The payday loan

(by Alan Champneys)

Loans-R-Us, a so-called pay-day loan shop, is offering three different interest rates

(c) 3500% per year, (a) 35% per month, (b) 1% per day,

Interest is charged at the begining of each period. So that if I borrow £1 for just one hour, I would pay pay £1.01 if I were to choose interest rate (a), £1.35 under rate (b), and £37 under rate (c). Each time interest is charged, it is calculated on the whole amount owed, including any unpaid interest.

Calculate the total amount I would owe under each interest rate, if I were to borrow £1 for a whole year, under each interest rate scheme. Which interest rate should I choose?

Suppose I choose rate (c) and borrow £1 for 5 years. How much would I owe at the end of 5 years?

What if I were to only pay back after 10 years? where could I find such money from?

Solution

This question is designed so that some students will jump to the wrong to the wrong conclusion, as follows. Paying back 1% of £1 each day wil result in a penny a day or £3.65 over a year. Paying back 35 pence per month will result in £4.20 over a year, whereas the high anual rate will result in £35 over a year. So clearly the daily rate is best, right?

But, of course, compound interest does not work like that. Using interest rate (a) over one year, clearly I pay back a total of

$$\text{(a):}\ \pounds 36 \quad = \pounds 35\,\text{interest} \ + \ \pounds 1$$

Instead, under (b) I owe $\pounds(1+0.35)$ in the first month and $\pounds[(1+0.35)+0.35(1+0.35)]$ in the second month. So this is a total of $\pounds(1+0.35)(1+0.35) = 1.82$ after two months, not $1 + 0.35 + 0.35 = 1.70$. So after two months I owe $\pounds(1 + 0.35)^2$. Thus after 12 months I owe

$$\text{(b):}\ \pounds(1 + 0.35)^{12} \quad = \pounds 36.64.$$

Proceeding similarly for (c) after 365 days I owe

$$\text{(c):}\ \pounds(1 + 0.01)^{365} \quad = \pounds 37.78.$$

So, actually, the annual interest rate gives me the least to pay back. If I kept things going for 5 years, I would then owe

$$\pounds(1 + 0.01)^{5\times 365} \quad = \pounds 77.003 \times 10^7 \approx \pounds 770\,\text{million}$$

Which is quite a lot of money based on a £1 loan and a "1%" interest rate! Actually, because of leap years, its actually slightly more than this either

$$(1 + 0.01)^{(4\times 365)+366} = 777\,\text{million} \ \text{ or } \ (1 + 0.01)^{(3\times 365)+(2\times 366)} = 785\,\text{million}\ .$$

And after 10 years (assuming two leap years):

$$\pounds(1 + 0.01)^{3652} \approx 5.929 \times 10^{15} = 6048\,\text{billion}, \quad \text{or } \pounds 6.084\,\text{quadrillion}$$

But this could never be repaid, because it is about 100 times more than the total amount of money estimated to exist in the global economy (about 100 trillion US dollars) !! This calculation just shows the problem with "payday loan" companies. It is also why, by law, all loans must specify an equivalent *Anuual Percentage Rate* (APR). This calculation shows that the APR of a loan that is advertised as '1% per day' is actually 3678 % , which does not sound so appealing.