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Bilateral Control with Vertical Contracts

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Abstract

A supplier is known to be subject to opportunism when contracting secretly with downstream competitors, particularly when downstream firms have "passive beliefs". We stress that in many situations, an equilibrium with passive beliefs may not exist and passive beliefs appear less plausible than "wary beliefs", introduced by McAffee and Schwartz, that account for multilateral deviations. We show that in a broad range of situations, equilibria with wary beliefs exist and reflect opportunism. Last, we confirm the insight, derived by O'Brien and Shaffer using a more ad-hoc equilibrium concept, that RPM eliminates the scope for opportunism.

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1 Introduction

When an upstream firm (a manufacturer, say) supplies several downstream competitors (e.g., retailers), it has an interest to restrict its supply so as to maintain high prices and profits, which it can then share with the downstream firms. However, if the upstream firm can deal secretly with one downstream competitor, it then has an incentive to "free-ride" on the other competitors and is thus tempted to supply additional quantities. Hart and Tirole (1990) (hereafter HT) have been the first to formally study this potential opportunism and show that it may prevent the upstream firm from fully exerting its market power. This insight, developed in a context where downstream firms compete à la Cournot, has since been confirmed by O'Brien and Shaffer (1992) (hereafter OS) for the case of Bertrand competition and by McAfee and Schwartz (1994) (hereafter MS) for alternative reactions to contract "renegotiations." This inability to exert full market power gives the upstream firm an incentive to reduce downstream competition, e.g., by favoring (or integrating) one competitor over its rivals. OS have pointed out that Resale Price Maintenance (RPM), whereby the retail price of a product is set by the manufacturer rather than by the retailer, also allows a manufacturer to eliminate such risk of opportunism; the idea is that, through RPM, a manufacturer can squeeze retail margins (by setting retail prices close to wholesale ones); then, when dealing with one retailer, the manufacturer has no longer an incentive to free-ride on the others' sales, since the manufacturer obtains the entire margin on these sales.

A key issue for the analysis of these situations is how downstream competitors react to "unexpected" (i.e., out-of-equilibrium) offers. Their willingness to accept such offers depends in turn on their beliefs regarding the offers made to their rivals. Intuitively, there is little scope for opportunism if the competitors are highly "cautious" about unexpected offers. If for example all competitors assume that the supplier is "flooding the market" whenever it proposes to supply below the monopoly price, they would respond negatively to such offers – and the supplier may thus be able to sustain the monopoly price. If instead competitors are more optimistic when receiving unexpected offers, they might be more receptive to "special deals", which in turn may exacerbate the supplier's temptation to flood the market. HT have argued that, in a Cournot-like context where first the upstream firm supplies given quantities at given prices, and then downstream firms compete for consumers, market competition, it is natural to assume that downstream competitors have "passive" or "market-by-market" beliefs, whereby they expect the supplier to stick to the equilibrium contracts with their rivals even if it makes them an out-of-equilibrium offer. The reason is that, in such a Cournot-like context, the quantity actually sold to one downstream firm does not directly affect the profit that the supplier derives from its contracts with the other firms. Therefore, there is arguably no reason to believe that a deviation on one contract would trigger a deviation on other contracts. Passive beliefs are also convenient in that they are usually easy to study, and they have been used as well by OS (in a slightly different way, as we explain below) and by MS.

We stress however below that the strategic "independence" between the contracts signed with the different competitors disappears when downstream competition is more Bertrand-like and/or when downstream firms find out which contracts were signed before actually competing in the final market. In all these cases, the contract signed with one competitor directly affects the profitability of the contracts signed with the other competitors. This has two implications. First, there may not exist any equilibrium with passive beliefs. The reason comes from the fact that, because of contract interdependency, the gain from a multilateral deviation may exceed the total gains of the unilateral deviations. Second, downstream firms should anticipate that, if the supplier deviates with one of them, it has an incentive to change the contracts offered to the others. Passive beliefs appear thus less plausible. We propose to consider instead the notion of *wary beliefs* introduced by MS: when it receives an unexpected offer, a downstream firm then anticipates that the supplier acts optimally with its rivals, given the offer just received. We provide two types of result. First, we show that there exists wary beliefs equilibria, even when passive beliefs equilibria fail to exist, and that these equilibria exhibit some degree of opportunism: the upstream firm does not fully exploit its market power, although it performs better than when downstream firms have passive beliefs. Second, in the Bertrand setting analyzed by OS, we confirm their insight regarding RPM: with RPM, there exists a wary beliefs equilibrium where the upstream firm fully exploits its market power.

2 Framework

The framework is a simplified version of the model proposed by OS. An upstream manufacturer M sells a product to final consumers through two differentiated retailers R_1 and R_2. The manufacturer produces with constant marginal cost c, while each retailer R_i operates at zero cost and faces a demand $D_i(p_1, p_2)$ that is differentiable, downward slopping in p_i and decreases when the two prices p_1 and p_2 increase uniformly.

To simplify exposition, we will assume that (i) demand is symmetric: $D_i(p_1, p_2) = D(p_i, p_j)$, with $\partial_1 D + \partial_2 D < 0 < \partial_2 D$, and (ii) when the wholesale price is set at marginal cost, price competition leads retailers to charge the same Bertrand price p^B, characterized by $p^B = \arg\max_p (p - c) D(p, p^B)$.

These assumptions imply that the inverse demand function is also symmetric: $P_i(q_1, q_2) = P(q_i, q_j)$ and differentiable. We will further assume that $\partial_1 P < \partial_2 P < 0$ and that, when the wholesale price is set at marginal cost, quantity competition leads the retailers to sell the same Cournot quantity q^C, characterized by $q^C = \arg\max_q \left(P\left(q, q^C\right) - c\right) q$. Last we suppose that, for any q_i and q_j such that $P(q_i, q_j) > 0$,

$$\partial_{11} P(q_i, q_j) q_i + 2\partial_1 P(q_i, q_j) < 0.$$

We model the interactions between the manufacturer and its retailers as a non-cooperative game:

1. M makes a take-it-or-leave-it offer to each retailer; each retailer only observes its own offer and decides whether to accept it or not. For the sake of exposition, we will focus on two-part tariffs, of the form $t_i(q_i) = f_i + w_i q_i$, which we will denote by $t_i = (f_i, w_i)$.[1]

2. The retailers who have accepted a contract in the first stage compete on the final market.

In the following sections, we analyze different situations with respect to the nature of downstream competition and the available information. We will consider both quantity and price competition on the downstream market; in the first case, the retailers set simultaneously the quantity they order and sell on the final market;[2] in the second case, retailers set retail prices and order quantities so as to satisfy demand. Following MS, we also consider two possible information structures: the accepted contracts can either be observed by both retailers before competing on the downstream market (*interim* observability game) or not (*interim* unobservability game).

3 Passive beliefs

Analyzing the equilibria of this game requires an assumption on how retailers revise their beliefs about the offers made to rivals, when receiving an "unexpected" (i.e., out-of-equilibrium) offer. We will suppose in this section that retailers do not revise their beliefs: that is, each retailer keeps assuming that the manufacturer offers the equilibrium

[1] In what follows, two-part tariffs are always part of a best response; an equilibrium in two-part tariffs is thus a "true" equilibrium, even considering unrestricted sets of contracts; however, there may exist additional equilibria in which two-part tariffs are not used.

