This paper contains 5 questions. Answer all questions.
You may attempt questions in any order.

Mark allocations are indicated next to each question.
The maximum for this paper is 50 marks.
Q1  (a) Express \((3 - \sqrt{5})^2\) in the form \(m + n\sqrt{5}\), where \(m\) and \(n\) are integers.  

(b) Hence express \(\frac{(3-\sqrt{5})^2}{1+\sqrt{5}}\) in the form \(p + q\sqrt{5}\), where \(p\) and \(q\) are integers.

Q2  The line \(AB\) has equation \(3x + 2y = 7\). The point \(C\) has coordinates \((2, -7)\).

(a)  i. Find the gradient of \(AB\).  

ii. The line which passes through \(C\) and which is parallel to \(AB\) crosses the \(y\)-axis at the point \(D\). Find the \(y\)-coordinate of \(D\).

(b) The line with equation \(y = 1 - 4x\) intersects the line \(AB\) at the point \(A\). Find the coordinates of \(A\).

(c) The point \(E\) has coordinates \((5, k)\). Given that \(CE\) has length 5, find the two possible values of the constant \(k\).

Q3  The triangle \(ABC\), shown in figure 1, is such that \(AB = 5\text{cm}\), \(AC = 8\text{cm}\), \(BC = 10\text{cm}\) and angle \(BAC = \theta\).

(a) Show that \(\theta = 97.9^\circ\), correct to the nearest 0.1\(^\circ\).

(b)  

i. Calculate the area of triangle \(ABC\), giving your answer, in \(\text{cm}^2\), to three significant figures.

ii. The line through \(A\), perpendicular to \(BC\), meets \(BC\) at the point \(D\). Calculate the length of \(AD\), giving your answer, in \(\text{cm}\), to three significant figures.
Q4 The functions f and g are defined with their respective domains by

\[ f(x) = x^2 \quad \text{for all real values of } x \]
\[ g(x) = \frac{1}{2x + 1} \quad \text{for all real values of } x, x \neq -0.5 \]

(a) Explain why f does not have an inverse. \[1 \text{ mark}\]

(b) The inverse of g is \( g^{-1} \). Find \( g^{-1}(x) \). \[3 \text{ marks}\]

(c) State the range of \( g^{-1} \). \[1 \text{ mark}\]

(d) Solve the equation \( fg(x) = g(x) \). \[3 \text{ marks}\]
Q5 Figure 2 shows part of a curve crossing the $x$-axis at the origin $O$ and at the point $A(8,0)$. Tangents to the curve at $O$ and $A$ meet at the point $P$, as shown.

![Figure 2]

The curve has equation

$$y = 12x - 3x^{\frac{5}{3}}$$

(a) Find $\frac{dy}{dx}$.  

[2 marks]

(b) i. Find the value of $\frac{dy}{dx}$ at the point $O$ and hence write down an equation of the tangent at $O$.  

[2 marks]

ii. Show that the equation of the tangent at $A(8,0)$ is $y + 8x = 64$.  

[3 marks]

(c) Find $\int \left(12x - 3x^{\frac{5}{3}}\right) \, dx$.  

[3 marks]

(d) Calculate the area of the shaded region bounded by the curve from $O$ to $A$ and the tangents $OP$ and $AP$.  

[7 marks]

End of paper