *Synchronization: A Universal Concept in Nonlinear Sciences*, Cambridge Univ. Press, 2001.

On synchronization analysis in networks with delay

C10.5
15:20

Philipp Hövel[1]*, Eckehard Schöll[1], Yuliya N. Kyrychko[2], Konstantin B. Blyuss[2]

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We investigate complete synchronization in delay-coupled networks of neural oscillators, which are modeled by FitzHugh-Nagumo systems. Based on a master stability function approach [1, 2], we consider first the network topology without delay. The master stability formalism separates the network topology, which is given by the adjacency matrix, from the local dynamics of the nodes by proper diagonalization. Originally, this technique determines an area in parameter space where the largest Lyapunov exponent is negative. Thus, the synchronization manifold becomes transversely stable.

In a second step, we include time delays in the coupling. In this case, the master stability formalism becomes more complicated. In our novel approach, the coupling delays are taken into account via a self-consistency condition. This leads to a reduction of the regime with negative Lyapunov exponents in comparison to the original undelayed synchronization manifold.

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[2] C.-U. Choe, T. Dahms, P. Hövel, and E. Schöll: *Controlling synchrony by delay coupling in networks: from in-phase to splay and cluster states*, Phys. Rev. E **81**, 025205(R) (2010).

Delay and symmetry in coupled oscillator networks

C10.6
15:40

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It is well known that interaction delays between coupled elements can induce multistability, transitions to high-dimensional chaos, synchronisation enhancement/ detriment or oscillation death. However, there is no clear picture yet of how delay effects relate to the underlying symmetry of the dynamical system. Are delays, for instance, always promoting the same type of influences or do they on the contrary result in specific consequences for specific symmetries? In this contribution we address how delays induce and affect transitions across the oscillation patterns allowed by different symmetries of coupled oscillators [1].

More specifically, we compare the influence of a coupling delay in two network motifs, namely a unidirectional and a bidirectional ring of three elements. We first consider networks of Stuart-Landau oscillators; the nodes are nonlinear elements and the coupling is linear. In the unidirectional ring without delay, the amplitudes start oscillating out-of-phase as the coupling between the oscillators increases. Introducing a delay in the coupling, the onset of oscillations is advanced, but the resulting pattern, out-of-phase oscillations, remains. If the three oscillators are coupled bidirectionally without delay, an increasing coupling strength induces anti-phase oscillations (where two oscillators are synchronised in-phase). In the corresponding delay-coupled network, the onset of oscillations is again advanced, but the resulting pattern is different: amplitudes oscillate in-phase with each other.

We also studied networks of Mackey-Glass oscillators, which are linear elements coupled in a nonlinear way. We observe the same influence of a coupling delay as above: in the two network motifs, the delay advances the bifurcation point. In a unidirectional ring the delay does not affect the symmetry of the dynamics, both with and without delay the elements oscillate out-of-phase with each other. In a bidirectional ring of Mackey-Glass oscillators the coupling delay has a symmetrising effect; without delay we observe an asymmetric steady state, when coupling is delayed, the elements oscillate in-phase.

These results can be generalised. First, we demonstrate analytically, by using the method of the pseudo-continuous spectrum [2], that a bifurcation on a steady state is always advanced, independent of the network structure. Secondly, we show that a symmetry-breaking bifurcation without delay can preserve symmetry in the long delay limit, depending on the eigenvalues of the adjacency matrix. A coupling delay has hence a destabilising effect in all networks, and a synchronising effect depending on network topology.

[1] M. Golubitsky and I. Stewart, “The Symmetry Perspective”, Birkhauser (2003)

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Contributed Talks C11: Micro-scale Systems (Victoria’s Room)

Nonlinear dynamics of the internal degrees of freedom and transport of benzene on graphite

C11.1
14:00

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The presence of internal degrees of freedom has been connected to the diffusion and friction of molecules on surfaces. Several mechanisms can be identified for such a link. The internal configuration of the molecule can affect the interaction with the substrate and thus the diffusion barrier. However, if they are chaotic, the internal degrees of freedom can also act as a deterministic noise [1].

In this work, the chaotic internal degrees of freedom of a benzene molecule adsorbed on a graphite substrate, their interplay with thermal noise, and their effects on the diffusion and drift, are investigated by making use of the presence of two different time scales. The fast chaotic internal degrees of freedom act as a finite heat bath which couples to the slow motion of the centre of mass. It is found that that the substrate temperature affects the dynamics of the internal degrees of freedom only weakly, yet still influences the friction and diffusion. The thermal noise ensures that correlation in the internal d.o.f. decays exponentially and that the dynamics are mixing. The invariant density of the molecule is affected and becomes equal to the thermal distribution. The total friction and total momentum diffusion consist of contributions from the thermal heat bath of the substrate, the finite heat bath of the molecule, and the decay of correlation in the slow dynamics.

In atomistic molecular dynamics simulations of the system, various internal degrees of freedom have been investigated separately. The torsion of the benzene molecule dominate the chaotic dynamics, and the associated d.o.f. act as a finite heat bath which supplies noise, as well as dampening. Without them, there is either anomalous super diffusion, or ballistic motion. The contributions from different degrees of freedom to diffusion and friction are identified numerically and found to be sufficiently large to account for the high friction found in experiments [2]. Based on the analytical and numerical results, some suggestions are made for experimental conditions under which the effects of internal degrees of freedom might be more directly observable.

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Chaotic scattering in astrodynamics

C11.2
14:20

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Problems in celestial mechanics has been studied deeply in context of chaotic behavior, predictability, and stability. However, many authors usually investigated the problems for that range of energy or Jacobian constant where the motion is bounded. In this study we show a more complex structure of the Hamiltonian phase space taking into account the phenomena of chaotic scattering, i.e. when the system is open. This extended investigation gives us a qualitative overview about the invariant chaotic saddle which is responsible for finite time irregular behavior.

Several well-known problems (the restricted three-body problem, the Sitnikov problem, Hill's equations, escapes from barred galaxies) will be presented. Numerical results confirm that an invariant fractal set can be found in the phase space of these systems. Moreover, due to the mixed phase space structure two different part of the chaotic saddle can be identified. The first, where the number of non-escaping trajectories decreases exponentially, corresponds to the hyperbolic part of the saddle. The other one includes the trajectories that may come close to the KAM islands, number of such orbits follow power law decay.

The dynamics of quantum equilibration

C11.3
14:40

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Recently, there has been significant progress in understanding the dynamics of equilibration in quantum theory. In [1, 2], we have shown that a quantum system that is initially out of equilibrium and is interacting with a much larger bath, will evolve towards an essentially static equilibrium state under very weak conditions.

In particular, we need only assume that the total Hamiltonian has non-degenerate energy gaps (which ensures interaction between all subsystems) and that the global state explores a sufficiently large space of states. Under these conditions, the state of the system will evolve towards a particular mixed state, and remain close to that state for almost all times, fluctuating extremely slowly about it. What are generally thought to be time fluctuations are just the result of quantum uncertainty (the probabilistic nature of the outcomes of quantum measurements), which would be present even if the state were absolutely time independent.

Note that this differs from the classical case, where the true state of the system at any given time is a pure state (a point in phase space), and the equilibrium state can only be obtained by time-averaging, or incorporating our own ignorance. The key difference in the quantum case is *entanglement*, which leads to objective uncertainty – even when we have complete knowledge of the global state, a system that is entangled with its environment will be best described by a mixed state.

Although these arguments cannot be applied directly to closed quantum systems, it is possible to show using the same assumptions that any state of a closed quantum system will appear to equilibrate with respect to all 'reasonable' measurements (strengthening the results of [3]).

Combining these results, we arrive at a broad and rigorous foundation for statistical mechanics, which helps explain the dynamical process of equilibration, and may be of great help in understanding more general non-equilibrium situations.

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Phase transitions induced by microscopic disorder

C11.4
15:00

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Based on the order parameter expansion [1, 2, 3, 4], we present an approximate method which allows us to reduce large systems of coupled differential equations with diverse parameters to three equations: one for the global, mean field, variable and two which describe the fluctuations around this mean value.

With this tool we analyze phase-transitions induced by microscopic disorder in three prototypical models of phase-transitions which have been studied previously in the presence of thermal noise: a set of globally coupled Φ^4-systems in the presence of both additive and multiplicative quenched noise and the canonical model for noise-induced phase transitions [5, 6].

The method allows to give closed equations for the critical disorder where macroscopic order is induced or destroyed by time independent local disorder. We analyze the limits of the approximation by comparing the results with the numerical solutions of the self-consistency equation which arises from the property of self-averaging. Finally, we carry on a finite-size analysis of the numerical results and calculate the corresponding critical exponents.

Numerical simulations suggest different universality classes for the different ways of applying disorder.

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Variational Formulation for the KPZ Equation: Consistency, Galilean-invariance violation, and fluctuation-dissipation issues in real-space discretization

C11.5
14:20

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We present a variational formulation for the Kardar-Parisi-Zhang (KPZ) equation that leads to a thermodynamic-like potential for the KPZ as well as for other related kinetic equations [1]. We prove some global shift invariance properties previously conjectured by other authors, and also show a few results about the form of the stationary probability distribution function for arbitrary dimensions. In addition, strong constraints are drawn for the choice of real-space discretization schemes, using the known fact that the KPZ equation results from a diffusion equation (with multiplicative noise) through a Hopf–Cole transformation. Whereas the nearest-neighbor discretization passes the consistency tests, known examples in the literature do not. We propose a consistent and highly accurate scheme, and emphasize the importance of the Lyapunov functional as a natural starting point for real-space discretization. Also, in the light of these findings, the mainstream opinion on the relevance of Galilean invariance and the fluctuation–dissipation theorem (peculiar of 1D) is challenged [2,3,4].

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Asymptotics of work distributions in driven non-equilibrium systems

C11.6
14:40

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In fluctuating non-equilibrium processes, as biological cells or molecular machines, the tails of probability distributions are of particular importance. This fact becomes evident in the fluctuation theorems. The Jarzynski equality $\mathrm{e}^{-\beta\Delta F} = \left\langle \mathrm{e}^{-\beta W} \right\rangle$ is a prominent example, showing that rare work values from the tail of their distribution dominate the average on the right hand side.

In general this tail is only poorly sampled in experiments or simulations. We therefore present a method to analytically determine an asymptote for the tails of the work distribution by using the method of optimal fluctuation.

We discuss the method for driven non-equilibrium systems described by either continuous Langevin equations or by a Markovian dynamics on a finite state space. In both cases we provide the leading exponential term of the asymptotics as well as the dominant pre-exponential factor. We then apply the method to various examples and compare the results with those from numerical simulations of the underlying stochastic dynamics.

Contributed Talks C12: Traffic and Granular Flow (Auditorium)

Mixing by cutting and shuffling in 3D tumbled granular flows

C12.1
14:00

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A field of dynamical systems called *piecewise isometries* (PWIs) has emerged in the last decade [1], yet practical applications of these discontinuous mappings remain scarce. We show that PWIs provide a natural mathematical framework for the kinematic continuum model of granular flow in a rotating spherical container, in a distinguished limit [5]. In this context, we present a new type of non-Euclidean PWI and study its ability to generate complexity; specifically, how "well" a PWI can mix and how this affects granular mixing in tumbled flows.