[2] In order to simplify the analysis, we assume that the retailers directly set quantities. As in HT, we could have assumed that the retailers order quantities first and then compete in prices.

contract to the rival retailer, even when receiving an out-of-equilibrium offer. This is the so-called "passive beliefs" or "market-by-market conjectures" assumption used both by HT and by MS.

3.1 Opportunism

Let us first determine the equilibrium with passive beliefs in a Cournot setting with *interim* unobservability. With passive beliefs each retailer R_i anticipates that its rival receives the equilibrium offer and thus put on the market the equilibrium quantity q_j^e. Therefore, in response to a contract t_i, R_i chooses a quantity

$$Q_i(w_i) = \arg\max_{q_i} \left(P_i\left(q_i, q_j^e\right) - w_i\right) q_i - f_i, \tag{1}$$

and accepts the contract as long as the corresponding profit is not negative. The manufacturer uses the franchise fees to extract all retail profits:

$$f_i = \left(P\left(Q_i(w_i), q_j^e\right) - w_i\right) Q_i(w_i),$$

and thus sets wholesale prices so as to maximize:

$$\max_{w_1,w_2} \left(P\left(Q_1(w_1), q_2^e\right) - c\right) Q_1(w_1) + \left(P\left(Q_2\left(w_2\right), q_2^e\right) - c\right) Q_2(w_2). \tag{2}$$

Each wholesale price w_i affects this profit only through $\left(P_i\left(Q_i\left(.\right), q_j^e\right) - c\right) Q_i\left(.\right)$, which by construction is maximized for $Q_i\left(c\right)$. Therefore, in equilibrium, the manufacturer charges wholesale prices equal to the marginal cost; there thus exists a unique equilibrium, which coincides with the standard Cournot equilibrium:

Proposition 1 *In a quantity setting with interim unobservability, there exists a unique equilibrium with passive beliefs. The manufacturer sets marginal transfer prices equal to marginal cost* $\left(w_i^e = w_j^e = c\right)$*, which leads to Cournot quantities and profit.*

This result, originally due to HT, is very intuitive. With passive beliefs each retailer R_i anticipates that its rival will stick to the equilibrium quantity q_j^e and is thus willing to pay up to $P\left(q_i + q_j^e\right)$ for any given quantity q_i. Since M can monitor the retail choice of q_i through the wholesale price w_i and recover any expected profit through the franchise fee f_i, it will "choose" q_i so as to maximize $\left(P\left(q_i + q_j^e\right) - c\right) q_i$, which is achieved for the Cournot best response to q_j^e. As in HT, the manufacturer is thus subject to opportunism and non linear wholesale contracts do not allow it to fully exploit its monopoly power.

3.2 Inexistence problems

Passive beliefs are convenient and usually lead to tractable results. They are also close in spirit to the "contract equilibria" introduced by Crémer and Riordan (1987) and used in a Bertrand setting by OS. This concept focuses on pairwise deviations: M and R_i sign the best contract, given the contract signed with R_j; in contrast, a perfect equilibrium with passive beliefs must also resist multilateral deviations, where the manufacturer simultaneously deviates with both retailers. Hence, any passive beliefs equilibrium is a contract equilibrium, but a contract equilibrium is not a passive beliefs equilibrium if it does not survive to multilateral deviations.

It is easy to check that multilateral deviations are not more relevant than unilateral deviations in the above Cournot setting where retailers never observe each other's contracts. The producer's profit is of the form:

$$(w_1 - c)\, q_1 + f_1 + (w_2 - c)\, q_2 + f_2, \tag{3}$$

where f_i and q_i denote respectively the fee paid and the quantity actually sold by R_i. Since R_i does not observe the offer t_j made to its rival before accepting or rejecting its own offer, f_i cannot depend on t_j. In addition, when retailers compete in a Cournot fashion and never observe each other's contracts, the actual quantity q_i also depends only on the offer t_i made to R_i, and not on t_j (it will of course depends on R_i's *belief* about t_j, but cannot depend on the *actual* t_j). Therefore, the two contracts affect the profit expression (3) in a separable way: the first two terms of the profit expression depend on t_1 only, while the other two terms depend on t_2 only. The impact of a multilateral deviation is thus simply the sum of the impacts of each unilateral deviation, implying that any contract equilibrium is also a perfect equilibrium with passive beliefs (that is, the two concepts coincide here).

We stress below that multilateral deviations may matter and prevent the existence of an equilibrium when retailers observe each other's contracts before choosing their quantities (*interim* observability) and/or when retailers compete in prices à la Bertrand. The quantity eventually sold by one retailer then depends on the offer actually made to the other retailer, thereby destroying the above-mentioned separability.[3]

Consider for example the case where retailers compete à la Bertrand and never observe each other's contracts. Then, the *price* charged by R_i still depends only on the offer made

[3] Segal and Whinston (2001) note a similar inexistence problem when the manufacturer faces non-constant returns to scale. There again, the quantity sold to one retailer affects the profit achieved with the other retailer and multilateral deviations become an issue.

to that retailer:

$$p_i = P(w_i) \equiv \arg\max_p (p - w_i) D\left(p, p_j^e\right), \tag{4}$$

but the *quantity* q_i eventually sold by R_i depends on both wholesale prices:

$$q_i = Q_i(w_1, w_2) \equiv D_i(P(w_1), P(w_2)).$$

Therefore, the profit expression (3) is no longer separable in w_1 and w_2: each wholesale price w_i has an effect on the wholesale revenue $(w_j - c) Q_j(w_1, w_2)$ generated by the other retailer and the impact of a multilateral deviation thus no longer adds-up those of unilateral deviations. Whenever this cross effect is sufficiently important, the manufacturer's objective is not concave and a multilateral deviation can be attractive even when unilateral deviations are not. The following proposition shows that indeed, multilateral deviations destroy the unique candidate equilibrium identified by OS when the two retailers are sufficiently good substitutes. Let

$$\varepsilon \equiv -\frac{p^B \partial_1 D\left(p^B, p^B\right)}{D\left(p^B, p^B\right)} \text{ and } \varepsilon_S \equiv \frac{p^B \partial_2 D\left(p^B, p^B\right)}{D\left(p^B, p^B\right)}$$

denote the direct and cross elasticities of the demand, evaluated at the Bertrand equilibrium. We have:

Proposition 2 *In a price setting with interim unobservability, there exists no perfect Bayesian equilibrium with passive beliefs when the elasticity of substitution is large enough, namely, if* $\varepsilon_S > \varepsilon/2$.

Proof. In the price setting, if there exists an equilibrium of the *interim* unobservability game with passive beliefs, this equilibrium is identical to the contract equilibrium characterized by OS. Hence the unique equilibrium involves differentiable wholesale tariffs, with marginal wholesale prices equal to the manufacturer's marginal cost, c and retail prices equal to p^B.[4]

Consider now a "double-sided deviation" based on wholesale prices (w_1, w_2); under passive beliefs, R_i is willing to pay up to

$$f_i = (P(w_i) - w_i) D\left(P(w_i), p^B\right),$$

[4] OS show that these are the only contract equilibria, without any prior restriction on the contracts. All equilibria lead to the same retail prices and quantities, and one of them involves two-part tariffs of the form $T_i(q_i) = \pi_i^B + cq_i$.

where $P(w) = \arg\max_{p} \pi_R(p, c) \equiv (p - w) D(p, p^B)$. The manufacturer's profit is:

$$\begin{aligned}\pi_P(w_1, w_2) \equiv \ & \{(w_1 - c) D(P(w_1), P(w_2)) + (P(w_1) - w_1) D(P(w_1), p^B) \\ & + (w_2 - c) D(P(w_2), P(w_1)) + (P(w_2) - w_2) D(P(w_2), p^B)\}\end{aligned}$$

