Identification of parameters in nonlinear viscoelastic, nonlinear viscoplastic material model

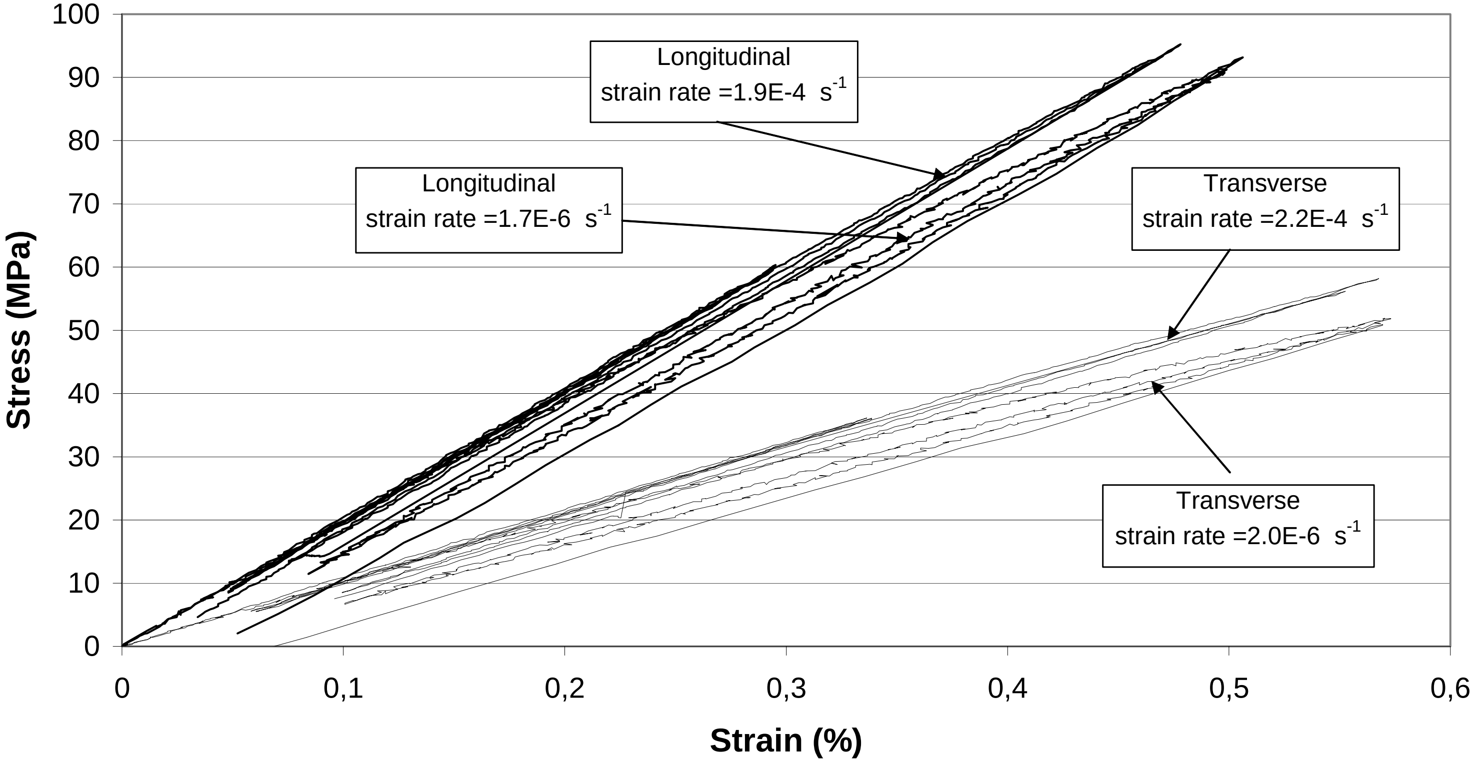
Janis Varna and Lars-Olof Nordin

Lulea University of Technology

Sweden

Wood composites:

non-linear and time-dependent materials:



Damage: is stiffness reducing?

Results from measurement of Young's modulus (GPa) before and after repeated loading to maximum stress:

Laminate	Direction	Never loaded	After 1:st loading	After 2:nd loading	After 3:rd loading
L1	L	20.6	20.6	20.7	20.7
L1	T	10.8	10.8	10.7	10.7
L3	L	20.5	20.6	20.5	20.5
L3	T	10.8	10.7	10.7	10.7

No reduction in Young's modulus

=> Micro damage excluded as reason for non-linear behaviour

Objectives

- To understand mechanics of the observed behavior
- To develop constitutive material model
- To develop methodology for experimental determination of stress dependent functions in the model

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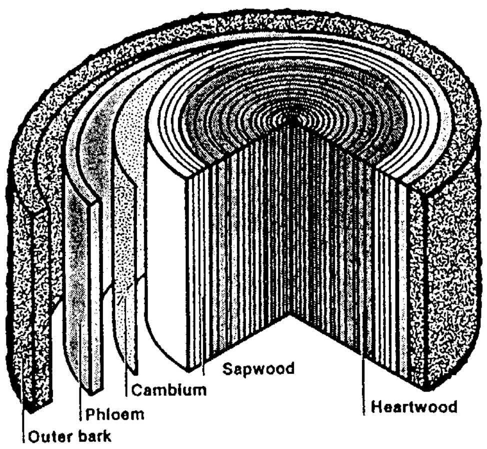
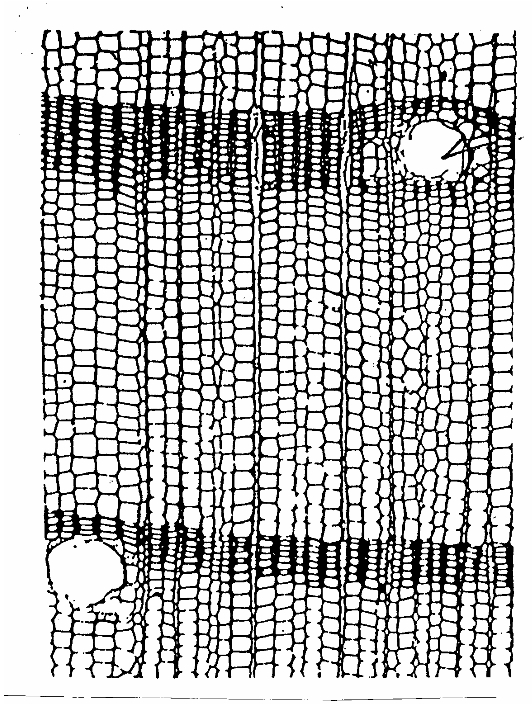
Outline of the presentation