We show that there exists an underlying mechanism, distinct from the stretching and folding ubiquitous in chaotic fluid mixing, termed *cutting and shuffling*, which can be formulated as a PWI, that leads to the efficient mixing of granular matter. Cutting and shuffling is a generic mechanism in the sense that it is purely kinematic: in 3D it arises from the ability to change the axis of rotation of the tumbler, while in 2D it is due to the shape of the container [3]. Consequently, cutting and shuffling and the corresponding PWIs framework make up the "skeleton" of tumbled granular flows.

In experiments, of course, stretching and folding is also present to enhance the effects of cutting and shuffling. Therefore, new diagnostics that can distinguish between these two types of mixing mechanisms are needed. PWIs generate complexity in a fundamentally different way than continuous chaotic dynamical systems because PWIs possess *no* positive Lyapunov exponents [4] and have *zero* topological entropy [5]. Therefore, the design and implementation of mixing and complexity measures is nontrivial, and new ones are required to properly quantify, e.g., sensitive dependence on initial conditions.

Finally, we consider whether granular mixing can, in some sense, be optimized using the properties of the corresponding PWI; there appears to be a choice of angles of rotation that maximize the amount of cutting and shuffling. We also consider the limitations of this mathematical formalism, specifically the cases where stretching and folding dominates cutting and shuffling, and vice versa.

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3D aspects of mixing and transport in tumbled granular flow

C12.2
14:20

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Mixing and transport in 3D flows is a difficult problem of significant practical and mathematical interest. While, some of the basic theory has been developed [1, 2], the exploration of the types of kinematic structures that can be observed in fully-3D flows has only just begun [3]. Laboratory experiments require visualization of 3D chaos, a nontrivial task [4], hence modeling and simulation is the first step in understanding 3D transport.

We study the kinematics of tumbled granular flow in a spherical container rotated about two distinct axes. This system possesses the convenient property of being able to "switch" between 2D motion (i.e., dynamics restricted to hemispherical shells) and fully-3D motion, depending on the choice of rotation rates about the axes [5]. In both cases, it is possible to compute *analytically* the action-action-angle transformation and period along a given trajectory from the governing piecewise-defined nonlinear dynamical system. Similarly, the exact location of normally-hyperbolic and normally-elliptic curves of period one points can be found and how these change as the flow becomes fully-3D investigated. This provides the basis for a 3D notion of an "island." Additionally, the specific mechanism by which a particle traverses the concentric hemispherical shells, on which trajectories are restricted in the 2D transport case, can be described fully in the 3D case.

When that dynamics are restricted to hemispherical shells, the 2D stable and unstable manifolds of curves of normally-hyperbolic periodic points can be computed by "stacking up" the 1D manifolds computed on a series of "nearby" shells. Though these are not fully-3D structures, their intersections present more complicated lobes dynamics than the intersection of 1D stable and unstable manifolds. Then, we perturb the system parameters and explore how the manifold structure changes in the fully-3D case. In this manner, a coherent structure in a 3D flow can be rigorously defined. Finally, we study the persistence of adiabatic structures (e.g., shells on which the dynamics appear to be ergodic), which are barriers to radial transport, as the flow is perturbed into the fully-3D regime, specifically the resonances and interaction of these adiabatic structures with "KAM tori" as in [6].

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Chaotic advection of inertial particles in gravitational field

C12.3
14:40

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The motion of inertial particles is investigated in a periodic flow in the presence of gravitation. The long term average motion is a uniform settling with a nontrivial settling velocity due to chaos. Diffusional behaviour also appears. Restriction of the flow to a finite column (regarded as a very simple "cloud" model) causes the dynamics to become transiently chaotic. The underlying invariant set, the chaotic saddle, together with its stable and unstable manifolds, exhibits a fractal structure. These structures depend on the column size. More detailed investigation of the saddle uncovers a nontrivial coarse grained density with a local maximum and an exponential tail in the vertical spatial dimension. Substitution of the chaotic effect of the flow with the effect of a stochastic force allows us to apply the Fokker–Planck equation in one spatial dimension. On the one hand, this approach explains the diffusional behaviour observed in the long term settling case perfectly; on the other hand, it also leads to remarkable results in connection with the distribution on the chaotic saddle of the finite column. These results confirm the apparent stochastic behaviour of chaotic phenomena. We note, however, that we have to apply the nontrival settling velocity in the Fokker–Planck equation rather than the theoretical one, which is a remaining trace of deterministic chaos.

Air traffic Complexity: a dynamical system approach

C12.4
15:00

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1 Problem statement

Air traffic density is expected to grow by a factor of 2-3 within 20 years, pushing the capacity of the system to its limits. Major projects have been initiated in order to develop new concepts for air traffic flow management (ATFM) and air traffic control (ATC). Innovative future systems will focus on accurate in time and space trajectory planning (so-called 4D contracts) and some delegation of separation task to aircraft. For the second point, it is needed to predict when and where it will be possible to let aircraft insure separation and to plan trajectories so as to minimize crossings of areas where such a procedure is not applicable. This is the purpose of complexity metrics that are expected to produce indicators of hardness to insure a conflict free traffic, regardless od the way the control is done. We present here a candidate complexity measure, relying on a dynamical system model, and that fulfills the previous requirement.

2 Complexity and Lyapunov exponents

A major issue in ATC is the ability to forecast the evolution of a given traffic in a short term (2-4 mn) to mid term (10-15 mn) time horizon. When such a forecasting is not possible, the situation has to be qualified as complex. On the other hand, well organized traffic is naturally conflict-free and highly predictible: in a such a case compexity is vanishing. We assume that aircraft trajectories are flow lines of an underlying (unknown) dynamical system and then produce complexity maps by computing lyapunov spectrum on points of a regular grid in airspace.

3 Vector field interpolation

From the previous section, it appears that estimating a realistic interpolating vector field from aircraft positions and velocities is central in complexity computation. The solution that will be presented is a 4D spline interpolant so that the reconstructed vector field X realizes the minimum of a functional criterion:

$$E(X) = \int_{\mathbb{R}} \int_{\mathbb{R}^3} \left\| \frac{\partial X}{\partial t}(t,x) \right\|^2 + \mu \|\Delta X(t,x)\|^2 dxdt \tag{1}$$

The possibility of having divergence-free 4D interpolants will be discussed too, with a comparison of the performance for complexity computation purpose.

4 Results

Practical computation of the complexity for large airspaces requires specially designed algorithms in order to cope with the heavy computational task. It is nevertheless possible to obtain complexity maps for country-sized airspaces. Such maps will be presented for a day of traffic over France, showing hot spots of complexity that can be related to known operational bottlenecks.

Few new models of crowd dynamics

C12.5
15:20

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Modelling of crowd dynamics is mainly based on the random walking models of a particle to be carried out in some area. The area might contain one, or several exits (absorbing states), and obstacles (inaccessible state). Basically, an area is a finite part of a rectangular lattice. Both analytical studies, and simulation are used to investigate the dynamics. A standard approach is based on the study of a random walking with gradient transfer of a particle observed within a “room”. More advanced approach may be based on the study of the behaviour of two-particles distribution function over the “room”.

Both simulation, and analytical studies may bring a lot concerning the behaviour of crowds. Meanwhile, a number of factors influencing the dynamics are still conspired form a researcher. We have implemented a series of models to figure out the specific issues in the crowd behaviour and the peculiarities in the space geometry on the crowd dynamics.

Firstly, we studied an impact of the geometry of an area on the dynamics of crowd evacuation, in simulations. Basically, a room is provided by the rectangular lattice, or some other periodical structure (maybe, a combination of few simple figures). To distinguish the effects resulted from the periodicity of a lattice, from those resulted from a randomized transfer itself, we studied the dynamics of a crowd occupying a room tiled with a non-periodic Penrose tiling.

Since Penrose tiling is not periodical, it makes a problem to introduce a visibility radius, that is a distance where an obstacle could be detected by a "person" (a particle). A visible distance is a parameter characterizing a person; one may study a behaviour of a population of persons polymorphic in this parameter. To resolve this problem, a dual approach has been realized: each obstacle has been supplied with a "shadow" visibility area that encapsulates an obstacle. The encapsulation consists from outer tiling cells conjoint to an obstacle; it is permeable for a particle, while the encapsulation affects the transition probability field, for that former. A series of comparative studies of the models differing in the tiling, only (with respect to the proximity of periodic and non-periodic geometry) has been carried out. A better evacuation for non-periodic (Penrose) tiling has been found.

Secondly, an effect of asymmetry in the space organization has been studied. To do that, a series of simulations has been carried out comparing the models with symmetrical organization of space (exits location, obstacle occurrence and form, etc.), and the models with small violations of the symmetry mentioned above. It was found that a break of symmetry eliminated a degeneracy in the dynamics, and evacuation gets better.

Thirdly, an impact of an attraction field (that is relevant to a distant orientation of a person) has been studied, as well, as an impact of a reflexive behaviour on the dynamics of evacuation has been studied. The reflexivity was modelled through a bias in the transition probability field.

Finally, the presented approach has been extended for a dynamics described by a two-particles probability distribution function, in the given area. Numerous simulations show a minor difference between the dynamics described by a single particle probability distribution function, and the two-particles one.

Characterization of the nontrivial and chaotic behavior that occurs in a simple bus traffic model

C12.6
15:40

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What happens in cities where vehicles are constantly interacting with traffic lights? Recently nontrivial and chaotic behavior of a minimalistic city traffic model has been reported in [2, 1]. Among other developments lower bounds in acceleration/brake ratios and the region in parameter space for which we observed nontrivial behavior was found. This parameter region may be related to the high sensitivity of traffic flow that eventually leads to traffic jams.

We follow the dynamics of one vehicle (a bus) moving through a sequence of traffic lights in one dimension, the bus is forced to stop between lights and wait still for a certain time. A bus in this sequence of traffic lights can have: (a) an acceleration a_+ until its velocity reaches the cruising speed $v_{\max}$, (b) a constant speed $v_{\max}$ with zero acceleration, or (c) a negative acceleration $-a_-$ until it stops. Therefore, we can write the equations of motion for the vehicle,

$$\frac{\mathrm{d}v}{\mathrm{d}t} = \begin{cases} a_+\,\theta(v_{\max} - v), & \text{accelerate} \\ -a_-\,\theta(v), & \text{break} \end{cases}, \tag{1}$$

where $\theta(x)$ is the Heaviside step function.

The bus must stop at a fixed point l_n after the $(n-1)$th traffic light for a time Γ_n. As it approaches the nth traffic light with velocity $v \leq v_{\max}$ the driver must make a decision depending on the sign of $\sin(\omega_n t + \phi_n)$ at the distance x_d (the last stopping point to arrive with null velocity at the traffic light).

The frequency ω_n and the phase ϕ_n at the n^{th} traffic light are used to control traffic. If $\sin(\omega_n t + \phi_n) > 0$ (green light) the driver continues to accelerate (or cruise at $v_{\max}$) through the traffic light. If $\sin(\omega_n t + \phi_n) \leq 0$ (red light) the driver starts braking with $-a_-$ until the light turns green again; at which point it starts accelerating with a_+.

We study two variants of the model in Eq. (1), one where we fix x_d (so the driver can modify a_-) and one where we fix a_-. The dynamics of the system are studied via a 2D map $(t_{n+1}, v_{n+1}) = M(t_n, v_n)$, where t_n (v_n) is the time (speed) at which the bus crosses the nth traffic light.