A bilateral deviation of the form $w_1 = w_2 = c + \varepsilon$ is profitable whenever $\partial_{11}^2 \pi_P(c, c) + \partial_{12}^2 \pi_P(c, c) > 0$. From the above expression,

$$\partial_{12}^2 \pi_P(c, c) = 2\partial_2 D(p^B, p^B) P'(c).$$

To compute $\partial_{11}^2 \pi_P(c, c)$, note that

$$\pi_P(w, c) = \pi_R(P(w), c) + \text{constant}.$$

Therefore,

$$\partial_1 \pi_P(w, c) = \partial_1 \pi_R(P(w), c) P'(w),$$

and thus (using $P(c) = p^B$, $\partial_1 \pi_R(P(c), c) = 0$ and $P'(c) = -\dfrac{\partial_{12}^2 \pi_R(p^B, c)}{\partial_{11}^2 \pi_R(p^B, c)}$)

$$\partial_{11}^2 \pi_P(c, c) = \partial_{11}^2 \pi_R(p^B, c) [P'(c)]^2 = -\partial_{12}^2 \pi_R(p^B, c) P'(c) = \partial_1 D(p^B, p^B) P'(c).$$

A bilateral deviation of the form $w_1 = w_2 = c + \varepsilon$ is thus profitable when

$$\partial_{11}^2 \pi_P(c, c) + \partial_{12}^2 \pi_P(c, c) = [\partial_1 D(p^B, p^B) + 2\partial_2 D(p^B, p^B)] P'(c) > 0,$$

that is (since $P'(c) > 0$), when

$$\frac{\varepsilon_S}{\varepsilon} = \frac{\partial_2 D(p^B, p^B)}{-\partial_1 D(p^B, p^B)} > \frac{1}{2}.$$

■

Thus, no equilibrium exists in the Bertrand-like framework with *interim* unobservability when the elasticity of substitution is higher than half of the direct elasticity at the Bertrand equilibrium prices. A similar observation applies when retailers observe each other's contracts before choosing their prices or quantities. The quantity q_i sold by R_i then depends again on the actual offers made to the two retailers,[5] and multilateral deviations may be profitable even when unilateral deviations are not. Suppose for example that demand is linear and given by:

$$P(q_1, q_2) = 1 - q_1 - \beta q_2.$$

[5] Denoting by $q^C(w_i, w_j)$ and $p^B(w_i, w_j)$ the standard Cournot quantities and Bertrand prices for wholesale prices w_i and w_j, the quantity sold by R_i is $q_i = q^C(w_i, w_j)$ when retailers compete in a Cournot fashion and $q_i = D(p^B(w_i, w_j), p^B(w_j, w_i))$ when they compete in a Bertrand fashion.

The parameter β reflects the degree of substitution between the two retailers: $\beta = 0$ corresponds to local monopolies, and $\beta = 1$ to perfect substitution. When retailers compete à la Bertrand and never observe each other's contracts, from the above proposition there is no equilibrium when $\beta > \frac{1}{2}$; when retailers observe each other's contracts (whether they compete in a Bertrand or Cournot fashion), Appendix A shows that there is similarly no equilibrium when $\beta > \widehat{\beta} \approx 0.806$.[6]

4 Wary beliefs

Passive or "market-by-market" beliefs are plausible in the above Cournot context.[7] The secrecy of the contracts then implies that, *from the point of view of the upstream monopolist*, the two retailers form two separate markets (even though retailers themselves perceive a strong interdependency). The producer has no incentive to change the offer to one retailer when altering the contract signed with the rival retailer: what matters is the retailer's anticipation about the quantity bought by its rival, not the quantity actually bought.

As already noted, this independence disappears when either retailers compete in a Bertrand fashion or when they observe rivals' contracts before ordering their own quantities. In these situations, modifying the contract offered to one retailer affects the other retailer's realized sales. Recognizing this point, MS suggested that retailers' beliefs should be consistent with the producers' incentives. As we will see, opting for consistent beliefs also helps restoring the existence of an equilibrium.[8]

[6] Caprice (2002) considers the case where contracts remain unobserved but retailers observe each other's acceptance decisions. This observability of acceptance decisions does not raise existence problems, but alters the equilibrium contracts when retailers have access to an alternative source of supply (a competitive but less efficient fringe, say); the manufacturer has then an incentive to lower the marginal wholesale price when acceptance decisions are observed: this makes one retailer "more aggressive" when the other refuses the contract, and thus reduces retailers' rents.

[7] The same applies to the Cournot-like context analyzed by HT, where the producer supplies given quantities to retailers before they compete for the final consumers; retailers are then capacity constrained by their previous orders and, in equilibrium, sell their quantities at the market price.

[8] There may also exist equilibria with other types of beliefs. For example, "symmetric conjectures", in which each retailer believes that the producer always treat both retailers in the same way (even out of the equilibrium), generates an equilibrium with monopoly prices (hence the producer can fully exploits its monopoly power under this particular type of belief). We show in this paper that monopoly prices cannot be sustained with wary beliefs. Segal and Whinston (2001) point out that allowing the producer to offer menus of contracts and choose quantities once retailers have accepted or rejected the offer reduces somewhat the set of possible equilibria for *any* belief, and that all equilibrium outcomes must converge towards the competitive one when a strict competitive equilibrium exists (that is, when the marginal cost

Note that retailers must form beliefs not only about the contract offered to their rivals, but also about the quantities or prices charged by their rivals (which in turn depend on rivals's beliefs). With passive beliefs, this issue is moot: since R_i believes that R_j received the equilibrium contract, it must believe that R_j anticipates that its rival also received the equilibrium contract and will thus sell the equilibrium quantity (in the Cournot-like setting) or charge the equilibrium price (in the Bertrand-like framework). R_i thus chooses the best response to the equilibrium quantity q_j^e or price p_j^e, given the contract offered to itself. However, if R_i believes that the producer also offered an unexpected offer to R_j, R_i is likely to anticipate a change in R_j's behavior. It is natural to assume that R_i will then expect R_j to optimally react to the producer's unexpected offer. MS thus proposed the notion of *wary beliefs*:

Definition 3 Wary beliefs

When R_i receives a contract t_i, it believes that:

1. *the manufacturer expects it to accept this contract,*
2. *the manufacturer offers R_j $(j \neq i)$ the contract $T_j(t_i)$ that is the best for the monopolist, among all contracts acceptable to R_j,*
3. *R_j reasons the same way.*

4.1 Quantity competition with *interim* unobservability

Let us first consider the Cournot-like framework in which retailers compete in quantities and where contracts are never observable. When being offered a contract $t_i = (f_i, w_i)$, R_i expects M to offer R_j a contract $T_j(t_i) = (F_j(t_i), W_j(t_i))$ and R_j to accept it. Then, since R_i's quantity constitute the best reply to rival's anticipated quantity, it must solve

$$Q_i(t_i) = \arg\max_{q_i}(P(q_i, Q_j(T_j(t_i))) - w_i)q_i.$$

In addition, R_i will accept a contract (w_i, f_i) if and only if the franchise fee f_i is lower than the retail expected profit, that is

$$f_i \leq (P(Q_i(t_i), Q_j(T_j(t_i))) - w_i)Q_i(t_i).$$

It remains to determine the retailers' beliefs $T_j(t_i)$. In the case of wary beliefs, when R_i is offered a tariff t_i it anticipates that M offers and R_j accepts a tariff $T_j(t_i)$ given by:

$$\begin{aligned} T_j(t_i) &= \arg\max_{(w_j, f_j)}(w_i - c)Q_i(t_i) + f_i + (w_j - c)Q_j(t_j) + f_j \\ \text{s.t.} &: f_j \leq (P(Q_j(t_j), Q_i(T_i(t_j))) - w_j)Q_j(t_j) \end{aligned}$$

of production is strictly increasing).

