Combinatorial algebraic geometry comprises studying algebro-geometric objects where geometric phenomena can be described by combinatorial data. Therefore, the study of these structures encompasses toric and tropical geometry.

Tropical geometry is a fast-growing field, and has found applications in many areas of pure and applied mathematics; see [MS15]. Tropical objects can be often obtained a certain degeneration of algebraic objects and, therefore, considered as combinatorial shadows of algebraic geometry objects. These combinatorial shadows are often simpler to study, yet retain an ample amount information from the geometry and topology of initial objects. See, for instance, the foundational works [Mik04, Mik05] and surveys [Vir08, IM12] on how this degeneration of algebraic varieties have played a fundamental role in application of tropical geometry to enumerative problems in algebraic geometry. See also [HK12, AHK18] for applications of tropical algebraic geometry in resolving forty-year-old problems in graph theory and combinatorics.

Further, tropical geometry can be viewed as the geometry over max-plus algebra, $(\mathbb{T}, \otimes, \oplus)$, where $\mathbb{T} = \mathbb{R} \cup \{-\infty\}$, and $\otimes$ and $\oplus$ are the usual sum and maximum, respectively. These operations can be obtained from the following multiplication and addition through the Maslov dequantisation: for $x, y \in \mathbb{R}$, let $x \otimes_h y := x + y$ and $x \oplus_h y := h \ln(\exp(x/h) + \exp(y/h))$, as $h \to 0$. See [Lit05] for a beautiful survey of this topic touching on questions in thermodynamics, classical and quantum physics, and probability.

Very recently, it has been observed that many ideas in tropical geometry can be reviewed by analysing the dynamics of the map

$$\Phi_m : (\mathbb{C}^*)^n \to (\mathbb{C}^*)^n$$

$$(z_1, \ldots, z_n) \mapsto (z_1^m, \ldots, z_n^m),$$

as the positive integer $m \to \infty$. The reader might consider the case $n = 1$, and observe that for any $z \in \mathbb{C}^*$, the sets $\{\Phi_m^{-1}(z)\}$, ‘converges’ to the same object! Analysis of the dynamics of $\Phi_m$, naturally falls into the realm of complex dynamics and analytic geometry, and the interplay between analytic geometry and tropical geometry has been mainly investigated through the notions of tropical currents. See for instance [Lag12, Bab14, BH17, Gub13, GK17, AB19, BGJ+21]. Our current PhD projects include, but not limited to, exploring the relations between tropical dynamics, and any of the following,

- Tropical and Toric Intersection Theory [Stu96, HS95, FS97, Mih21];
• Tropical Differential Algebra [Gri17, FGLH+20];
• Combinatorial Hodge Theory [HK12, AHK18];
• Random Holomorphic Dynamics [BCHM18].

Suitable candidates must be interested in algebraic geometry and differential geometry, but we do not generally expect any background in toric and tropical geometry in the beginning.

References


