QUANTITATIVE THERMOELASTIC STRESS ANALYSIS OF LAMINATED COMPOSITE COMPONENTS

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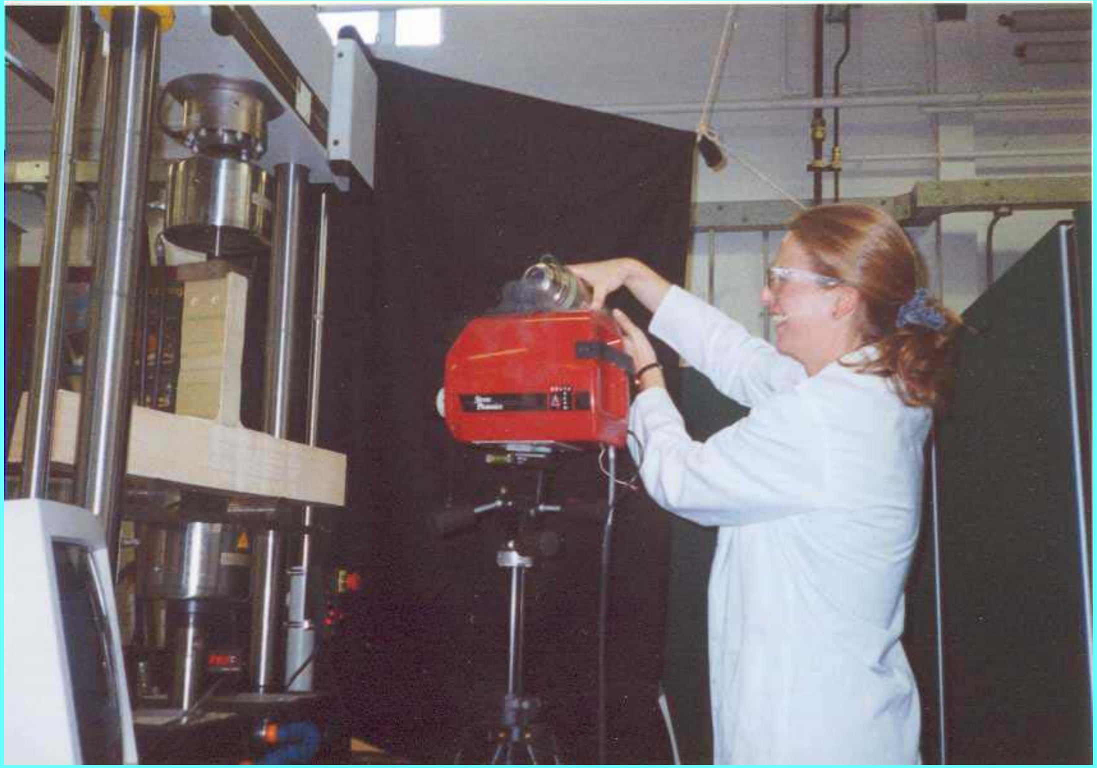