The solution to this program does not depend on t_i. Therefore, the conjectures formed by retailer R_i are independent of the contract it received and thus $T_j(t_i) = t_j^*$, the equilibrium offer. In this Cournot framework with *interim* unobservability, wary beliefs are thus equivalent to passive beliefs:[9]

Proposition 4 *In the quantity setting with interim unobservability, wary beliefs are equivalent to passive beliefs; there thus exists a unique equilibrium with wary beliefs characterized by*

$$w_i^{C,U} = w_j^{C,U} = c \text{ and } q_i^{C,U} = q_j^{C,U} = q^C$$

This equivalence is the underlying reason behind the plausibility of passive beliefs already noted by HT and recalled above for the Cournot setting with interim unobservability. We now show that this equivalence breaks down when contracts are interim observable or when firms compete in a Bertrand setting.

4.2 Interim observability

We assume in this section that contracts are *interim* observable: contract offers are initially secret (acceptance decisions are therefore based on beliefs) but retailers observe the accepted contracts before competing (in prices or in quantities) on the final market. The equilibrium of the retail competition subgame is therefore the solution of a standard Cournot ($q_i = q^C(w_i, w_j)$) or Bertrand-fashion ($p_i = p^B(w_i, w_j)$) competition game for which the firms face costs equal to w_i and w_j. In what follows, we denote by $q^R(w_i, w_j)$, $p^R(w_i, w_j)$ and $\pi^R(w_i, w_j)$, the retail quantity, price and profit emerging from the retail competition subgame:

- In the case of quantity competition:

$$q^R(w_i, w_j) = q^C(w_i, w_j) \text{ and } p^R(w_i, w_j) = P\left(q^R(w_i, w_j), q^R(w_j, w_i)\right).$$

- In the case of price competition:

$$p^R(w_i, w_j) = p^B(w_i, w_j) \text{ and } q^R(w_i, w_j) = D\left(p^B(w_i, w_j), p^B(w_j, w_i)\right).$$

[9] We denote the equilibrium retail price by $p^{\mathsf{i,j}}$, where $i = C$ (resp. B) in the Cournot-like (resp. Bertrand-like) case, and $j = O$ (resp. U) when contracts are *interim* observable (resp. unobservable). p^{B} and p^{M} are respectively the standard Bertrand and monopoly prices, and $p^{\mathsf{C}} = P\left(q^{\mathsf{C}}, q^{\mathsf{C}}\right)$ is the standard Cournot price.

- In both cases the retail profit is given by:

$$\pi^R(w_i, w_j) = \left(p^R(w_i, w_j) - w_i\right) q^R(w_i, w_j).$$

When being offered a contract $t_i = (f_i, w_i)$, R_i again expects M to offer and R_j to accept a tariff $T_j(t_i)$ given by:

$$\begin{aligned} T_j(t_i) &= \arg\max_{(w_j, f_j)} (w_i - c) q^R(w_i, w_j) + f_i + (w_j - c) q^R(w_j, w_i) + f_j \\ \text{s.t.} &: f_j \leq \pi^R(w_j, W_i(t_j)) \end{aligned}$$

Clearly, the solution of this program does not depend on f_i. And since the objective function is strictly increasing in f_j, the constraint must be binding. The rival's anticipated contract is thus given by:

$$W_j(w_i) = \arg\max_{w_j} (w_i - c) q^R(w_i, w_j) + (w_j - c) q^R(w_j, w_i) + \pi^R(w_j, W_i(w_j)) \tag{5}$$

and

$$F_j(w_i) = \pi^R(W_j(w_i), W_i(W_j(w_i))).$$

In contrast with the Cournot case with *interim* unobservability, the objective function in (5) is no longer separable in w_i and w_j. This implies that the beliefs will now depend on w_i. Wary beliefs thus differ from passive beliefs.[10]

M chooses wholesale prices w_1^* and w_2^* that maximize its profit given the acceptable franchise fees:

$$\begin{aligned} (w_1^*, w_2^*) = \arg\max_{(w_1, w_2)} & \left[(w_1 - c) q^R(w_1, w_2) + \pi^R(w_1, W_2(w_1))\right. \\ & \left. + (w_2 - c) q^R(w_2, w_1) + \pi^R(w_2, W_1(w_2))\right]. \end{aligned}$$

In particular, the equilibrium wholesale prices (w_1^*, w_2^*) constitute a fixed point of the functions (W_1, W_2):

$$w_1^* = W_1(w_2^*) \text{ and } w_2^* = W_2(w_1^*).$$

[10] MS adopt a different formulation, which relies on the retail equilibrium that would be generated by wholesale prices (w_1, w_2) if they were common beliefs. This formulation is well adapted to the case where contracts are observed at the retail competition stage but can be misleading otherwise. In particular, they assert that, with interim unobservability, wary beliefs equilibria coincide with passive beliefs equilibria. This is correct when retailers compete à la Cournot, because in that case the producer's profit is separable in w_1 and w_2 (implying that $W_j(w_i)$ is independent from w_i). However, as we will show later, with Bertrand downstream competition the producer's profit is no longer separable (w_1 affects R_1's retail price, which thus affects the producer's sales through R_2), and wary beliefs equilibria differ from passive beliefs equilibria.

In the remainder of this section, for tractability we assume that demand is linear:

$$P(q_1, q_2) = 1 - q_1 - \beta q_2 \Leftrightarrow D(p_1, p_2) = \frac{1 - \beta - p_1 + \beta p_2}{1 - \beta^2}.$$

The following proposition provides some characterization of wary beliefs equilibria. We first focus on beliefs W_1 and W_2 that are polynomial functions of, respectively, w_2 and w_1. We then consider symmetric equilibria, without restriction on beliefs.

Proposition 5 *When contracts are interim observable, wary beliefs no longer coincide with passive beliefs. If demand is linear and retailers have wary beliefs, then: (i) there exists a unique equilibrium with polynomial beliefs and this equilibrium is symmetric; and (ii) in any symmetric equilibrium, the equilibrium retail price is strictly lower than the monopoly price and the manufacturer therefore does not obtain the monopoly profit.*

Proof. See Appendix B. ■

Therefore, in contrast to the case of passive beliefs, there always exists an equilibrium with wary beliefs. Retailers being now more "suspicious" about the manufacturer's behavior, out-of-equilibrium offers are more likely to be rejected, which ensures the existence of an equilibrium. In the set of polynomial beliefs, this equilibrium is unique (beliefs are then affine functions of wholesale prices) and symmetric. However, if wary beliefs reduce the scope for opportunism, they do not completely eliminate this problem and the manufacturer cannot maintain monopoly prices. The opportunism problem is thus "robust", in the sense that it does not critically depend on a particular choice of equilibrium concept (contract equilibrium, passive or wary beliefs equilibria, and Cournot or Bertrand retail competition).

