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The weightless girl 1

(by Alan Champneys)

Some people say gravity is caused by the Earth's rotation. But a little thought shows that centrifugal force acts outwards not towards the ground. (Think of a wet dog spinning its body to throw off excess water).

Lauren, a girl of mass M, is standing on the equator where acceleration due Centrifugal force will make experience a weight that is slightly less than Mg. But how much less?

Some useful facts

- Centrifugal force $= mr\omega^2$, where r is distance to spin axis and ω is rotation speed in radians/s ($2\pi\times$ revs/s)
- The radius of the earth is $6371\ km$. The earth spins one revolution per 24 hours. $g =9.81$.

First, what is the earth's rotation speed in radians per second?

$\omega =$ revs/hour $=$ rad/s

Hence calculate Laruen's centrifugal force as a function of her mass M. What percentage of her gravitational weight is lost due to centrifugal force?

centrifugal force $=$ $\times M =$ % of Mg

Solution

You may need to talk about the difference between centrifugal force (which is experienced only if you are in a rotating frame) and centripetal acceleration (that enables the body to rotation to occur).

Let $\Omega = 1/24$ be Earth's rotation in revolutions per hour. We want ω, the same quantity, but in radians per second. There are 3600 seconds in an hour. So

$$\omega = \frac{2\pi\Omega}{3600}$$

Hence

$$\omega = \frac{2\pi}{24 \times 3600} = 7.2722 \times 10^{-5} \text{ rad/s}$$

Suppose Lauren's weight is M kg. The she will experience

$$\begin{aligned}
\text{centrifugal force} &= Mr\omega^2 \\
&= m6.371 \times 10^6 (7.72722 \times 10^{-5})^2 \\
&= 0.0337 \times M
\end{aligned}$$

This we should compare with her gravity force

$$\text{gravitational force} = Mg = 9.81 \times M$$

So the fraction of gravitational force that is counterbalanced by centrifugal force is

$$\frac{Mr\omega^2}{Mg} = \frac{0.0337M}{9.81M} = \frac{0.0337}{9.81} = 0.0034$$

Hence only about 0.3% of Lauren's weight is compensated by centrifugal force. Or, put another way, the centrifugal force she feels is about

$$\frac{1}{0.0034} \approx 291 \quad \text{times weaker than gravity}$$

The weightless girl 2

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So, the centrifugal force experienced by Lauren, standing at the equator, is about 291 times weaker than gravity.

Now imagine that the Earth was spinning faster. Centrifugal force increases with rotation speed, but gravity is unaffected. But how much faster would it to spin in order for Lauren to feel weightless. That is, is there a critical rotation speed ω_c for which centrifugal force exactly balances gravity.

The critical angular velocity (in radians per second) is

$$\omega_c = \qquad\qquad \text{rad/s}$$

which is $\qquad$ times faster than the earth currently spins

How long would a day last on such a fast rotating Earth?

$$\text{day length} = \qquad\qquad \text{hours}$$

Solution

To find ω_c we need to set

$$Mr\omega_c^2 = Mg$$

this gives

$$\omega_c = \sqrt{\frac{g}{r}} = \sqrt{\frac{9.81}{6.371 \times 10^6}} = 1.24 \times 10^{-3}\,\text{rad/s}$$

But dividing by the current rotation speed, this gives

$$\frac{\omega_c}{\omega_{\text{Earth}}} = \frac{1.24 \times 10^{-3}}{7.2722 \times 10^{-5}} = 17.0633$$

Thus, the Earth only need spin about 17 times faster!

Actually we could have worked this out because we needed

$$\frac{\omega_c^2}{\omega_{\text{Earth}}^2} = \frac{291}{1}$$

and $\sqrt{291} = 17.06$

On such an Earth, a day would last $24/17$ hours = 1 hour and 24 minutes.

Further information

There are many possible more realistic extensions to this problem ….

What if Lauren were standing in Bristol, namely at 51° North? What about at the North Pole?

What if Lauren was standing on the moon who's gravity is about 80 times less than Earth but who's radius is about a quarter of Earth's?

At what height (in radial distance from the centre of the Earth) can a satellite be geostationary? That is, a distance for which the sate-lite can orbit at precisely the Earth's rotation speed, with centrifugal force precisely balancing gravity, so that the satellite appears from Earth to be in the same place. (To answer this you will need to use Newton's law of gravitational force between two bodies who's centres of mass are a given distance r apart).

How would the required distance from the Earth change if the satellite were over the equator or if it were above Bristol?

Geostationary satellites are extremely important in telecommunications and in GPS systems. Can you think why?