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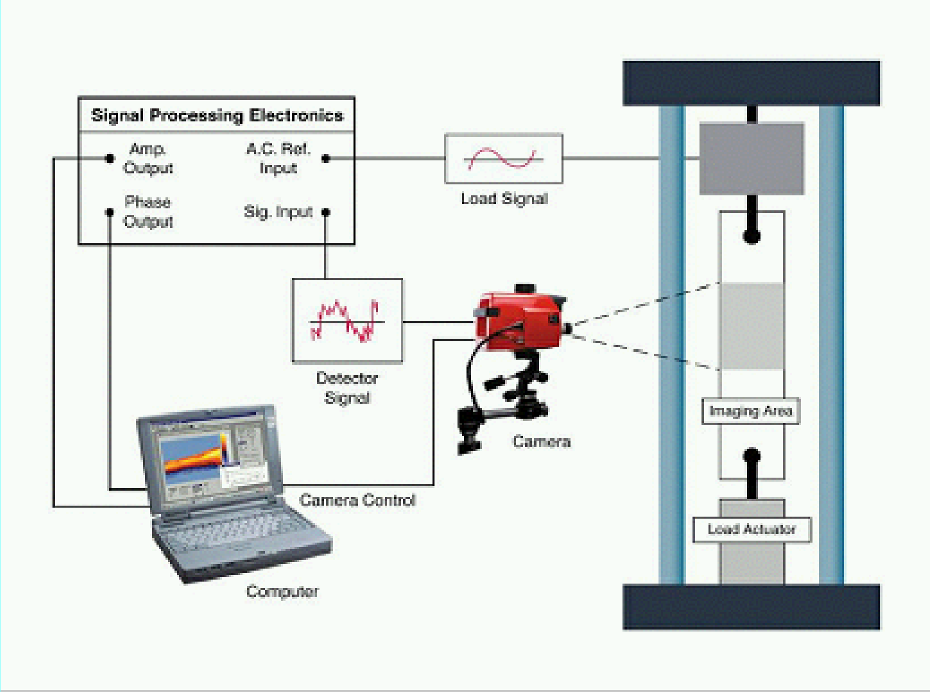
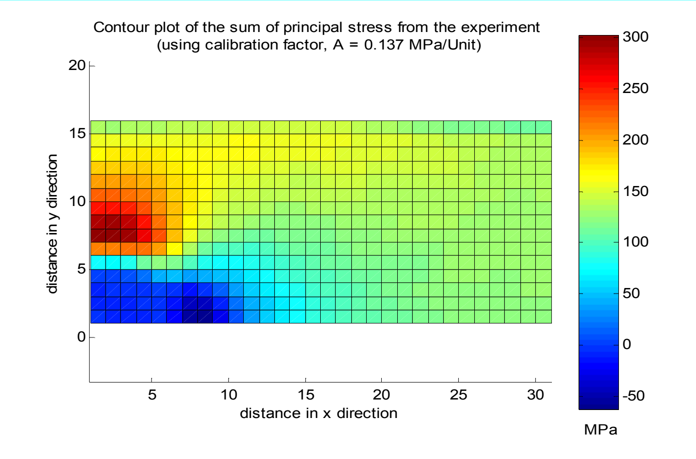
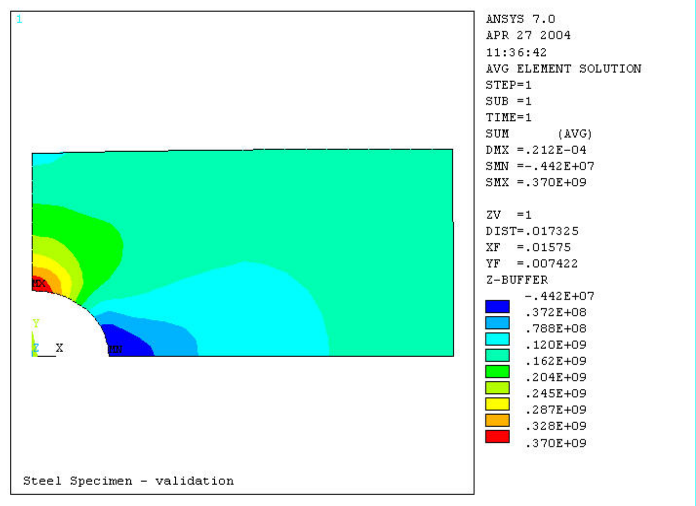
Summary

- TSA-brief introduction
- General thermoelastic theory for orthotropic materials
- Motivation
- Notation for a general laminate
- Revised theory for a general laminate
- Test specimens and material properties
- Calibration Results
- Next steps

Thermoelastic Stress Analysis







$$\Delta(\sigma_1 + \sigma_2) = AS$$

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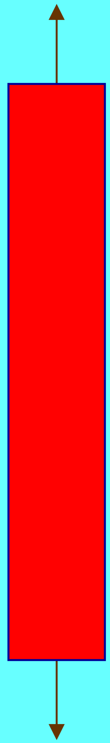
Thermoelastic effect



$$\Delta T = -\frac{T}{\rho C_{\varepsilon}} \sum_{i=1,2,3} \frac{\partial \sigma_{ij}}{\partial T} \varepsilon_{ij} + \frac{Q}{\rho C_{\varepsilon}}$$

$$\Delta T = -\frac{\alpha}{\rho C_{p}} T \sum_{i=1,2,3} \sigma_{ii}$$

$$\boxed{\Delta T = -\frac{\alpha}{\rho C_{p}} T \Delta(\sigma_1 + \sigma_2)}$$

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Theory for specially orthotropic material

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_6 \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_6 \end{bmatrix}$$

Thermoelastic equations:

