Watering a sports field

(by Martin Homer)

This is an open-ended challenge, there are no right or wrong answers, it can be used as a brainstorming exercise or carried out over a prolonged period of time, like a project.

During a long, hot summer, cricket fields need regular watering. To make sure the grass is in optimal condition, your local club wants to use an automatic pop-up sprinkler system. But how should the sprinklers be laid out to ensure the best possible watering for least cost and inconvenience?

Ideally, every bit of the field would be watered equally, and the club particularly wants to avoid over- and under-watering the grass. Wasting water would be bad, too.

Most pop-up sprinklers can water either a whole circle, or a segment of a circle. You can assume that the club opts for a sprinkler with an 11m coverage radius.

Solution

First of all, what size and shape is your cricket field? Choose something simple to start with, and you can always make it more complicated later. There aren't any rules to precisely define the size of a field, but you should be able to find a good starting point online. It's probably easiest to start with a circular field.

What different layouts for the sprinkers could you choose? It's probably best to start with regular ones first. How many do you need to guarantee the whole field is watered?

To make the problem easier, it might be worth thinking about shapes that tessalate, in a way that circles don't. Squares and hexagons are an obvious starting point, since both tessalate. There are two sizes that might be worth considering:

1. the biggest square/hexagon that fits inside the sprinkler circle (*circumcircle*),
2. the smallest square/hexagon that fits outside the sprinkler circle (*incircle*).

How big do they need to be in each case? And how many do you need to cover the field?

The first guarantees that the whole cricket field will be watered (but with overlaps), while the second will leave gaps between the watered circles. Neither is ideal, though: overlaps result in over-watering, and wasted water, while gaps mean that some grass will not be watered at all. Can you measure the overlaps/gaps between the watered circles? How much area is overwatered/missed? It might help to work out the areas of the circle, inscribed, and escribed squares and hexagons.

Is there a compromise solution between the fully covered solution, and the one with gaps?

What other factors might come into play in practice?