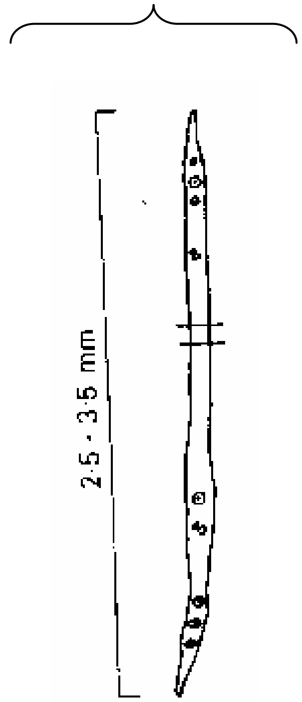
- Wood structure and paper fiber composites
- Nonlinear viscoelastic, viscoplastic material model
- Data reduction methodology in viscoelasticity
- Model of plasticity
- Conclusions

Wood





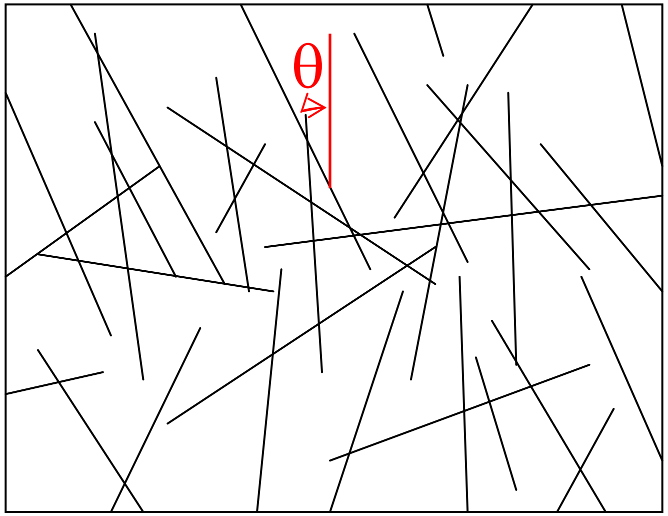
Softwood fiber

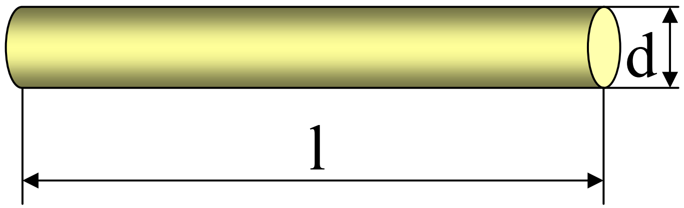


(L/d ≈ 100)

Wood composites

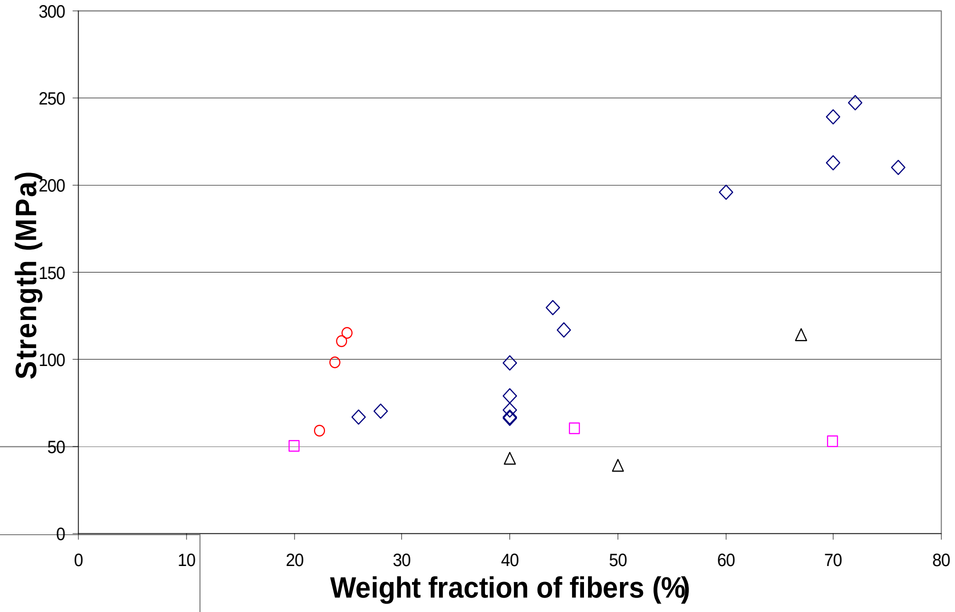
- Anisotropic fibers
  - Wood anisotropic
  - Fiber bundles or single fibers
  - Cells collapsed or tubular
- Short fiber composites- dependent on
  - Fiber orientation distribution
  - Fiber aspect ratio

