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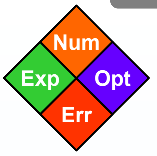
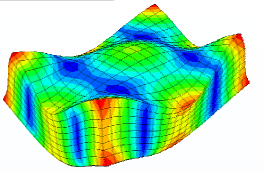


A Mixed Numerical-Experimental Identification Method for Evaluating the Constitutive Parameters of Composite Laminated Shells



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Motivations

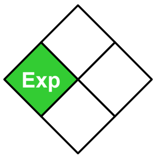
Classical characterisation of composites requires

- at least 6 constitutive parameters : E_1, E_2, G_{12}, G_{13}, G_{23}, ν_{12}
- usually 5 static tests for their identification
- a large number of specimens in order to reduce scattering
- one set of specimens for each test
- long, expensive and often destructive processes

Mixed numerical-experimental identification procedures

- are based on modal analysis
- have the potential to reduce the testing to one specimen, without any damage
- provide a full characterisation of elastic properties in a fast and inexpensive manner

Experimental modal analysis



Contact-free modal measurements

- Free-free boundary conditions
- Scanning laser vibrometer for high sensitivity measurements of FRFs and mode shapes

Calibrated acoustic excitation system

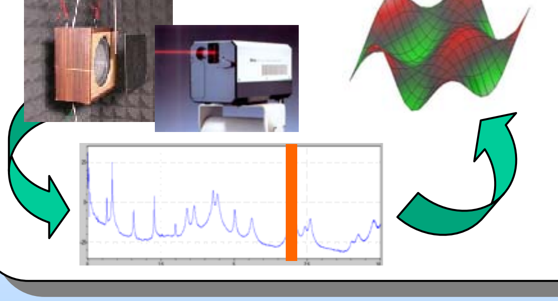
- Microphone as excitation signal reference
- Acoustic source optimization

Wide frequency range

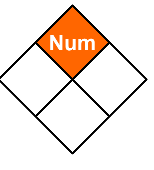
- From 50 Hz to 15 kHz (between 10 and 25 modes are measured)

MDOF curve fitting

- High quality eigenfrequencies & mode shapes extraction even with damped or close modes



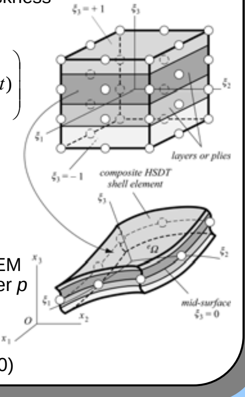
Numerical modal analysis



***p*-order HSDT shell finite element (PSDT model)**

- Improvement of the finite element by Surana & Sorem (1990)
- Based on a Taylor development of the through-the-thickness displacement up to an order p
- Most general ESL-HSDT model
- Linear to cubic triangular or quadrangular elements
- 3 (p+1) DOF per node
- Accurate modelling of transverse shear deformations
- Exact Gaussian integration through the thickness
- Full 3D elasticity stress-strain law

$$ {}^e\mathbf{u}^h(\boldsymbol{\xi},t) = \sum_{i=1}^{\xi_n} \left({}^a h_i(\xi_1,\xi_2) \sum_{j=0}^{p} \frac{\xi_3^j}{j!}\, {}^e\mathbf{q}_i^{(j)}(t) \right) $$



Validation

- Vs analytical and experimental results
- Comparison with commercial FEM codes

Results

- Excellent precision wrt analytical ESL and Layerwise theories, experiments and 3D FEM
- Fast convergence when increasing the order p
  - p=3 sufficient for relatively thick shells ($15<a/h<25$)
  - p=5 necessary for thick shells ($a/h<15$)
- Accurate even with very thick shells ($a/h<10$)

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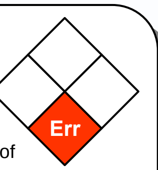
Modal error norms and objective function



