Prediction of Impact-Induced Fibre Damage in Circular Composites Plates

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• Objectives

- **The aim of this work is to develop an impact damage model for preliminary design analysis that avoids the need of a dynamic finite element analysis of the structure.**
  - **- Prediction of damage area based on fibre breakage length**
  - **- Prediction of reduced damage area properties**

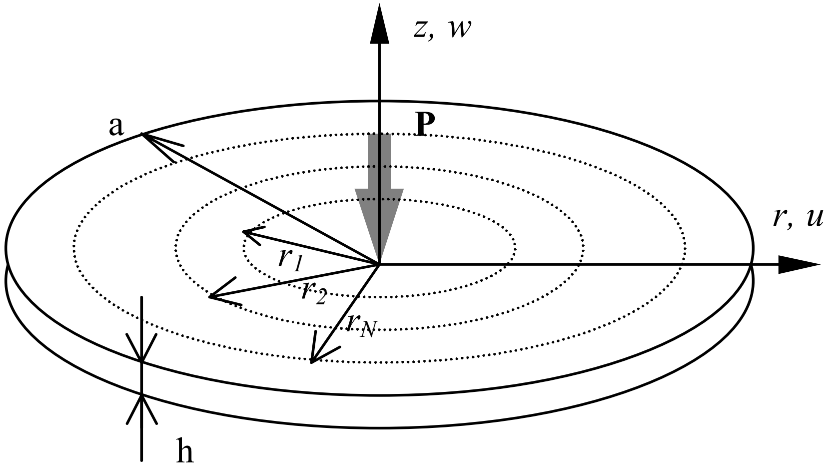
- These results could then be used as basic input data for predicting the compressive after impact (CAI) strength with the Soutis-Fleck model.
- A commercial programme, Maple VI is used to run the present model and FE analysis together with experimental data to evaluate its accuracy.

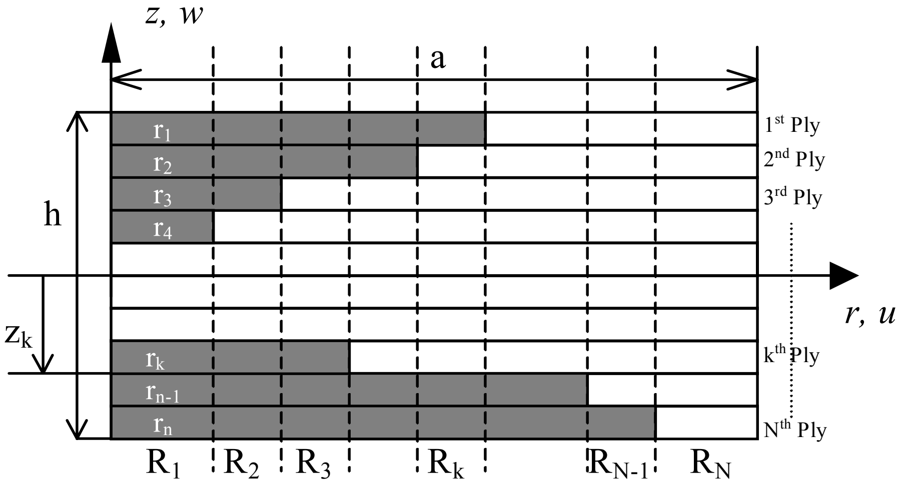
• Impact Damage Modelling

- Concept

- The impact damage model is based on the concept that the low velocity impact response is similar to the deformation due to a static transverse load.
- Damage in composite plates induced by low velocity impact can be studied by treating the response to the impact as static global bending.
- By neglecting the inertia forces from the plate the problem could be reduced to a static equivalent one.
- By considering degraded stiffness in the plate with increasing loads, idealised damage accumulation has been introduced using Rayleigh-Ritz method applied to principle of virtual displacement ($\bar{\omega}$).
- Deflection is finite and geometrical non-linearity is considered.

- Clamped circular plate with fibre damage radius r_1, r_2, r_K,…..r_N at each ply.





- Boundary Condition (Fully Clamped)

$$w_{r=a} = \frac{dw_{r=0}}{dr} = \frac{dw_{r=a}}{dr} = 0\,,\quad w_0^{r_1} = w_d^{r_1}\,, \frac{dw_0^{r_1}}{dr} = \frac{dw_d^{r_1}}{dr}\,,\quad w_0^{r_k} = w_d^{r_k}\,, \frac{dw_0^{r_k}}{dr} = \frac{dw_d^{r_k}}{dr}$$

- Total Internal Energy Expression

Total Elastic Energy (U_T) = Total Stretching Energy + Total Bending Energy

$$U_T = U_{ST} + U_{BT} = \int_0^a \int_0^{2\pi} (U_S + U_B)\, r\, d\theta\, dr = \int_0^a \left(A\left(\left(\frac{d^2 w}{dr^2}\right)^2 + \left(\frac{1}{r^2}\frac{dw}{dr}\right)^2 \right) + B\left(\frac{1}{r}\frac{d^2 w}{dr^2}\frac{dw}{dr}\right) + C\left(\frac{dw}{dr}\right)^4 \right) r\,dr$$

where: $A = \frac{\pi}{8}(3D_{11} + 3D_{22} + 2D_{12} + 4D_{66})$, $B = \frac{\pi}{4}(D_{11} + D_{22} + 6D_{12} - 2D_{66})$, $C = \frac{\pi}{32}(3A_{11} + 3A_{22} + 2A_{12} + 4A_{66})$.

