
FAILURE CRITERIA FOR PREDICTION OF TRANSVERSE MATRIX CRACKING IN COMPOSITES

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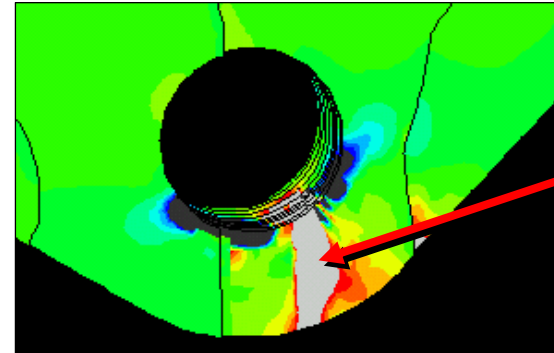
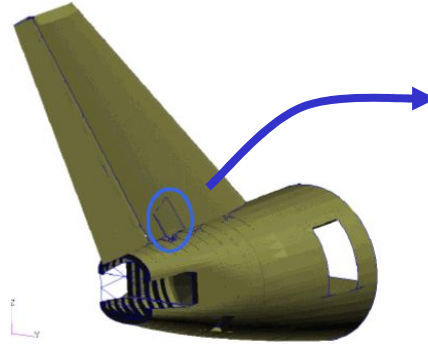
Comptest 2004
Bristol, 21-23 September 2004

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- **Failure criterion for matrix cracking under in-plane shear and transverse tension.**
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Introduction

Failed composite lug

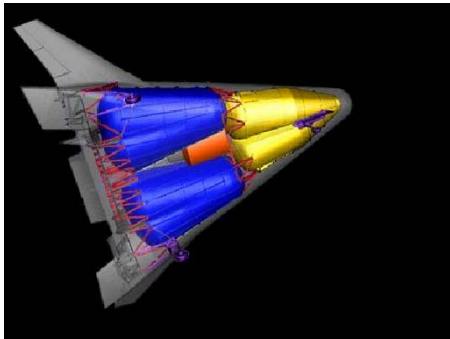


Clevage
Fracture

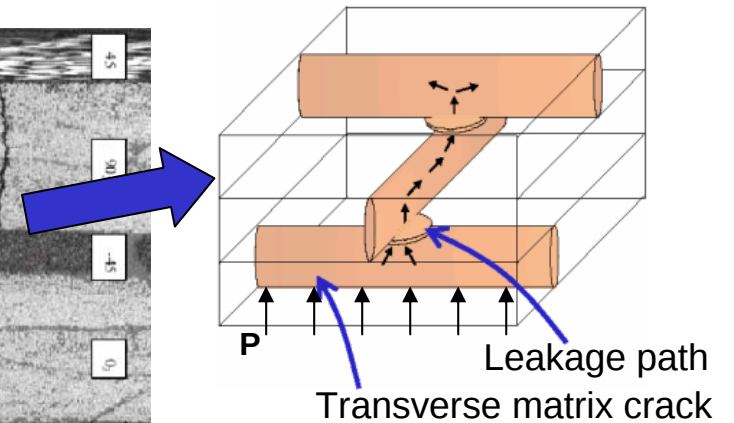
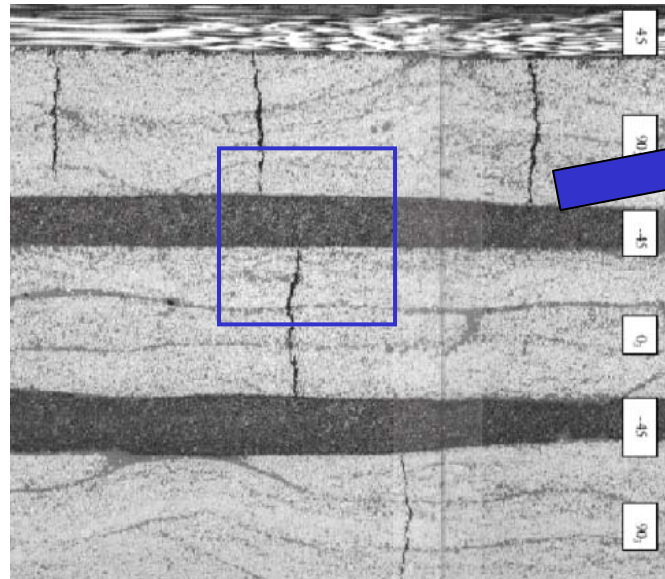
- Failure Criteria are used for predicting damage initiation and, when used in a Continuum Damage Model, final failure.
- Composites have multiple damage modes; each requires a different criterion.
- Failure prediction methodologies and criteria still under debate; main unresolved issues:
 - ✓ in-situ effects.
 - ✓ effects of σ_{12} on fibre kinking.
 - ✓ increase of apparent shear strength under moderate compressive σ_{22}