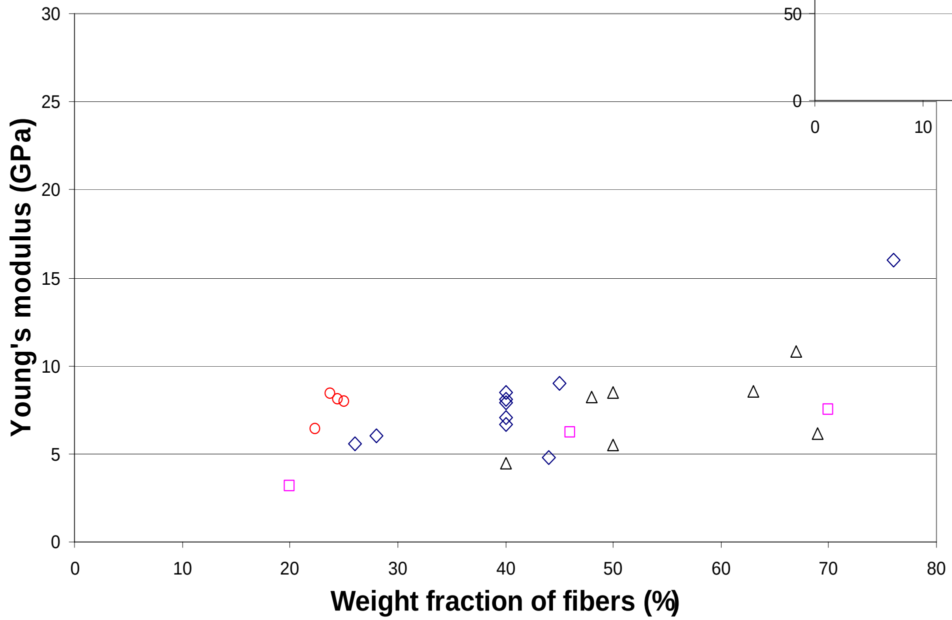




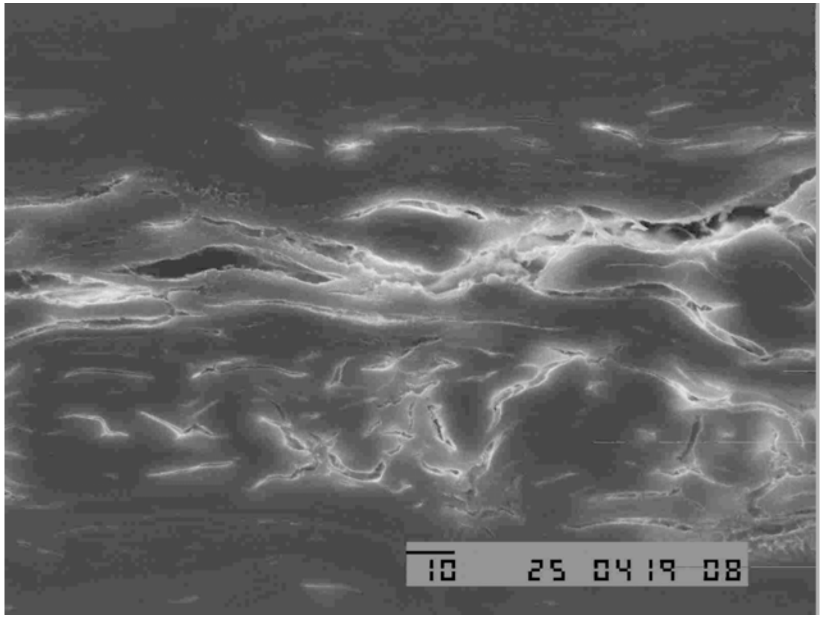
Wood composites

- Large variation in mechanical properties



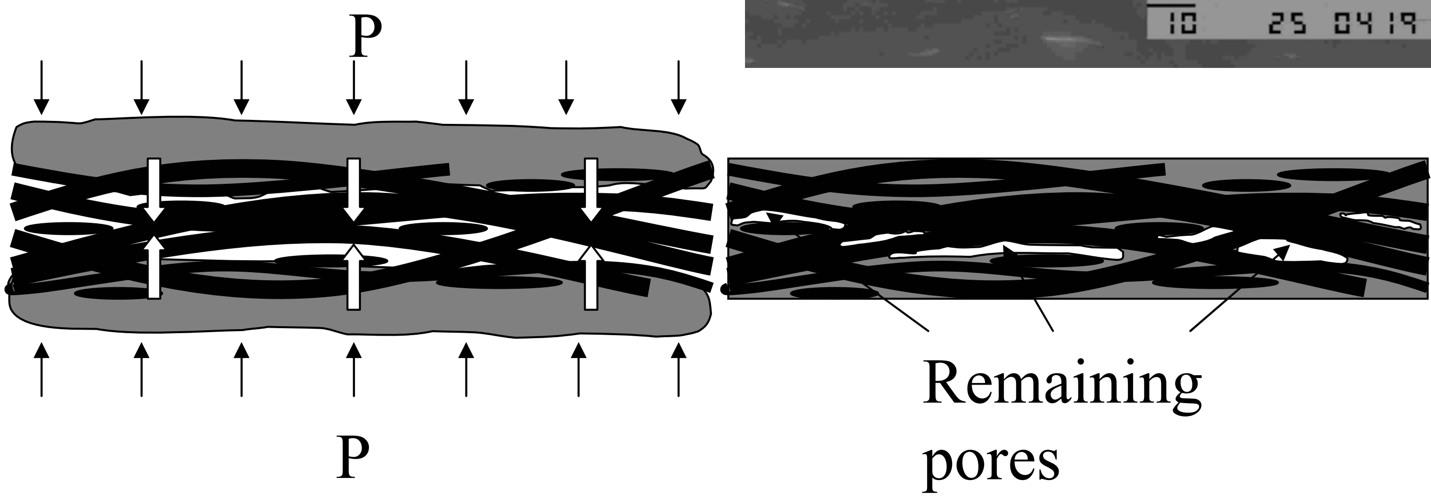


Porosity



Impregnated paper

before pressing:



Material model:

Non-linear viscoelastic and non-linear viscoplastic

constitutive law (Schapery):

$$\varepsilon_i = \varepsilon_i^{el} + b_{ij}\int_o^{\psi} \Delta S_{jk}(\psi - \psi')\frac{d}{d\psi'}(a_{42}\sigma_k)d\psi' + \varepsilon_i^{pl}(t,\sigma_k)$$

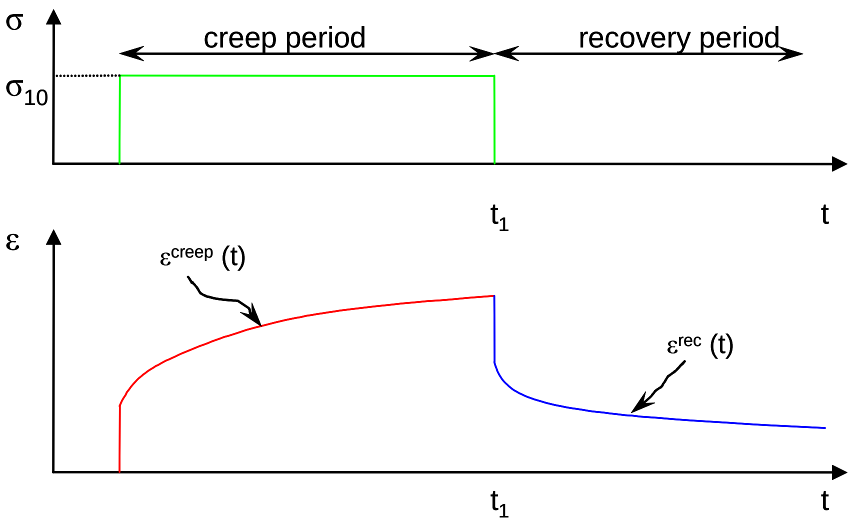
where $\psi = \int_0^t a_{21}\,dt'$, $\psi' = \int_0^{\tau} a_{21}\,dt'$ and

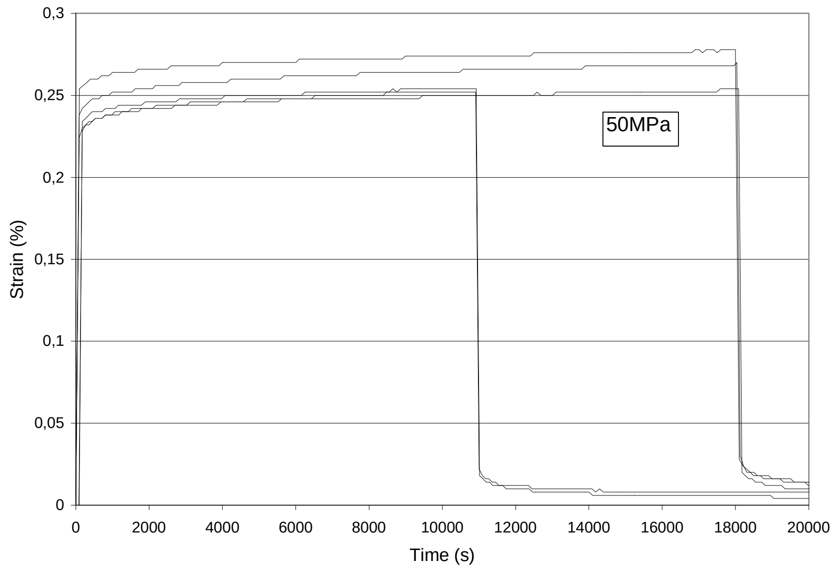
$$\Delta S_{ik}(\psi) = \sum_m C_{ik}^m \left(1 - \exp\left(-\frac{\psi}{\tau_m}\right)\right)$$

