University of BRISTOL
Bristol Composites Institute (ACCIS)

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ADVANCED COMPOSITES COLLABORATION FOR INNOVATION & SCIENCE

# Physically Based Modelling of Fatigue in Composites

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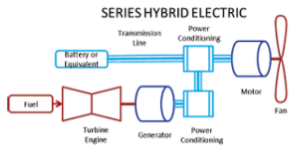
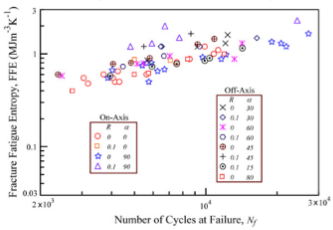
This Royal Academy of Engineering Senior Research Fellowship, supported by Rolls-Royce plc, will deliver a completely novel numerical framework for predicting the fatigue endurance of composite materials and structures, with particular emphasis on applications in hybrid electric propulsion systems. The key aim is to provide designers with revolutionary validated simulation tools, based on a probabilistic multi-scale implementation of the finite-element method, to robustly predict fatigue damage from physical first principles, via the combination of the concepts of fatigue fracture entropy and cohesive cracking. These tools will allow adopting a ground-breaking "virtual design" approach to mitigate fatigue in fibre-reinforced plastics, cutting the time to market for novel lightweight products and boosting the world-wide competitiveness of the UK composites sector.

(1) For fatigue "the cumulative entropy generation is constant at the time of failure and is independent of geometry, load and frequency" (Naderi, 2010)
(2) "Fatigue crack propagation in quasi-brittle materials can be described as a continuous sequence of fracture onset steps within the process zone" (Allegri, 2020)



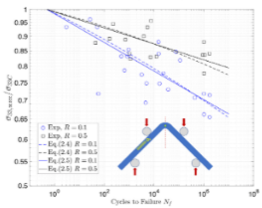

***Fatigue Fracture Entropy***

$$\gamma_f = \int_0^{t_f} \left( \frac{w}{T} + \frac{e_d}{T} + \frac{k}{T^2} \nabla T \cdot \nabla T \right) dt$$



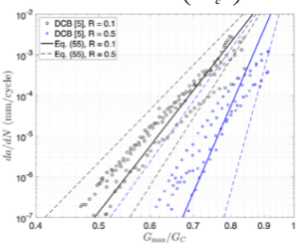

***Engineering S-N Curves***

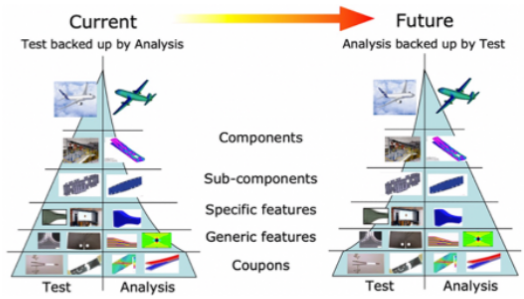
$$\frac{\sigma_{max}}{\sigma_C} = \left( \frac{1}{2N_f} \right)^{\frac{1}{b(R)}}$$




***Paris Law***

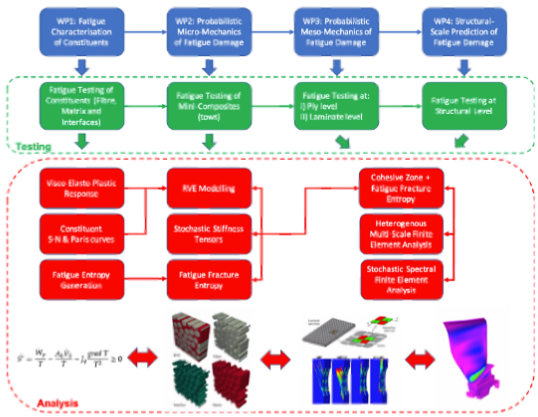
$$\frac{da}{dN} = \ell_{ch} C\left[b(R)\right] \left( \frac{G_{max}}{G_c} \right)^{b(R)/2}$$



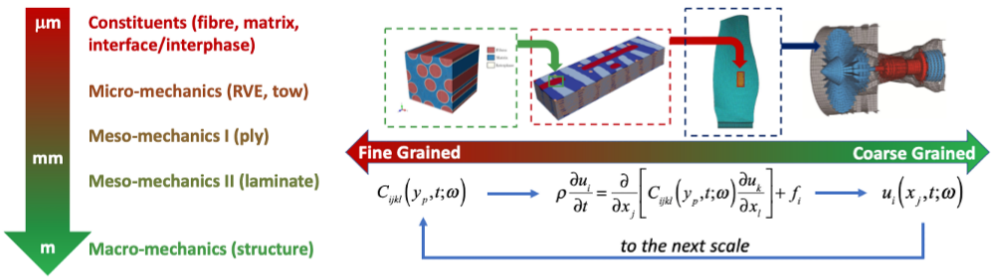




Naderi M et al, (2010). On the Thermodynamic Entropy of Fatigue Fracture. Proc R Soc A; 466: 423-38
Allegri G, (2020). A Unified Formulation for Fatigue Crack Onset and Growth via Cohesive Zone Modelling. J Mech Phys Solids; 138: 103900.

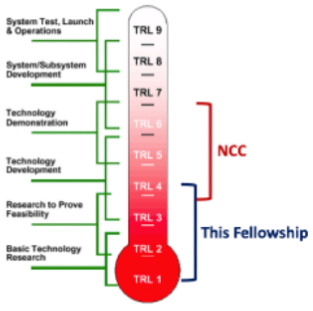
Fatigue Fracture Entropy + S-N Curves (onset) + Paris Law (propagation) → ***Physically Based Description of Fatigue***




**Stochastic Finite Element Method (SFEM) coupled with Heterogeneous Multi Scale Method (HMM)**




Abdulle A et al, (2012). The Heterogeneous Multiscale Method. Acta Numerica; 1: 1-87.
Ghanem RG & Spanos PD, (1991). Stochastic Finite Elements: A Spectral Approach, Springer-Verlag, Berlin.




**Validated Virtual "Lifing" of Composite Structures**

| Aerospace | Automotive | Renewables | Civil | Marine |
|---|---|---|---|---|
| • Turbofans<br>• Generators<br>• Ducted Fans<br>• Pressure Vessels<br>• Airframes<br>• Helicopter rotors | • Chassis<br>• Suspensions<br>• Pressure Vessels<br>• Generators<br>• Impact absorbing structures | • Blades (wind & tidal)<br>• Pylons<br>• Generators<br>• Transmission towers | • Bridge decks<br>• Strengthening & repair of RC<br>• Joining | • Hulls<br>• Bearings<br>• Propellers<br>• Generators<br>• Pressure Vessels |

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