- Total External Energy (Work)

$W = Pw_0$ where P = point load at the center of circular plate center

w_0 = displacement at r=0

- Assumed Displacement Shape Function for Clamped Boundary Condition

$$w = w_0\left(1 - \frac{r^2}{a^2} + 2\frac{r^2}{a^2}\log\frac{r}{a}\right),\quad w' = w_0\left(\frac{4r}{a^2}\log\frac{r}{a}\right),\quad w'' = w_0\left(\frac{4}{a^2}\log\frac{r}{a} + \frac{4}{a^2}\right)$$

- Equating the total internal energy to the external energy (Equilibrium State U_T-W=0)

$$\int_0^a\left(A\left(\left(\frac{d^2w}{dr^2}\right)^2 + \left(\frac{1}{r^2}\frac{dw}{dr}\right)^2\right) + B\left(\frac{1}{r}\frac{d^2w}{dr^2}\frac{dw}{dr}\right) + C\left(\frac{dw}{dr}\right)^4\right)rdr - Pw(0) = 0$$

Introducing the principle of virtual displacement (PVD) and substituting the shape function to the total internal energy, the total internal energy and the work are expressed as:

$$U_T = \int_0^a\left(2A\left(w''\overline{w}'' + \frac{1}{r^2}w'\overline{w}'\right) + \frac{B}{r}\left(w''\overline{w}' + w'\overline{w}''\right) + 4Cw'^3\overline{w}'\right)rdr = \left(A\frac{16}{a^2}w_0 + C\frac{256}{81a^2}w_0^3\right)\overline{w}_0$$

$W = P\overline{w}_0$ where $\overline{w}_0$ = virtual displacement at r=0

- General Displacement Equation for Clamped Circular Composite Plate

$$U_T = W \Longrightarrow \left(A\frac{16}{a^2}w_0 + C\frac{256}{81a^2}w_0^3 \right)\overline{w_0} = P\overline{w_0} \qquad \therefore \qquad P = \left(A\frac{16}{a^2}w_0 + C\frac{256}{81a^2}w_0^3 \right)$$

• Damage Accumulation Process

The process for modelling damage accumulation is based on the degraded stiffness region of the circular plate. When a certain load is applied to the plate the extent of damage i.e. r_1, r_2, r_3….r_N at each ply is determined by using a maximum strain criterion and also by the number of failed plies.

$$U_T = \int_0^{r_4}\left(A_1\left((w'')^2 + \left(\frac{1}{r^2}w'\right)^2 \right) + B_1\frac{1}{r}w''w' + C_1 w'^4 \right) rdr + \int_{r_4}^{r_3}\left(A_2\left((w'')^2 + \left(\frac{1}{r^2}w'\right)^2 \right) + B_2\frac{1}{r}w''w' + C_2 w'^4 \right) rdr + \cdots$$

$$\cdots + \int_{r_2}^{r_1}\left(A_k\left((w'')^2 + \left(\frac{1}{r^2}w'\right)^2 \right) + B_k\frac{1}{r}w''w' + C_k w'^4 \right) rdr + \cdots + \int_{r_n}^{a}\left(A_N\left((w'')^2 + \left(\frac{1}{r^2}w'\right)^2 \right) + B_N\frac{1}{r}w''w' + C_N w'^4 \right) rdr$$

where A_k, B_k and C_k are degraded membrane and bending stiffnesses at region R_k which is situated between r_2 and r_1 along radius r.

• Prediction of Impact Damage

In order to predict damage area a maximum failure strain criterion is used. Ply damage occurs if any radial strain value (ε_r) along the radius r exceeds its ultimate tensile strain value ε_{xx}^{T} or compressive strain value ε_{xx}^{C}. It is assumed that the ply damage has a circular shape with the radius r, which is a fibre damage length, due to the axisymmetric out-of-plane displacement. The maximum strain criterion is formulated below:

$$\varepsilon_r^k = \varepsilon_{Sr} + \varepsilon_{Br} = \frac{1}{2}\left(\frac{dw}{dr}\right)^2 + Z_k\left(\frac{d^2w}{dr^2}\right), \qquad \frac{\varepsilon_r^k}{\varepsilon_x^{T/C}} \geq 1$$

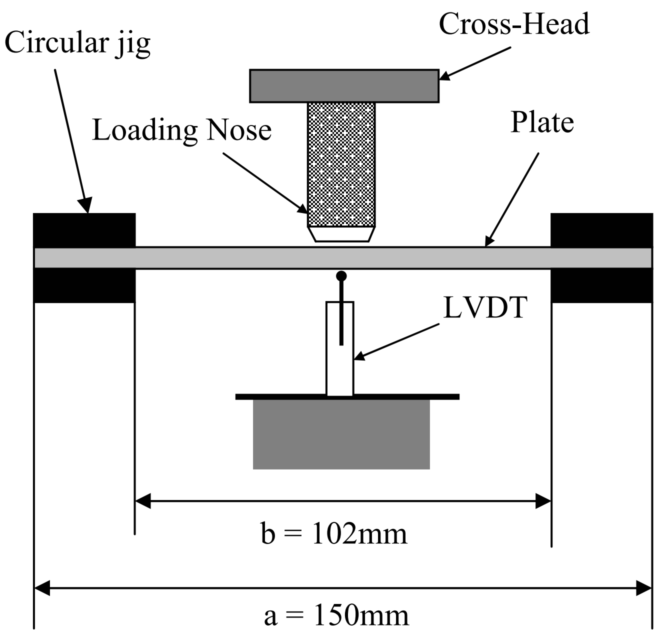
where ε_r^k: Radial strain in k^th ply, Z_k: Distance from the middle plane of the plate to the k_{th} ply and $\varepsilon_x^{T/C}$: ultimate unidirectional tensile or compressive failure strain.