4.3 Price competition with interim unobservability

Let us finally consider the remaining case, where retailers compete in prices and never observe each other's contracts. When being offered a contract $t_i = (f_i, w_i)$, R_i expects M to offer R_j a contract $T_j(t_i) = (W_j(t_i), F_j(t_i))$ and R_j to accept it. Then, since R_i's price constitute the best reply to rival's anticipated price, it must solve

$$P_i(t_i) = \arg\max_{p_i} (p_i - w_i) D(p_i, P_j(T_j(t_i))).$$

In addition, R_i accepts a contract (f_i, w_i) if and only if the franchise fee f_i is lower than the retail expected profit, that is

$$f_i \leq (P_i(t_i) - w_i) D(P_i(t_i), P_j(T_j(t_i))).$$

With wary beliefs, when R_i is offered t_i it anticipates that M offers and R_j accepts the wholesale contract $T_j(t_i)$ given by:

$$\begin{aligned} T_j(t_i) &= \arg\max_{(w_j, f_j)} (w_i - c)D(P_i(t_i), P_j(t_j)) + f_i + (w_j - c)D(P_j(t_j), P_i(t_i)) + f_j. \\ \text{s.t.} &: f_j \leq (P_j(t_j) - w_j)D(P_j(t_j), P_i(T_i(t_j))) \end{aligned}$$

In contrast with the previous cases, we cannot rule out that the beliefs are function not only of the wholesale price w_i, but also of the franchise fee f_i – in which case the participation constraint is not guaranteed to be binding. This potential dependence of beliefs on franchise fees is however rather artificial; it comes from the fact that R_i's beliefs $T_j(t_i)$ affects its price response $P_i(t_i)$, which in turn affects the determination of $T_j(.)$. If we restrict attention on beliefs that do not depend on franchise fees, we obtain similar results to those obtained for the case of *interim* observability.

Proposition 6 *When contracts are* interim *unobservable and retailers compete à la Bertrand, wary beliefs do not coincide with passive beliefs. If demand is linear and retailers have wary beliefs that only depend on wholesale prices, then: (i) there exists a unique equilibrium with polynomial beliefs, and this equilibrium is symmetric; and (ii) in any symmetric equilibrium, the equilibrium retail price is lower than the monopoly price.*

Proof. See Appendix C. ■

4.4 Price comparisons

Wary beliefs mitigate somewhat the scope for opportunism. In particular, they eliminate the equilibrium inexistence problem. We also show in appendices B and C that, while wary beliefs coincide with passive beliefs in the case of Cournot competition and interim unobservability, in all other cases wary beliefs (symmetric) equilibrium prices are above the level achieved in passive beliefs equilibria (when they exist). The intuition is that when a retailer is offered a higher wholesale price than expected, with wary beliefs he anticipates that the other retailer also receives a higher wholesale price,[11] and is thus willing to pay a higher franchise fee than with passive beliefs; this, in turn, encourages the manufactuer to offer higher wholesale prices, leading to higher retail prices as well.

[11] That is, $W_i' > 0$. The expression of the manufacturer's profit is of the form

$$\pi_P = (w_1 - c)\, q_1^R(w_1, w_2) + (w_2 - c)\, q_2^R(w_2, w_1) + f_1(w_1) + f_2(w_2),$$

so that the optimal w_2 indeed increases with w_1 (given the linearity of demand and of retail behavior, $\partial_{12}^2 \pi_P = \partial_1 q_2^R + \partial_2 q_1^R > 0$).

The above propositions however show that wary beliefs do not entirely eliminate the scope for opportunism, since prices remain below the monopoly level. In addition one can check that, for the unique equilibrium with polynomial beliefs, and for any $0 < \beta < 1$ and $0 \leq c < 1$ (with $p_P^{i,j}$ denoting the candidate equilibrium retail price with passive beliefs):

$$p_P^{C,O} < p^B = p_P^{B,U} < p_B^{B,O} < p^C = p^{C,U} = p_P^{C,U} < p^{C,O} < p^{B,O} < p^{B,U} < p^M.$$

Thus, with wary beliefs prices are lower with retail Cournot competition than with retail Bertrand competition. This comes from the fact that, while price competition is more intense than quantity competition *for given wholesale prices*, the manufacturer's opportunism is moderated in the Bertrand setting, leading to higher wholesale prices – and sufficiently higher to offset the lower retail margin. Consider for example the *interim* unobservability case. When retailers compete in quantities, wary beliefs then coincide with passive beliefs and lead to the standard Cournot outcome. In the Bertrand setting, the actual quantities sold by the two retailers are interdependent: retailers anticipate that their rival will be charged a higher price when they are themselves charged a higher price, which in turn encourages the manufacturer to maintain relatively higher prices. A similar argument applies when the contracts are *interim* observable, although actual quantities are now interdependent in both settings. The gap between the equilibrium prices under Cournot and Bertrand is reduced, since contract interim observability makes the outcome more competitive in Bertrand but less competitive in Cournot.

5 Application: the anticompetitive impact of Resale Price Maintenance

5.1 Contract equilibrium

As mentioned, OS have shown that, when retailers compete in prices and never observe their rivals' contracts, in all "contract equilibria" the manufacturer's opportunism leads retailers to charge the standard Bertrand price $\left(p^B\right)$. OS have also shown that RPM, in the form of a price ceiling, solves the opportunism problem:

Proposition 7 (O'Brien and Shaffer, RJE 1992) *In all contract equilibria, the manufacturer charges a marginal price equal to its marginal cost and retailers charge the Bertrand price* $\left(p_i^* = p^B\right)$*. When Resale price maintenance (or a price ceiling) is allowed, there exists a contract equilibrium where the manufacturer maintains monopoly prices* $\left(p_i = p^B\right)$ *and achieves the monopoly profit.*

As in HT for Cournot downstream competition, the scope for opportunism comes from the fact that, when negotiating with R_i, M has an incentive to free-ride on R_j's margin. A solution to this free-riding problem is to squeeze R_j's margin, which can be achieved by imposing a price ceiling equal to the (marginal) wholesale price: this removes the manufacturer's incentives to engage in opportunism and restores the credibility of monopoly prices.

5.2 Wary beliefs

As already discussed, the concept of "contract equilibrium" is intuitive but does not coincide with the equilibrium of a well-defined game. In addition, the related notion of passive beliefs equilibrium is subject to inexistence problems, which potentially limits the relevance of the analysis. However, using wary beliefs, one can check that the insight of OS is robust.

Suppose that contracts consist of a two-part tariff, together with an imposed retail price. There is thus no actual retail competition; and when receiving an offer $(f_i, w_i; p_i)$, R_i must anticipate not only the two-part tariff t_j offered to its competitor, but also the imposed retail price p_j. We have:

Proposition 8 *In the price competition setting with RPM contracts, there exists an equilibrium with wary beliefs where prices and the producer's profit are at the monopoly level.*

Proof. See Appendix D. ■

The intuition is the same as in OS's analysis of contract equilibria. The manufacturer's opportunism problem arises when retail margins are strictly positive. Then, when M signs a contract with R_1, M is interested by the quantity sold through R_2 because of its margin $w_2 - c$ but does not take into account R_2' margin $p_2 - w_2$. Whenever this retail margin is positive, M does not entirely internalize the effect of a cut in price p_1, which leads to prices lower than the monopoly level. With RPM, M can however set both the wholesale prices and the retail prices at the monopoly level, thereby eliminating retailers' margins and the source of its opportunistic behavior.