Stress
$$\sigma_i = \sum_{i=1}^{6} Q_{ij}(\varepsilon_j + \alpha_j \Delta T)$$

Strain
$$\varepsilon_i = \sum_{j=1}^{6} S_{ij}\varepsilon_j + \alpha_i \Delta T$$

$$\Delta T = -\frac{T}{\rho C_\varepsilon}\sum_{i=1}^{6}\frac{\partial \sigma_i}{\partial T}\varepsilon_i + \frac{Q}{\rho C_\varepsilon}$$

$$\Delta T = \frac{T}{\rho C_\varepsilon}\{(-Q_{11}\alpha_1 - Q_{12}\alpha_2)(S_{11}\sigma_1 + S_{12}\sigma_2 + \alpha_2\Delta T) +$$

$$(-Q_{12}\alpha_1 - Q_{22}\alpha_2)(S_{21}\sigma_1 + S_{22}\sigma_2 + \alpha_2\Delta T)\} + \frac{Q}{\rho C_\varepsilon}$$

Introducing relationships between specific heats

$$C_p - C_\varepsilon = -\frac{T}{\rho}\sum_{i=1}^{6}\frac{\partial \sigma_i}{\partial T}\frac{\partial \varepsilon_i}{\partial T}$$

$$\Delta T = -\frac{T}{\rho C_p}(\alpha_1\sigma_1 + \alpha_2\sigma_2) + \frac{Q}{\rho C_p}$$

$$S = \frac{(\alpha_1\sigma_1 + \alpha_2\sigma_2)}{A^*}$$

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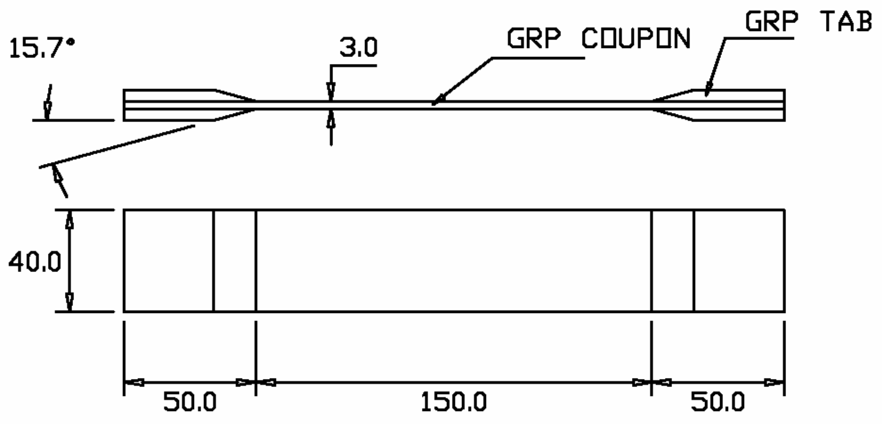
Test specimens