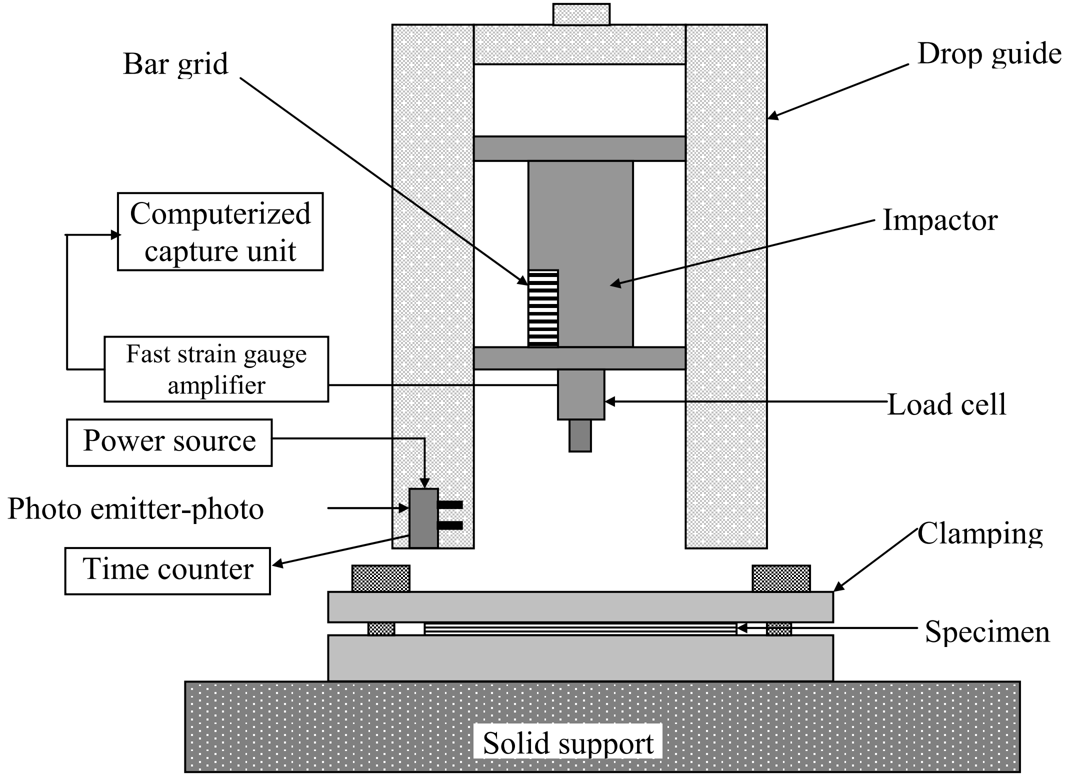
.• Model Set-up for Validation

Material and Lay-up: IM7/8552 - $[45/-45/0/90]_{3s}$.

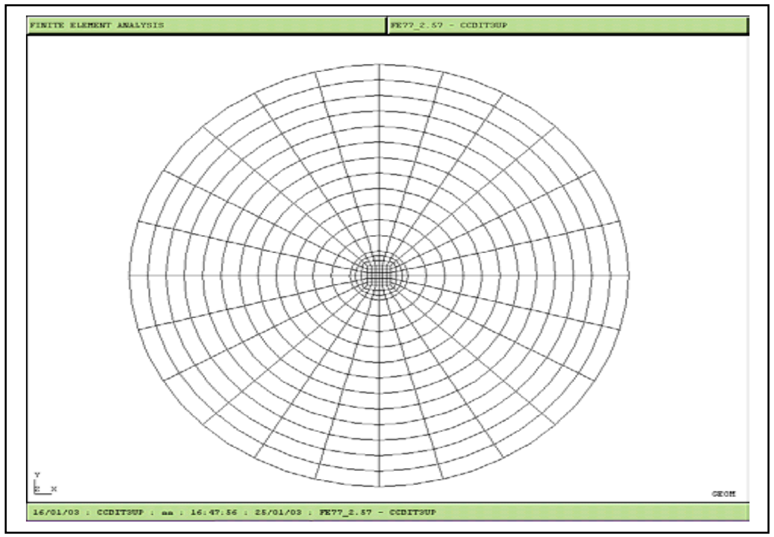
- Experimental Test Setup (Loading Nose Radius: 6mm, Circular Plate Radius:51mm)

Quasi-Static Test **Impact Test**





- Finite Element Mesh for Non-linear Static and Dynamic Analysis



Surface Quadrilateral 8-Node Elements With 396 Elements

- Damage Prediction Model

- Fibre damage modelling at ply level in 3mm thick composite circular plates was performed.
- The plates were 102 mm in diameter, fully clamped.
- The material properties (IM7/8552) and stacking sequences are given below:

Material Properties: E_{11} = 155 GPa, E_{22} = 10 GPa, ν_{12} = 0.31, G_{12} = 4.5 Gpa,