We focus our study of the bus dynamics on the effect of the stopping point (l_n), in particular the conditions that lead to chaotic behavior. We also compare with the car model presented in [2] focusing on the effect that green waves on one system have on the other system.

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Poster Session II (Recital Room)

Quantum Classical Correspondence for a non-Hermitian Bose-Hubbard Dimer

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Recently there has been considerable interest in non-Hermitian mean-field theories for the phenomenological description of decay, scattering and transport behaviour of cold atoms. However, hitherto it has often been disregarded that the presence of the non-Hermiticity will in general modify the mean-field approximation. In the present talk we provide a derivation of the mean-field dynamics for a non-Hermitian two-mode Bose-Hubbard dimer, which serves as a model for a BEC in a leaking double-well trap. We show that the equations of motion can be obtained from a generalised canonical structure including a metric gradient flow. The interplay of nonlinearity and non-Hermiticity introduces a qualitatively new behavior to the mean-field dynamics: The presence of the non-Hermiticity promotes the self-trapping transition, while damping the self-trapping oscillations, and the nonlinearity introduces a strong sensitivity to the initial conditions in the decay of the normalization. The full many-particle dynamics shows a rich variety of breakdown and revival as well as tunneling phenomena on top of the mean-field structure.

Random dynamical models from time series

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Construction of parametrized models (*global reconstruction*) of deterministic dynamical systems from time series has been broadly discussed in the literature for the recent 20 years. A mathematical apparatus substantiating such a possibility has been developed. Different methods of constructing models of evolution operators have been proposed; recently basic limitations have been understood and formulated. In particular, the authors of some works demonstrated that these approaches can be used for prediction of changes in the qualitative behavior of a weakly nonautonomous system for times longer than the duration of the observed time series.

We formulate a consistent Bayesian approach to modeling stochastic (*random*) dynamical systems by time series and implement it by means of artificial neural networks. A feasibility of this approach for both, creating models adequately reproducing the observed stationary regime of system evolution, and predicting changes in qualitative behavior of a weakly nonautonomous stochastic system is demonstrated on model examples. It is shown that some basic limitations arising in the case of deterministic systems may be reduced substantially for stochastic systems. In particular, we demonstrate a successful prognosis of complication of system's behavior as compared to the observed one, which is impossible in principle for deterministic dynamical systems.

A generic stochastic model for crystal nucleation

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We propose a generic stochastic model for crystal nucleation. It is generally known that first-order phase transitions occur by nucleation mechanism, and both the nucleus, a cluster of molecules or atoms, and the nucleation work, a energy barrier to the phase transition, are basic thermodynamic quantities in the theory of nucleation [1]. However, the critical nucleus formation is statistically a random event with a probability largely determined by the nucleation work. Another purpose of this study is to get new insights into microscopic explanations of stochastic models which may be compared with the agent-based computational models, and to bridge the gap between agent-based models (ABM) and stochastic processes. The application of the proposed model refers to the nucleation process, a widely spread phenomenon in both nature and technology, which may be considered as a representative of the aggregation phenomena in complex systems. The discovery of the generation and extinction of crystal nuclei at very low temperatures [2, 3, 4] suggests that stochastic generation of crystal nuclei would be considered as the result of fluctuation of complex cluster structure of the supercooled liquid. The role of both heterogeneity and the interface between clusters in the enhancement of nucleation rate has still to be explained. For instance, it was observed that nuclei could almost always be formed near the surface of the cluster instead of in the interior, and one factor favoring nucleation near the surface would be the greater freedom of motion and, hence, a larger nucleation probability. This is surprising because it is known that the surface layers of the nuclei tend to be disordered and melt at significantly lower temperatures than their cores. Finally, we have shown that while the number of particles at the liquid-cluster interface increases, the stability of the entire system decreases simultaneously, and the nucleus formation would be definetely enhanced due to the displacement of the bifurcation point in the region of smaller clusters.

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Velocity distributions of foraging bumblebees in the presence of predators

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We analyse changes in the flight behaviour of foraging bumblebees under varying environmental conditions, measured in a laboratory experiment by Ings and Chittka[1]. We estimate parameters for different plausible velocity distributions by maximising their likelihood and compare their goodness of fit by applying the Akaike Information Criterion. Using Quantile-Quantile-plots we check for deviations between the estimated probability distributions and the data. We also discuss differences in these distributions for different individual bumblebees. On this basis, we look for systematic changes of the distributions due to the presence of different kinds of artificial spiders.

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Stochastic effects in epidemic models with permanent and long lasting immunity

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In this talk, we will show that the simplest stochastic epidemiological models with spatial correlations exhibit two types of oscillatory behavior in the endemic phase. In a large parameter range, the oscillations are due to resonant amplification of stochastic fluctuations, a general mechanism first reported for predator-prey dynamics [1]. In a narrow range of parameters that includes many infectious diseases which confer long lasting immunity the oscillations persist for infinite populations [2]. This effect is apparent in simulations of the stochastic process in systems of variable size, and can be understood from the phase diagram of the deterministic pair approximation equations. Finally, we will discuss the relevance of the stochastic and deterministic pair approximation models to understand the behavior of simulations on networks of homogeneous degree [3, 4] and consider applications of the underlying methods to seasonally forced epidemic models.

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Using extreme value theory to determine transport statistics of a disordered Hamiltonian system

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We combine the treatment of deterministic chaos in Hamiltonian systems with aspects of the theory of disordered systems for a simple two-dimensional twist map.

Motivated by applications from plasma turbulence, we replace the cosine potential of the well-known Chirikov-Taylor standard map by random one-dimensional analytic potentials with spatial disorder but periodic boundaries.

Structures in phase space include nested island hierarchies, chaotic seas and invariant tori. But while each disorder realization has its specific KAM behavior, the ensemble of systems has to be treated statistically. A fundamental domain of controllable size in phase space enables us to investigate the limit of a disordered system of infinite size using extreme value theory. We obtain distributions of critical perturbation amplitudes, which in turn allow us to conclude about transport exponents.

Chaotic advection in the free atmosphere

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Based on the freely available dataset ERA-Interim containing measured wind speeds of the atmosphere at different altitudes, represented on a regular grid, we carried out simulations of passively advected particles. To illustrate the chaotic nature of the advection dynamics, the local Lyapunov exponent of particle pairs, and the effective fractal dimensions of pollutant droplets were determined. The effective dimensions grow in time and tend to 2 corresponding to perfect mixing. To our knowledge, this is the first study in which the topological entropy of material lines is computed, and a typical value of 0.4 1/day is found. We also investigated the go-around times within one hemisphere of initially localized pollutant distributions. These times are on the order of a month with a nontrivial geographical distribution and seasonal dependence. The spreading of the gases produced by volcanic eruptions are also studied. A comparison of our simulations with satellite data shows that the spreading can faithfully be modeled as passive advection of nondiffusive tracers.

Bifurcations with impact and friction; why it is easier to drag chalk than push it

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Newtonian impact and coulomb friction, when treated seperately are known to lead to consistent formulations in terms of piecewise-smooth dynamical systems. Nevertheless, so-called discontinuity-induced bifurcations such as grazing and sliding bifurcations can occur. This talk considers what happens when friction is taken into account during an impact event. The analysis is restricted to 2D, where the canonical example is that of a slender body allowed to contact a rigid frictional surface (a piece of chalk on a blackboard). First it is shown how to derive consistent impact laws that generalise the coefficient of resitution law. These lead to the possibilities of discontinuity-induced bifurcations as the sequences of sticking or slipping changes during an impact event. Next it is shown that for contacting motion the so-called Painleve paradox of non-uniqueness can be resolved by smoothing and passing to the limit. However, there remains the possibility of reverse chatter, where infinite numbers of impacts accumulate in reverse time, that cannot be ruled out at transition points between stick and slip. The existence of such motion shows the fundamental non-uniqueness in forward time simulations of rigid formulations of impact with Coulomb friction.

Design of optimal entrainment for limit-cycle oscillators

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A theory for obtaining waveform for the effective entrainment of a weakly forced oscillator is presented. Phase model analysis is combined with calculus of variation to derive a waveform with which entrainment of an oscillator is achieved with minimum power forcing signal. Optimal waveforms are calculated from the phase response curve and a solution to a balancing condition. The theory is tested in chemical entrainment experiments in which oscillations close to and further away from a Hopf bifurcation exhibited sinusoidal and higher harmonic nontrivial optimal waveforms, respectively.

Analysis of Grazing Bifurcations within a Discontinuity-geometry Framework

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A periodically-forced impact oscillator (PFIO) is a mathematical model that can be applied in a wide range of real-world applications, such as rattle in gear systems. This model has two fundamental components, a differential equation that models the system in free flight between impacts and a discrete reset rule to model the behaviour of the system at impact. The discontinuities introduced into the model by the reset rule mean that any solution to the differential equation with appropriate initial conditions now only applies up to the next impact, whereupon the reset rule defines initial conditions for a new free flight. A PFIO is an example of a piecewise-smooth system, and in such a system conventional analytic and numeric techniques do not always give a full picture of the qualitative changes in behaviour under parameter variation.

This overall study is exploring the use of a discontinuity-geometry framework [1], which is based on the form and properties of a number of system-specific geometric objects within a three-dimensional representation space. As this framework is not widely known we will start with a brief introduction to it, with definitions of the relevant geometric objects.

If the impacting surface of a PFIO is sufficiently far from the natural centre of the oscillation there will be no interaction between the surface and the dynamics of the system, and thus the only motion will be periodic non-impacting orbits. However, as the impacting surface is brought ever closer to this centre, eventually impacts will start to occur and periodic orbits with impacts may appear. The first point of contact between the non-impacting orbit and the impact surface is often referred to as a *grazing bifurcation*. The exact dynamic behaviour born at a grazing bifurcation will depend on the other parameters of the system, and we will use discontinuity geometry to explore and explain some of the different dynamic behaviours manifested by a damped harmonic oscillator around such a bifurcation.

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Covariant Lyapunov analysis of chaotic Kolmogorov flows

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Fluid turbulence is a typical example of chaotic dynamical systems. However, it is not well understood how and what sort of propeties of the dynamical system characterizes physical properties of turbulence including the Kolmogorov scaling laws and the intermittency.

Hyperbolicity is one of the fundamental properties of dynamical systems. A dynamical system is called to be hyperbolic if the tangent space of the phase space can be decomposed into the stable and unstable directions (Oseledec decomposition), i.e. the stable and unstable manifolds intersect at nonzero angles. When a dynamical system possesses hyperbolic property, its theoretical treatment is easier in general compared with non-hyperbolic cases. Moreover , hyperobolicity is deeply connected to central concepts of the dynamical system theory, such as structural stability.

Although hyperbolicity is an important concept in the dynamical system theory, we know little about the hyperbolicity of important physical systems including fluid motions governed by the Navier-Stokes equations. Here, we study numerically the hyperbolicity of the fluid system which is governed by the Navier-Stokes equation on a two-dimensional torus (Kolmogorov flows).