Modal error norms based on

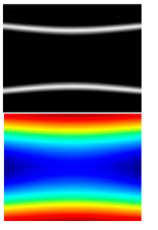
- Measured and computed eigenfrequencies

$$ \varepsilon_i^f(\mathbf{x}_k) = (\tilde{\omega}_i^k - \omega_i)/\omega_i $$

- Diagonal and off-diagonal terms of the MAC matrix μ_{ij} of the experimental and numerical mode shapes

$$ \mu_{ij}^k(\mathbf{x}_k) = \frac{[(\tilde{\boldsymbol{\varphi}}_i^k)^{\mathrm{T}}\boldsymbol{\varphi}_j]^2}{[(\tilde{\boldsymbol{\varphi}}_i^k)^{\mathrm{T}}\tilde{\boldsymbol{\varphi}}_i^k][(\boldsymbol{\varphi}_j)^{\mathrm{T}}\boldsymbol{\varphi}_j]} \qquad \varepsilon_i^d(\mathbf{x}_k) = 1-\mu_{ii}^k \qquad \varepsilon_i^o(\mathbf{x}_k) = \sum_{\substack{j=1\\ j\neq i}}^{q} \mu_{ij}^k $$

- Nodal lines correlation norm obtained by a 2D spline interpolation of mode shapes and an image correlation technique



$$ \varepsilon_i^n(\mathbf{x}_k) = \frac{1}{n^2}\sum_{r=1}^{n}\sum_{s=1}^{n}[f(\tilde{I}_{rs,i}^k) - f(I_{rs,i})]^2 $$

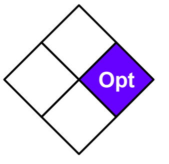
$$ \begin{aligned} f(A) &= 0 \text{ if } \mathrm{abs}(A) \geq \delta \\ &= 1-\mathrm{abs}(A)/\delta \text{ if } \mathrm{abs}(A) < \delta \end{aligned} $$

Global objective function

- Combination of modal error norms
- Weights determined empirically
- Sensitive to all the parameters

$$ \boldsymbol{\varepsilon}(\mathbf{x}_k) = \{\alpha^f\boldsymbol{\varepsilon}^f, \alpha^d\boldsymbol{\varepsilon}^d, \alpha^o\boldsymbol{\varepsilon}^o, \alpha^n\boldsymbol{\varepsilon}^n\}^{\mathrm{T}} $$

Optimization



Algorithms

- Levenberg-Marquardt nonlinear least squares optimization algorithm : robust and stable, super-linear convergence rate if small residual
- Direct finite difference computation of the error norm derivatives

Results

- Robust and effective optimization method
- Convergence in 3 to 6 iterations
- Very small error residual

$$ \underset{\mathbf{x}_k}{\text{minimize}}\ \varepsilon(\mathbf{x}_k) \quad \text{with } \varepsilon(\mathbf{x}_k) = \frac{1}{2}\|\boldsymbol{\varepsilon}(\mathbf{x}_k)\|_2^2 $$

Validation

Specimen

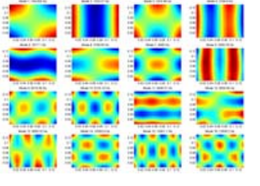
- Carbon-epoxy UD plate, 135x135x8.3 mm, a/h = 16
- Orthotropic material: E_1, E_2, G_{12}, G_{23}, G_{31}, ν_{12}

Classical characterization

- Tensile test ASTM D3039
- Three point bending
- Torsion of a rectangular coupon

Mixed identification method

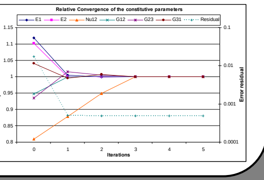
- Modal analysis and MDOF curve fitting (13x13 measurement grid)
- Finite element model (15x15 quadratic PSDT shell elements with p=3)



Convergence

- Convergence of all moduli in 3 iterations
- Slower evolution of Poisson's ratio
- Good reproducibility (<2%)
- Excellent robustness even with 50% perturbations on initial parameters

Parameter		*Static*	*Std*	*Mixed Ident.*	*% diff*
E_1	(GPa)	89.7	6.7%	89.4	-0.4%
E_2	(GPa)	7.42	5.9%	7.26	-2.2%
ν_{12}	(-)	0.36	4.4%	0.37	3.4%
G_{12}	(GPa)	3.56	9.1%	3.7	3.7%
G_{23}	(GPa)	2.11	1.0%	2.25	6.7%
G_{31}	(GPa)	3.28	3.1%	3.36	2.6%



Conclusions

The proposed mixed identification method leads to a precise determination of all the 6 elastic properties of composite plates with a single vibration test. Poisson's ratio is generally accurately identified but is sensitive to the quality of measured mode shapes. The transverse shear moduli can be determined accurately on relatively thick specimens (a/h ~ 15).

CompTest2004, Bristol (21-23 september)