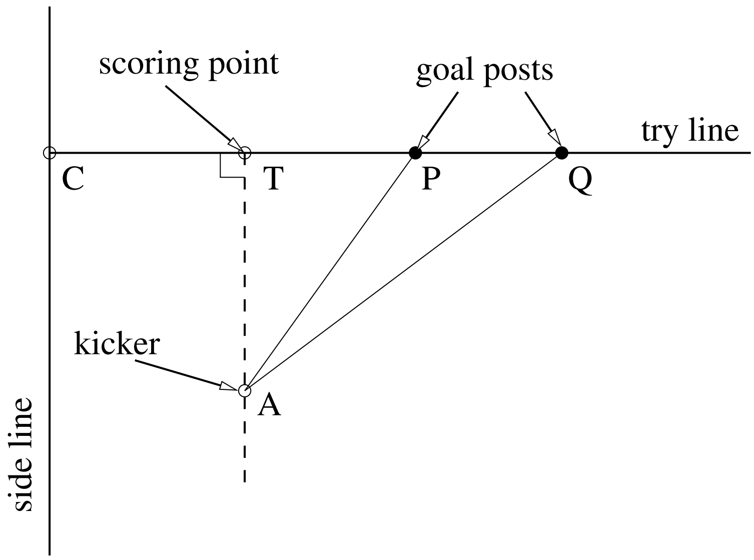
Rugby kicking 1

(by Alan Champneys)

In rugby, when a try is scored, a conversion kick has to be taken form a perpendicular line that intersects the try line where the try was scored (the dashed line in the diagram). The kicker is trying to get the ball through the posts. The kicker is free to choose the point A on the dashed line. But where should A be chosen to maximise the angle <PAQ between the goalposts?



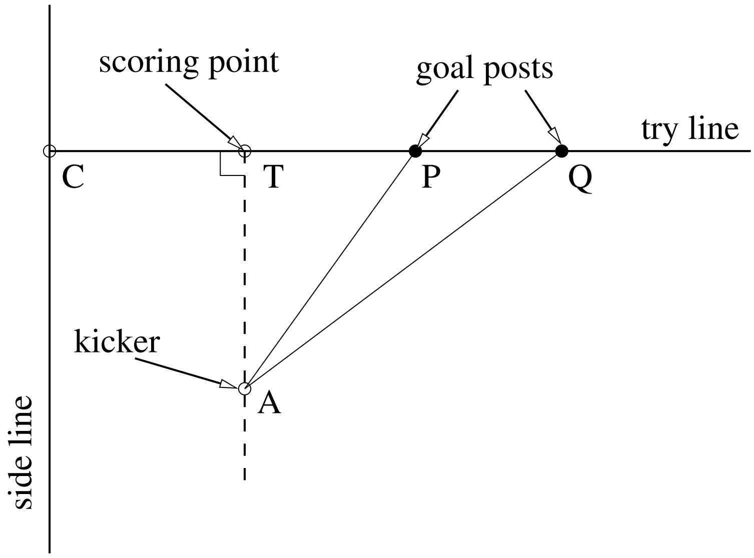
What happens to the angle <PAQ if A is chosen just next to T? What happens to the angle if A is chosen to be at the far end of the pitch?

We are going to find a formula for chosing the optimal point A.

Rugby kicking 2

(by Alan Champneys)

Let's start with some numbers. For a typical rugby pitch, the width is 70 m and the goal width PQ is 5.6 m. Given these dimensions (assuming the goal is in the middle of the try line) calculate the distances CP.



As an example, let's suppose the try is scored at 10 m from the side line. That is, CT= 10 m. Calculate the distances TP and TQ.

Suppose the kicker chooses to kick from 10 m away from the try line. That is, TA= 10 m. Use trigonometry to calculate

tan(< TAP) =   and   tan(< TAQ) =

Hence, calculate the angle <PAQ

for TA= 10 m: < PAQ =

Rugby kicking 3

(by Alan Champneys)

Now suppose the kicker instead stands 30 m away from T (with again the try being scored at TC$= 10$ m).

Repeating the calculation for TA$= 30$ m:

$\tan(< \mathrm{TAP}) =$ and $\tan(< \mathrm{TAQ}) =$

Hence

for TA$= 10$ m: $< \mathrm{PAQ} =$

Which is the better position to kick from; 10 m or 30 m?

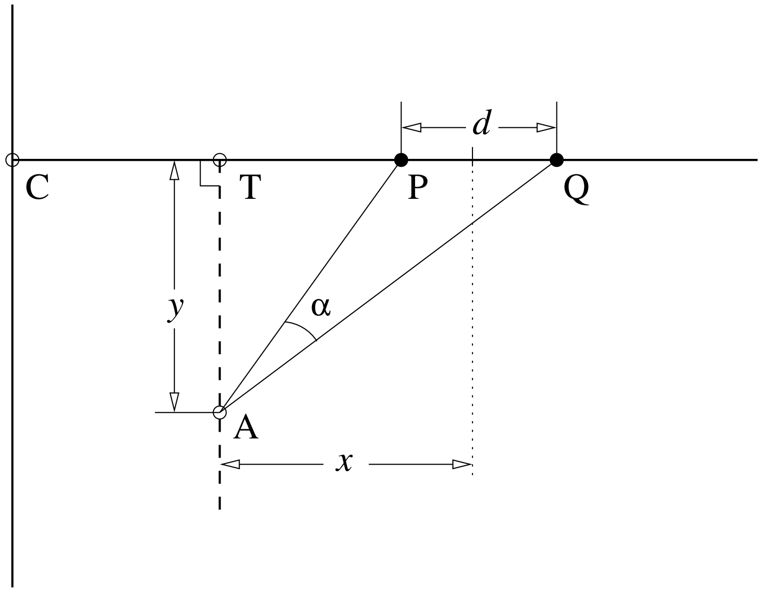
What about if they stood right at the far end of the pitch, TA$= 100$ m?