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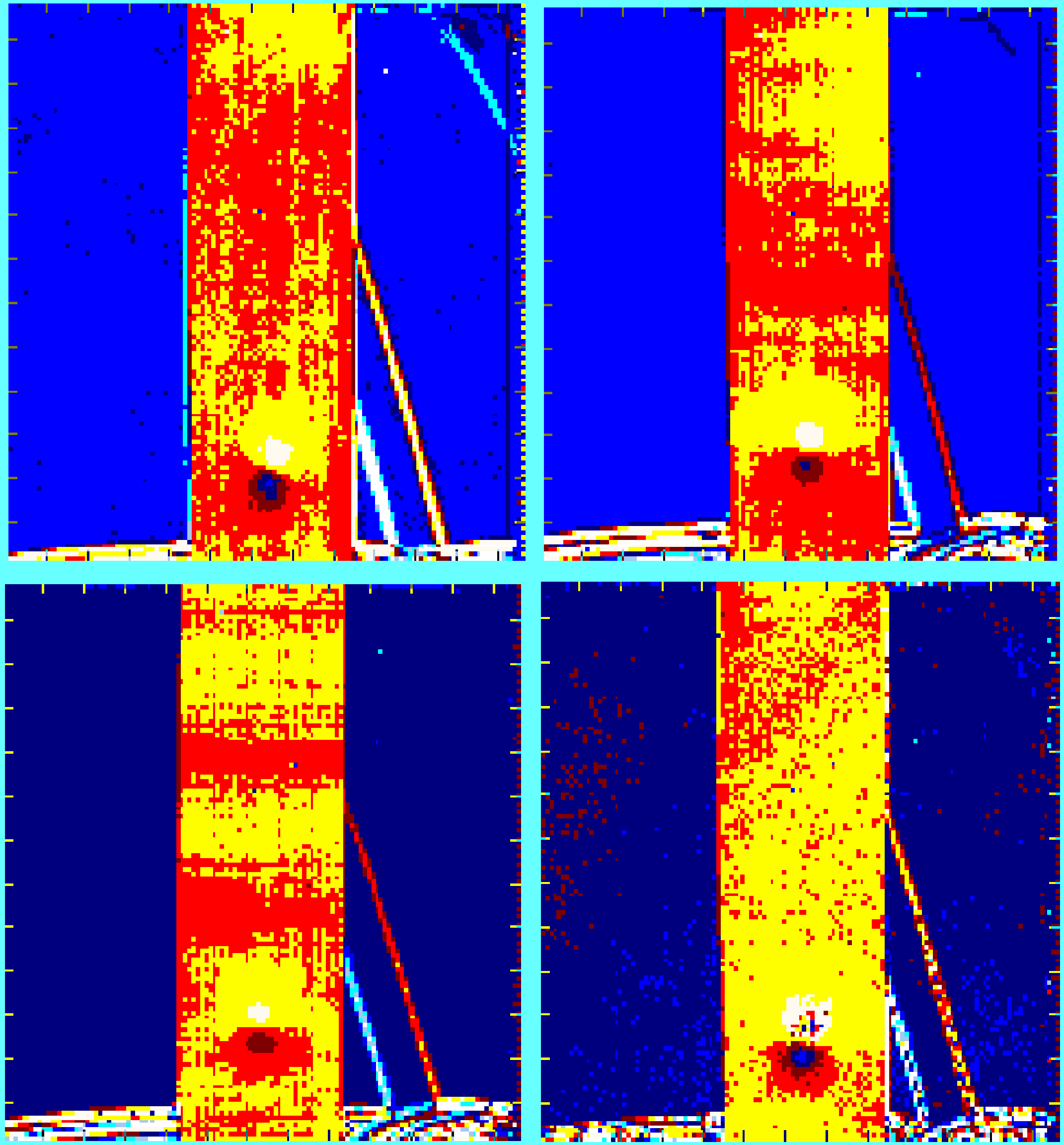
Results for a constant applied stress level



$[0]_{13S}$
Termed UD

$[[0/90]_6[0]\ [0/90]_6]_s$
Termed 0/90

$[[90/0]_6[0]\ [90/0]_6]_s$
Termed 90/0

$[[+/-\ 45]_{13s}$
Termed +/- 45

$[[0]_6[90/0/90]\ [0]_6]_s$
Termed mixed

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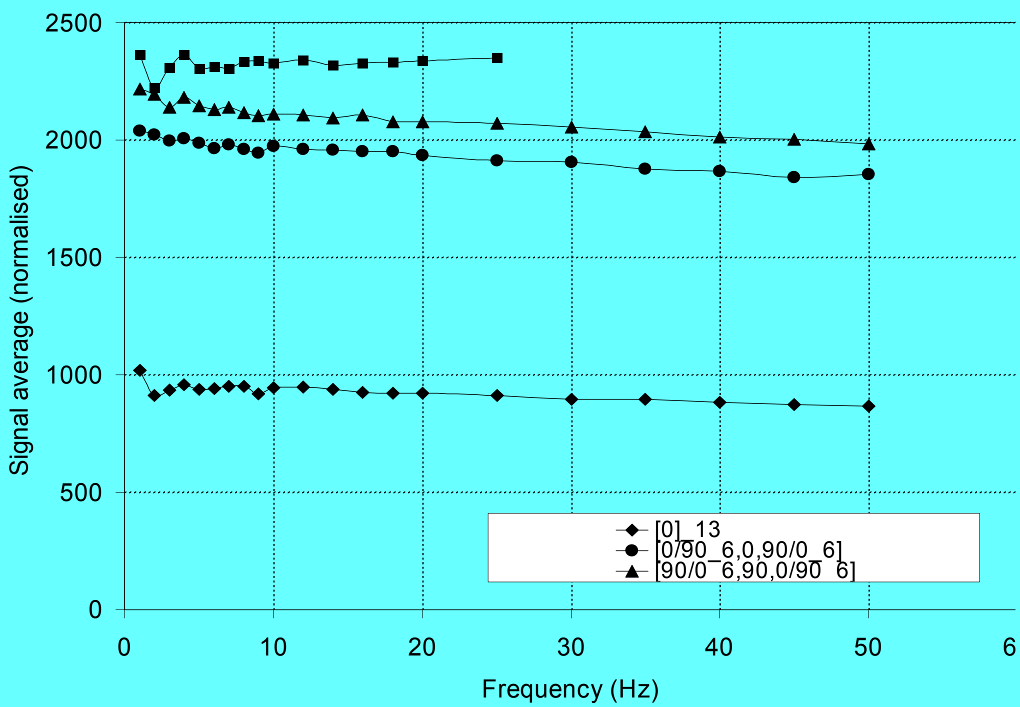
Thermoelastic response



