

## Focused Research - Number Theory in Function Fields (UK) Schedule

April 3-5, 2017 University of Bristol

### Monday, 3/4/17

<b>9:00-9:30</b>	Coffee/Tea
<b>9:30-10:15</b>	Zeev Rudnick
<b>10:15-11:00</b>	Chris Hall
<b>11:00-11:30</b>	Coffee/Tea
<b>11:30-12:15</b>	Dan Carmon
<b>12:15-14:00</b>	Lunch
<b>14:00-14:45</b>	Ezra Waxman
<b>14:45-15:30</b>	Julio Andrade
<b>15:30-15:45</b>	Coffee/Tea
<b>15:45-18:00</b>	Open Discussion

### Tuesday, 4/4/17

<b>9:00-9:30</b>	Coffee/Tea
<b>9:30-10:15</b>	Hung Bui
<b>10:15-11:00</b>	Ofir Gorodetsky
<b>11:00-11:30</b>	Coffee/Tea
<b>11:30-12:15</b>	Brad Rodgers
<b>12:15-14:00</b>	Lunch
<b>14:00-14:45</b>	Alexandra Florea
<b>14:45-15:30</b>	Tom van Overbeeke
<b>15:30-16:00</b>	Coffee/Tea
<b>16:00-18:00</b>	Open Discussion
<b>18:30-21:00</b>	Conference dinner

### Wednesday, 5/4/17

<b>9:00-9:30</b>	Coffee/Tea
<b>9:30-10:15</b>	Lior Bary-Soroker
<b>10:15-11:00</b>	Efrat Bank
<b>11:00-11:30</b>	Coffee/Tea
<b>11:30-12:15</b>	Alexei Entin
<b>12:15-14:00</b>	Lunch
<b>14:00-14:45</b>	Patrick Meisner
<b>14:45-15:15</b>	Coffee/Tea
<b>15:15-17:00</b>	Open Discussion

## 1. TITLES AND ABSTRACT

Zeev Rudnick

Title: Angles of Gaussian primes.

Abstract:

Fermat showed that every prime  $p = 1 \pmod{4}$  is a sum of two squares:  $p = a^2 + b^2$ , and hence such a prime gives rise to an angle whose tangent is the ratio  $b/a$ . Hecke showed, in 1919, that these angles are uniformly distributed, and uniform distribution in somewhat short arcs was given in by Kubilius in 1950 and refined since then. I will discuss the statistics of these angles on fine scales and present a conjecture, motivated by a random matrix model and by function field considerations. (joint work with Ezra Waxman).

Ezra Waxman

Title: Direction of Gaussian Primes for the Rational Function Field.

Abstract:

Hecke proved that the Gaussian primes are equidistributed across sectors of the complex plane, by making use of (infinite) Hecke characters and their associated L-functions. In this talk I will present a function field model for the problem of the distribution of angles of Gaussian primes. The model will yield an analogue to Hecke's equidistribution theorem. By applying a recent result of N. Katz concerning the equidistribution of "super even" characters (the function field analogues to Hecke characters) I will provide a result for the variance of function field Gaussian primes across sectors; a computation whose analogue in number fields is unknown beyond a trivial regime.

Alexandra Florea

Title: Densities in the hyperelliptic ensemble.

Abstract:

I will talk about the 1-level density and the pair correlation of zeros of quadratic Dirichlet L-functions over function fields. I will explain how one can obtain several lower order terms for both of these statistics, when the Fourier transform of the test function is sufficiently restricted.

Dan Carmon

Title: Computing the Galois group for the Bateman-Horn extension via existence of transpositions.

Abstract:

Let  $F_i(t, x) \in \mathbb{F}_q[t][x]$  be fixed irreducible polynomials in two variables. An analogue of the Bateman-Horn problem over the rational function field is to evaluate the asymptotic probability that  $F_i(t, f(t))$  are all simultaneously irreducible, where  $f(t)$  ranges over all polynomials in  $t$  of degree at most  $n$ , as  $q^n$  tends to infinity. For  $n$  fixed and  $q$  large, the problem has been solved by Entin under certain restrictions on  $n$  and the polynomials  $F_i$ . We were able to simplify and extend Entin's results to a larger family of  $F_i$  and  $n$ , by directly proving that the relevant monodromy group contains a transposition.

The talk will begin with a review of Entin's previous results and methods, followed by details of the proof that the monodromy contains transpositions.

Brad Rodgers

Title: Decompositions of arithmetic functions and sums against characters.