Ultimate tensile failure strain, = 1.54 %,

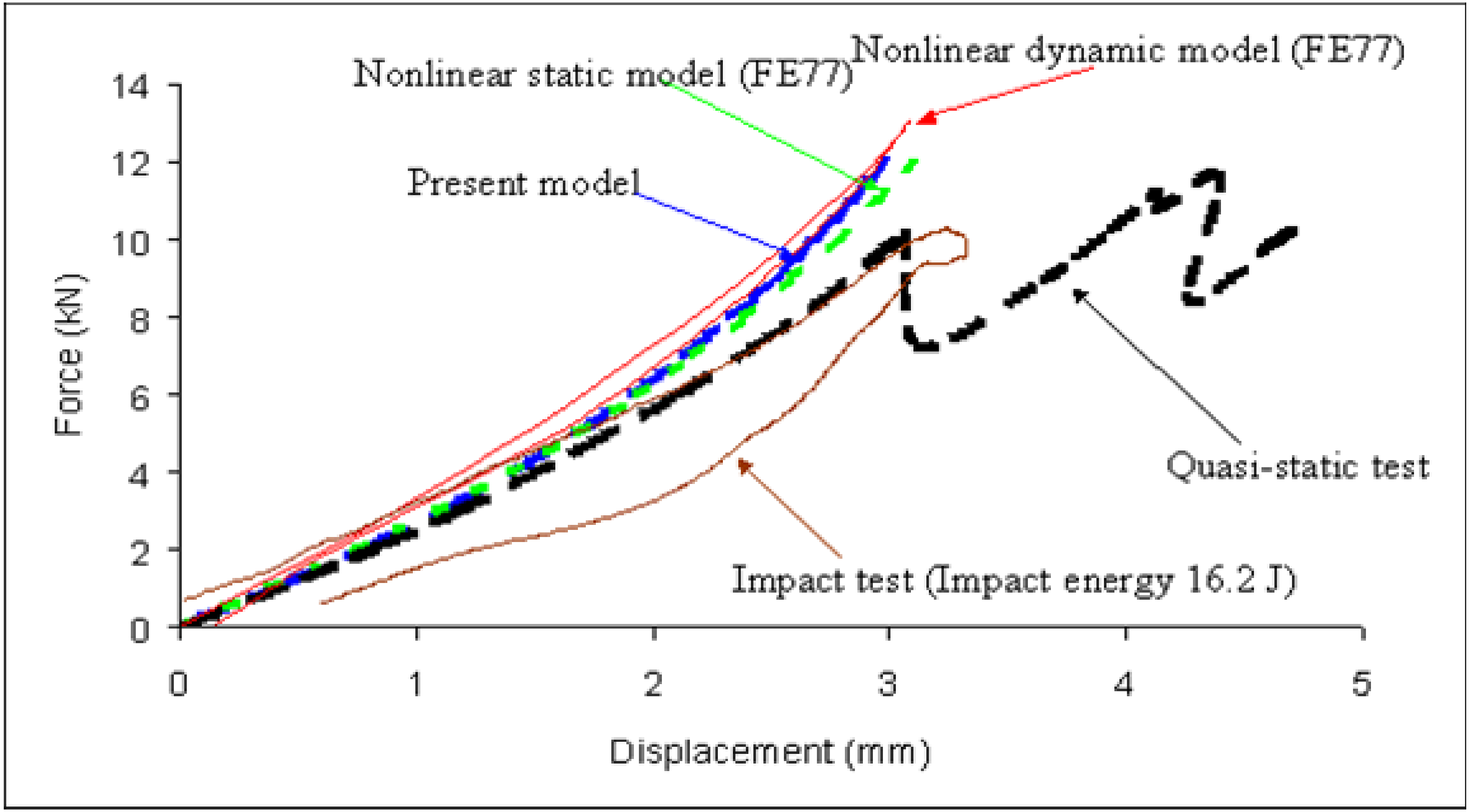
Ultimate compressive failure strain, = 1.1% ,

Stacking Sequences: $[45/-45/0/90]_s$ and $[45/-45/0/90]_{3s}$.

It is assumed that the first ply (top ply) has already failed due to the contact force of the impactor. (The damaged region is regarded as being equivalent to the impactor radius, 6mm.)