To evaluate the hyperbolicity, we use the method of covariant Lyapunov vectors(CLV) developed recently by Ginelli et al (2007)[1], which gives the Lyapunov vectors tangent to the stable and unstable manifolds. Applying a Fourier-spectral numerical scheme to the full Navier-Stokes equation, we obtain the covariant Lyapunov vectors along the orbit of a chaotic solution, and evaluate the distribution of the angles between the stable and unstable manifolds on the orbit in order to evaluate the hyperbolicity. It is found by gradually increasing the Reynolds number that the zero angle is not found at the onset of a chaotic solution implying that the system is hyperbolic. However, as the Reynolds number is increased, the distribution extends toward the zero angle, i.e., the angles between the stable and the unstable manifolds become smaller, and at a certain Reynolds number, the distribution of the angles is observed to reach the zero angle meaning that the system becomes non-hyperbolic. This observation suggests that the Navier-Stokes turbulence on the torus may be a non-hyperbolic system.

Hyperbolicity is an important property of dynamical systems in theoretical point of view, but its relation to physical properties is not clear. Kuptsov et al. studied hyperbolicity of the coupled Ginzburg-Landau equation using the method of covariant Lyapunov vectors, and suggested that the property of non-hyperbolicity is related to the emergence of spatiotemporal chaos [2]. Also in our study, the 2nd Lyapunov exponent is found to change its sign in the vicinity of the transition point from hyperbolic to non-hyperbolic system.

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Fermi acceleration on a dissipative bouncer model under scaling analysis.

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The phenomenon of Fermi acceleration is adressed for the problem of a classical and dissipative Bouncer model. It basically consists of a particle of mass m, that collides with a rigid and periodically moving wall under a constant gravitacional field [1]. The dynamics of the model, in both complete and simplified versions is obtained by the use of a two-dimensional nonlinear map. The dissipation is introduced using a restitution coeficient on the periodically moving wall.

Using scaling argunents, we describe the behaviour of the avarage chaotic velocities on the model as a function of the number of collisions with the moving wall and as function of time [2]. We consider variations of the two control parameters; therefore critical exponents are obtained. We show that formalism can be used to describe the occurence of a phase transition from limited to unlimited energy growth as the restitution coeficient approaches the unity. The formalism can be used to characterize the same transition in two-dimensional time-varying biliard problems.

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A Kicked SIRS Model with Logistic Growth

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The dynamics of epidemics is usually based on the susceptible, exposed, infective and recovered continuous-time model (SEIR) [1], or on the simpler SIR model. Both models have endemic equilibria that are asymptotically stable, or the disease dies out. Since oscillations are observed in the incidence of many infectious diseases (like measles, mumps, rubella and influenza), is of interest to determine how oscillating solutions can be introduced in the models. In the continuous-time framework, to account for oscillations the transmission rate was allowed to vary seasonally or spatial heterogeneity was included [2, 3]. Instead, in discrete time, such studies were not satisfactorily performed. Recently, we proposed a discrete-time version of a SIRS model which exhibits oscillations [4]. It was based on the observation that the inclusion in a SIR model of a positive feedback, from the removed class to the susceptible one (in a very narrow range of the corresponding control parameter), has the effect of enhancing periodically the spread of disease.

We added to this SIRS model (i) a variable population size (assumed as logistic) and (ii) seasonal variability (introduced by a sequence of kicks, acting on the infection probability). We consider a population consisting of susceptible, infected and recovered (usually, permanently immune) individuals S, I, R. The discrete-time *SIRS* model has the form

$$\begin{aligned} S_{n+1} &= qS_n + cR_n \\ I_{n+1} &= (1-q)S_n + bI_n, \\ R_{n+1} &= (1-c)R_n + (1-b)I_n \end{aligned} \tag{1}$$

where, during each sampling interval n, q denotes the probability that a susceptible avoids the infection, b is the proportion of individuals, which remain infected ($0 < b < 1$), c ($0 < c < 1$) a fraction of recovered individuals, which lose immunity. The probability q is an arbitrary function $0 < q(S, I) < 1$ with the property $q(S, 0) = 1$, and it depends on the particular form of propagation of the disease. The probability q is modelled as $q = 1 - pI/N$, where $N = S + I + R$ is the total population, but other different forms can be assumed. We consider either a fixed population N or logistic growth of the susceptible group: $S = \lambda S(1 - S/M)$, where λ and M are the logistic parameters. The probability p can be perturbed by a sequence of kicks of amplitude k and period T: $p = p_0 + k\delta_{n,T}$, where (i) $\delta_{n,T} = 1$ if n/T is an integer and 0 if not; or (ii) $\delta_{n,T} = 1$ for $n/T, n/T + 1, ..., n/T + m$ (n/T and m are integer).

Investigations of the parameter space show the experience of many different effects: (i) seasonal fluctuations, (ii) configurations of equilibrium and chaos, (iii) regularization of chaotic growth. Therefore, we believe that the model is a useful tool for describing a wide class of infectious diseases and population dynamics.

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Functionality and Speciation in Boolean Networks

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Boolean Networks have been used to model Genetic Regulatory Networks since Stuart Kauffmann proposed them as a model in the 1960s. Early work focused on how the topology of a network influenced its dynamics. We investigate the inverse problem asking which network topologies satisfy a specified dynamic.

In earlier work by A. Wagner et al [1, 2] a biological function or cell process was specified by an initial condition $\underline{v}(0)$ and an end point $\underline{v}_\infty$ in the expression state space. By so specifying a biological function one can then ask which networks perform this function.

Our view is that in many cases a more appropriate means for defining a biological function would be by specifying the entire path $\{\underline{v}(0), \underline{v}(1), ..., \underline{v}(T)\}$. We will report on how these two contrasting definitions of biological functionality lead to divergent results for their respective functional topologies, particularly regarding the implications for neutral evolution, multi-functionality and speciation.

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Quasiperiodic driving of coupled nonlinear systems

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Strange Nonchaotic Attractors (SNAs) are created by the quasiperiodic driving of nonlinear dynamical systems. In present work we study the transmission effect of quasiperiocdic drive on coupled nonlinear dynamical system. We have considered [1] a system of mutually coupled Rössler oscillators. Among all coupled oscillators, one is direct subject to driving. The dynamics of other oscillators which are in effect indirectly forced is particularly observed in this work. The transmission effect of quasiperiodic drive is apparent here, since strange nonchaotic motion is seen in subsystems that are not directly modulated by the drive. Synchronization in its various forms is common in coupled nonlinear systems; the phase synchronization of subsystems and the effect of quasiperiodic drive on it is observed in the system under study. We find the occurrence of imperfect phase synchronization with forcing, in this type of synchronization the system transits from one phase synchronized state to another, with arbitrary phase slips. Phase stability of coupled system over different initial conditions under identical forcing is observed as a general property of strange nonchaotic motion.

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Chaotic layer width in space-periodic Hamiltonian systems under adiabatic driving

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Calculation of the width of the separatrix chaotic layer is one of the main problems in the theory of Hamiltonian chaos. Recently, it has been shown that the width of the separatrix chaotic layer diverges under adiabatic sinusoidal driving, and the formula for the layer width has been obtained [1]. Here we generalize the results of [1] and derive the formula that is valid for a wide class of time-periodic adiabatic forces. The validity is well confirmed by numerical simulations with different kinds of external forces. Diverse applications of this result are discussed.

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Dynamics and ordered states in low Reynolds number swimmers solutions.

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Self propelling particles [1] constitute an intriguing realization of soft active systems. Here microscopic constituents can drive themselves mechanically by the uptake of energy and show a rich variety of collective behavior, including dynamical order-disorder transitions and pattern formation on various scales. Biology provides us with an important example: collective phenomena are observed for instance when a large number of microorganisms swim together in biological fluids and show an high level of organization in ordered structures [2, 3]; Another important example is given by artificial swimmers driven chemically [4]. Motivated by this we investigate analytically the existence of a macroscopically ordered phase in systems of mechanical swimmers with only hydrodynamic coupling using methods from non-equilibrium statistical mechanics.

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Dynamical properties for a classical particle confined in an infinitely deep box of potential containing a periodically oscillating square well

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Some dynamical properties for a classical particle confined in an infinitely deep box of potential containing a periodically oscillating square well are studied. The potential $V(x,t)$ is given by

$$V(x,t) = \begin{cases} \infty, \text{ if } x \leq 0 \text{ or } x \geq (a+b) \\ V_0, \text{ if } 0 < x < \frac{b}{2} \text{ or } (a+\frac{b}{2}) < x < (a+b) \\ V_1\cos(\omega t), \text{ if } \frac{b}{2} \leq x \leq (a+\frac{b}{2}) \end{cases},$$

where the control parameters a, b, V_0, V_1 and ω are constants. The dynamics of the problem is described by a two-dimensional area preserving mapping on the variables energy and time [1]. Considering the symmetry of the problem, the mapping is constructed upon entrance of the particle in the oscillating square well. Along the region where the potential is constant and equal to V_0 and the region where the potential is time dependent, the velocities of the particle are constants, because there are no forces acting in the particle. However, when the particle moves from one region to another, it experiences and abrupt change in its kinetic energy. It is interesting to define dimensionless variables since there are too many control parameters which are not all of them relevant to describe the dynamics, five in total, namely a, b, V_0, V_1 and ω. We define the following control parameters $\delta = V_1/V_0$, $r = b/a$, $N_c = \omega/(2\pi)\ (a/\sqrt{2V_0/m})$ and new variables $e_n = E_n/V_0$ and measure the time in terms of the number of oscillations of the moving well, $\phi = \omega t$. The phase space of the model is of mixed type in the sense that Kolmogorov-Arnold-Moser (KAM) islands are observed surrounded by a chaotic sea which is characterized by a positive Lyapunov exponent. The size of the chaotic sea depends on the control parameters and is limited by a set of invariant tori (also called as invariant spanning curves) which prevents the energy of the particle to growth unlimited. Thus, if the law which controls the time perturbation of the moving well is smooth enough, Fermi acceleration (unlimited energy growth of the particle) is not observed. Average properties of the chaotic sea at low energy level were investigated under scaling approach as function of this three control parameters. We found critical exponents near a transition from integrability to non-integrability. We validate our scaling hypotheses by a merger, after a suitable axis rescaling, for different curves of the deviation of the average energy as function of n (entrance number of the particle in the oscillating square well) onto a single and universal plot [2]. Such kind of behavior is typical of systems experiencing phase transition. We found also the Lyapunov exponents and showed that the system studied has chaotic components.

Fixed points were also obtained and characterized. The formalism is robust and can be extent to many different kinds of mappings.

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Study of transient nuclei near freezing

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The molasses tail in dense hard core fluids is investigated by extensive event-driven molecular dynamics simulation through the orientational autocorrelation functions. Near the fluid-solid phase transition, there exist three regimes in the relaxation of the pair orientational autocorrelation function, namely the kinetic, molasses (stretched exponential), and diffusional power decay. The density dependence of both the molasses and diffusional power regimes are evalutated and the latter compares with theoretical predictions in three dimensions. The largest cluster at the freezing density of only a few sphere diameter in size persist for only 30 picoseconds ($\sim 2.8 \times 10^{-11}$[s]). The most striking observation through the bond orientatinal order parameter is the dramatic increase of the cluster size as the freezing density is approched.

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Reproducing the effects of traffic congestion by means of thermodynamics-like particle model

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Many contemporary treatises dealing with physics of traffic discuss the possibility for application of thermodynamical approaches to the traffic modeling. These attempts were partially successful predominantly if the local thermodynamics was adopted. Concretely, introducing the socio-physical particle scheme with mutual interactions described by the repulsive forces among the subsequent elements one can obtain surprisingly good analytical estimations for microscopical traffic quantities or their statistical distributions. In spite of the fact that the initial scheme is of thermodynamical origin the alternative formulation of the model leads to the effects of crowding, which opens new perspective in freeway traffic simulations.

