Micro-Scale Analysis of Progressive Static Damage in CMCs

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This PhD project investigates the damage behaviour of Ceramic Matrix Composites (CMCs) at the micro-scale. This level of analysis entails representative elementary volumes (RVES) located within single fibrous tows. In SiC-SiC CMCs both the fibres and the matrix have comparable failure strains, hence damage appears in complex “diffuse” patterns of micro-cracks, originating from defects such as voids. An RVE generation algorithm originally developed for organic matrix composites has been adapted to CMC, including compliant fibre coatings that promote toughness. A micro-scale homogenization framework, based on periodic boundary conditions (PBCs), has been implemented. Abaqus FE results are in good agreement with Mori-Tanaka theory for the linear elastic regime.

Random Fibres placement Algorithm:

Pathan, Tagarielli, Patsias and Baiz-Villafranca, Comp Part B 2016

- Generation of random seeds in the plane
- Coordinate optimization: L-BFGS-B Quasi-Newton optimization solver

\[ F = \sum_{k=1}^{N_s} a_k \]

\[ a_i = \left( b_i - \beta_i \right) \]

\[ \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \geq R_i + R_j + l_{\text{min}} \quad \forall \ i, j \in N \quad i \neq j \]

\[ -R_i - l_{RVE}/2 \leq (x_i, y_j) \leq R_i + l_{RVE}/2 \]

In the algorithm, a condition for enforcing the periodicity of the fibre at the boundary has been applied.

Finite element implementation:

Implementation of a Python code for the generation of the RVE in Abaqus CAE includes:

- Geometry generation
- Mesh periodicity enforcement
- Periodic Boundary Conditions for Abaqus/Standard

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Homogenization Framework:

Denoting any possible composite realization at a given \( \delta \) as:

\[ B_\delta = \{ B_\delta(\omega) ; \omega \in \Omega \} \]

Considering a single realisation \( B_\delta(\omega) \):

- in absence of body forces,
- subject to specific boundary conditions,
- stress and strain field that can be averaged over the volume as:

\[ \bar{\sigma}_\delta(\omega) = \frac{1}{V_\delta} \int_{V_\delta} \sigma(\omega, x) \, dV \]

\[ \bar{\varepsilon}_\delta(\omega) = \frac{1}{V_\delta} \int_{V_\delta} \varepsilon(\omega, x) \, dV = \varepsilon^0 \]

The problem is the definition of \( \delta \) big enough to pass from a random field of stiffness to an effective Hooke’s law:

\[ \bar{\sigma}_\delta = C_{\text{eff}} \bar{\varepsilon}_\delta \]

where the dependence on randomness and spatial fluctuation has been removed.

In order to satisfy the energetic macro-homogeneity, Hill-Mandel PBCs have been applied:

\[ u(x + L) = u(x) + \varepsilon^0 \cdot x \]

\[ L = l_n \]

Homogenization Results:

The results obtained with this homogenization framework have been compared also with Voigt-Reuss upper and lower bound, as well as a three-phases Mori-Tanaka model.

Further works:

- Effect of microstructural imperfection on the mechanical properties of the fibre-tow scale RVE.
- Introduction of a discrete damage to reproduce the failure behaviour (via phase field theory).