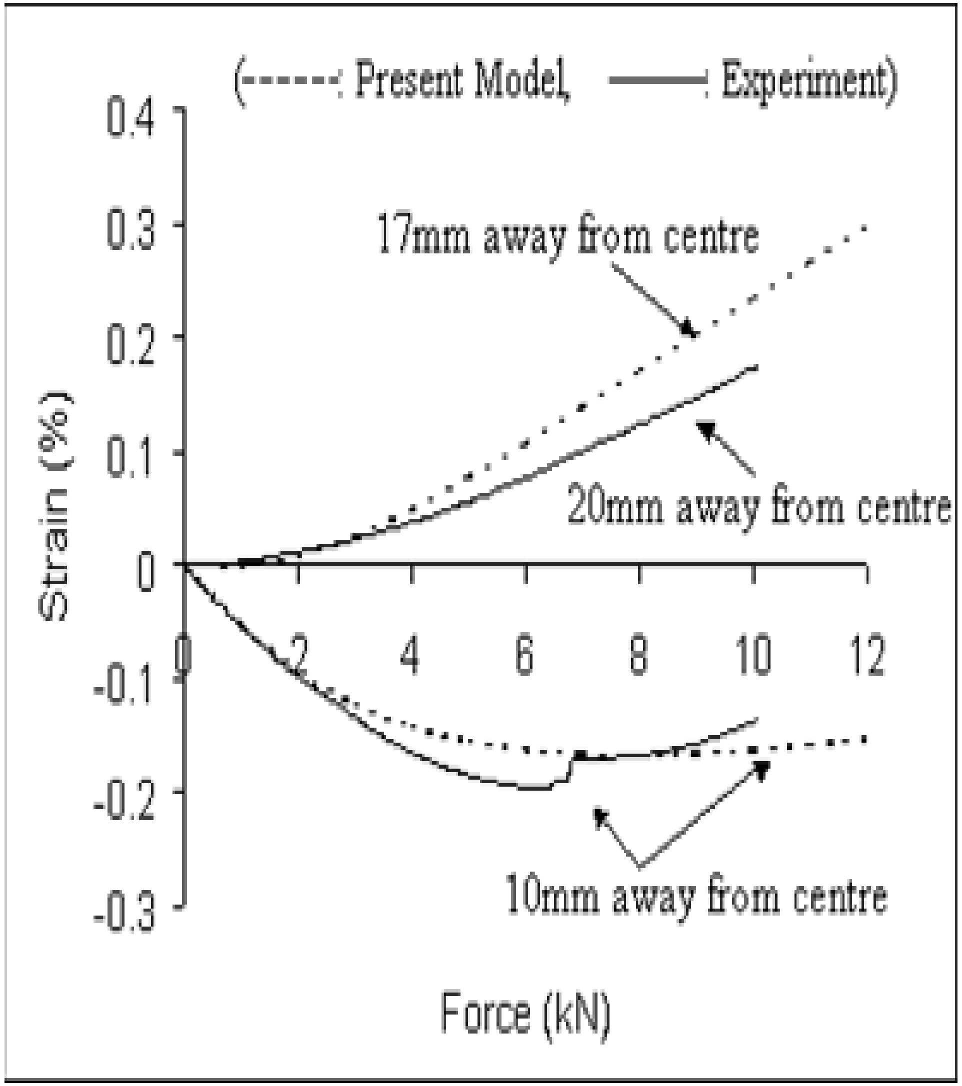
.• Results

- Un-degraded Deflection Comparison between Model, FE and Experimental Results

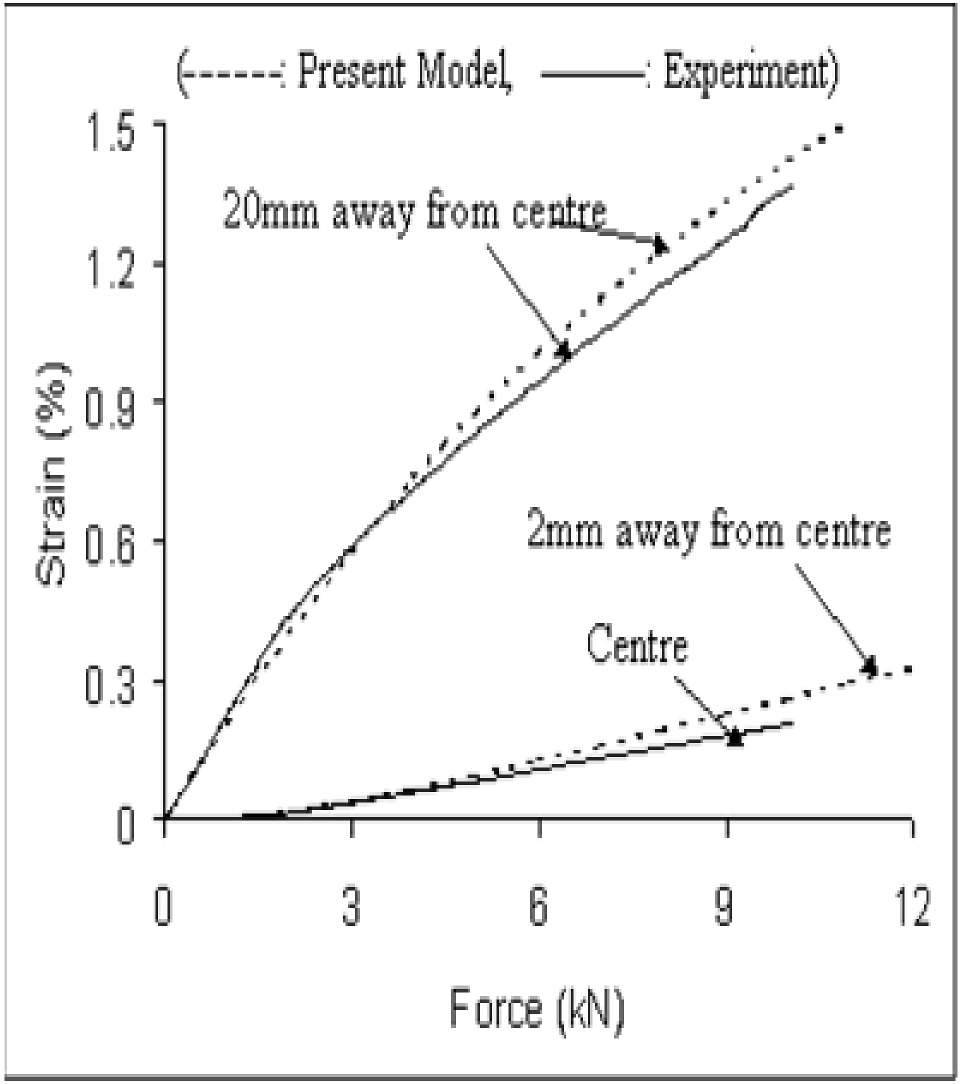


<u>Force-central deflection curves for a 3mm thick IM7/8552 [45/-45/0/90]$_{3s}$ circular plate</u>

