

The payday loan 1

(by Alan Champneys)

Loans-R-Us, a so-called pay-day loan shop, is offering three different interest rates

(a) 3500% per year, (b) 35% per month, (c) 1% per day,

Interest is charged at the beginning of each period. So that if I borrow £1 for just one hour, under interest rate (a) I would pay £36, under (b) I would pay .35, and under (c) I pay £1.01.

Each time interest is charged, it is calculated on the whole amount owed, including any unpaid interest.

Calculate the total amount I would owe under each interest rate, if I were to borrow £1 for a whole year, under each interest rate scheme, assuming that I don't pay anything back until the end of the year.

Which interest rate should I choose?

Solution

This question is designed so that some students will jump to the wrong to the wrong conclusion, as follows:

Paying back 1% of £1 each day will result in a penny a day or £3.65 over a year. Paying back 35 pence per month will result in £4.20 over a year, whereas the high annual rate will result in £35 over a year. So clearly the daily rate is best, right?

But, of course, compound interest does not work like that. Using interest rate (a) over one year, clearly I pay back a total of

$$(a): = £36 = £35 \text{ interest} + £1$$

Instead, under (b) I owe £(1+0.35) in the first month and £[(1+0.35)+0.35(1+0.35)] in the second month. So this is a total of £(1+0.35)(1+0.35) = 1.82 after two months, not 1 + 0.35 + 0.35 = 1.70. So after two months I owe £(1 + 0.35)². Thus after 12 months I owe

$$(b): £(1 + 0.35)^{12} = £36.64.$$

Proceeding similarly for (c) after 365 days I owe

$$(c): £(1 + 0.01)^{365} = £37.78.$$

So, actually, the annual interest rate gives me the least to pay back.

The payday loan 2

(by Alan Champneys)

You should have found that the amounts you pay back under each rate are:

$$(a) \text{ 3500\% per year Total} = \pounds 36, \quad (1)$$

$$(b) \text{ 35\% per month Total} = \pounds(1 + 0.35)^{12} = \pounds 36.64, \quad (2)$$

$$(c) \text{ 1\% per day Total} = \pounds(1 + 0.01)^{365} = \pounds 37.78. \quad (3)$$

So you should choose option (c) even though it looks like the worst deal.

This is because of the exponential growth of compound interest, and shows how so-called "pay day loan" companies prey on the poor and vulnerable to make huge profits. It is also why, by law, all loans must specify an equivalent *Annual Percentage Rate* (APR). This calculation shows that the APR of a loan that is advertised as '1% per day' is actually 3678 % , which does not sound so appealing

But it gets worse. Suppose I choose rate (c) and borrow $\pounds 1$ for 5 years. How much would I owe at the end of 5 years?

What if I were to only pay back after 10 years? where could I find such money from?

Solution

If I kept things going for 5 years, I would then owe

$$\pounds(1 + 0.01)^{5 \times 365} = \pounds 777.003 \times 10^7 \approx \pounds 7770 \text{ million}$$

Which is quite a lot of money based on a $\pounds 1$ loan and a “1%” interest rate! Actually, because of leap years, its actually slightly more than this either

$$(1 + 0.01)^{(4 \times 365) + 366} = 777 \text{ million} \quad \text{or} \quad (1 + 0.01)^{(3 \times 365) + (2 \times 366)} = 785 \text{ million} .$$

And after 10 years (assuming two leap years):

$$\pounds(1 + 0.01)^{3652} \approx 5.929 \times 10^{15} = 6048 \text{ billion,} \quad \text{or} \quad \pounds 6.084 \text{ quadrillion}$$

But this could never be repaid, because it is about 100 times more than the total amount of money estimated to exist in the global economy (about 100 trillion US dollars) !!

This calculation further shows the problem with “payday loan” companies.