LaRC03
Failure Criteria

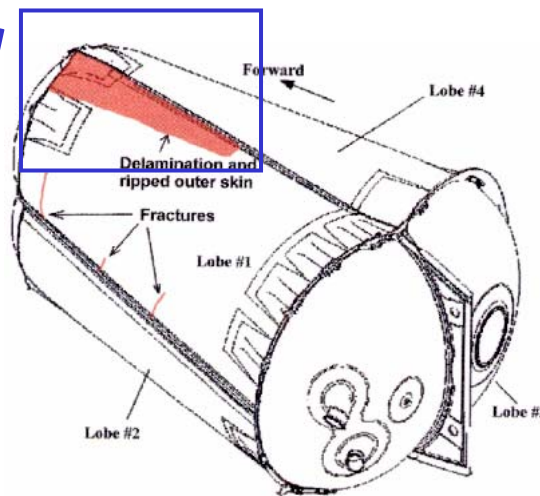
Introduction



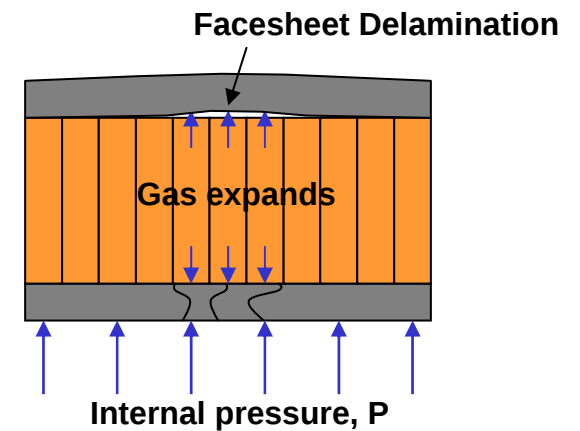
X-33
RLV



Matrix cracks provide primary leakage path in composite tanks



Crypumping in sandwich core



Tank failed during testing. X-33 Program cancelled by NASA

Introduction

Transverse Tension Matrix Cracking

- Typically considered a benign damage mode
- Important for leakage, damage initiation

Issues/Difficulties Predicting Transverse Matrix Cracks

- In-situ matrix strengths are a function of ply thickness

Typically: $Y_{UD}^T \leq Y_{is}^T \leq 4Y_{UD}^T$ ← tension

$$S_{UD}^L \leq S_{is}^L \leq 4S_{UD}^L \quad \leftarrow \text{shear} \quad UD=\text{unidirectional}; is=\text{in-situ}$$

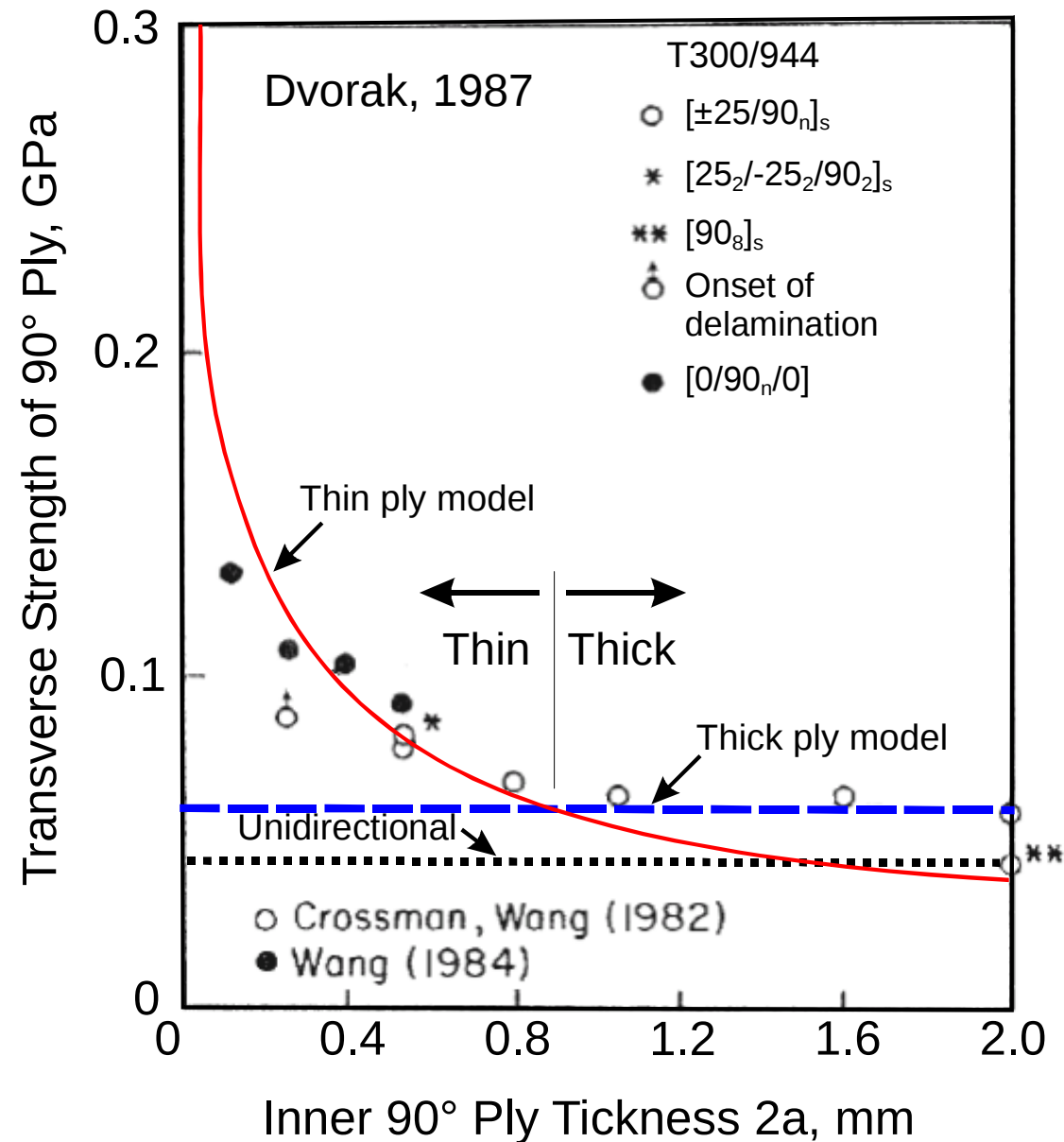
- No fracture mechanics basis in Hashin's criterion