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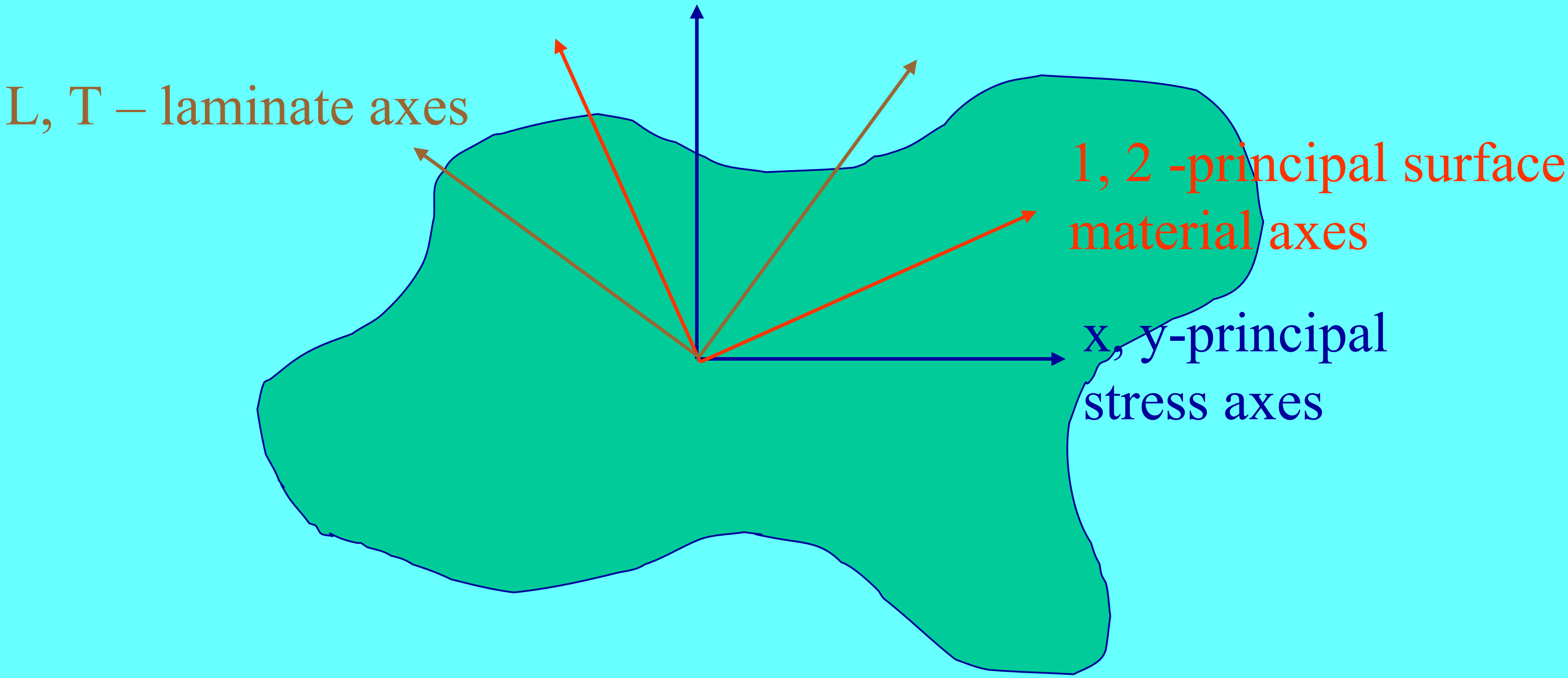
$$S = \frac{(\alpha_1\sigma_1 + \alpha_2\sigma_2)}{A^*}$$

