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Identification of the elastic constants of composite materials using deflectometry

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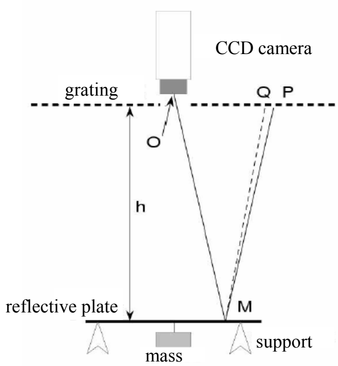
Design of a simple mechanical test for identifying the elastic parameters of anisotropic materials.

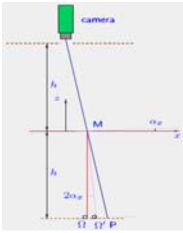
Bending tests on square plates → Heterogeneous response involving all the unknown parameters → Full-field curvature measurement → Application of the Virtual Fields Method ⇨ Stiffnesses

→ DIFFICULT TASK

Proposed solution: direct full-field slope measurements with deflectometry.

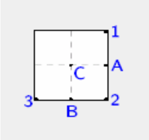
→ Only one differentiation is needed for measuring curvatures.



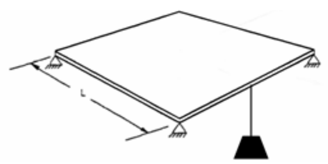


- **Performances**
  - Grid period: 8 mm ± 8 µm ;
  - Distance grid / specimen: 1.1 m ± 1 mm
  - Resolution: 7.3 µrad (73 nm per cm)

Three types of loading:

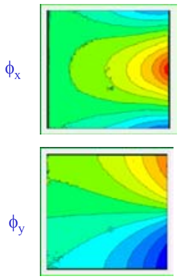


example loading in A:

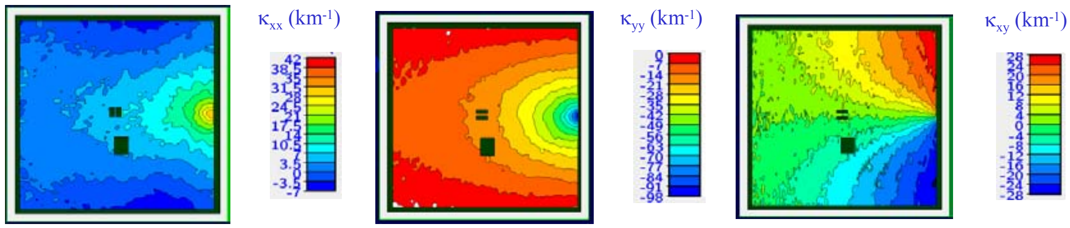


- **Measurement of curvatures**
  - best linear fit over 3 x 3 pixels subimages
  - Resolution : ~ 1 km^{-1}

Slope fields:



Curvature fields:



Identification results

- **Virtual fields method**
  - Constitutive equation (orthotropic elasticity)

$$\begin{pmatrix} M_x \\ M_y \\ M_s \end{pmatrix} = \begin{bmatrix} D_{xx} & D_{xy} & 0 \\ D_{xy} & D_{yy} & 0 \\ 0 & 0 & D_{ss} \end{bmatrix} \begin{pmatrix} \kappa_x \\ \kappa_y \\ \kappa_s \end{pmatrix}$$

  - Principle of virtual work

$$\underbrace{-\int_S \vec{M}.\vec{\kappa}^* \, dS}_{\text{Internal virtual work : } W_i^*} + \underbrace{\sum_{i=1}^{n} F_i . w_i^*}_{\text{External virtual work : } W_e^*} = 0$$

  - Choice of virtual fields :

1: $\kappa_x^* = 1, \kappa_y^* = 0, \kappa_s^* = 0$
2: $\kappa_x^* = 0, \kappa_y^* = 1, \kappa_s^* = 0$
3: $\kappa_x^* = 0, \kappa_y^* = 0, \kappa_s^* = 1.$

  - Equations :

1: $D_{xx} k_{xm} + D_{xy} k_{ym} = m_1 g w_1^*$
2: $D_{xy} k_{xm} + D_{yy} k_{ym} = m_1 g w_2^*$
3: $D_{ss} k_{sm} = m_1 g w_3^*$

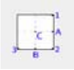
Engineering constants :

$$E_{xx} = \frac{D_{xx}}{e^3/12}(1 - \nu_{xy}\nu_{yx})$$

$$E_{yy} = \frac{D_{yy}}{e^3/12}(1 - \nu_{xy}\nu_{yx})$$

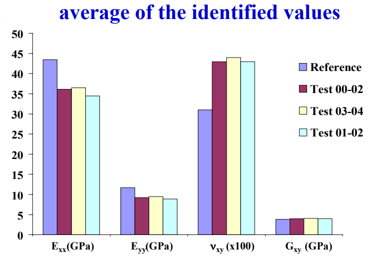
$$G_{xy} = \frac{D_{ss}}{e^3/12} \quad \nu_{xy} = \frac{D_{xy}}{D_{yy}} \quad \nu_{yx} = \frac{D_{xy}}{D_{xx}}$$

Orthotropic material : 2 tests required (glass-epoxy unidirectional laminate)



00 = (m_1,0,0),
01 = (0,m_1,0),
02 = (0,0,m_1),
03 = (m_1,0, m_1),
04 = (0, m_1,m_1)
m_1 = 90.9 g

average of the identified values



- **Summary**
  - Deflectometry: robust, sensitive, unexpensive…
  - Scatter and differences not totally understood: need for more insight in measurements.
  - VFM: convenient way to process FFM.

- **Further work**
  - Extension to thick plates: identification of G_{xz} and G_{yz}.
  - Use heterogeneous virtual fields.
  - Application to damping measurements by harmonic inertial loading →