Previous work

- Parvisi, Wang, etc. ('78-'84): identified in-situ effect in tensile tests
- Dvorak, Laws, Tan ('86-'89): fracture mechanics predictions of in-situ strength
- Shahid & Chang ('95): PDCOMP extension of Tan

Introduction

In-situ effect:

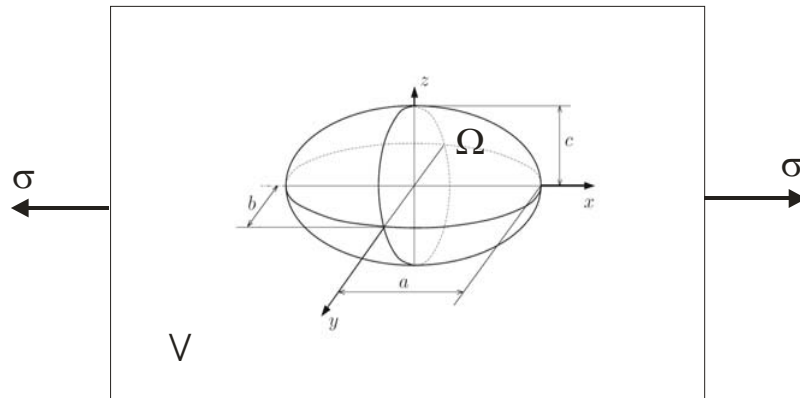


Objectives

- To develop a model able to predict the in-situ strengths.
- To develop a novel failure criterion for matrix transverse cracking under transverse tension and in-plane shear.
- Validate the models proposed by comparing predictions with experimental data.

Approach

Eshelby's inhomogeneity problem for isotropic materials (1957)



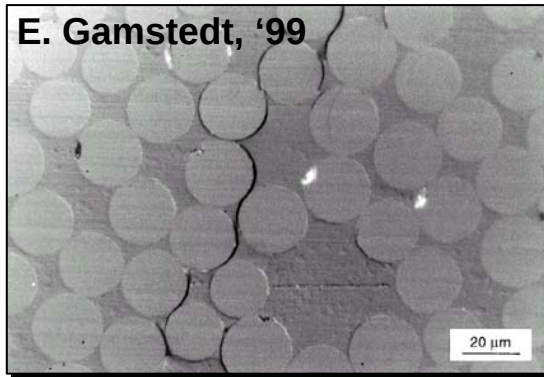
$$\begin{aligned} C^V &\neq C^\Omega && \text{(inhomogeneity)} \\ C^\Omega &\rightarrow 0 && \text{(cavity)} \\ c \rightarrow \infty; \frac{a}{b} &\rightarrow 0 && \text{(slit crack)} \end{aligned}$$

- ✓ Elastic field far from inclusion.
- ✓ Stress & strain tensors outside the inclusion.
- ✓ Interaction energy.

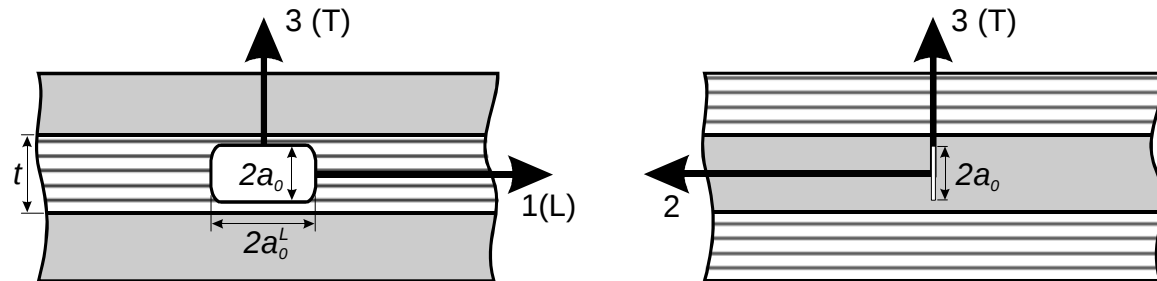
Inhomogeneity problem for orthotropic materials:

- Kinoshita (1971), Faivre (1971), Laws (1977).

Approach



Material defects can propagate in longitudinal (L) and transverse (T) directions.



$$E_{\text{int}} = \frac{1}{2} \pi a_0 \left(\Lambda_{22}^0 \sigma_{22}^2 + \chi(\gamma_{12}) \right)$$

