

Dutch book 1

(by Filippo Simini)

In sporting bets, the return of a winning bet is calculated multiplying the stake by the ‘odds multiplier’.

The following table shows the odds multipliers of two bookmakers for the same game, which has two possible outcomes: victory of the home team (Win) or defeat of the home team (Lose).

	Bookmaker 1	Bookmaker 2
Win	$w_1 = 3$	$w_2 = 2.5$
Lose	$l_1 = 1.5$	$l_2 = 2$

For example, a bet of £10 with Bookmaker 1 on the victory of the home team would return $£10w_1 = £30$ if the home team wins (and zero if the home team lose). Note that odd ratios are always larger than 1: $l_1, l_2, w_1, w_2 > 1$.

Suppose you have £1 to bet and you think the home team will lose the game, which Bookmaker should you pick to maximise your return? How much will you gain if you win, and what will be your loss if you lose?

(Note that here we are ignoring any betting fee).

Solution

The Bookmaker offering the highest odds ratio for the selected outcome should be chosen. So, if $l_1 > l_2$ Bookmaker 1 should be chosen, otherwise if $l_1 < l_2$ Bookmaker 2 should be chosen.

In this case $l_2 = 2 > 1.5 = l_1$. If the home team loses, you win the bet and get $l_2 = £2$ back. So, since you paid £1 for the bet, your gain is $l_2 - 1 = £1$. If the home team wins, you lose the bet and get nothing back, so you lose the £1 of the stake.

Given that bookmakers compete with each other to attract more bets and that betters would prefer to place their bet with the bookmaker that offers the highest odds ratio (and hence the highest potential return), bookmakers tend to increase their odds ratios above the values of their competitors.

Dutch book 2

(by Filippo Simini)

Consider the odds ratios of part 1:

	Bookmaker 1	Bookmaker 2
Win	w_1	w_2
Lose	l_1	l_2

where $w_1 > w_2$ and $l_2 > l_1$.

Suppose you have £1 to bet, but you are not sure of the outcome of the game, so you decide to bet on both outcomes.

Assume you bet £ x on the victory of the home team and £ $y = (1 - x)$ on the defeat of the home team.

What would be your total gain (or loss) if the home team wins? And if it loses?

Solution

If the home team wins you obtain w_1x from the winning bet with Bookmaker 1 and you lose 1, the total stake:

$$R_w = w_1x - 1$$

If the home team loses you obtain $l_2(1 - x)$ from the winning bet with Bookmaker 2 and you lose 1, the total stake:

$$R_l = l_2(1 - x) - 1$$

Dutch book 3

(by Filippo Simini)

In gambling, a “Dutch book” is a set of odds and bets which guarantees a profit, regardless of the outcome of the gamble.

Assuming you bet $\pounds x$ on the victory of the home team and $\pounds(1 - x)$ on the defeat of the home team, your have

$$R_w = w_1x - 1$$

if the home team wins, and

$$R_l = l_2(1 - x) - 1$$

if the home team loses.

In this situation, is it possible to create a Dutch book and gain some money irrespective of the outcome of the game?

Under which conditions on the odds ratios w_1 and l_1 would it be possible to find a value of x which always guarantees a profit?

Solution

It is possible to create a Dutch book if both returns are positive: $R_w = w_1x - 1 > 0$ and $R_l = l_2(1 - x) - 1 > 0$.

The figure below shows that this is possible if the two lines

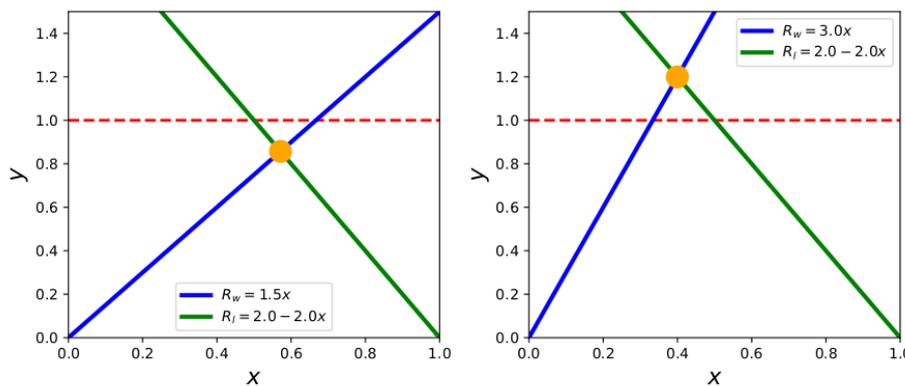
$$y = w_1x$$

and

$$y = l_2 - l_2x$$

intersects in a point (x^*, y^*) such that $y^* > 1$.

In the case on the left, there is no x for which both lines are above $y = 1$ (i.e. $R_w > 1$ and $R_l > 1$ simultaneously). In the case on the right, there is a range of x values for which both lines are above 1, so creating a Dutch book is possible.



In formulae, the point x^* where the two lines intersect is the solution of the equation: $w_1x^* = l_2 - l_2x^*$, which yields

$$x^* = \frac{l_2}{w_1 + l_2}$$

Betting $\pounds x^*$ ensures that you will get the same return irrespective of the game's outcome. This return will be positive if $R_w = w_1x^* - 1 = w_1 \frac{l_2}{w_1 + l_2} - 1 > 0$, which yields

$$1 > \frac{1}{w_1} + \frac{1}{l_2}.$$

With odds w_1 and l_2 satisfying this condition it is possible to create the Dutch book.