We introduce the spatially discrete, time continuous particle system whose intelligent elements are interconnected by the psychological short-ranged interactions and stochastically influenced by the nonzero level of psychological strain. Dividing the circular lattice into the regions of different psychological strains we detect the strong traffic congestion propagating through the entire system. As follows from the relevant statistical analysis of traffic data the both macro- and microscopical structures of the cellular model introduced are in a perfect agreement with those observed in freeway samples.

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Dynamical Scar States and Quantum Fidelity of Chaotic Billiards

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It is now getting possible to fabricate nearly nano-sized structures in the semiconductor heterojunctions. Then the dynamical behavior of electrons inside the structures has been attracting attention of many researchers. Recently we have found the dynamical scar states in the time-evolution of the electron wavepacket[1].

The time-evolution is calculated by discretized time-dependent Schrödinger equation in the 2D mesh points[2], on which the nanodevices are modeled. Setting the initial state as the Gaussian wavepacket, we calculate the time-average of the absolute square of the time-evolving wave functions at each mesh point. Then the scar can been found in its average, if it is launched along an unstable periodic orbit of chaotic billiard. We shall call this scar-like state as a dynamical scar state. Then the quantum fidelity is evaluated to measure the robustness of the states between the orginal chaotic billiard and the distorteds. The robustness is important to realize the quantum computation. We treat two types of distorted billiards: a bent billiard and a billiard with impurities. The former one has a bottom which is bent a bit as a perturbation. The latter one has Gaussian shaped impurities inside it.

Even enlarging the perturbation, in the bent billiard the dynamical scars are found to be remakably stable. It is also observed that the regimes of the fidelity eventually transfer from the perturbation(PT), to the Fermi-golden rule(FG) and the Lyapunov(L) regimes as thoretically expected[3], keeping the scars are sound. Then in the strong semiclassical(SC) regime the scars gradually diffuses all around the billiard. However, the decay rates of the fidelity are suppressed with the appearance the dynamical scar states. In the FG regime, the dependence of the perturbation parameter seems just linear, not square. In the L regime, the decay rate is much smaller than the Lyapunov exponent. On the other hand, this kind of suppression has not been found, when the scars are not apparent[4]. Such sutuation can be realized in the billiard with impurities. The effect of several impurities destroys the scar states easily.

The suppression of the decay of dynamical quantum states in chaotic billiards are found, if the perturbation keeps the dynamical scars clear. We also can identify four kinds of regimes(PT, FG, L, SC), however, the properties of the FG and the L regimes are different from the orginal definition of the regimes, espesially in the bent billiard.

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Dynamical bottlenecks to energy flow in Hamiltonian systems: the periodic orbit perspective

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Vibrational energy in molecular or atomic systems flows unevenly among the degrees of freedom, and the relaxation towards the equilibrium proceeds slower than predicted by statistical theories. The bottlenecks of transport have been related to transitions across the so-called dynamical barriers in phase space. Since relaxation to equilibrium is related to rates of processes, it is necessary to understand the specific pathways of energy redistribution among the degrees of freedom in such systems. The transition state theory predicts well the transition rates in the limit of large barriers. For a Hamiltonian of the so-called crossed fields problem, we use the periodic orbit theory to investigate finer details of transitions when the barrier is weak.

Models of optimal migration with various patterns of information access

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The most wide-spread approach to model a spatially distributed community is based on *reaction–diffusion* equations. It has severe disadvantage constraining the beings under consideration within the aimless random transfer in space, and total lack or a memory; it is absolutely beyond a biological sense. Here we propose a non-diffusive model of spatially distributed dynamics of a population based on Haldane principle. In general, this principle means that beings transfer themselves in space not randomly, but in a manner to maximize net reproduction [1, 2].

The model is a system of two equations in discrete time, with two variables:

$$N^{(i)}(n+1) = [a^{(i)} - b^{(i)}N^{(i)}(n)]N^{(i)}(n); \quad i = 1, 2\,;$$

here $N^{(i)}$ is the subpopulation abundance in i-th station, $a^{(i)}$ is a relevant fertility factor, and $b^{(i)}$ is parameter describing the specific area necessary for a normal reproduction *per capita*; n is a generation number. A migration starts, when net reproduction in the immigration station exceeds that latter in the station of emigration, with respect to the transfer cost, e. g.

$$a^{(1)} - b^{(1)}N^{(1)}(n) < p \times \Big(a^{(2)} - b^{(2)}N^{(2)}(n)\Big)\,.$$

Information access is another important issue, in modelling of spatially distributed communities. Here we consider some models of the dynamics of a community inhabiting two stations, for three patterns of information access:

- global information access. Beings have a complete information towards the abundances in each station, and the environmental conditions in them (expressed in the parameters $a^{(i)}$ and $b^{(i)}$, as well as the transfer cost p. The migration increases the net reproduction, with respect to the transfer cost p;
- local information access. The beings know the abundance and environmental conditions in the habitation station only. Transfer cost might be unknown to them. A possible way to model the spatial dynamics here is the implementation of threshold migration, e. g., a migration may start as soon, as the descendant number exceeds the optimal abundance and $N^{(i)}(n) \geq a/(2b)$, and
- total lack of information. This case is rather proximal to a diffusion approximation, while is not completely equivalent.

Both single population model, and prey–predator community are considered. Evolution optimality (Haldane's principle) here means the maximization of an average (over two stations) net reproduction. Relevant models for the community of two, or several species has been implemented. A number of computation experiments has been carried out.

A problem of multi-station model implementation is also discussed.

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Localized modes in the lattice with nonadiabatic nonlinear response

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The delay of the material response to an applied signal is a significant effect in many settings. In this context, it is relevant to mention the spreading of the electron wave packet in chains with non-adiabatic electron-phonon interactions, where the transition from localized to delocalized states is affected by the time-delayed nonlinear response of the medium [1]. In the framework of the light transmission through an array of waveguides, the coupling of the electronic oscillations in the dielectric material to the electromagnetic wave is, in general, non-adiabatic too. In the latter case, the light transmission through the lattice can be modeled by the discrete nonlinear Schrödinger equation with a non-instantaneous cubic nonlinearity:

$$i\frac{d\psi_n(t)}{dt} + C\left[\psi_{n+1}(t) + \psi_{n-1}(t) - 2\psi_n(t)\right] - \kappa|\psi_n(t-\tau)|^2\psi_n(t) = 0,$$

where n is the discrete coordinate, τ the delay time, C the coupling constant, and κ the nonlinearity parameter ($\kappa = -1$ and $+1$ correspond to the self-attraction and self-repulsion, respectively). Subject of our work are the existence, stability and dynamical properties of localized modes in this model. The numerical analysis is carried out using a properly modified delay-differential equation solver package DDE-BIFTOOL v. 2.00 [2].

In discrete lattices with the on-site non-instantaneous nonlinear response, bright solitons can be created by the modulational instability of continuous waves, which occurs in the same parameter region as in the lattice with the instantaneous nonlinearity. On the other hand, we demonstrate that the stability of solitons is affected by the delayed response in three ways: (i) the solitons are destroyed when the delay time exceeds a certain critical value; (ii) in the case of a very fast nonlinear response, the instability growth rate of inter-site solitons (centered between two adjacent lattice sites) *decreases* in comparison with the instantaneous model; (iii) the type of the instability (exponential/oscillatory) changes for medium values of the delay time. The unstable inter-site and on-site solitons (those centered on lattice site) with very close values of the total power in the fast-responding media evolve into localized breathing modes when their amplitude is slightly perturbed. They can also be transformed into moving localized modes by the application of a kick. However, larger values of the delay time enhance the temporal correlation between time-distant events, thus suppressing the mobility of the lattice solitons.

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Bifurcation structure of zonal flow solutions on a rotating sphere

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Among fluid motions governed by the Navier-Stokes equations, two-dimensional viscous incompressible dynamics with doubly periodic boundary conditions (Kolmogorov problem on a torus) has been considered to be a typical and fundamental example providing mathematical details of solution structure, including successive bifurcations leading to turbulent flows.

In 1960s, Iudovich[1] studied the stability of steady solutions in the Kolmogorov problem, found an analytical formula for the critical Reynolds number, and proved the global stability of a 2-jet solution. Recently, based on numerical results, Kim and Okamoto[2] conjectured an interesting property of the Kolmogorov flow that there may exist a stable unimodal solution only with one pair of positive and negative vortices for any Reynolds number against the forcing of larger wavenumber.

Here we consider two-dimensional fluid motion on a sphere. This problem is similar to the Kolmogorov problem in the points that the flow domain is a compact region with no boundary, but the topology of the domain is simpler (genus 0). This problem is also related to large-scale flows of planetary atmospheres, where we often observe long-lasting zonal flows like those on the Jupiter under the effect of planetary rotation. Two dimensional Navier-Stokes flow on a rotating sphere is a simplest model for those phenomena. In this paper, therefore, we study in detail the bifurcation structure of solutions in the cases both with and without rotation, and a comparison is made between viscous and inviscid cases.

Along the same line with the Kolmogorov problem, we adopt the forcing realizing the 3-jet flow of the spherical harmonics Y_3^0, because 2-jet flow is proved to be globally stable as in the Kolmogorov problem.

In the non-rotating case, as the Reynolds number increases, the 3-jet solution becomes unstable and bifurcates to a supercritical longitudinally traveling wave solution, which further bifurcates to a second supercritical longitudinally traveling wave solution. The second traveling wave solution finally becomes Hopf unstable. In our sphere case no unimodal solution nor stable solution is found at high Reynolds number in contrast with the torus case of Kim and Okamoto[2].

In the rotating case, when the absolute value of the rotation rate Ω is relatively small, the critical mode in the non-rotating case is replaced by another modes dependent on the rotation rate, which finally becomes Hopf unstable as the Reynolds number is increased, leading to somewhat different bifurcation diagram. However, if $|\Omega|$ is larger than critical values ($\Omega > \Omega_1 > 0$ or $\Omega < \Omega_2 < 0$), the zonal flow of Y_3^0 is found to be linearly stable even at high Reynolds numbers. It is of interest that these critical values Ω_1, Ω_2 are clearly different from those for inviscid case found by Baines[3]. The range of Ω giving unstable zonal flows for the viscous case is broader than that for the inviscid case. This seeming contradiction is resolved by the observation that at the value of Ω of inviscid stability, the viscous linear growth rate converges to zero as the Reynolds number is increased, even when the value of Ω belongs to the viscous unstable region.

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Onset of turbulence in spatiotemporal systems

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The onset of turbulence in spatiotemporal systems is a long-standing problem of paramount importance, which has been intensively studied in the past decades. Here we show the existence of two-state on-off intermittent behavior at the onset of turbulence in a spatially extended dynamical systems. As a representative example, we consider the damped and forced drift wave equation.

$$\phi_t + a\phi_{txx} + c\phi_x + f\phi\phi_x + \gamma\phi = -\epsilon\sin(Kx - \Omega t). \tag{1}$$

For magnetically confined fusion plasmas $\phi(x,t)$ is the non-dimensional electric potential of a drift wave, and the constants a, c, and f stand for plasma and wave parameters, and we introduced a phenomenological linear damping term with coefficient γ. The effect of other possibly relevant modes is represented by a time periodic driving with amplitude ϵ, wave number K and frequency Ω .