$$\Lambda_{22}^0 = 2 \left(\frac{1}{E_2} - \frac{\nu_{21}^2}{E_1} \right)$$

$$\chi(\gamma_{12}) = 2 \int_0^{\gamma_{12}} \sigma_{12} d\gamma_{12}$$



Transverse: **unstable growth**

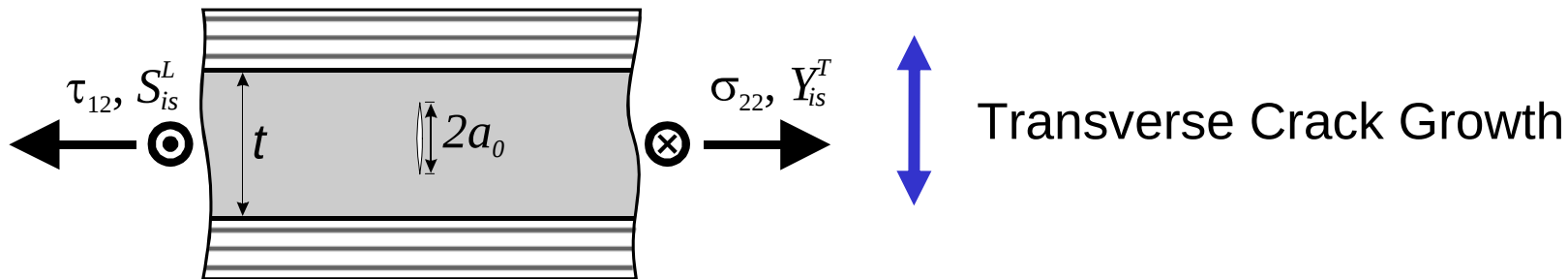


Longitudinal: **stable growth**

$$G(T) = \frac{1}{2} \frac{\partial E_{\text{int}}}{\partial a_0} = \frac{\pi a_0}{2} \left(\Lambda_{22}^0 \sigma_{22}^2 + \chi(\gamma_{12}) \right)$$

$$G(L) = \frac{E_{\text{int}}}{2a_0} = \frac{\pi a_0}{4} \left(\Lambda_{22}^0 \sigma_{22}^2 + \chi(\gamma_{12}) \right)$$

Thick Plies $2a_0 \ll t$



In-situ strengths:

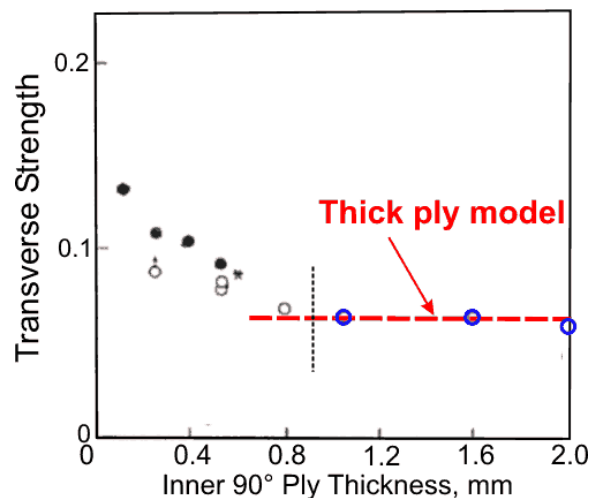
$$G_{Ic}(T) = \frac{\pi a_0}{2} \Lambda_{22}^0 (Y_{is}^T)^2$$

$$\gamma_{12} = \frac{1}{G_{12}} \sigma_{12} + \beta \sigma_{12}^3 \quad \rightarrow$$

$$Y_{is}^T = \sqrt{\frac{2G_{Ic}(T)}{\pi a_0 \Lambda_{22}^0}}$$

$$G_{IIc}(T) = \pi a_0 \int_0^{\gamma_{12}^u} \sigma_{12} d\gamma_{12}$$

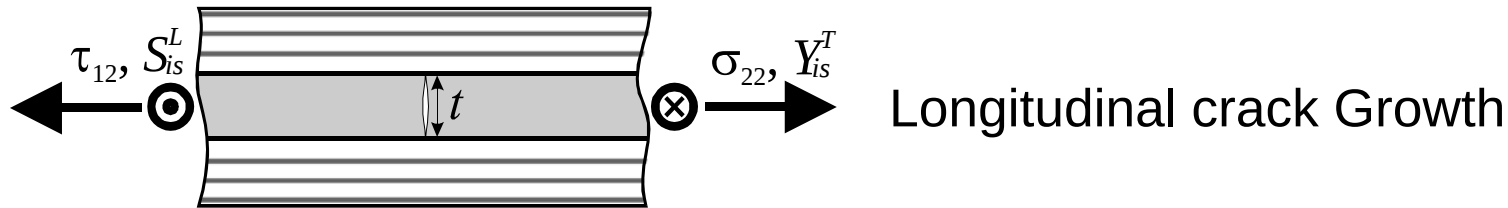
$$G_{IIc}(T) = \pi a_0 \left[\frac{(S_L^{is})^2}{2G_{12}} + \frac{3}{4} \beta (S_L^{is})^4 \right]$$



Defect size a_0 is assumed to be a material property

In-situ strengths of thick plies are independent of ply thickness t

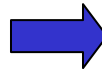
Thin Plies $2a_0 = t$



In-situ strengths:

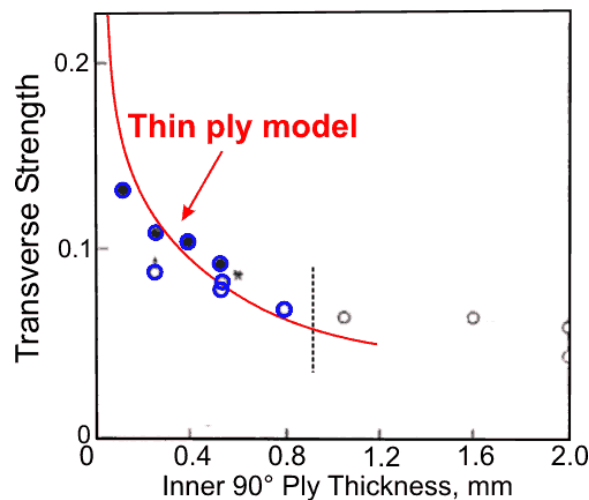
$$G_{Ic}(L) = \frac{\pi t}{8} \Lambda_{22}^0 (Y_{is}^T)^2$$

$$G_{IIc}(L) = \frac{\pi t}{4} \int_0^{\gamma_{12}^u} \sigma_{12} d\gamma_{12}$$



$$Y_{is}^T = \sqrt{\frac{8G_{Ic}(L)}{\pi t \Lambda_{22}^0}}$$

$$G_{IIc}(L) = \pi t \left[\frac{(S_L^{is})^2}{8G_{12}} + \frac{3}{16} \beta (S_L^{is})^4 \right]$$

