What is a complex system?

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Complex systems research is becoming ever more important in both ٠ the natural and social sciences. It is commonly implied that there is such a thing as a complex system, different examples of which are studied across many disciplines. However, there is no concise definition of a complex system, let alone a definition on which all scientists agree. We review various attempts to characterize a complex system, and consider a core set of features that are widely associated with complex systems in the literature and by those in the field. We argue that some of these features are neither necessary nor sufficient for complexity, and that some of them are too vague or confused to be of any analytical use. In order to bring mathematical rigour to the issue we then review some standard measures of complexity from the scientific literature, and offer a taxonomy for them, before arguing that the one that best captures the qualitative notion of the order produced by complex systems is that of the Statistical Complexity. Finally, we offer our own list of necessary conditions as a characterization of complexity. These conditions are qualitative and may not be jointly sufficient for complexity.

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- Hence, the fundamental foundational question in this domain is: What is complexity?
- In particular, is there one kind of complexity for all the sciences or is complexity domain-specific?

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- 5. "Complexity in natural landform patterns is a manifestation of two key characteristics. Natural patterns form from processes that are nonlinear, those that modify the properties of the environment in which they operate or that are strongly coupled; and natural patterns form in systems that are open, driven from equilibrium by the exchange of energy, momentum, material, or information across their boundaries."

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Conditions for Complexity

- Nonlinearity
- Feedback
- Spontaneous Order
- Robustness and lack of central control
- Emergence
- Hierarchical Organisation
- Numerosity

Complexity, Information and Probability

Complex systems are often characterized in information-theoretic terms. This is based on the idea that the order of a complex system can be understood as maintained by the internal processing of information. Whether or not it makes sense to talk about physical systems containing and processing information independently of our representations of them is a vexed issue.

Kolmogorov complexity

Shannon entropy cannot express the notion of randomness, order, or complexity of a single object. It can only express properties of a total set of sequences under some distribution. Combining notions of computability and statistics one can express the complexity of a single object. This complexity is the length of the shortest binary programme from which the object can be effectively reconstructed (see e.g. Li and Vitnyi 2009). It is known as algorithmic complexity or Kolmogorov complexity and was formulated independently by Solomonoff, Kolmogorov and Chaitin in the midsixties (Kolmogorov 1965, 1983).

Complexity between order and randomness

• One of the most intriguing ideas is that of complexity as what lies between order and randomness. The latter two notions are here meant in the sense of algorithmic complexity theory. Kolmogorov produced a measure for the complexity of a string of numbers. The idea is that to the extent that strings can be compressed they are ordered and to the extent that they cannot be compressed they are random. It is important that this measure is independent of the source of the string. For example, a string of apparently random numbers might in fact have been generated from the decimal expansion of pi according to some rule, but this fact does not show up in the string itself. Systems of interest to complexity scientists are neither completely ordered nor completely random. What is needed then would seem to be a measure of complexity different from the Kolmogorov complexity.

Measures of Complexity

- Logical Depth
- Thermodynamic Depth
- Statistical Complexity

Two distinctions

- Computable versus non-computable
- Statistical versus deterministic

 Complex System (physical account) A complex system is an ensemble of many elements which are interacting in a disordered way, resulting in robust organisation and memory.