Functions to define: ε_i^{el} C_{ik}^m b_{ij} a_{21} a_{42} $\varepsilon_i^{pl}(t,\sigma_k)$

Methodology: viscoelasticity

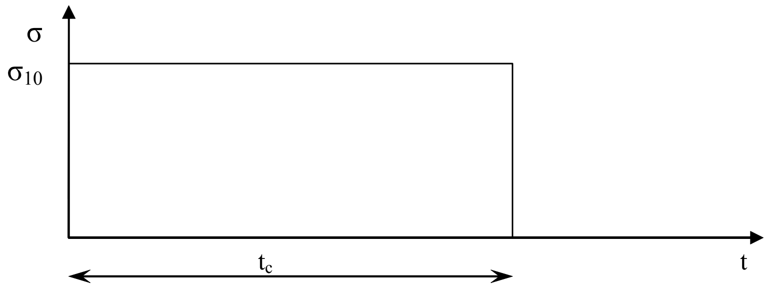
Creep - recovery test
at several stress levels



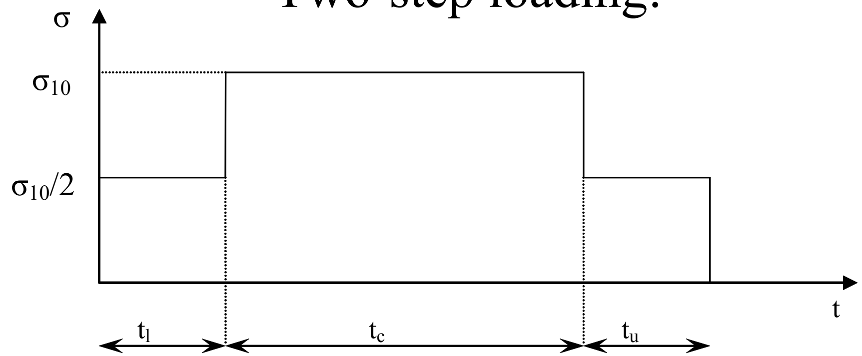


Problem: viscoelastic deformation during time of loading and unloading leads to *errors* in determined material functions

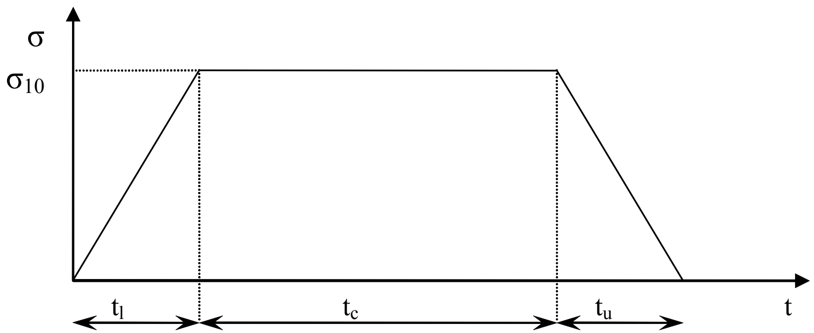
Heaviside step loading:

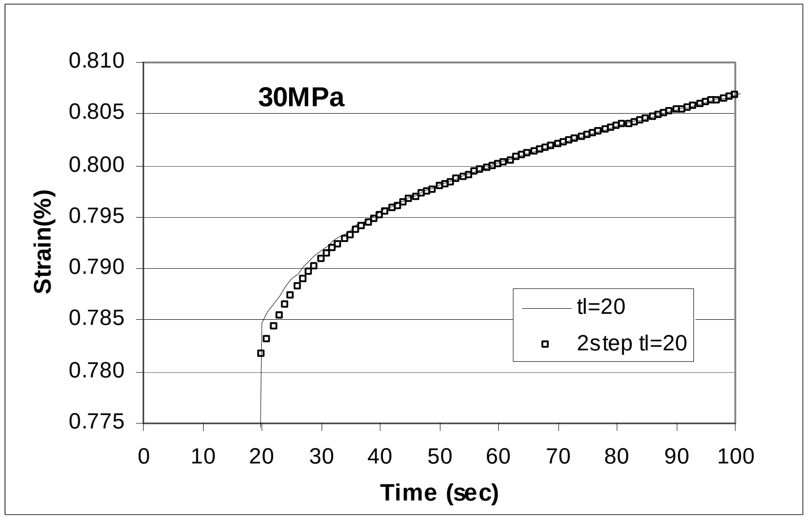


Two-step loading:



Real loading requires certain time for application and removal of load:



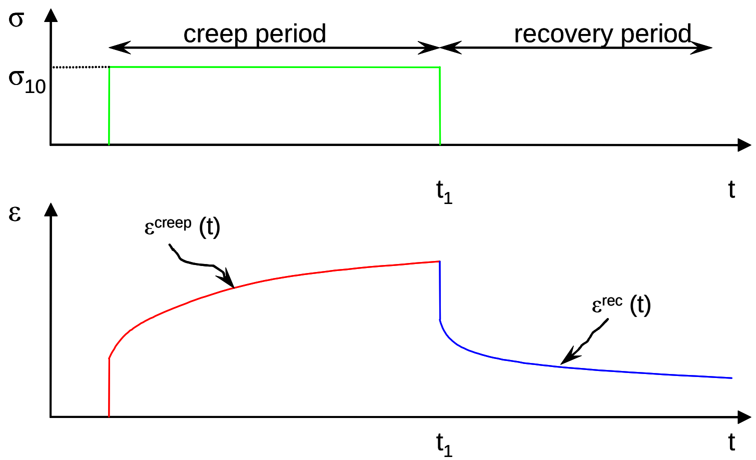


Viscoelasticity: identification

Creep - recovery test

$$\varepsilon = \varepsilon^{el} + b \int_{o}^{\psi} \Delta S(\psi - \psi') \frac{d}{d\psi'}\left(a_{42}\sigma\right) d\psi'$$

$$\Delta S(\psi) = \sum_m C^m \left(1 - \exp\left(-\frac{\psi}{\tau_m}\right)\right)$$



Parameter identification –2- step model

1. Least square fit to recovery curve

$$\varepsilon^{rec} = \sigma \sum_m A^m \exp\left(-\frac{t - t_1}{\tau_m}\right) \qquad A^m = C^m \phi_m$$

Known function: $\phi_m = \phi_m\left(a_{42}(\sigma), a_{21}(\sigma), a_{42}(\sigma/2), a_{21}(\sigma/2)\right)$

$$C^m \qquad f_1\left(a_{42}(\sigma), a_{21}(\sigma)\right) = 0$$

Comparison with ”true” functions

One-step model

Table 3. C_{11}^{m} values determined from tests with varying rates of load increase.