- Strain Comparison Between Model and Experimental Results



(a) Bending strain at the top surface



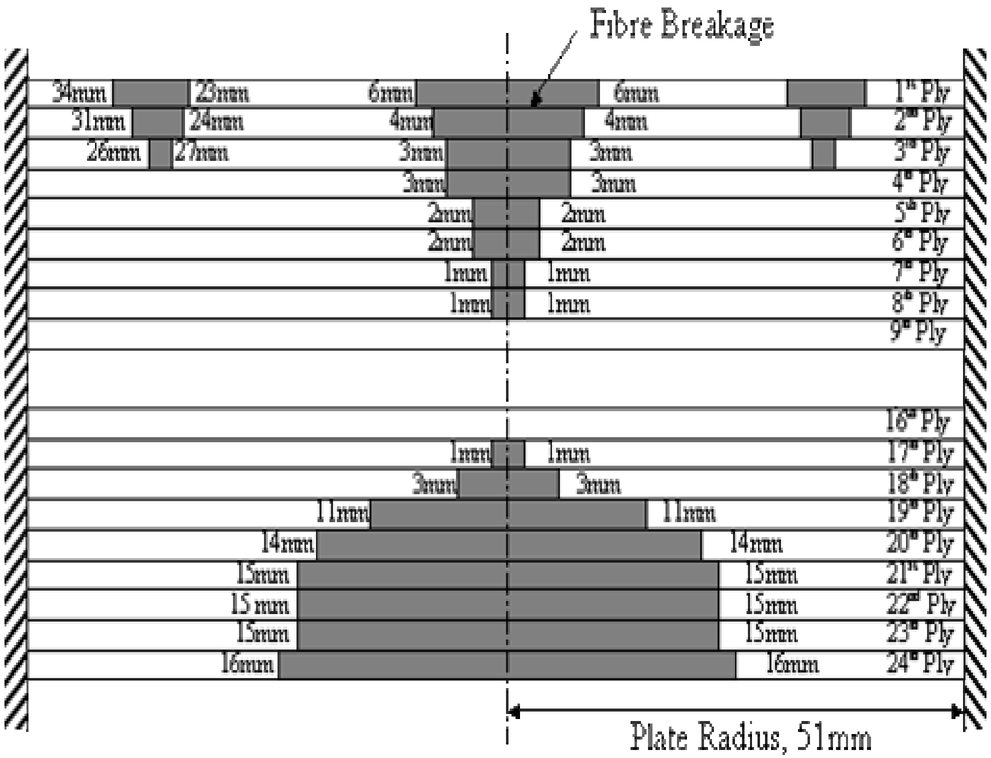
(b) Bending strain at the bottom surface

Comparison between predicted and measured strains: (a) at the top surface and (b) bottom surface of a 3mm thick IM7/8552 $[45/-45/0/90]_{3s}$ circular plate

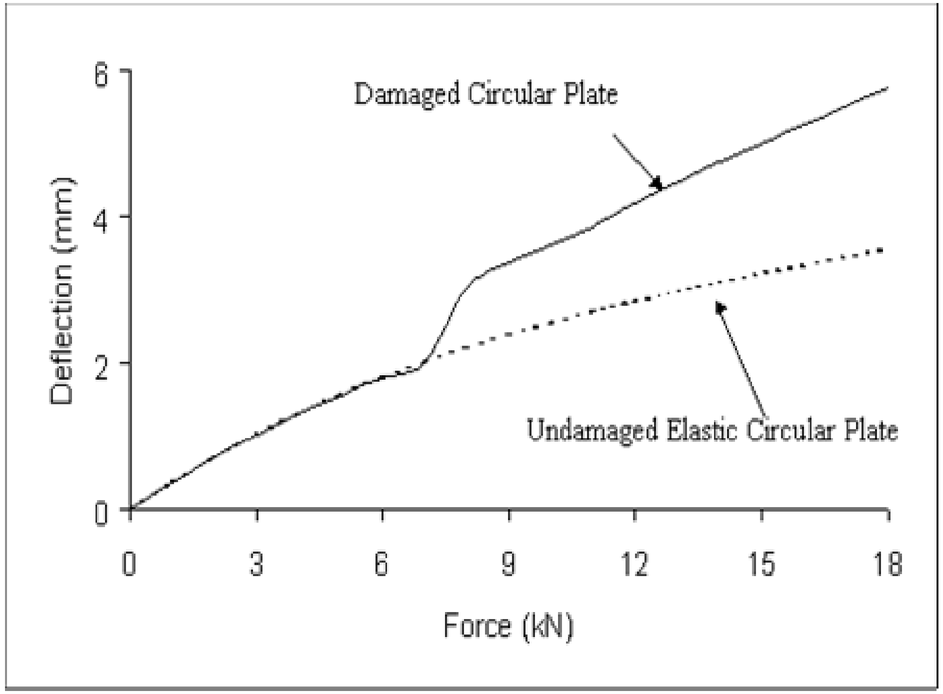
- Damage Area Predictions in a 3mm Thick Circular Plate

(a) Through-thickness (Z-r plane) fibre damage

(Degraded Elastic Moduli: E_x = 19.8 GPa, E_y = 19.8 GPa, G_{xy} = 7.5 GPa, ν_{xy} = 0.31 and ν_{yx} = 0.31)

