

Dutch book 1

(by Filippo Simini)

In sporting bets, the return of a winning bet is calculated multiplying the stake by the ‘odds multiplier’.

The following table shows the odds multipliers of two bookmakers for the same game, which has two possible outcomes: victory of the home team (Win) or defeat of the home team (Lose).

	Bookmaker 1	Bookmaker 2
Win	$w_1 = 3$	$w_2 = 2.5$
Lose	$l_1 = 1.5$	$l_2 = 2$

For example, a bet of £10 with Bookmaker 1 on the victory of the home team would return $£10w_1 = £30$ if the home team wins (and zero if the home team lose). Note that odd ratios are always larger than 1: $l_1, l_2, w_1, w_2 > 1$.

*Suppose you have £1 to bet and you think the home team will lose the game, which Bookmaker should you pick to maximise your return?
How much will you gain if you win, and what will be your loss if you lose?*

(Note that here we are ignoring any betting fee).

Dutch book 2

(by Filippo Simini)

Consider the odds ratios of part 1:

	Bookmaker 1	Bookmaker 2
Win	w_1	w_2
Lose	l_1	l_2

where $w_1 > w_2$ and $l_2 > l_1$.

Suppose you have £1 to bet, but you are not sure of the outcome of the game, so you decide to bet on both outcomes.

Assume you bet £ x on the victory of the home team and £ $y = (1 - x)$ on the defeat of the home team.

What would be your total gain (or loss) if the home team wins? And if it loses?

Dutch book 3

(by Filippo Simini)

In gambling, a “Dutch book” is a set of odds and bets which guarantees a profit, regardless of the outcome of the gamble.

Assuming you bet $\pounds x$ on the victory of the home team and $\pounds(1 - x)$ on the defeat of the home team, your have

$$R_w = w_1x - 1$$

if the home team wins, and

$$R_l = l_2(1 - x) - 1$$

if the home team loses.

In this situation, is it possible to create a Dutch book and gain some money irrespective of the outcome of the game?

Under which conditions on the odds ratios w_1 and l_1 would it be possible to find a value of x which always guarantees a profit?