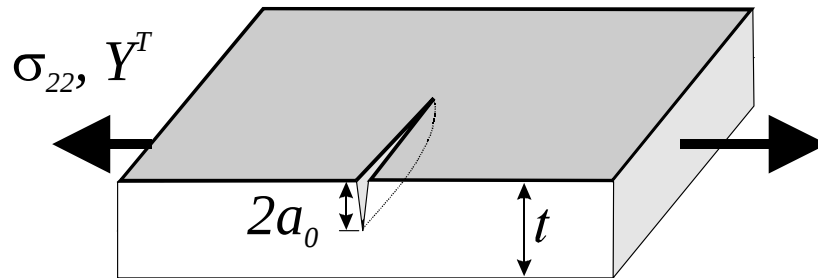


In-situ strengths of thin plies

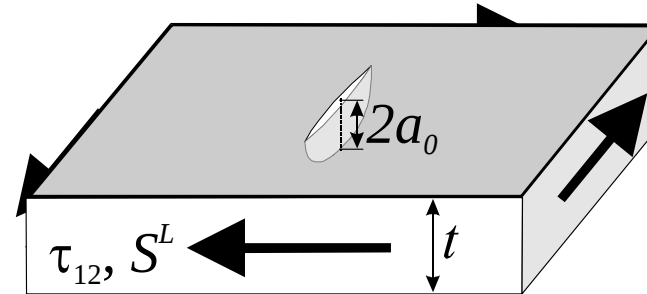
- depend on ply thickness t
- do not depend on defect size a_0

Unidirectional Laminates

Tension test coupon



Shear test coupon



Classic Free Edge Crack
Solutions (Tada):

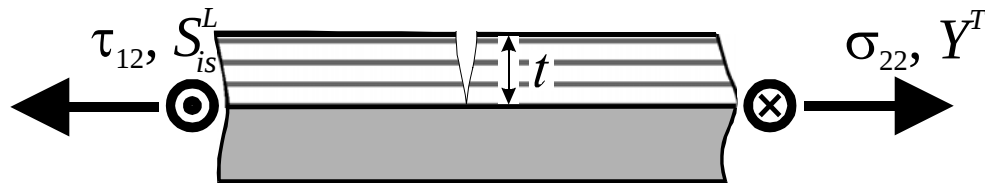
$$G_{Ic}(T) = 1.12^2 \pi a_0 \Lambda_{22}^0 (Y^T)^2$$

$$G_{IIc}(T) = 2\pi a_0 \int_0^{\gamma_{12}^u} \sigma_{12} d\gamma_{12}$$

Substituting $G_{Ic}(T)$ and $G_{IIc}(T)$ into expressions for thick plies:

$$\begin{cases} Y_{is}^T = 1.12 \sqrt{2} Y^T \\ \frac{(S^L)^2}{G_{12}} + \frac{6}{4} \beta (S^L)^4 = \frac{(S_{is}^L)^2}{2G_{12}} + \frac{3}{4} \beta (S_{is}^L)^4 \end{cases}$$

Thin Outer Plies $2a_0 = t$



Longitudinal crack growth

Free Edge Crack Solutions (Tada):

$$Y_{is}^T = 1.79 \sqrt{\frac{G_{Ic}(L)}{\pi t \Lambda_{22}^0}}$$

$$\frac{(S_o^L)^2}{4G_{12}} + \frac{3}{8} \beta (S_o^L)^4 = \frac{G_{IIc}(L)}{\pi t}$$

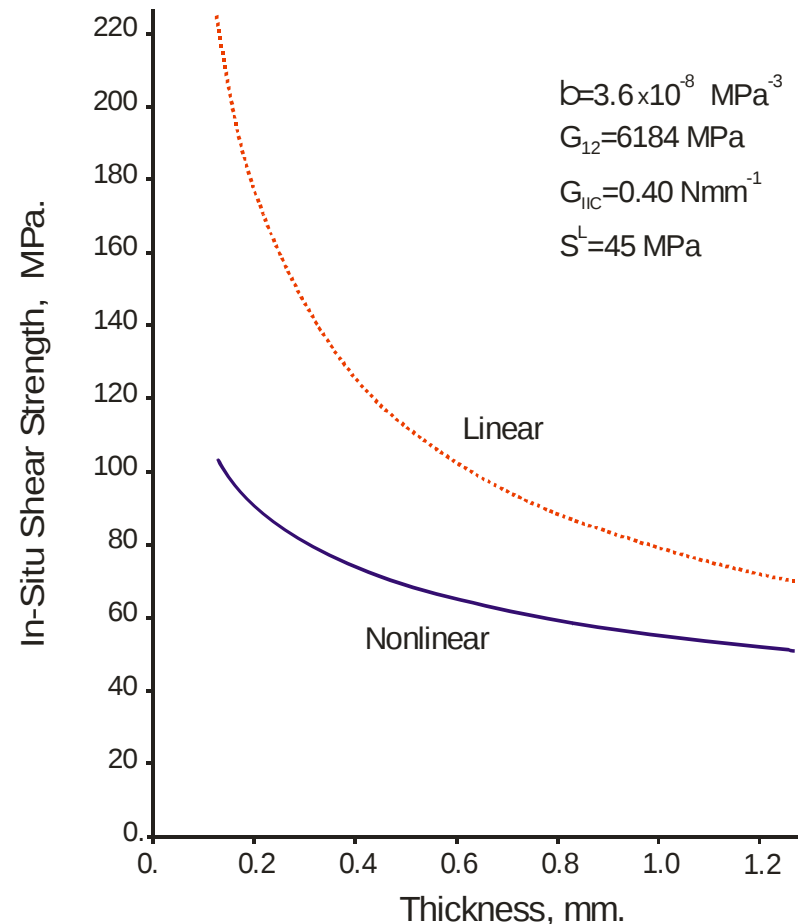
General solution for in-situ shear strengths