The two states are stationary solutions corresponding to different wave energies. In the language of (Fourier mode) phase space, these states are embedded in two invariant manifolds that become transversely unstable in the regime where two-state on-off intermittency sets in. In this case the trajectories wander through the available phase space volume approaching the vicinity of both manifolds in an erratic way.

Here we show that the distribution of laminar duration sizes (when the system stays near an invariant manifold) is compatible with the similar phenomenon occurring in time only in the presence of noise. In extended system the noisy effect is provided by the spatial modes excited by the perturbation. We show that this intermittency is a precursor of the onset of strong turbulence in the system.

Non-Markovian stochastic Liouville equation and anomalous relaxation kinetics

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The kinetics of phase and population relaxation in a quantum system induced by noise with anomalously slowly decaying correlation function $C(t) \sim (wt)^{-\alpha}$, with $0 < \alpha < 1$, is analyzed within the renewal approach (RA) which allows one to describe anomalous (non-Markovian) features of the noise [1].

The Hamiltonian of the system is taken in the form $H = H_0 + V(t)$, where H_0 is the time independent part and $V(t)$ is the fluctuating interaction which models the effect of the noise. Fluctuations of $V(t)$ are described within the RA which models the fluctuation process by a set of sudden changes (jumps) of $V(t)$ whose statistics is described the probability density function $W(t)$ of waiting times. The considered case of anomalously slow fluctuations with the above mentioned heavy-tailed correlation function $C(t) \sim (wt)^{-\alpha}$ corresponds to the very slowly decreasing $W(t) \sim 1/(wt)^{1+\alpha}$ and strongly non-Markovian character of fluctuations. The kinetics of $V(t)$ changes is controlled by the operator $\hat{P}$ of jumps between states j corresponding to different interactions $V = V_j$ (with zero mean: $\langle V \rangle_j = 0$).

Within the RA the non-Markovian stochastic Liouville equation for the density matrix $\rho(t)$ of the system is derived [2] which results in the following expression for Laplace transform $\hat{\widetilde{\rho}}(\epsilon) = \int_0^\infty dt\, \hat{\rho}(t) e^{-\epsilon t}$:

$$\hat{\widetilde{\rho}}(\epsilon) = [\hat{\Omega}_\epsilon + \hat{\Lambda}\widetilde{M}(\hat{\Omega}_\epsilon)]^{-1}, \text{ where } \widetilde{M}(\epsilon) = \epsilon\widetilde{W}(\epsilon)/[1 - \widetilde{W}(\epsilon)], \tag{1}$$

$\hat{\Lambda} = 1 - \hat{P}$ is the operator of jump-like fluctuations, and $\hat{\Omega}(\epsilon) = \epsilon + i\hat{H}$, with $\hat{H} = [H, \dots]$ being the commutator of the Hamiltonian of the system.

The analysis with the use of this expression demonstrates a number of peculiarities of the relaxation kinetics. The peculiarities are studied with the the two-level quantum model, as an example. In particular, the kinetics is shown to be anomalously slow. Moreover for $\alpha < 1$ in the most interesting limit of short characteristic time of fluctuations w^{-1} the kinetics, i.e. the time evolution of the density matrix $\rho(t)$ of the system, is independent of w (!) [2]: $\hat{\widetilde{\rho}}(\epsilon) = \langle \hat{\Omega}^{\alpha-1}(\epsilon) \rangle \langle \hat{\Omega}^{\alpha}(\epsilon) \rangle^{-1}$. Here the brackets $\langle \dots \rangle$ denote the average over the equilibrium state of the relaxation operator $\hat{\Lambda}$.

As α grows form 0 to 1 the regime of the limiting relaxation (for $w \to \infty$) changes from static to fluctuation narrowing one. Simple analytical expressions are obtained describing the specific features of the kinetics. It is found, in particular, that the relaxation kinetics is long time tailed $\sim 1/t^\alpha$. In some limits the theory predicts relaxation kinetics described by the Mittag-Leffler function $E_\alpha[-(wt)^\alpha]$. Following a number of recent works (for review see ref. [1]) this kinetics can be thought as a result of the anomalous Bloch equation with fractional derivative in time.

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How to construct the mechanics of the structured particles

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The mechanics of structured particles (SP) is more general than the mechanics of unstructured material points (MP) or hard bodies. Really, at the MPs motion in non-homogeneity space and their interaction the energy of motion changes only, while for the SP internal energy varies also. As usually the change of SP internal energy is described empirically by the classical mechanics for MP. So a question arises, how to construct the rigorous mathematical description of SPs dynamics within the frames of the Newtonian mechanics. We found the answer on this question by obtains the equation of SPs motion when SP representing as an equilibrium system of potentially interacting MPs which is equilibrium in a thermodynamic limit.

It turns out that under certain conditions dynamics of such systems is irreversible [1-3]. These conditions are formulated as follows: 1). the energy of an equilibrium subsystem must be presented as a sum of internal energy and the energy of SP motion as a whole. 2). each material point in the system must be connected with a certain equilibrium system independent of its motion in space. 3). during all the process the subsystems are considered to be equilibrium.

The first condition is necessary to introduce internal energy in the description of system dynamics as a new key parameter characterizes energy variations in equilibrium systems. The second condition enables not to redefine equilibrium system after mixing of material points. The last condition is taken from thermodynamics. It is equivalent to the condition of weak interactions in the equilibrium system, which do not violate equilibrium of SP. Moreover, it implies that each SP contains so many elements that it can be described using the concept of equilibrium system.

In this report we consider derivation of the equitation of motion of two equilibrium of interacting SP. With the help of this equation it is shown how the mechanism of friction can be explained in classical mechanics. It is shown also how based on the hypothesis of local equilibrium, which enables to represent nonequilibrium systems as an ensemble of equilibrium subsystems, one can generalize the obtained results for two interacting SP. It is also shown how Lagrange, Hamilton and Liouville equations for non-equilibrium systems are derived from the equation of motion of a set of SP. We consider how such equations are different from their canonic prototypes for the system of MP. We consider why the SP dynamics is determined by the two types of symmetries: the symmetry of space in which the SP motion and internal symmetry of distributions of elements of SP. It is shown how the main equation of thermodynamics can be derived from the equation of SP interaction and how the concept of entropy arises in classical mechanics.

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Effect of negative resistance in the transport of a dimer system

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The one-dimensional as well as the two-dimensional overdamped Langevin dynamics of a dimer system consisting of two harmonically interacting components are studied. Both components are absorbed at the same spatially periodic substrate potential and are coupled to the same external heat bath. In contrast to previous works, we consider the impact of an inhomogeneous forcing, viz., an external localized point force applied at only one of the two components, on the dynamic of the dimer system. For the one-dimensional case, two accurate approximations for the center of mass mobility and its diffusion coefficient are obtained for weak and strong couplings. It turns out that the mobility of a dimer as a function of the competing length scales of the system, that are the period of the substrate potential and the equilibrium distance between the two constituents, shows a resonance behavior. More precisely there exist a set of optimal parameter values maximizing the mobility. Interestingly, while in the one-dimensional case the mobility as a function of the noise strength is a monotonic function of the latter in 2D we found the effect of negative resistance, i.e., the mobility possesses a minimum at a finite value of the noise strength for a given overcritical external force magnitude.

Function projective synchronization of chaotic systems through Open-Plus-Closed-Loop coupling

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Chaotic systems are characterized by being extremely sensitive to initial conditions, deterministically random and hence ultimately unpredictable. Since its introduction by Pecora and Carrol in 1990, chaos synchronization has received increasing attention due to its theoretical challenge and its great potential applications in secure communication, nano oscillators, chemical reactions, biological systems and so on.

Amongst all the types of chaos synchronization, a general form of synchronization called function projective synchronization(FPS) where the master and slave vectors synchronize up to a scaling function $\alpha(t)$, has been especially extensively studied during recent years. This feature could be used to get more secure communication in application to secure communications, because it is obvious that the unpredictability of the scaling function in FPS can additionally enhance the security of communication. Modified function projective synchronization(MFPS) is also proposed, where the responses of the synchronized dynamical states synchronize up to a desired scaling function matrix $\Lambda(t)$.

I. Grosu et.al., developed a powerful method of control: the open-plus-closed-loop (OPCL) method[1, 2]. This method gives precise driving for any continuous system in order to reach any desired dynamics.

In this presentation we present our studies on FPS and MFPS of a few chaotic and hyperchaotic systems. Using OPCL control method we design controllers for the FPS and MFPS of Lorenz system, Rossler system, hyperchaotic Chen system and Lu system[3, 4]. The OPCL control to obtain FPS is designed as shown below

A chaotic or hyperchaotic driver is defined by

$$\dot{y} = f(y) + \Delta f(y), y \epsilon R^n, \tag{1}$$

where $\Delta f(y)$ contains the mismatched terms which is zero for identical oscillators. It drives another chaotic or hyperchaotic oscillator $\dot{x} = f(x), x \epsilon R^n$ to achieve a goal dynamics $g(t) = y(t)$. After coupling, the response system is given by

$$\dot{x} = f(x) + D(x, g), \tag{2}$$

where the coupling function is defined as

$$D(x, g) = \dot{g} - f(g) + (H - \frac{\partial f(g)}{\partial g})(x - g), \tag{3}$$

$\frac{\partial f(g)}{\partial g}$ is the Jacobian of the dynamical system and H is an arbitrary constant Hurwitz matrix $(n \times n)$ whose eigen values all have negative real parts.

Numerical simulations are performed to verify the effectiveness of the proposed method. A secure communication scheme based on MFPS is proposed in theory.

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A Kernel-based Modelling for Reconstruction of Nonstationarity in Complex Dynamical Systems

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In real-world complex systems, we encounter the nonstationarity in their dynamics due to the existence of external perturbation and/or internal change of the control parameters of the systems. Especially, they often have multiple time scales, i.e., the slow-mode dynamics acts as external perturbation for the fast-mode dynamics during the period of observations[1].

In this study, we propose a method to model the above nonstationarity by constructing a set of time varying functions $\{f_t\}, f_t : \mathbb{R}^d \to \mathbb{R}$ which minimize the following cost function:

$$J = \sum_{t=2}^{T} \{(y_t - f_t(\boldsymbol{x}_{t-1}))^2 + \lambda \|f_t\|_{\mathcal{H}}^2 + \nu \|f_t - f_{t-1}\|_{\mathcal{H}}^2\}, \tag{1}$$

where $\{y_t\}_{t=1-d}^{T}$ is the observed scalar time series of the system, $\boldsymbol{x}_{t-1} = (y_{t-1}, ..., y_{t-d})$ is the time-delay embedding vector at time $t-1$, $\mathcal{H}$ is some function space which f_t belongs to, and λ and ν are hyper-parameters controlling weights of each terms. In the right hand side of Eq. (1), the first term is the sqare error of the prediction $f_t(\boldsymbol{x}_{t-1})$ from true value y_t, the second term is a penalty to avoid overfitting, and the third penalty term is also introduced to enforce the smoothness in the change from f_{t-1} to f_t which represents the slowly varying nonstationarity of the system.