6 Concluding remarks

The above analysis shows that wary beliefs provide a reasonable alternative to passive or market-by-market beliefs whenever the contract actually offered to one downstream firm affects the upstream monopolist's incentives when dealing with the other downstream firm. Whenever such interdependence arises: (i) an equilibrium with passive beliefs may not exist, due to the fact that multilateral deviations may become attractive; and (ii) passive beliefs differ from and are arguably less plausible than wary beliefs. We also show that an equilibrium with wary beliefs exists and reflects the "opportunism" problem generated by contract secrecy.

The analysis also shows that, while the contract equilibrium concept adopted by O'Brien and Shaffer is debatable, their insight is robust: when considering equilibria with wary beliefs, RPM allows the manufacturer to solve its commitment problem and maintain monopoly prices. Another way to test the robustness of this insight consists in reverting to public contracts, but assuming that the manufacturer deals with the retailers in sequence, as in the following three-stage game:

1. M publicly offers R_1 a wholesale two-part tariff $t_1(q) = f_1 + w_1 q$, which R_1 publicly accepts or refuses.

2. M publicly offers R_2 a wholesale two-part tariff $t_2(q) = f_2 + w_2 q$, which R_2 publicly accepts or refuses.

3. Retailers who have accepted a contract set their prices and order quantities so as to satisfy demand.

It can then be checked that the vertically integrated outcome cannot be supported in equilibrium, for the same reason as before: when negotiating R_2's contract, M has an inventive to free-ride on R_1's retail margin and generate a lower price $p_2 < p_2^m$. RPM again restores monopoly profits: if contracts include an imposed retail price, the equilibrium involves $w_1 = p_1 = p_1^m$, which induces $p_2 = p_2^m$ and thus allows the manufacturer to generate and recover the monopoly profit. Hence RPM restores the vertically integrated outcome that would otherwise be eroded by competition. As in OS, a price ceiling suffices to obtain this result, by removing the manufacturer's incentives for opportunism.[12]

[12] An industry-wide retail price floor (applying to both retailers) would also solve the manufacturer's commitment problem (see Rey and Tirole (1997)).

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A Interim observability and passive beliefs

We show that there is no passive beliefs equilibrium when contracts are interim observable and the substitutability parameter is large $\left(\beta > \widehat{\beta}\right)$.

A.1 Quantity competition

At the last stage of the game, each retailer sets the quantity it buys and resells, having observed the contract received (and accepted) by its competitor. Given the linear demand, the retail equilibrium is unique and "symmetric"; quantities and retail profits are given by:

$$q_i = q^C(w_i, w_j) = \frac{2-\beta-2w_i+\beta w_j}{4-\beta^2}, \tag{6}$$

$$\pi_i^R = \pi^C(w_i, w_j) = \left(\frac{2-\beta-2w_i+\beta w_j}{4-\beta^2}\right)^2. \tag{7}$$

When receiving an offer (f_i, w_i), R_i anticipates that R_j received the equilibrium offer and therefore faces a marginal cost equal to w_j^*. R_i thus accepts the offer if

$$f_i \leq \pi^C\left(w_i, w_j^*\right).$$

M chooses w_1^* and w_2^* so as to maximize its profit given the acceptable franchise fees: $(w_1^*, w_2^*) = \underset{(w_1,w_2)}{\arg\max}\,\pi_P(w_1, w_2)$, where

$$\begin{aligned}\pi_P(w_1, w_2) &= (w_1-c)\,q^C(w_1, w_2) + (w_2-c)\,q^C(w_2, w_1) \\ &\quad + \pi^C(w_1, w_2^*) + \pi^C(w_2, w_1^*).\end{aligned}$$

First-order conditions lead to a unique candidate equilibrium:

$$w_1^* = w_2^* = \frac{-\beta^2+\left(4-\beta^2\right)c}{2\left(2-\beta^2\right)},$$

while second-order derivatives are given by

$$\partial_{11}^2\pi_P = \frac{-4\left(2-\beta^2\right)}{\left(4-\beta^2\right)^2} \text{ and } \partial_{12}^2\pi_P = \frac{2\beta}{4-\beta^2}.$$

Second-order conditions ($|\partial_{11}^2\pi_P| \geq \partial_{12}^2\pi_P$) are therefore satisfied only if $\beta \leq \widehat{\beta}$, where $\widehat{\beta}$ is the unique solution between 0 and 1 of the equation $4-4\beta-2\beta^2+\beta^3=0$. For $\beta > \widehat{\beta}$, the manufacturer's profit function is not concave and there thus exists no equilibrium with passive beliefs.

A.2 Price competition

The proof is similar for the case of price competition. Equilibrium retail prices and profits are given by:

$$p_i = p^B(w_i, w_j) = \frac{(1-\beta)(2+\beta) + 2w_i + \beta w_j}{4-\beta^2}, \quad (8)$$

$$\pi_i^R = \pi^B(w_i, w_j) = \frac{\left((1-\beta)(2+\beta) - \left(2-\beta^2\right) w_i + \beta w_j\right)^2}{\left(4-\beta^2\right)^2\left(1-\beta^2\right)}. \quad (9)$$

The manufacturer's profit π_P thus becomes, as a function of wholesale prices

$$\begin{aligned} \pi(w_1, w_2) &= (w_1 - c) D\left(p^B(w_1, w_2), p^B(w_2, w_1)\right) + \pi^B(w_1, w_2^*) \\ &\quad + (w_2 - c) D\left(p^B(w_2, w_1), p^B(w_1, w_2)\right) + \pi^B(w_2, w_1^*). \end{aligned}$$

The candidate equilibrium is:

$$w_1^* = w_2^* = \frac{\beta^2 + \left(4-\beta^2\right) c}{4},$$

and second-order derivatives are:

$$\partial_{11}^2 \pi_P = \frac{-4\left(2-\beta^2\right)}{\left(4-\beta^2\right)^2\left(1-\beta^2\right)} \text{ and } \partial_{12}^2 \pi_P = \frac{2\beta}{\left(4-\beta^2\right)\left(1-\beta^2\right)}.$$

Second-order conditions are therefore again satisfied only if $\beta \leq \widehat{\beta}$.

B Interim observability: proof of Proposition 5

This Appendix studies wary beliefs equilibria for the case where contracts are interim observable.

B.1 Quantity competition

When being offered a wholesale price w_i, R_i expects M to offer and R_j to accept a wholesale price $W_j(w_i)$ and a franchise fee $F_j(w_i)$. R_i thus accept the contract (f_i, w_i) if $f_i \leq \pi^R(w_i, W_j(w_i))$. Wary beliefs must satisfy:

$$F_j(w_i) = \pi^C(w_j, W_i(w_j))\big|_{w_j = W_j(w_i)}. \quad (10)$$

and

$$W_j(w_i) = \arg\max_{w_j} \left[(w_i - c) q^C(w_i, w_j) + (w_j - c) q^C(w_j, w_i) + \pi^C(w_j, W_i(w_j))\right].$$

The first-order condition characterizing the beliefs is thus:

$$\begin{aligned} -(2-\beta)\left(\beta^2-\left(4-\beta^2\right)c\right)+2\beta\left(4-\beta^2\right)w_i-4\beta W_i\left(W_j\left(w_i\right)\right) \\ -4\left(2-\beta^2\right)W_j\left(w_i\right)+2\beta\left(2-\beta-2W_j\left(w_i\right)+\beta W_i\left(W_j\left(w_i\right)\right)\right)W_i'\left(W_j\left(w_i\right)\right) &= 0. \end{aligned} \tag{W_j}$$

Given the acceptable franchise fees characterized by (10), M chooses w_1^* and w_2^* so as to maximize:

$$\begin{aligned} \pi_P\left(w_1,w_2\right) = \; & \left[(w_1-c)q^C\left(w_1,w_2\right)+(w_2-c)q^C\left(w_2,w_1\right)\right. \\ & \left.+\pi^C\left(w_1,W_2\left(w_1\right)\right)+\pi^C\left(w_2,W_1\left(w_2\right)\right)\right] \end{aligned}$$

Note that beliefs satisfy:

$$\partial_1\pi_P\left[W_1(w),w\right]=0 \text{ and } \partial_2\pi_P\left[w,W_2(w)\right]=0 \tag{11}$$

and that equilibrium wholesale prices (w_1^*,w_2^*) satisfy $w_1^*=W_1\left(w_2^*\right)$ and $w_2^*=W_2\left(w_1^*\right)$.

