COMPARISON THEORY FOR MCMC ALGORITHMS

(with application to Doubly-Intractable Posteriors)

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1. INTRODUCTION

Bayesian statistics is a popular framework for carrying out statistical inference wherein uncertainty about parameters is encoded in a probability distribution called the posterior distribution. In all but the simplest cases, the posterior distribution does not lie in a standard parametric family; as such, in order to answer inferential questions, it is necessary to make use of numerical methods.

Among these methods, perhaps the most widely-used is Markov chain Monte Carlo (MCMC), a class of stochastic algorithms for generating approximate samples from the posterior distribution through an iterative approach (see e.g. [4] for some introduction). Given approximate samples of sufficient quality, one can then use these to form { statistical estimators, uncertainty sets, etc. }. It is thus of key practical and theoretical interest to identify algorithms in this class which can converge effectively, with rigorous complexity bounds depending favourably on problem parameters (e.g. dimension, number of observations, etc.).

For ‘nice’ statistical models in which covariates and observations are fully-observed, there are several ‘default’ MCMC approaches for which convincing complexity analyses are now available, see e.g. [2] for some overview. Nevertheless, there remains a variety of relevant statistical models which lack this tractability, which cover diverse applications in epidemiology, network analysis, spatial statistics, and beyond. These ‘intractable’ models have necessitated the development of more refined MCMC solutions, for which quantitative theoretical validation has thus far been more limited.

2. MINI-PROJECT

The plan for this mini-project would be to undertake a theoretical study of MCMC algorithms for posterior simulation in a specific class of intractable models, known variously as the “doubly-intractable” or “unnormalised likelihood” setting (see e.g. Chapter 5 of [3]), with a goal of obtaining quantitative guarantees on the convergence properties of some specific algorithms.

Recent work [1] has demonstrated that for MCMC algorithms developed for a related class of intractable models (so-called ‘pseudo-marginal’ MCMC), the technique of Markov chain comparison theorems offers an attractive route to proving robust and interpretable estimates on their convergence behaviour. For the present project, we would hope to adapt and extend these theoretical tools to handle models with unnormalised likelihoods, and the algorithms designed for that setting.
3. **Supervisor**

My research interests centre around the design and analysis of stochastic algorithms, with applications to problems in modern statistics and machine learning. I am particularly interested in Monte Carlo methods, such as Markov Chain Monte Carlo and Sequential Monte Carlo. I enjoy trying to understand the convergence and complexity behaviour of these algorithms, with the hope that the theoretical insights gained in doing so can guide their practical implementations to be more automatic, robust, and efficient.

**References**