$$S_{is}^L = \sqrt{\frac{\left(1 + \beta \phi G_{12}^2\right)^{1/2} - 1}{3\beta G_{12}}}$$

Thick ply: $\phi = \frac{12(S^L)^2}{G_{12}} + \frac{72}{4}\beta(S^L)^4$

Thin ply: $\phi = \frac{48G_{IIC}}{\pi t}$

Thin outer ply: $\phi = \frac{24G_{IIC}}{\pi t}$



Mixed-Mode Criterion for Matrix Cracking

Wu and Reuter:

Experimental tests on composite specimens under mode I, mode II and mixed-mode I & II loading.

Fracture surface topography depends on the type of loading.

Hahn Criterion:
$$(1-g) \frac{K_I}{K_{Ic}} + g \left(\frac{K_I}{K_{Ic}} \right)^2 + \left(\frac{K_{II}}{K_{IIc}} \right)^2 \leq 1$$

Using LEFM:
$$(1-g) \sqrt{\frac{G_I}{G_{Ic}}} + g \frac{G_I}{G_{Ic}} + \frac{G_{II}}{G_{IIc}} \leq 1 \quad g = \frac{G_{Ic}}{G_{IIc}}$$

Substituting the stresses into the expressions for: $G_I, G_{II}, G_{Ic}, G_{IIc}$

**LaRC03 criterion
for matrix tensile
cracking:**

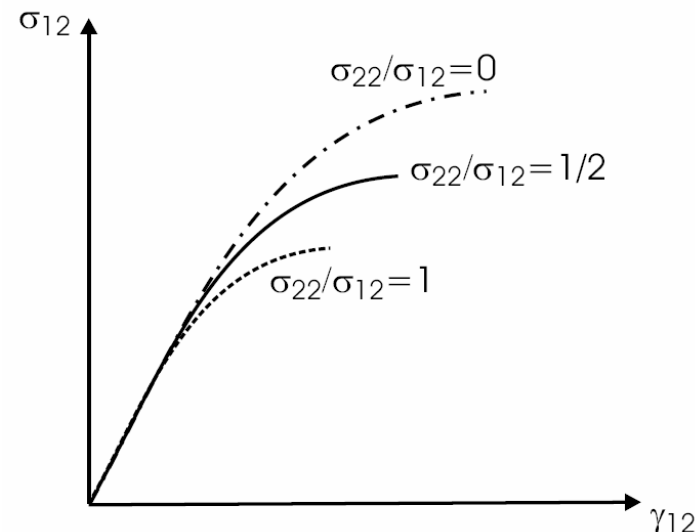
$$(1-g) \frac{\sigma_{22}}{Y_{is}^T} + g \left(\frac{\sigma_{22}}{Y_{is}^T} \right)^2 + \left(\frac{\sigma_{12}}{S_{is}^L} \right)^2 \leq 1$$

Mixed-Mode Criterion for Matrix Cracking

Non-linear behavior:

LEFM not applicable for relating K to G.