B.1.1 Polynomial beliefs

We focus here on polynomial wary beliefs, of the form

$$W_1(w)=\sum_{k=0}^{n_1}\omega_{1,k}w^k \text{ and } W_2(w)=\sum_{k=0}^{n_2}\omega_{2,k}w^k,$$

and characterize the unique perfect Bayesian equilibrium with such beliefs. We first show below that beliefs are affine and symmetric. We then check that there exists a unique equilibrium and that it satisfies $q^*>q^M$.

- **Any polynomial wary belief is affine** $(n_1=n_2=1)$.

  Regrouping the terms in (W_j) yields:

$$\begin{aligned} \underbrace{-(2-\beta)\left(\beta^2-\left(4-\beta^2\right)c\right)+2\beta\left(4-\beta^2\right)w_i}_{\deg=1}-\underbrace{4\left(2-\beta^2\right)W_j\left(w_i\right)}_{\deg=n_j} \\ -\underbrace{4\beta W_i\left(W_j\left(w_i\right)\right)}_{\deg=n_in_j}+\underbrace{2\beta(2-\beta)W_i'\left(W_j\left(w_i\right)\right)}_{\deg=(n_i-1)n_j} \\ -\underbrace{4\beta W_j\left(w_i\right)W_i'\left(W_j\left(w_i\right)\right)}_{\deg=n_in_j}+\underbrace{2\beta^2W_i\left(W_j\left(w_i\right)\right)W_i'\left(W_j\left(w_i\right)\right)}_{\deg=(2n_i-1)n_j} &= 0 \end{aligned}$$

This implies $n_j\geq 1$. And $n_i>1$ would imply $2n_i-1\geq\max\left(n_i,3\right)$, in which case the last term would dominate, a contradiction. (W_1) and (W_2) thus impose $n_1=n_2=1$, that is, the beliefs are of the form $W_i\left(w\right)=\omega_{i,0}+\omega_{i,1}w$.

- **Any equilibrium with affine wary beliefs is such that $\omega_{1,1} = \omega_{2,1}$.**

Focusing on the linear terms, (W_1) and (W_2) impose:

$$-\beta\left(4-\beta^2\right)+\left(2\left(2-\beta^2\right)+4\beta\omega_{2,1}-\beta^2\omega_{2,1}^2\right)\omega_{1,1} = 0, \quad (12)$$
$$-\beta\left(4-\beta^2\right)+\left(2\left(2-\beta^2\right)+4\beta\omega_{1,1}-\beta^2\omega_{1,1}^2\right)\omega_{2,1} = 0. \quad (13)$$

Substracting these two conditions yields:

$$\left(\omega_{1,1}-\omega_{2,1}\right)\left(2\left(2-\beta^2\right)+\beta^2\omega_{1,1}\omega_{2,1}\right)=0. \quad (14)$$

Differentiating (11) with respect to w implies:

$$\partial_{11}^2\pi_P\omega_{1,1}+\partial_{12}^2\pi_P = 0, \quad (15)$$
$$\partial_{12}^2\pi_P+\partial_{22}^2\pi_P\omega_{2,1} = 0. \quad (16)$$

The second-order conditions of the manufacturer's program impose $\partial_{11}^2\pi_P, \partial_{22}^2\pi_P \leq 0$. Therefore,

$$\omega_{1,1}\omega_{2,1}=\frac{\left(\partial_{12}^2\pi_P\right)^2}{\partial_{11}^2\pi_P\partial_{22}^2\pi_P}\geq 0$$

and (14) thus imposes $\omega_{1,1} = \omega_{2,1} = \omega_1$. Condition (12) thus simplifies to

$$\beta\left(4-\beta^2\right)+\left(-2\left(2-\beta^2\right)-4\beta\omega_1+\beta^2\omega_1^2\right)\omega_1=0 \quad (17)$$

- **There exists a unique ω_1^* satisfying (17) and second-order conditions.**

The second-order cross derivative of the manufacturer's profit π_P is positive:

$$\partial_{12}^2\pi_P=\frac{2\beta}{4-\beta^2}>0.$$

The second-order conditions of the manufacturer's program are $\partial_{11}^2\pi_P \leq 0$ and $\partial_{11}^2\pi_P + \partial_{12}^2\pi_P \leq 0$. Together with $\partial_{12}^2\pi_P > 0$ and (15), they imply $0 \leq \omega_1 \leq 1$. Since the left-hand side of (17) is a third-degree polynomial $\phi(\omega_1)$ satisfying

$$\phi(-\infty)<0, \phi(0)>0>\phi(1)=-(1+\beta)(2-\beta)^2 \text{ and } \phi(+\infty)>0,$$

(17) has a unique solution ω_1^* in $[0,1]$.

- **There exists a unique equilibrium, which is symmetric.**

Focusing on the constant terms and using $\omega_{1,1} = \omega_{2,1} = \omega_1$, conditions (W_1) and (W_2) impose:

$$\begin{aligned}(2-\beta\omega_1^*)\,\omega_1^*\omega_{1,0} - \left(4-\beta^2\right)\omega_{2,0} &= -\frac{(2-\beta)\left(\beta^2-\left(4-\beta^2\right)c-2\beta\omega_1^*\right)\omega_1^*}{2\beta},\\ (2-\beta\omega_1^*)\,\omega_1^*\omega_{2,0} - \left(4-\beta^2\right)\omega_{1,0} &= -\frac{(2-\beta)\left(\beta^2-\left(4-\beta^2\right)c-2\beta\omega_1^*\right)\omega_1^*}{2\beta}.\end{aligned}$$

Substracting these two conditions leads to:

$$\left(4-\beta^2-2\omega_1^*+\beta\omega_1^{*2}\right)(\omega_{1,0}-\omega_{2,0}) = 0.$$

$\omega_1^* \in [0,1]$ thus implies $\omega_{1,0} = \omega_{2,0} = \omega_0^*$ and the above two conditions reduce to:

$$\omega_0^* = \frac{(2-\beta)\left(\beta^2-\left(4-\beta^2\right)c-2\beta\omega_1^*\right)\omega_1^*}{2\beta\left(4-\beta^2-2\omega_1^*+\beta\omega_1^{*2}\right)}.$$

This shows that there exists a unique solution to the overall program, and that this solution is symmetric. The equilibrium wholesale price is defined by

$$w^* = W(w^*) = \omega_0^* + \omega_1^* w^* \Leftrightarrow w^* = \frac{\omega_0^*}{1-\omega_1^*}.$$

B.1.2 Any symmetric equilibrium price satisfies $q^* > q^M$

We show here that the quantity is strictly lower than the monopoly level in any symmetric equilibrium (without any prior restriction on beliefs). The proof is in two steps: we first rule out quantities below the monopoly level using the second-order conditions of the manufacturer's program, and then the monopoly level itself by exhibiting a profitable deviation.