τ_m (s)	Input (1/Pa)	$t_l = t_u = 0$ (1/Pa)	$t_l = t_u = 10s$ (1/Pa)	$t_l = t_u = 20s$ (1/Pa)	$t_l = t_u = 30s$ (1/Pa)
3	8.145E-13	8.156E-13	3.293E-13	1.706E-13	1.143E-13
10	5.379E-12	5.378E-12	3.757E-12	2.570E-12	1.882E-12
30	-7.458E-13	-7.449E-13	-6.547E-13	-5.624E-13	-4.869E-13
100	8.630E-12	8.630E-12	8.295E-12	7.900E-12	7.531E-12
300	1.900E-12	1.900E-12	1.875E-12	1.844E-12	1.814E-12
1000	7.429E-12	7.428E-12	7.397E-12	7.358E-12	7.324E-12
3000	-1.076E-12	-1.076E-12	-1.073E-12	-1.070E-12	-1.070E-12
10000	1.807E-11	1.807E-11	1.802E-11	1.797E-11	1.800E-11

Two-step model

Table 8. C_{11}^{m} values determined from tests with varying rates of load increase.

τ_m (s)	Input (1/Pa)	$t_l = t_u = 0$ (1/Pa)	$t_l = t_u = 10s$ (1/Pa)	$t_l = t_u = 20s$ (1/Pa)	$t_l = t_u = 30s$ (1/Pa)
3	8.145E-13	8.156E-13	6.358E-13	3.407E-13	2.285E-13
10	5.379E-12	5.378E-12	5.493E-12	4.527E-12	3.586E-12
30	-7.458E-13	-7.449E-13	-7.629E-13	-7.433E-13	-7.119E-13
100	8.630E-12	8.630E-12	8.709E-12	8.687E-12	8.652E-12
300	1.900E-12	1.900E-12	1.906E-12	1.906E-12	1.904E-12
1000	7.429E-12	7.428E-12	7.436E-12	7.436E-12	7.436E-12
3000	-1.076E-12	-1.076E-12	-1.076E-12	-1.076E-12	-1.076E-12
10000	1.807E-11	1.807E-11	1.807E-11	1.807E-11	1.807E-11

Viscoelasticity: identification (cont.)

Requirements

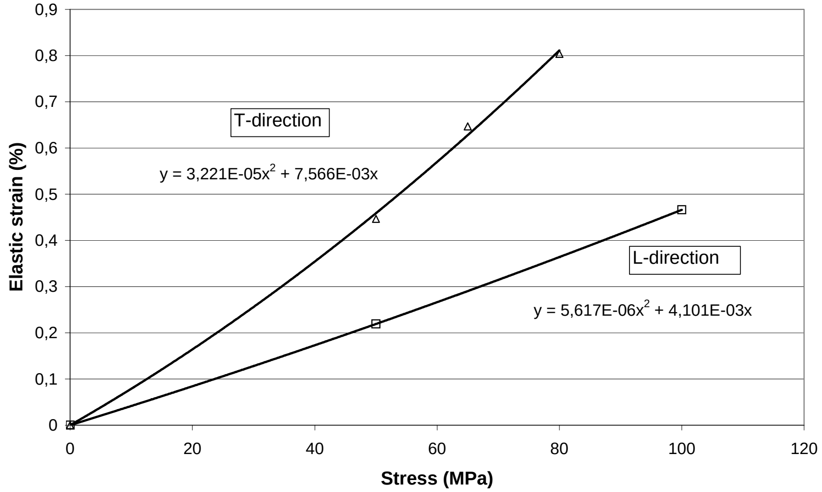
- The simulated creep strain at the final time instant $t = t_1$ equals to the experimental.
- The shape of the simulated creep curve is adjusted to go "as close as possible" to the experimental points (least squares)

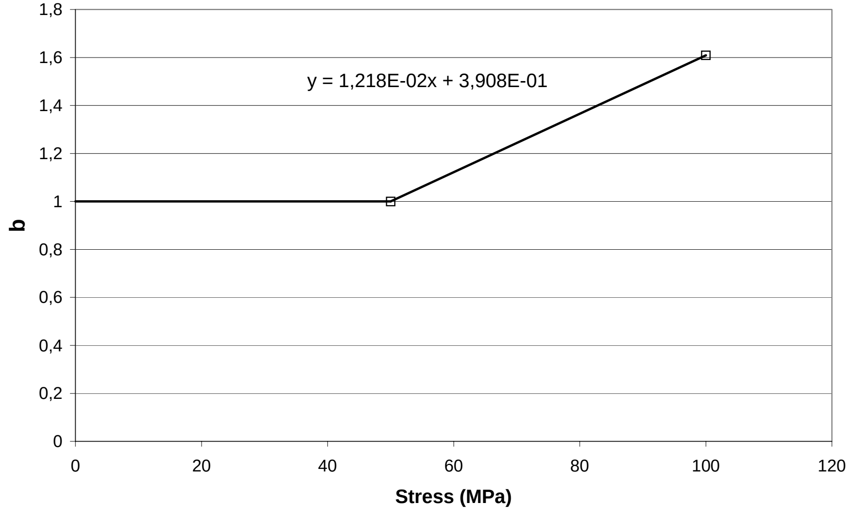
$$\varepsilon_{\exp}^{creep}(t_1) = \varepsilon_{simul}^{creep}(t_1)$$

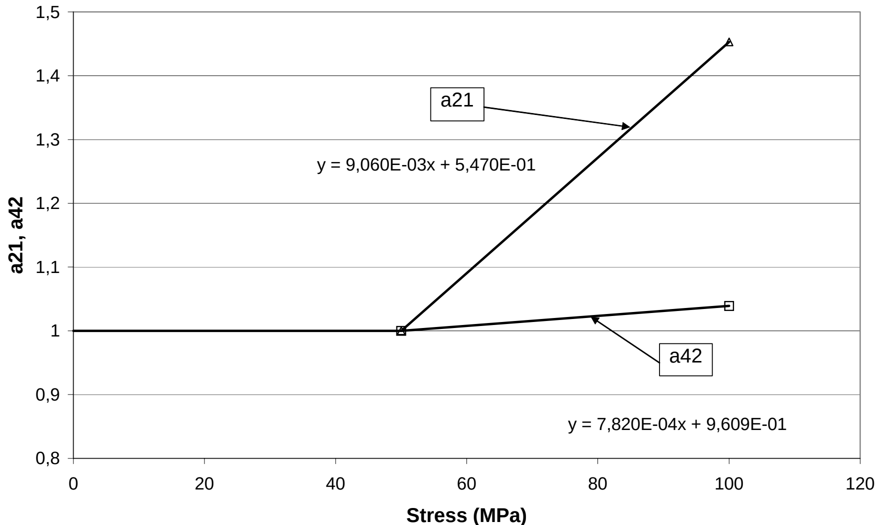
$$f_2\left(\varepsilon^{el}, b, a_{42}(\sigma), a_{21}(\sigma)\right) = 0$$