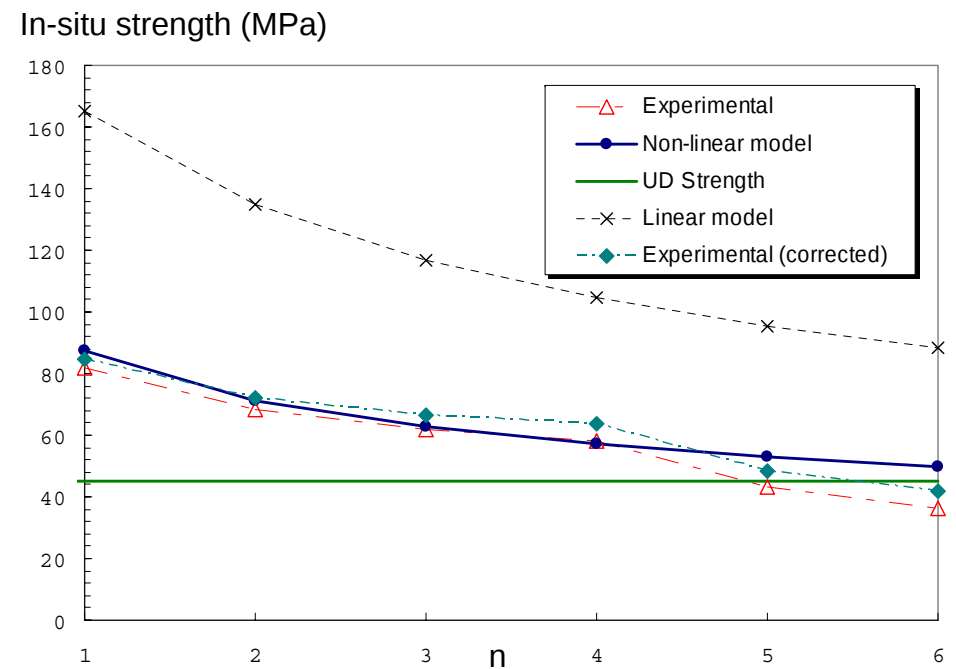
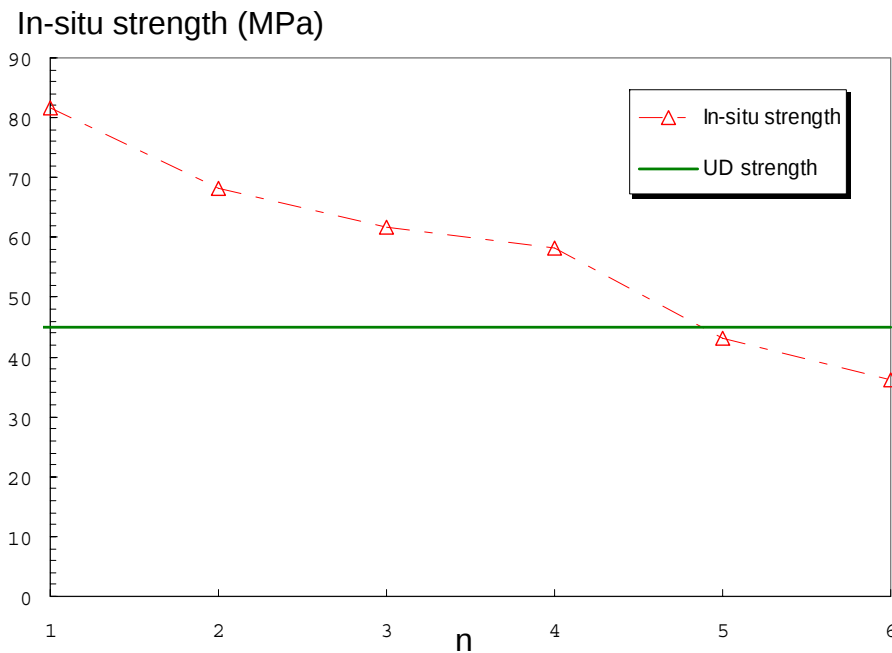
$$(1-g) \frac{\sigma_{22}}{Y_{is}^T} + g \left(\frac{\sigma_{22}}{Y_{is}^T} \right)^2 + \frac{\chi(\gamma_{12})}{\chi(\gamma_{12}^u)} \leq 1$$



Verification Problems

CASE 1: In-situ shear strength

Chang and Chen experiments in $(0_n/90_n)_s$ CFRP laminates (1987).



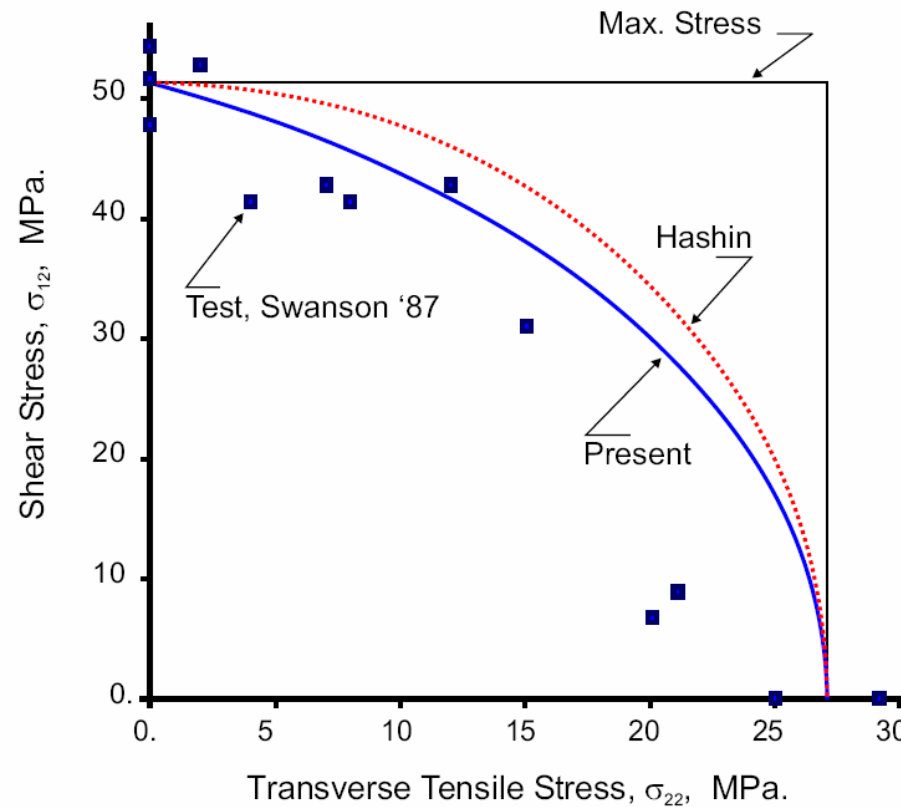
Verification Problems

CASE 2: New failure criterion applied to unidirectional laminates

Experimental data:

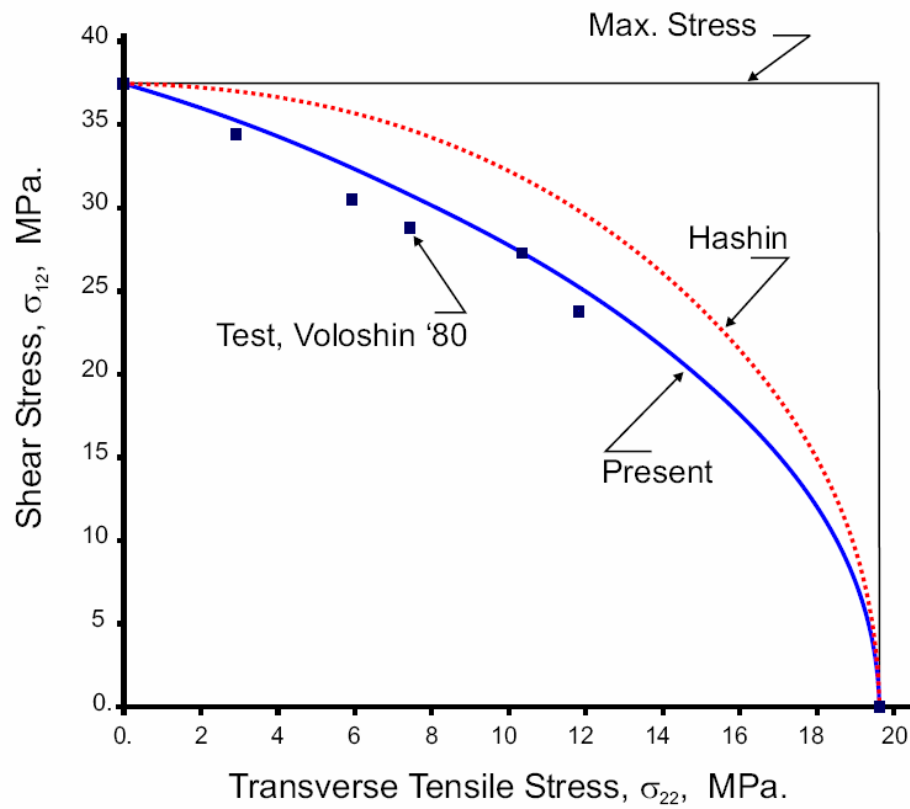
- AS4-55A, Swanson (1987).
- E-glass-LY556, Soden (1998).
- Scotchply, Voloshin (1980).

AS4-55A

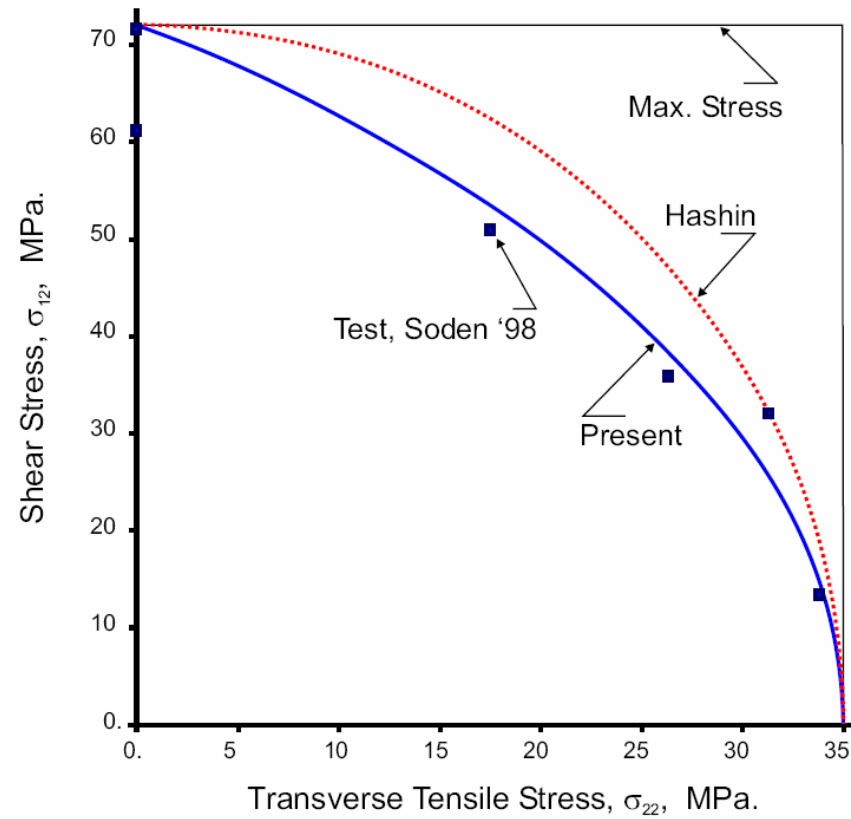


Verification Problems

Scotchply



E-glass-LY556



Conclusions

Conclusions

- New model, based on non-linear shear behavior, was developed for the prediction in-situ shear strengths.
- New criterion for tensile matrix cracking developed based on ply-level fracture mechanisms.
- New criterion uses easily measured unidirectional strength properties and ply configuration to calculate in-situ strengths.
- Predictions are in excellent correlation with experimental results.