As a function f_t, we here consider the following linear combination of kernels:

$$f_t(\boldsymbol{x}) = \sum_{m=1}^{M} \alpha_m(t) k(\boldsymbol{x}, \boldsymbol{c}_m), \tag{2}$$

where $k(\cdot,\cdot) : \mathbb{R}^d \times \mathbb{R}^d \to \mathbb{R}$ is a kernel function, $\{\boldsymbol{c}_m\}_{m=1}^{M}$ are reference points in the d-dimensional state space, and $\{\alpha_m(t)\}_{m=1}^{M}$ are the corresponding parameters at time t. If the kernel function $k(\cdot,\cdot)$ is positive definite such as the Gaussian function, the best regression function is expressed as the form of Eq. (2) among the Hilbert space $\mathcal{H}$ defined by the given $k(\cdot,\cdot)$[2]. By assuming the type of f_t as Eq. (2), minimization of Eq. (1) is reduced to a simple linear equation about $T \times M$ parameters $\{\{\alpha_m(t)\}_{m=1}^{M}\}_{t=1}^{T}$, whose values are estimated from sample $\{y_t\}_{t=1-d}^{T}$.

The dynamics of $\alpha(t) = (\alpha_1(t), ..., \alpha_M(t)) \in \mathbb{R}^M$ reflects the nonstationarity of the system. So, if a sequence of functions $\{f_t\}$ well capture the nonstationarity, this nonstationarity is reconstructed in a subspace of the M-dimensional space by performing some dimensionality reduction method such as principle component analysis (PCA) for T-samples of $\alpha(t)$. In the talk, we give a detailed mathematical formulation of the above proposed method and show some illustrative examples.

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A new time–delay chaotic circuit

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The presence of time–delay in dynamical systems is particularly significant, since it may represent a source of instability or, in general, may induce undesired oscillations and, thus, poor performance [1]. However, the existence of time–delay in nonlinear dynamical systems, by making the system infinite–dimensional, may be beneficial, allowing the occurrence of complex dynamics like chaos [2]. Furthermore, delay plays an important role in controlling the behavior of dynamical systems. Delayed signals, in fact, are used for chaos control according to a technique, proposed in [3], based on the OGY method [4].

This paper aims to introduce a new nonlinear circuit based on simple circuital components. In particular, the presence of a time–delay in the proposed circuit makes the system infinite–dimensional and, thus, able to show complex behavior, like limit cycles and chaos.

We selected the architecture of the circuit considering that, in order to ensure the emergence of chaos keeping simple the final configuration, at least three blocks are necessary: a nonlinearity, an integrator, and a time–delay.

The nonlinearity has been chosen as a piece–wise linear (PWL) function, which can be realized with few circuital components exploiting the saturations given by the voltage supply.

With this choice, the system dynamics can be written as:

$$\dot{x} = -ax(t) - bh(x(t-\tau)) \tag{1}$$

where x is the circuit state variable, $h(x)$ is the nonlinear PWL function, τ is the time–delay, and a and b are system parameters.

The implementation of the delay is based on the cascade of multiple Sallen–Key active filters. This allows the delay to be easily tuned by changing the number of cascaded filters. This possibility allows the investigation of the different dynamical behaviors emerging from the considered circuit.

The circuit designed following this scheme has been implemented with off-the-shelf discrete components. Experimental results confirmed the presence of chaos for suitable values of the delay τ.

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Wave processes in the system of coupled oscillators appearing in neurodynamics

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In [1] the following delay differential equation is proposed to describe the electrical activity of neural cells:

$$u' = \lambda\big[-1 + f_{\mathrm{K}}(u(t-1)) - f_{\mathrm{Na}}(u)\big]u, \tag{1}$$

where $u(t)$ is a membrane potential of a neuron and functions $f_{\mathrm{Na}}(u) = \beta f_2(u)$, $f_{\mathrm{K}}(u(t-1)) = \alpha f_1(u(t-1))$ define potassium and sodium currents, respectively. Let $f_j(0) = 1$ and $f_j(u) = \sum\limits_{k=1}^{\infty} \frac{a_{jk}}{u^k}$ as $u \to +\infty$ $(j = 1, 2)$. The factor $\lambda > 0$ is defined by the speed of electrical processes in the system and is considered large.

We note that the potassium current depends on delayed membrane potential, and that the delay is normalized to one for the purpose of convenience. Given $\alpha-\beta-1 > 1$ and λ large enough, the equation (1) possesses orbitally asymptotically stable cycle $u = u_*(t, \lambda)$ (see [1]).

We consider the problem of weak electric interaction in a chain of N identical oscillators of the form (1). In the system

$$u_j' = \lambda\big[-1-f_{\mathrm{Na}}(u_j)+f_{\mathrm{K}}(u_j(t-1))\big]u_j + D(u_{j+1}-2u_j+u_{j-1}), \quad j = 1, \ldots, N \tag{2}$$

we put $u_0 = u_1$, $u_N = u_{N+1}$, factor $D > 0$ of order one defines the coupling of neurons, and $\lambda >> 1$. Obviously, the system (2) possesses the synchronous cycle $u_1 \equiv \cdots \equiv u_N = u_*(t, \lambda)$. Our main result states that for a suitable choice of parameters α, β, and D and for arbitrary $\lambda >> 1$, this system, along with the stable synchronous cycle, possesses as well at least N orbitally asymptotically stable nonuniform cycles.

Asymptotical analysis of the system (2) shows that behavior of coordinates $y_j = \ln(u_{j+1}) - \ln(u_j)$, $j = 1, \ldots, N-1$ can be defined by coarse solutions of the system with an impulse action

$$\begin{aligned} &y_j' = D(\exp(y_{j+1}) + \exp(-y_j) - \exp(y_j) - \exp(y_{j-1})),\ j = 1, \ldots, N-1\\ &y_j(1+0) = y_j(1-0) + \alpha y_j(0), \quad y_j(\alpha+0) = (1+\beta)y_j(\alpha-0),\\ &y_j(\alpha+1+0) = y_j(\alpha+1-0) + \alpha y_j(\alpha),\ y_j(T_0+0) = (1+\beta)y_j(T_0-0). \end{aligned} \tag{3}$$

where $y_0 = y_N = 0$, $T_0 = \alpha+1+(\beta+1)/(\alpha-\beta-1)$ is a first approximation of a period of a relaxation cycle $u_*(t, \lambda)$. Using the solution $y_*(t, z) = (y_1(t, z), \ldots, y_{N-1}(t, z))$ of the problem (3) with initial value $y_j(0) = z_j$, $z = (z_1, \ldots, z_{N-1})$ we can define the map

$$z \to \Psi(z) \equiv y_*(t, z)\big|_{t=T_0+0}, \tag{4}$$

which hyperbolic fixed points correspond to cycles of the problem (2). We show that for a suitable choice of parameters α, β, D the map (4) possesses at least N stable fixed points.

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A Mathematical Model of Congestion Control and Route Allocation in a Network with Multiple Sources

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We investigate the trade-off between utility and robustness in a model of congestion control where there can be multiple routes between a source and destination in the network. The model contains a random route allocation scheme for each source-destination pair s where the degree of randomness and the distribution of traffic on paths is controlled by h_s, the entropy of the route distribution. For small values of h_s, network utility is maximized if traffic is allocated to the cheapest routes. This is the basis of the current OSPF protocol. However there are undesirable side-effects such as lack of robustness if the favored route is interrupted. In other instances the network becomes unstable as traffic shifts from overloaded routes to less crowded ones. Route allocations corresponding to larger values of h_s avoid these drawbacks at the cost of reduced utility. A derivation of the model equations from a constrained maximal utility problem along with a detailed analysis of two sample topologies with one source destination pair was carried out in [1, 2]. The minimum value of h_s for which one can achieve a high utility depends on the network topology. In this presentation we extend our results to topologies with multiple source destination pairs.

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Deterministic diffusion in Nosé-Hoover systems

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Though Nosé-Hoover system is widely used for molecular dynamics simulation(MD), its dynamical properties are not well understood. The lack of knowledge on this system sometimes causes troubles when we carry out MD. For example, it is often assumed that Nosé-Hoover system is ergodic, and have a unique invariant

measure. However, this assumption seems incorrect for some cases. Numerical simulations and mathematical analysis give strong evidence that Nosé-Hoover themostat coupled with integrable system is not ergodic[1, 2].

Recently, several new simulation methods called "accelerated molecular dynamics" have developped[3]. These methods, developped to simulate the dynamics including "rare events" such as chemical reaction, are based on the transition state theory and Arrhenius equation. So a natural question arises: Does Nosé-Hoover system satisfy transition state theory?

To answer this question, we invesitigate the transition dynamics of Nosé-Hoover system. First, we studied the deterministic diffusion process in the following 1D Nosé-Hoover system.

$$\frac{dx}{dt} = p, \tag{1}$$

$$\frac{dp}{dt} = -\sin(x) - \frac{\eta p}{Q}, \tag{2}$$

$$\frac{d\eta}{dt} = p^2 - T, \tag{3}$$

where T and Q are constants, which represent the temperature and coupling with thermostat.

The numerical simulation shows that the diffusion of this system strongly depends on Q, T, and the initial condition. We will show the details of this chaotic diffusion and discuss on its mechanism using Poincaré map. The result of simulation in higher-dimension systems will also be presented.

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Numerical Modeling of Tsunami Wave Propagation

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Tsunamis are colossal water waves generated as a result of an abrupt displacement of substantial amount of water. Such waves often travel in open ocean over vast areas transferring huge amounts of energy. On impact with the shores due to the decreasing depth of the ocean floor their amplitudes suddenly become much higher. Thus, they represent a natural hazard that poses a threat to many coastal communities throughout the world. Warning systems are dependent upon the accuracy of mathematical models that give insight into the wave dynamics and make possible the simulation of the three stages of tsunami formation, propagation, and run-up. Essentially, it is in the propagation stage that our current interest lies. An important characteristic of the tsunami wave in this stage is that its height is relatively small and its wavelength is typically much greater than the depth of the water it travels through. As a result, the tsunami wave generally complies with the shallow water wave theory, Casulli[2]. The vertical components of acceleration and velocity of the water particles do not affect the pressure, which is hydrostatic, Gisler [1]. Let us give the the nonlinear two-dimensional conservative form of the shallow water equations in the Cartesian coordinate system (x, y). The equations include a continuity equation, momentum equations in the x and y directions and the motion of the water is given as a function of time t, Stoker [3].

$$\frac{\partial \vec{U}}{\partial t} + \frac{\partial \vec{F}}{\partial x} + \frac{\partial \vec{G}}{\partial y} = \vec{S} \tag{1}$$

Here $\vec{U}$ is the vector of conserved variables, while $\vec{F}$ and $\vec{G}$ are flux vectors. $\vec{S}$ represents a source term that could include the Coriolis force and a bottom friction parameter. The above are vectors of the flow depth h, which is the height of the water perturbation above the normal sea level, and the flow velocity (u, v):

$$\vec{U} = \begin{pmatrix} h \\ hu \\ hv \end{pmatrix}, \vec{F} = \begin{pmatrix} hu \\ hu^2 + \frac{1}{2}gh^2 \\ huv \end{pmatrix}, \vec{G} = \begin{pmatrix} hv \\ huv \\ hv^2 + \frac{1}{2}gh^2 \end{pmatrix}, \vec{S} = \begin{pmatrix} 0 \\ gh\frac{\partial H}{\partial x} - \frac{\tau_x}{\rho} \\ gh\frac{\partial H}{\partial y} - \frac{\tau_y}{\rho} \end{pmatrix} \tag{2}$$

(τ_x, τ_y) is the bottom shear stress, g is the acceleration due to gravity, ρ is the density of water and H is the water depth under unperturbed conditions.