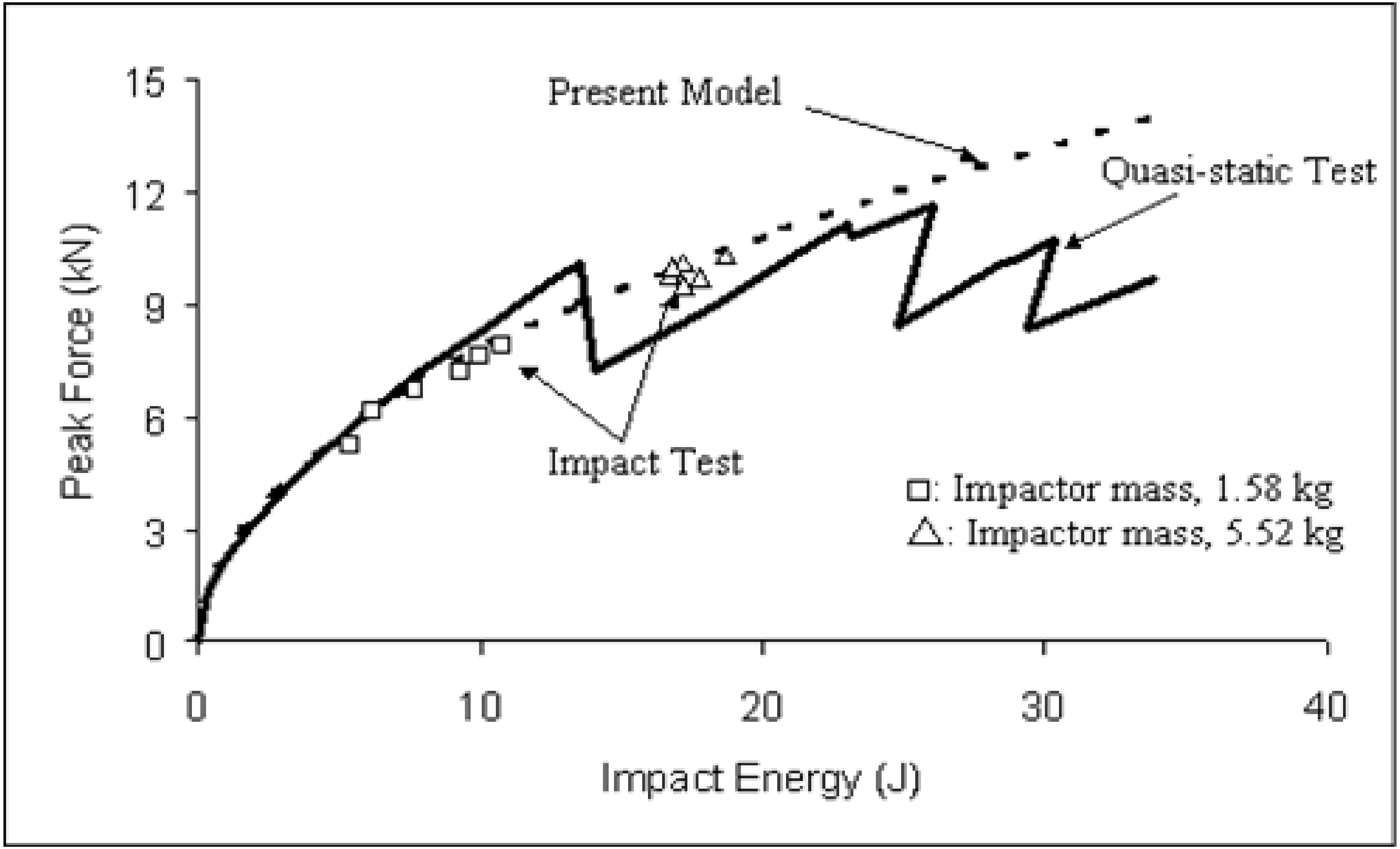


(b) Central deflection versus applied force



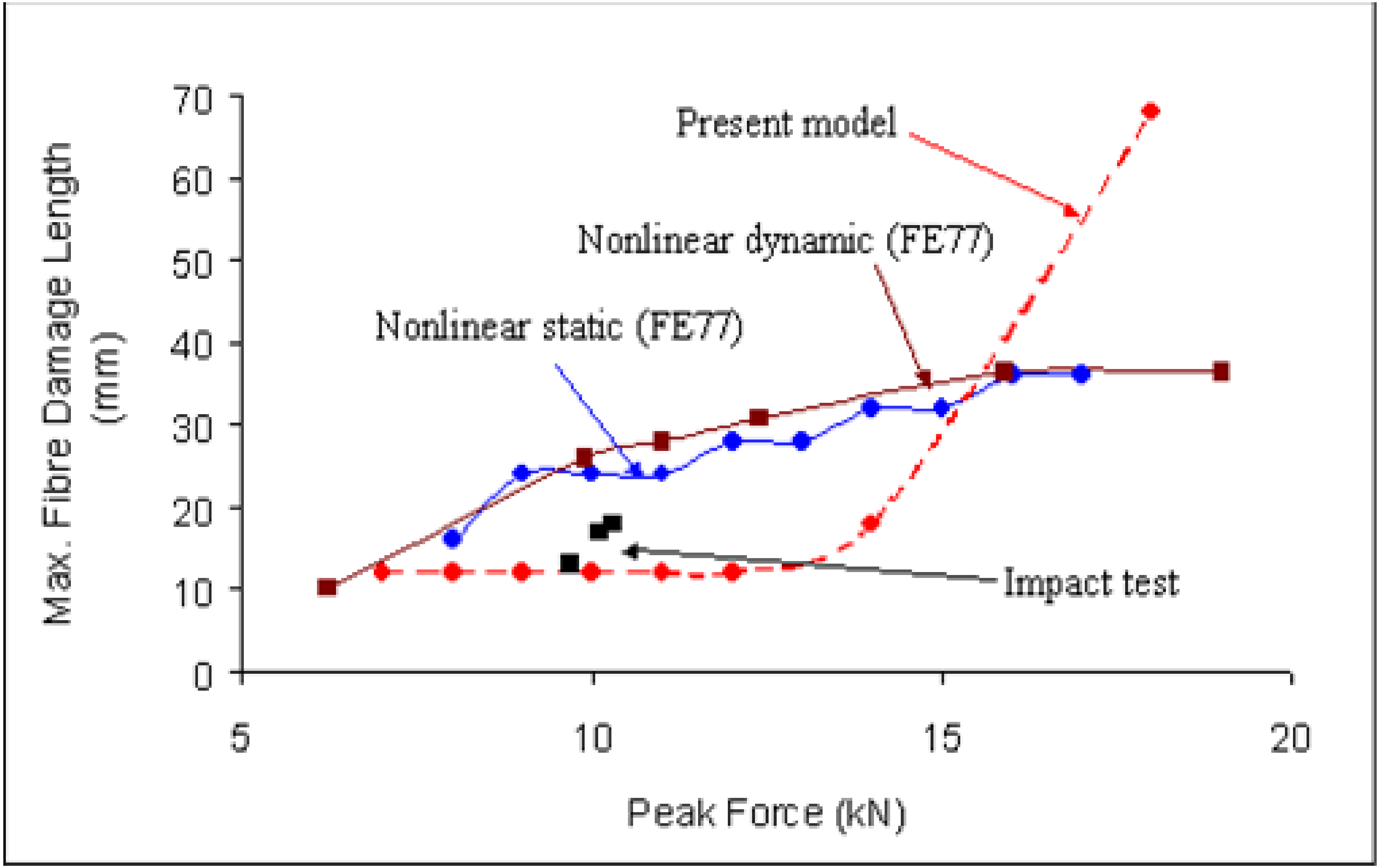
<u>Through-thickness (Z-r plane) fibre damage in a 3mm thick $[45/-45/0/90]_{3s}$ circular plate; applied load 18 kN (a) and central deflection versus applied force (b)</u>

