Moving Mount Fuji

(by Thilo Gross)

How many dump trucks would you need to move mount Fuji[1]?

This is a classic job-interview question and we can find an answer by a so-called a Fermi estimate: We break the answer into little pieces, each easier than the question as a whole.

The volume of a pyramid is a a third of the volume of the box into which the pyramid would fit. Fuji is a mountain, not a pyramid, but we can use this formula to roughly estimate its volume

$$V =$$

So given this volume we can estimate the approximate mass of mount Fuji

$$M =$$

How many trucks would it need to move this mass?

$$N =$$

[1] You may asume that the mountain is about 4km tall

Solution

The purpose of this worksheet is to provide a gentle introduction to Fermi estimates. The origins of this question are not completely clear but it became popular after it has been in the standard pool for job interview questions at Microsoft for several years.

We are given that Fuji is 4km high and from the photo it's about twice as wide at the base. Using the "pyramid approximation" we obtain

$$V = \frac{8 \cdot 8 \cdot 4}{3} \text{ km}^3 \approx 85 \text{ km}^3 = 85 \cdot 10^9 \text{ m}^3$$

From somewhere I remember that the density of rock is about 3 kg/l which means 3 tonnes per cubic meter. Hence

$$M \approx 255 \cdot 10^9 \text{ t}$$

As a check I googled for typical mountain masses and various sources say $3 \cdot 10^{14}$ kg, which is in very good agreement with our estimate. A large road-going dump truck can carry approximately 30 tonnes of material. So we need approximately

$$N \approx 10^{10}, \qquad = \text{about 10 billion}$$

trucks. Of course we used wild approximations, but we can be fairly sure that the result is within the right order of magnitude. The Fermi estimate shows that moving a mountain would be huge project. If Japan devoted a fraction of its GDP to this task it may be able to build the road infrastructure and a million trucks, each of which would have to do 10.000 trips to the mountain.

If you want to challenge your students you can ask them to Fermi estimate as well how many large trucks exist in the world. There are many ways in which progress can be made, for example by estimating how many truck drivers exist, or by considering that the world burns about 10 gigatonnes of fossil fuel per year and a least the coal included in this number has been on a truck at some point.

Fermi Challenge

(by Thilo Gross)

Here are three quick questions. Try to use Fermi estimates to find answers.

I want to build a classical redbrick house in Bristol. It will be a 2 bedroom property. How many bricks do I need?

$$A =$$

A plumber in London cleans out the drains of a laundrette. This yields enough small change to fill a 5l bucket. Estimate the value of the change (in pound), assuming a typical mixture of coins.

$$B =$$

Consider a city with about 1 million residents, e.g. greater Manchester. How many playgrounds could we build on an area that is as large as the combined area taken up by parking spaces in the city?

$$C =$$

Now multiply your answers

$$A \cdot B \cdot C =$$

Solution

This worksheet is intended as a competition between groups of students. It assumes that the students are already familiar with Fermi estimates, e.g. they may have previously done our worksheet “Moving Mount Fuji”.

The first question can be answered by first estimating the total length of wall (perhaps 80 m for a 6 m by 8 m house, including some meters of double wall on the front and back and interior walls), then the total area of walls (we could assume 3 m average height of walls), and then dividing by the area of the side of a brick (about 180 cm^2).

The second question is more tricky. Here it can be useful to just take whatever coins you have in your pocket. Count their value and estimate their volume.

For the third question, you can estimate the number of cars (it’s more than 1 car per 2 people in the UK.) Most of the time these cars sit around parked (about 23.5h per day) so we need at least as many parking spaces as cars, perhaps a bit more. Multiply by the area of typical parking space (ca. 15 m^2) and divide by the size of a typical playground (ca. 400 m^2).

Obviously there are no definitive answers and alternative paths to solutions are possible. The multiplication of results in the last step is done to determine the winner of the competition. If you have an odd number of groups then the group which the result of $A \cdot B \cdot C$ is the median has almost certainly the best estimate. For an even number of groups the group for which the result is closest to the geometric mean of results has almost certainly the best estimates (unless one group is particularly far off the mark, in such a case ignore that group).