Fairly advantageous numerical methods such as the finite volume method, Allen [4], have not been extensively explored when it comes to the solution of the aforementioned system of equations. This makes the attempt of modeling tsunamis using this particular method relevant. Our intention, is to produce a computer code for the simulation starting with a one-dimensional dam-break problem for which there exists an exact solution and after that extending it to a two-dimensional version. A number of recent studies have focused on the Indian Ocean tsunamis from 2004. In view of the above, the goal of our work will be to test the results of the numerical model we intend to develop against results from previous models based on simulations of the tsunami waves that occurred in the Indian Ocean and, in addition, to simulate tsunami waves initiated by an earthquake on the same fault-line but of higher magnitude that could eventually result in a bigger amount of displaced water.

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Friction controlled bending solitons as folding pathway toward colloidal clusters

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We study the conformational transition of an ensemble of magnetic particles from a linear chain to a compact cluster when subjected to an external magnetic field modulation. We show that the transient dynamics induced by switching the field from static to rotating is governed by the relative friction of adjacent particles in the chain. Solid particles show bending solitons counter-propagating along the chain while buckling of the chain is the mechanism preferred by ferrofluid droplets. By combining real-space experiments with numerical simulations we unveil the underlying mechanism of folding pathways in driven colloidal system.

Vibration analysis of a turbocharger model

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Vibrations are a major problem in many mechanical systems, especially within the field of machinery dynamics e.g. heat exchangers, compressors, pumps and turbines (in general). The complexity of these problems are often induced by nonlinear contact forces such as fluid-struture interactions. In these systems it is crucial to exhibit the necessary qualitative and quantitative knowledge about the vibrations, e.g. resonance or hysteresis effects that may lead to large amplitudes in order to avoid complications or as a worst case scenario uncontrollable complete failures with destruction of machinery as a consequence.
Driven rotating turbomachinery is particularly vulnerable to vibrations and often failures are more critical because these systems contain huge amounts of kinetic energy.
This motivates the need for predictions such that design changes e.g. geometry, material properties (internal as well as external) etc. can be chosen on a proper basis in order to prevent failures.

A different vibration analysis of a turbocharger model is proposed. The method is able to extract how the maximal vibration amplitude depends on model parameters. It is basically a predictor-corrector with an optimization step — the method is dependent on periodicity, a constraint which is necessary because the method is based on the continuation of periodic orbits. (This does not necessarily exclude the lowest degree of quasiperiodic responses.)
In the current example, the method will be applied to a finite element model of a turbocharger which has a quasiperiodic response. The sole nonlinearity of the model comes from fluid-structure interactions in the oil-film bearings.

The results will show how the maximal amplitude of the turbocharger depends on the oil viscosity of the bearings.
The method is applicable to any system which exhibits periodic/low-dimensional behaviour.

Furthermore dimension reduction approaches to analyse the low-dimensional macroscopic (e.g. amplitude) behaviour of a large finite element model are discussed.

The Impact of the Glial K^+ Spatial Buffering on the Nernst Membrane Potentials

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Maintenance of normal brain function is critically dependent on the regulation of potassium concentration in extracellular space (K^+_{ex}). Small variations of K^+_{ex} lead to changes in the membrane potential of neurons and to alterations in neuronal ring thresholds and the release of neurotransmitters ([1, 3]). It was suggested by Kufflers group ([2]) that when a local increase of K^+_{ex} occurs in neural tissue, astrocytes take up these ions, distribute them via gap junction-coupled cell syncytium, and extrude the ions at sites in which extracellular potassium concentration is low. This process, termed K^+ spatial buffering, is essential for the stabilization of the extracellular ion homeostasis and the modulation of neuronal membrane potentials.

The membrane resting potential of neurons, as an electrochemical equilibrium, is commonly given by the steady state solution of the Nernst-Planck equation:

$$\partial_t c(x,t) = \partial_x \left(-D(\partial_x c(x,t) + \frac{zF}{RT} c(x,t) \partial_x \phi(x,t)) \right). \tag{1}$$

In order to investigate the role of the glial K^+ spatial buffering on the membrane potentials, the Nernst-Planck equation has been modied by adding a glial feed back function $f(c) = a - b\partial_x c$ to its right hand side.

Mathematical analysis of the steady state of this equation suggests that K^+ spatial buffering has a signicant impact on the basic membrane properties, especially the neuronal resting membrane potentials.

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Periodic orbits in open systems

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Open dynamical systems are those with one or more holes through which trajectories can escape. The initial condition is typically distributed uniformly within the system (escape problem) or at a hole (transport problem), with the aim of calculating the survival or transmission/reflection (respectively) probabilities as a function of time. Periodic orbits have long been used successfully in both classical and quantum contexts to calculate properties of strongly chaotic open systems, for example the "escape rate", which is the rate of exponential decay of the survival probability.

I will present very recent progress in several directions, obtaining "local" escape rates for strongly chaotic systems with small holes, also detailed expressions for the leading escape and transport behaviour of integrable, intermittent and mixed dynamics. Properties of periodic orbits, both entering and avoiding the hole(s), feature strongly in this work. There are unexpected connections with number theory, as well as applications for transport of atoms, electrons, light and sound in various cavities. I will discuss both previously published work [1, 2, 3] and new results on both intermittent and mixed systems demonstrating connections with a non-standard Diophantine approximation problem and strongly asymmetric transport properties.

[1] Bunimovich, L. A. & Dettmann, C. P. [2005], "Open circular billiards and the Riemann hypothesis," *Phys. Rev. Lett.* **94** 100201.

[2] Bunimovich, L. A. & Dettmann, C. P. [2007], "Peeping at chaos: Nondestructive monitoring of chaotic systems by measuring long-time escape rates," *EPL*, **80** 40001.

[3] Dettmann, C. P. & Georgiou, O. [2009], "Survival probability for the stadium billiard" *Physica D* **238**, 2395-2403.

How well can one resolve the phase space of a chaotic map?

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All physical systems are affected by some noise, which limits the resolution that can be attained in partitioning their phase space. For chaotic systems this resolution depends on the interplay of the local stretching/contracting effects of the deterministic dynamics and the smearing due to noise. Our goal is to determine the 'finest attainable' partition for a given chaotic dynamical system and a given weak additive white noise. The strategy is to find local solutions to the Fokker-Planck equation all over the phase space, and use their overlaps as the criterion for an optimal partition. The method is validated by estimating time averages of dynamical observables such as escape rates and Lyapunov exponents of unimodal maps.

Destruction of transport barriers in geophysical jets with Rossby waves

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A dynamical model of a geophysical zonal jet current in the ocean or the atmosphere with two propagating Rossby waves is considered. Attention is concentrated on destruction of a central barrier preventing cross-jet transport. We develop a method for computing a central invariant curve which is an indicator of existence of the barrier. Breakdown of this curve under a variation of the Rossby wave amplitudes and onset of chaotic cross-jet transport happen due to specific resonances producing stochastic layers in the central jet. The main result is that there are resonances breaking the transport barrier at unexpectedly small values of the amplitudes. This phenomenon may have serious impact on mixing and transport in the ocean and the atmosphere. The effect can be observed in laboratory experiments with azimuthal jets and Rossby waves in rotating tanks under specific values of the wave numbers that are predicted in the theory.

[1] M.Yu. Uleysky, M.V. Budyansky, S.V. Prants, *Phys. Rev. E* **81**, 017202 (2010).

Plenary Talk I8 (Auditorium)

Dark Solitons and Vortices in Bose-Einstein Condensates: Oscillations and Precessions, Dynamics and Interactions in an Ultracold World

I8
17:30

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In this talk, we will present an overview of recent theoretical, numerical and experimental work concerning the static, stability, bifurcation and dynamic properties of coherent structures that can emerge in one- and higher-dimensional settings within Bose-Einstein condensates at the coldest temperatures in the universe (i.e., at the nanoKelvin scale). We will discuss how this ultracold quantum mechanical setting can be approximated at a mean-field level by a deterministic PDE of the nonlinear Schrodinger type and what the fundamental nonlinear waves of the latter are. Then, we will try to go to a further layer of simplified description via nonlinear ODEs encompassing the dynamics of the waves within the traps that confine them, and the interactions between them. Finally, we will attempt to compare the analytical and numerical implementation of these reduced descriptions to recent experimental results and speculate towards a number of interesting future directions within this field.

Friday, 10th September 2010

Plenary Talk I9 (Auditorium)

The Multiscale Dynamics of Sprites and Lightning

I9
9:00

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Since 1990 tens of kilometers large Transient Luminous Events (sprites, elves, halo's, jets etc.) were discovered high above thunderclouds. In 1994 a satellite found that gamma-ray flashes come not only from the cosmos, but also from earth, more precisely from thunderclouds; they are now called Terrestrial Gamma-Ray Flashes. Observation campaigns for Transient Luminous Events and Terrestrial Gamma-Ray Flashes are now conducted from ground, from observatories on high mountains, from aircraft and from space.

I will review phenomena and the state of theory: First, linear phenomena like elves, halo's and run-away electron avalanches have to be distinguished from strongly nonlinear phenomena like sprites, jets and lightning strokes. I will then focus on sprites; overall they have a characteristic "fractal" tree structure, each branch of the tree has a particular inner layered structure, and in some cases in the tips of the branches, the stochastic particle nature of free electrons has to be taken into account. This multiscale dynamics can be elucidated only through combining numerical and analytical tools: Simulations reveal dynamic structures, and analysis then eliminates the structures on the smallest scale and formulates new models on a larger scale. I will illustrate this interplay of analysis and simulations on three examples in "thunderstorm" physics: density approximations for particle dynamics, moving boundary approximations for ionization fronts and electrodynamic characterizations of complete discharge branches.

Consecutively, for halo's and sprite discharges, theory now begins to span the full range from microscopic particle collisions up to discharge phenomena of tens of kilometers in size. This is the first time that lightning related phenomena are all the way tracked down to their physical basis.

Scientific and popular literature can be found on http://www.cwi.nl/∼ebert.

Plenary Talk I10 (Auditorium)

Effects of network structure in simple models of gene regulation

I10
10:30

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Boolean networks have been proposed as potentially useful models for genetic control. An important aspect of these networks is the stability of their dynamics in response to small perturbations. Previous approaches to stability have assumed uncorrelated random network structure. Real gene networks typically have nontrivial topology significantly different from the random network paradigm. To address such situations, we present a general method for determining the stability of large Boolean networks of any specified network topology and predicting their steady-state behavior in response to small perturbations. Additionally, we generalize to the case where individual genes have a distribution of expression biases, and we consider a nonsynchronous update, as well as extension of our method to non-Boolean models in which there are more than two possible gene states. We find that stability is governed by the maximum eigenvalue of a modified adjacency matrix, and we test this result by comparison with numerical simulations. We also discuss the possible application of our work to experimentally inferred gene networks, and propose that a dynamical instability in the gene regulatory network may be a causal mechanism associated with some cancers.