- Peak Force Comparison

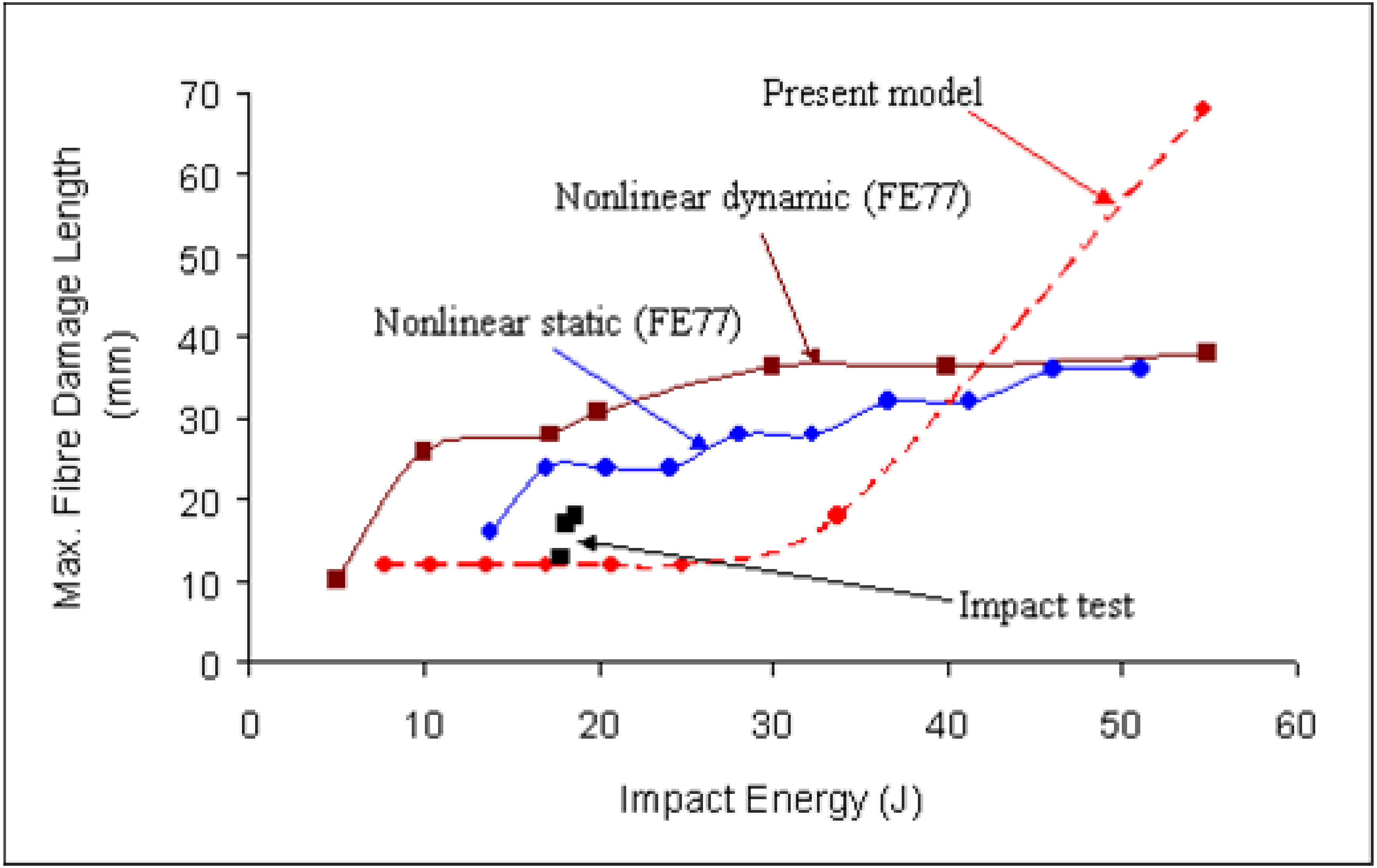


Peak contact force against impact energy, predicted and measured values, for clamped IM7/8552 $[45/-45/0/90]_{3s}$ circular plates of 102 mm diameter.

- Damage Area Comparison between Model, FE and Experimental Results



(a) Fibre damage length versus peak force



(b) Fibre damage length versus impact energy

Impact damage in an IM7/8552 ($[45/-45/0/90]_{3s}$ circular plate. (a) Fibre damage length versus peak force and (b) Fibre damage length versus impact energy

• Concluding Remarks

- In order to predict fibre damage area, the number of failed plies and the reduction in stiffness from an impact event, an analytical model has been produced using a simple non-linear approximation method (Rayleigh-Ritz method) and the principle of virtual work .

- The model successfully predicts the central deflection of undamaged and damaged plates due to a transverse load when compared to FE values.

- The model successfully predicts strains at the top and bottom ply when compared to experimental values.

- The reduction in stiffness of the impact damage site has been predicted.

- The model presented here can save significant running time, compared with FE non-linear static or dynamic solutions.