Abstract:

In a function field setting, we will review some of the interesting phenomena that come into play when analyzing the statistics of sums of arithmetic functions in random short intervals, random sparse arithmetic progressions, and against random characters, along with a decomposition of arithmetic functions that helps elucidate some of this phenomena.

Alexei Entin

Title: Decomposition statistics over function fields, a geometric view.

Abstract:

we show how many recent results concerning the decomposition statistics of polynomials and divisors over  $F_q$  in the  $q \rightarrow \infty$  limit can be approached through a unified geometric framework. This allows to easily reproduce, strengthen and generalize these results.

Efrat Bank

Title: Primes in short intervals on curves over finite fields.

Abstract:

We prove an analogue of the Prime Number Theorem for short intervals on a smooth proper curve of arbitrary genus over a finite field. Our main result gives a uniform asymptotic count of those rational functions, inside short intervals defined by a very ample effective divisor  $E$ , whose principal divisors are prime away from  $E$ . In this talk, I will discuss the setting and definitions we use in order to make sense of such count, and will give a rough sketch of the proof. This is a joint work with Tyler Foster.

Hung Bui

Title: Hybrid Euler-Hadamard formula and moments of L-functions

Abstract: Keating and Snaith used random matrix theory to predict the moments of various families of L-functions. However, the arithmetic factors were missing in the random matrix model and had to be added in an ad-hoc way. In this talk, we shall discuss the model of Gonek, Hughes and Keating where the arithmetic factors appear naturally.

Patrick Meisner

Title: Number of Points on Curves over Finite Fields

Abstract: Classical results due to Katz and Sarnak show that the number of points on a curves in families over  $F_q$  is distributed as the trace of a random matrix if we fix the genus and let  $q$  tend to infinity. We discuss what happens if we fix  $q$  and let the genus,  $g$ , of the curve tend to infinity.

Julio Andrade

Title: Truncated Product Representations for L-functions in Function Fields.

Abstract: In this talk, I will describe some recent results involving approximations of quadratic Dirichlet L-functions over function fields by truncations of their Euler products. This is a joint work with Steve Gonek and Jon Keating.

Ofir Gorodetsky

Title: New Results on Shifted Correlation of Functions on  $F_q[T]$ .

Abstract: We present new large- $q$  results on the the number of twin primes in the ring  $F_q[T]$ . More generally, our results concern shifted correlation of a wide class of arithmetic functions on  $F_q[T]$ . Our methods involve L-Functions, and some old and new equidistribution results concerning the relevant L-Functions and their zeroes. This is joint work with Will Sawin.

Lior Bary-Soroker

Title: Twin Prime Polynomials

Abstract: In recent years, an asymptotic formula for the number of  $N=N(n,q)$  of twin prime polynomials of the form  $f, f+h$  was established when  $q$  is much larger than  $n$  (here  $f$  is taken from the set of monic polynomials of degree  $n$  over the finite field with  $q$  elements):  $N = q^n/n(1 + O(q^{-1/2}))$ . This formula agrees with the standard heuristic, but the downfall is that it encodes no arithmetic information of  $h$ . The focus of the talk is to discuss the way the arithmetic information is encoded in the error term and to suggest an approach to understand better the error term. Some results for small  $n$  will be presented.

Chris Hall

Title: Variance of Sums in Arithmetic Progressions and Higher Degree L-functions

Abstract: The von Mangoldt function is one of several arithmetic functions one can attach to the Riemann zeta function, and we would like to understand how sums of it behave in arithmetic progressions. Keating and Rudnick considered an analogous problem over  $F_q(t)$  and proved that the variance of the sums (across conjugacy classes) exhibited dichotomous behavior. Their results pertain to the zeta function of the affine line  $A^1/F_q$ , a rank-one L-function, and one can consider analogues of the von Mangoldt function (and sums in arithmetic progressions) for higher-rank L-functions. We will describe these analogues and state results for the variance of the sums. This is joint work with J.P. Keating and E. Roditty-Gershon.

Tom van Overbeeke

Title: The Euler totient function in short intervals

Abstract: In this talk we study the variance of the Euler totient function (normalized to  $\phi(n)/n$ ) in the integers  $\mathbb{Z}$  and the polynomials  $F_q[T]$ . It turns out that in  $\mathbb{Z}$ , under some assumptions, the variance of the normalized Euler function becomes constant. This is supported by several numerical simulations. Surprisingly, in  $F_q[T]$ , the analogue does not hold: due to a high amount of cancellation, the variance becomes much smaller and dependent on the size of the interval.