$$f_3\left(\varepsilon^{el}, b, a_{42}(\sigma), a_{21}(\sigma)\right) = 0$$

Functions ε^{el}, b, a_{21} and a_{42}

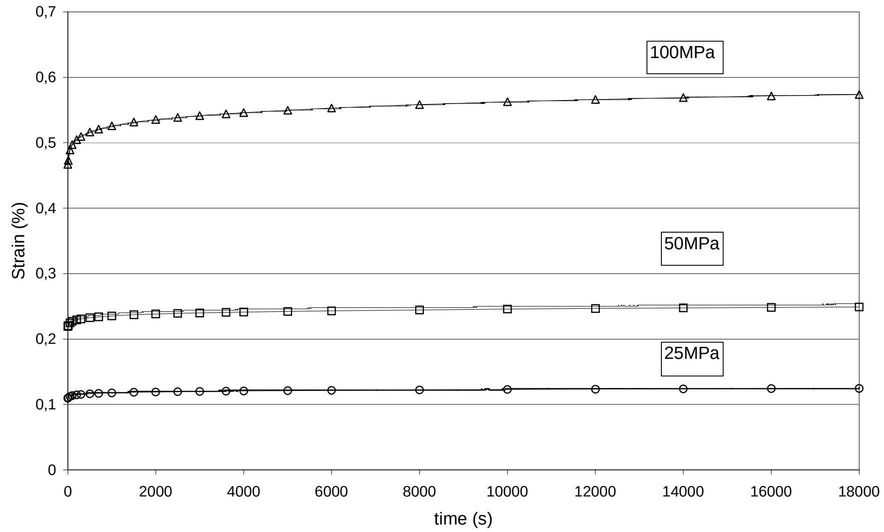




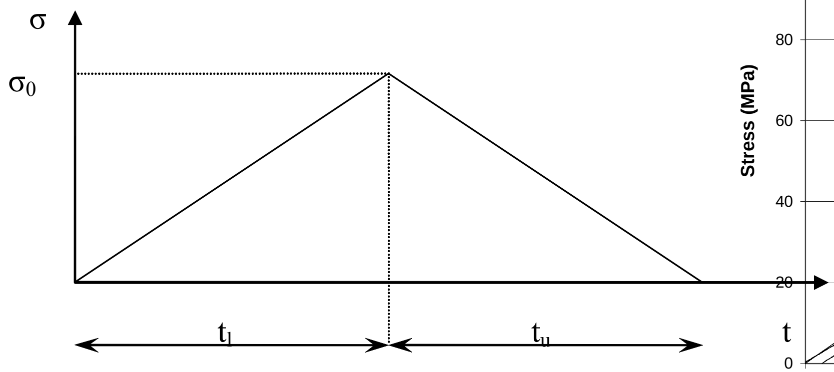


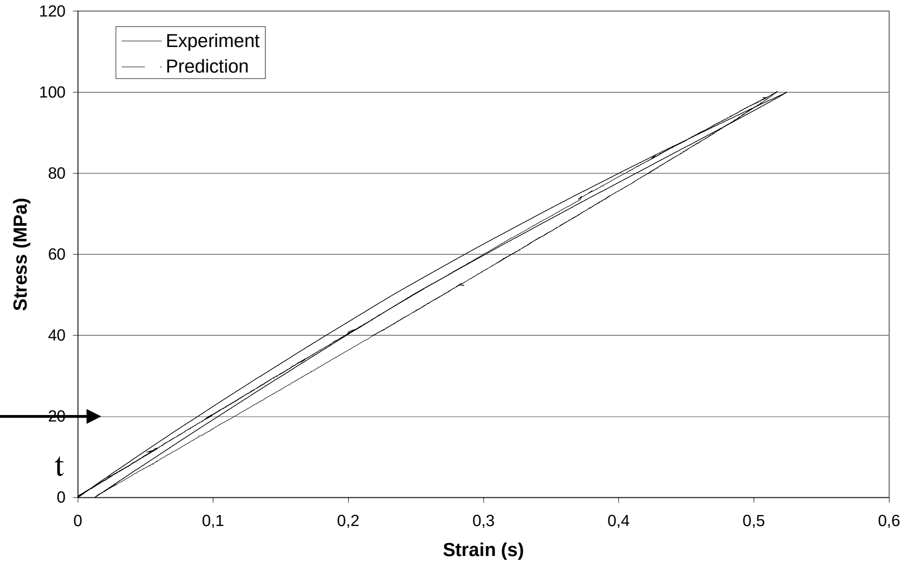
Validation

Validation of data reduction, predicted and experimental creep curve:



Simulation of experimental slow (0.035MPa/s) load ramp





Simplified models

•Model 1 $a_{42} = 1$ → infinite

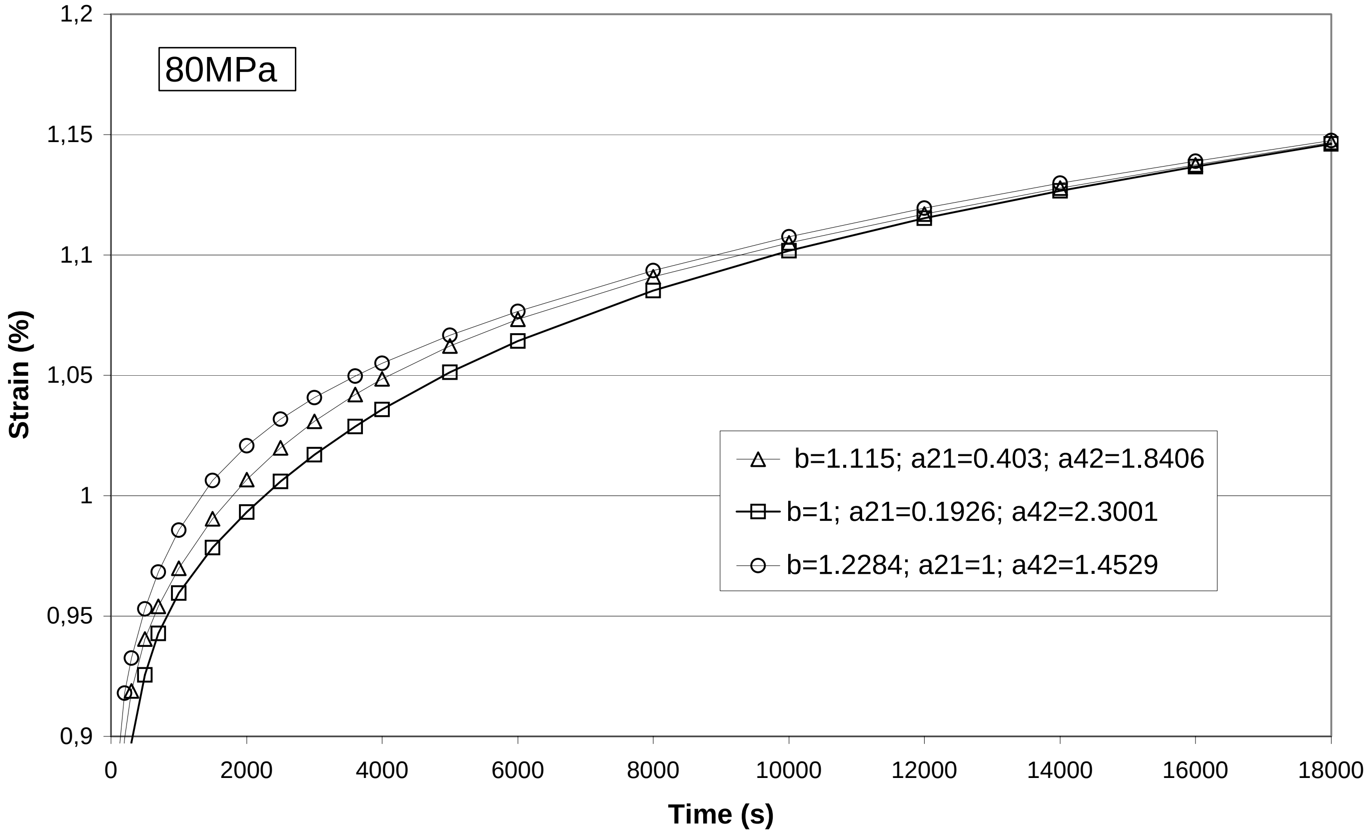
•Model 2 b=1

a_{21}

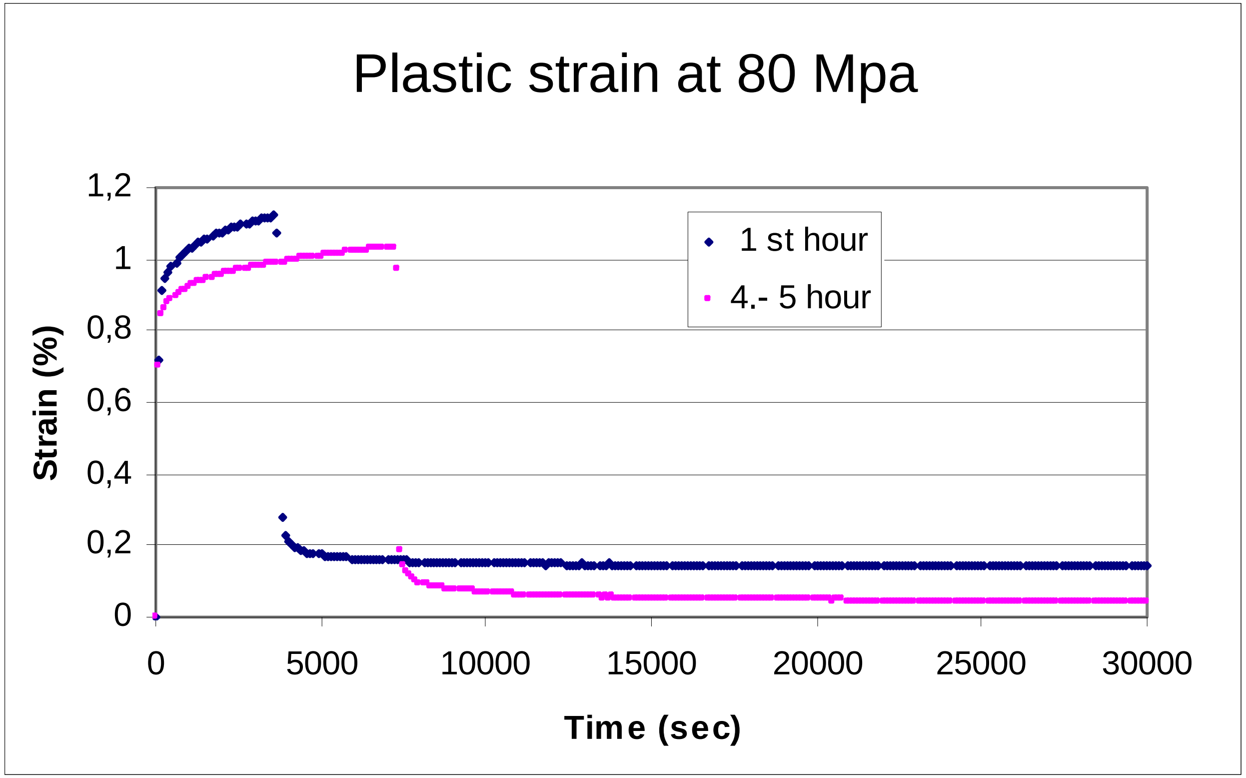
•Model 3 $a_{21} = 1$

$$\varepsilon = \varepsilon^{el} + b \int_{o}^{\psi} \Delta S \left(\psi - \psi'\right) \frac{d}{d\psi'} \left(a_{42}\sigma\right) d\psi'$$

Simplified models



Irreversible strains in compression



Plastic strain model

Tuttles (1993) :

$$\varepsilon_T^{pl} = \left\{ \int_0^t \left[\sigma_T(\tau)\right]^N d\tau \right\}^n$$

Creep conditions:

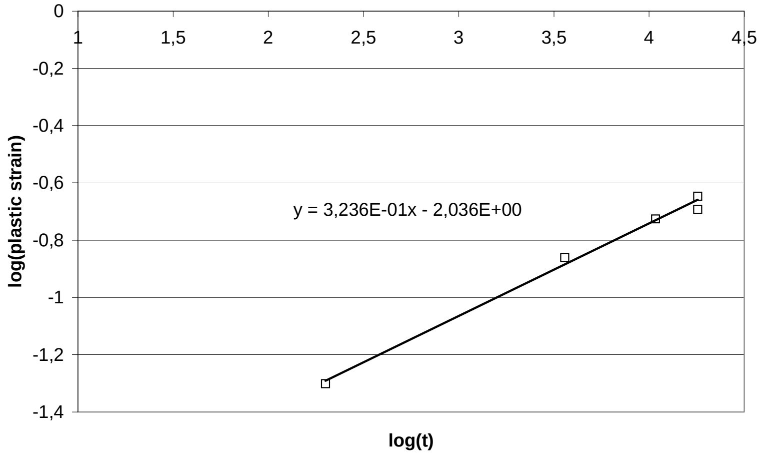
$$\varepsilon_{pl} = C\,\sigma_{10}^{N\cdot n}\,t^n$$

Identified constants:

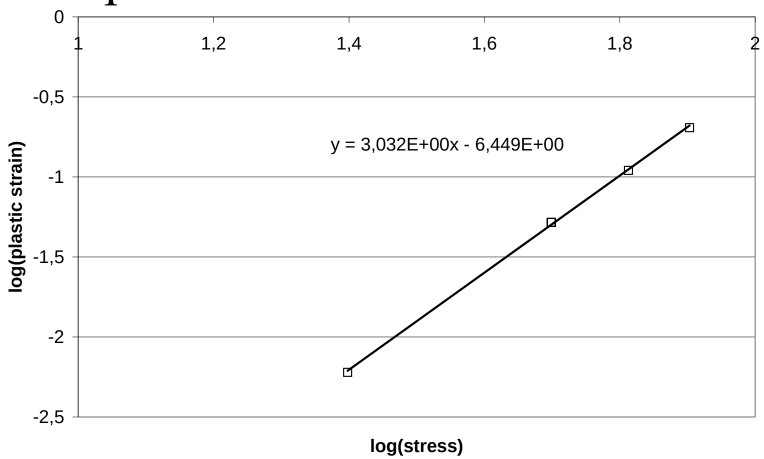
n=0.324

N=9.357

Experiment constant stress 80MPa:



Experiment constant time 5h:



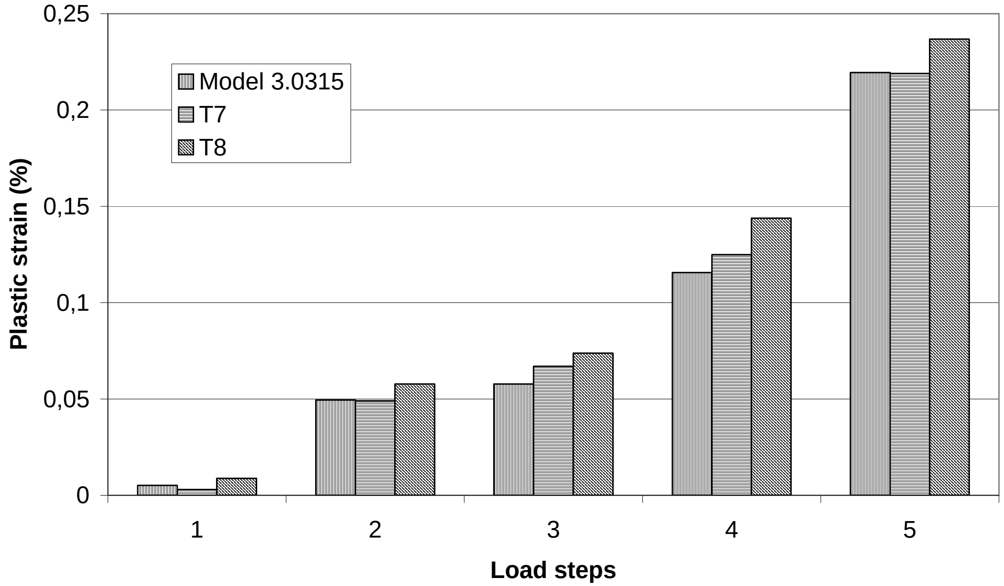
Validation of plasticity model

<u>Experimental</u>: loading of same specimen in steps with intermediate recovery

- 1 step: 25MPa, 5h
- 2 step: 50MPa, 5h
- 3 step: 50MPa, 3h
- 4 step: 65MPa, 5h
- 5 step: 80MPa, 5h

Modeling same load history:

$$\varepsilon_{pl}(t_1+t_2+\dots t_k) = C\left\{\sigma_{10}^{N} t_1 + \sigma_{20}^{N} t_2 + \dots \sigma_{k0}^{N} t_k\right\}^{n}$$



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Determination of viscoelastic functions

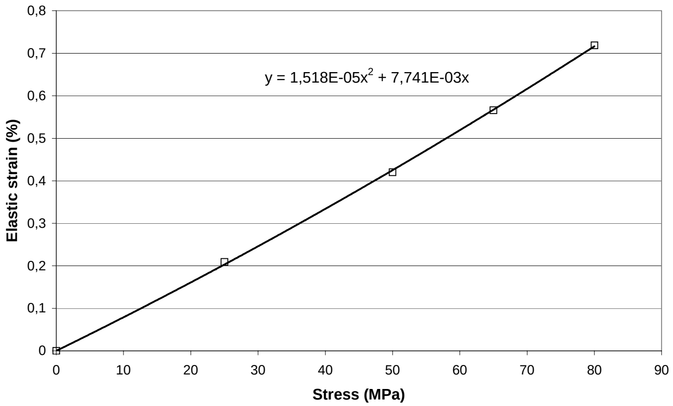
Test of damage development:

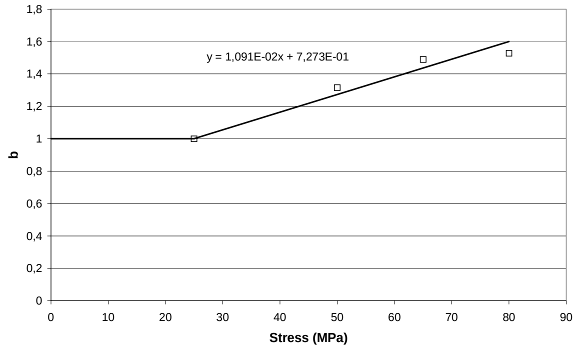
Direction	Never loaded	After 1:st loading	After 2:nd loading	After 3:rd loading
T	10.9	10.8	10.7	10.8

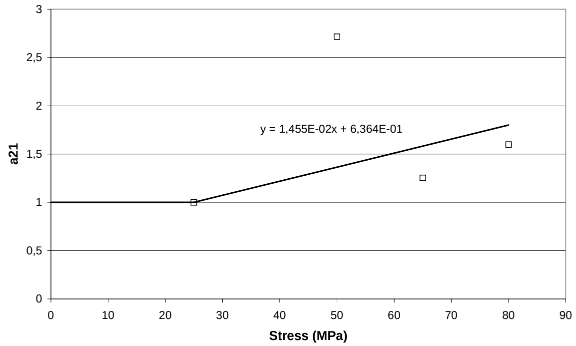
Two-step model with corrections for plastic strains:

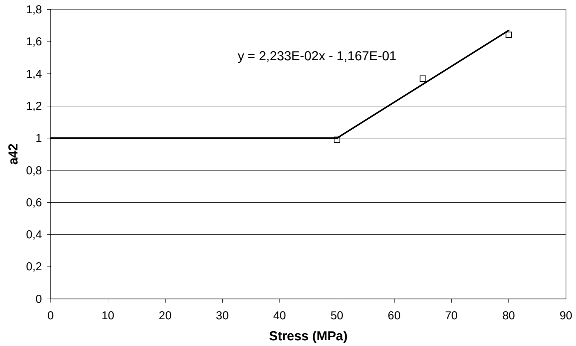
Table 2. Coefficients C_{11}^{m} (Pa^{-1})

τ_m (s)	T-direction
50	3.350E-12
300	8.070E-13
900	3.739E-12
6000	2.982E-12
30000	4.733E-12









Conclusions

- Non-linear viscoelastic non-linear viscoplastic model based on Schapery was developed and successfully applied for paper fiber composites
- Methodology for viscoplasticity characterization developed
  - The time and stress dependence of viscoplastic strains described by Tuttles model
  - Identification of constants by creep-recovery tests with constant stress or constant time
- Non-linear viscoelastic functions determined with two-step model and with corrections for the viscoplastic strains
- Material is Moisture sensitive (current work)
  - Softens cell wall material
  - Affects adhesion between fiber and matrix