Can you see that there must be an intermediate distance (between 0 and 100 m) such that the angle <PAQ is maximised?

Rugby kicking 4

(by Alan Champneys)

Now we are going to try to generalise using algebra.



Suppose that the post width d and the distance x of T from the centre line of the pitch are fixed. We want to find the optimum value of y, which maximises α.

Previously we calculated the angle PAQ $= \alpha$ for some given values of d, x and y. Repeat the calculation to find a general expression for

$$\alpha = \arctan(\qquad\qquad\qquad\qquad) - \arctan(\qquad\qquad\qquad\qquad).$$

Rugby kicking 5

(by Alan Champneys)

The function from the previous part can be written

$$\alpha = \arctan\left(\frac{2x+d}{2y}\right) - \arctan\left(\frac{2x-d}{2y}\right)$$

Taking the realistic value for the goal width $d = 5.6\,\text{m}$, and taking $x = 25$ m (the same position of T used in parts 2 and 3) find a table of values of α for $y = 0$ m, 5 m, 10 m, 15 m, etc. up to 50 m:

y (m)	0	5	10	15	20	...
α (°)						

Plot these values on a graph of α versus y. Estimate the value of y for which α is maximised.

An A-level extension

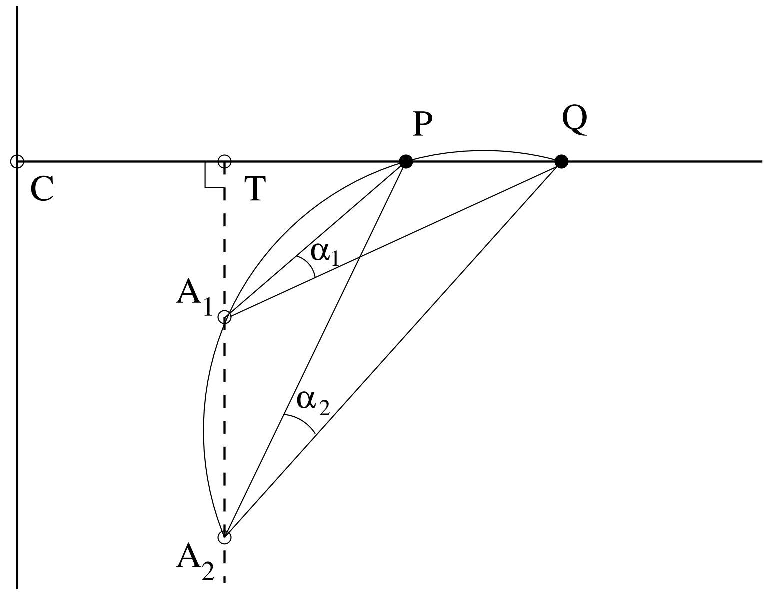
Use calculus to find the maximum of α as a function of y

But we are going to find another way ...

Rugby kicking 6

(by Alan Champneys)

We are now going to use geometry. Consider the two possible kicking positions A_1 and A_2, depicted in the diagram below.



Note that A_1 and A_2 lie on the same circle through the posts P and Q. Hence what can you say about the angles α_1 and α_2? Why?

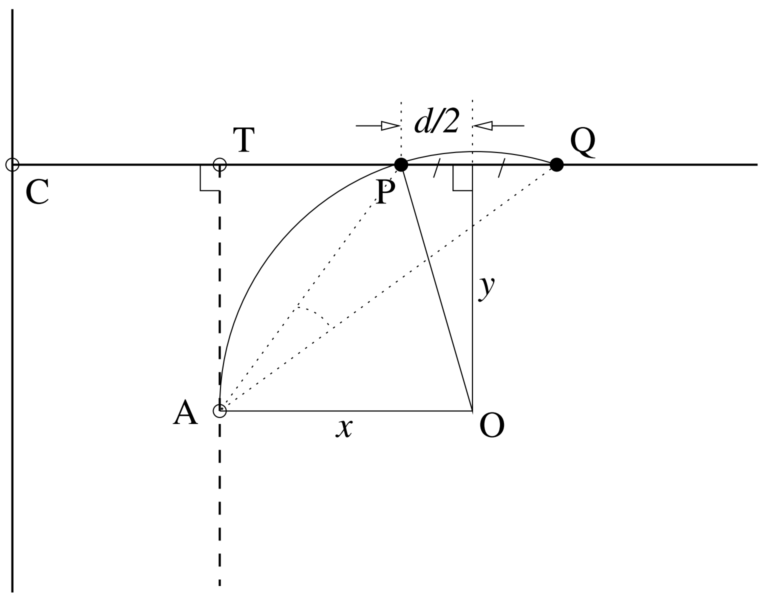
What happens to the angles α_1 and α_2 if you make the circle larger or smaller?

Can you draw the circle on which the optimum kicking point on the dashed line must lie?

Rugby kicking 7

(by Alan Champneys)

Now we are going to use this touching circle to calculate the optimal distance y. Consider this diagram



What is the distance OP?

Hence use Pythagoras' Theorem to find an expression for y.

$$y =$$

Congratulations you have found the optimal kicking distance!

What does curve of y against x look like? Can you sketch it?
What does it mean?