Reference axes in a composite material



$$S = \frac{(\alpha_1 \sigma_1 + \alpha_2 \sigma_2)}{A^*}$$

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Axes in the specimens

- UD (1, 2), (x, y) and (L, T) are coincident
- 0/90 as above
- Mixed as 0/90
- 90/0 as above but the (1, 2) are rotated by 90^o
- +/- 45 as above but the (1, 2) are rotated by 90^o
- Mixed as 0/90

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Thermoelastic response in terms of stresses in the direction of the laminate axes:

$$S = \frac{(\alpha_x \sigma_x + \alpha_y \sigma_y + \alpha_{xy} \sigma_{xy})}{A^*}$$

Thermoelastic response in terms of the strain in the principal surface material directions:

$$S = \frac{\{(Q_{11}\alpha_1 + Q_{12}\alpha_2)\varepsilon_1 + (Q_{12}\alpha_1 + Q_{22}\alpha_2)\varepsilon_2\}}{A^*}$$

Thermoelastic response in terms of the strain in the direction of the laminate axes:

$$S = \frac{1}{A^*}[\{(Q_{11}\alpha_1 + Q_{12}\alpha_2)\cos^2\theta + (Q_{12}\alpha_1 + Q_{22}\alpha_2)\sin^2\theta\}\varepsilon_L + \{(Q_{11}\alpha_1 + Q_{12}\alpha_2)\sin^2\theta + (Q_{12}\alpha_1 + Q_{22}\alpha_2)\cos^2\theta\}\varepsilon_T + \{(Q_{11}\alpha_1 - Q_{12}\alpha_2)\cos\theta\sin\theta - (Q_{12}\alpha_1 + Q_{22}\alpha_2)\cos\theta\sin\theta\}\gamma_{LT}]$$

Strategy

Determine A^* for the laminates and in each case the value should be the same as

$$A^* = \frac{DRF\rho C_p}{Te}$$

This will be done by calculating A^*S using the formula given in the previous slides and estimated material property values.

Material properties

Specimen Lay-up	α_{LT} (x 10^{-6}/°C)	α_{LT} (x 10^{-6}/°C)	ν_{LT}	E_{LT} (MPa)
$[0_{13}]$	6	37	0.25	32816
$[0_5/90/0/90/0_5]$	11	30	0.20*	30237
$[(0/90)_6/0/(0/90)_6]$	19	22	0.09	23367

Specimen Lay-up	α_1 (x 10^{-6}/°C)	α_2 (x 10^{-6}/°C)	ν_{12}	E_1 (MPa)
$[0_{13}]$	6.0	37	0.25	32816
$[0_5/90/0/90/0_5]$	6.0	37	0.25	32816
$[(0/90)_6/0/(0/90)_6]$	6.0	37	0.25	32816

Loading and TSA readings

Specimen Lay-up	Load (kN)	Stress (MPa)	Disp. Amp. (mm)	S_{LOAD}	S_{STRAIN}
$[0_{13}]$	8±4	57±28	0.22	585	845
$[0_5/90/0/90/0_5]$	8±4	57±28	0.22	765	999
$[(0/90)_6/0/(0/90)_6]$	8±4	57±28	0.22	1136	1136

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Calculation of A^*

$[0_{13}]$	$[0_5/90/0/90/0_5]$	$[(0/90)_6/0/(0/90)_6]$
5.86E-07	5.39E-07	4.71E-07
5.87E-07	6.82E-07	5.96E-07
5.57E-07	5.34E-07	4.71E-07
5.58E-07	6.76E-07	5.96E-07

$[0_{13}]$	$[0_5/90/0/90/0_5]$	$[(0/90)_6/0/(0/90)_6]$
5.86E-07	9.98E+08	5.87E-16
5.87E-07	9.97E+08	5.88E-16
5.57E-07	1.52E+09	3.68E-16
5.58E-07	1.51E+09	3.69E-16

Conclusions

- Calibration of thermoelastic signal is possible if all material property values are known
- The stress in the surface ply of a calibration specimen needs to be calculated accurately
- More work required on laminates that do not have a 0^{o} surface ply.