- **In any symmetric equilibrium with wary beliefs, the quantity is not lower than the monopoly quantity**.

In a symmetric equilibrium, (W_i) simplifies to:

$$\begin{aligned}&-(2-\beta)\left(\beta^2-\left(4-\beta^2\right)c\right)+2\beta\left(4-\beta^2\right)w-4\beta W(W(w))\\ &-4\left(2-\beta^2\right)W(w)+2\beta\left(2-\beta-2W(w)+\beta W(W(w))\right)W'(W(w)) = 0.\end{aligned} \tag{18}$$

Using $w^* = W(w^*)$, this yields

$$W'(w^*) = \frac{\beta^2-\left(4-\beta^2\right)c+2\left(2-\beta^2\right)w^*}{2\beta(1-w^*)}. \tag{19}$$

Differentiating (11) with respect to w further implies:

$$\partial_{11}^2\pi^P(w^*,w^*)\,W'(w^*) + \partial_{12}^2\pi^P(w^*,w^*) = 0. \tag{20}$$

The second-order conditions of the manufacturer's maximization program are $\partial_{11}^2 \pi^P (w^*, w^*) \leq 0$ and $\partial_{11}^2 \pi^P (w^*, w^*) + \partial_{12}^2 \pi^P (w^*, w^*) \leq 0$. Together with $\partial_{12}^2 \pi^P = \frac{2\beta}{4-\beta^2} > 0$ and (20), they imply $0 < W'(w^*) \leq 1$. Together with (19), this defines an interval for the equilibrium wholesale price:

$$\frac{-\beta^2 + (4 - \beta^2) c}{2(2 - \beta^2)} < w^* \leq \frac{\beta + (2 + \beta) c}{2(1 + \beta)} = w^M, \tag{21}$$

where w^M is the wholesale price that would drive retailers to the monopoly outcome:

$$q^R (w^M, w^M) = q^M \Leftrightarrow w^M = \frac{\beta + (2 + \beta) c}{2(1 + \beta)}.$$

This establishes $w^* \leq w^M$ and thus $q^* \geq q^M$, $p^* \leq p^M$. Note that the lower bound defined by (21) coincides with the equilibrium wholesale price with passive beliefs (and interim observability, as here). Thus, the equilibrium with wary beliefs is less competitive than the one with passive beliefs.

- **The monopoly quantity is not an equilibrium quantity.**

The gain from a symmetric deviation: $(w^M + \varepsilon, w^M + \varepsilon)$ is:

$$\delta(\varepsilon) = \pi_P(w^M + \varepsilon, w^M + \varepsilon) - \pi_P(w^M, w^M).$$

If w^M is a symmetric equilibrium wholesale price, $\delta'(0) = 0$. Evaluating (19) and differentiating (18) at $w = w^M$ yields:

$$W'(w^M) = 1 \text{ and } W''(w^M) = \frac{2(2 - \beta)(1 + \beta)^2}{\beta(2 + \beta)(1 - c)}.$$

(20) thereby implies

$$\partial_{11}^2 \pi_P (w^M, w^M) + \partial_{12}^2 \pi_P (w^M, w^M) = 0 \tag{22}$$

and thus $\delta''(0) = 0$. It is thus necessary to compute the third derivative, which using symmetry is given by:.

$$\delta'''(0) = 2\partial_{111}^3 \pi_P(w^M, w^M) + 6\partial_{112}^3 \pi_P(w^M, w^M).$$

Note that $\partial_{12}^2 \pi_P = \dfrac{2\beta}{4 - \beta^2}$ implies $\partial_{112}^3 \pi_P = 0$. In addition, differentiating $\partial_1 \pi_P (W(w), w) = 0$ yields

$$\partial_{12}^2 \pi_P (W(w), w) W'(w) + \partial_{22}^2 \pi_P (W(w), w) = 0,$$

and differentiating again at $w^M = W\left(w^M\right)$ leads to (using $W'\left(w^M\right) = 1$)

$$\partial^3_{111}\pi_P\left(w^M, w^M\right) + \partial^2_{11}\pi_P\left(w^M, w^M\right) W''\left(w^M\right) = 0. \tag{23}$$

Combining (22) and (23), we have

$$\delta'''(0) = 2\partial^2_{12}\pi_P(w^M, w^M) W''\left(w^M\right) = \frac{8(1+\beta)^2}{(2+\beta)^2(1-c)} > 0,$$

implying that a deviation $(w^M + \varepsilon, w^M + \varepsilon)$ is profitable for small positive values of ε.

B.2 Price competition

The first-order conditions (W_i) characterizing the beliefs become:

$$\begin{aligned} &(2+\beta)(1-\beta)\left(\beta^2 + \left(4-\beta^2\right)c\right) + 2\beta\left[\left(4-\beta^2\right)w_i - \left(2-\beta^2\right)W_i\left(W_j\left(w_i\right)\right)\right] \\ -4\left(2-\beta^2\right)W_j\left(w_i\right) &+ 2\beta\left[(2+\beta)(1-\beta) - \left(2-\beta^2\right)W_j\left(w_i\right) + \beta W_i\left(W_j\left(w_i\right)\right)\right] W_i'\left(W_j\left(w_i\right)\right) = 0. \end{aligned} \tag{W_i}$$

B.2.1 Polynomial beliefs

Retracing the same steps as above, it can be checked again that the only wary beliefs equilibrium is symmetric and involves affine beliefs, of the form $W(w) = \omega_0^* + \omega_1^* w$, where ω_1^* is the unique solution in $[0,1]$ to:

$$\beta\left(4-\beta^2\right) + \left(2\left(2+\beta^2\right) + 2\beta\left(2-\beta^2\right)\omega_1 + \beta^2\omega_1^2\right)\omega_1 = 0, \tag{24}$$

and ω_0^* is given by:

$$\omega_0^* = \frac{(2+\beta)(1-\beta)\left(\beta^2 + \left(4-\beta^2\right)c + 2\beta\omega_1^*\right)\omega_1^*}{2\beta\left(4-\beta^2-\left(2-\beta^2-\beta\omega_1^*\right)\omega_1^*\right)}. \tag{25}$$

The equilibrium wholesale price is again defined by

$$w^* = W(w^*) = \omega_0^* + \omega_1^* w^*,$$

and the equilibrium retail price is:

$$p^* = p^B(w^*, w^*) = \frac{(1-\beta)(1-\omega_1^*) + \omega_0^*}{(2+\beta)(1-\omega_1^*)}.$$

B.2.2 Any symmetric equilibrium price is below the monopoly price

As for the quantity competition case, we use the second-order conditions of the manufacturer's program to rule out any price strictly higher than the monopoly price. We then show that the monopoly price cannot be sustained at the equilibrium.

- **In any symmetric equilibrium with wary beliefs, the equilibrium retail price does not exceed the monopoly price.**

At a symmetric equilibrium, (W_i) reduces to (W_i) simplify to:

$$\begin{aligned} & (2+\beta)(1-\beta)\left(\beta^2+\left(4-\beta^2\right)c\right)+2\beta\left(\left(4-\beta^2\right)w-\left(2-\beta^2\right)W\left(W\left(w\right)\right)\right) \\ & -4\left(2-\beta^2\right)W\left(w\right)+2\beta\left((2+\beta)(1-\beta)-\left(2-\beta^2\right)W\left(w\right)+\beta W\left(w\right)\right)W'\left(W\left(w\right)\right) = 0. \end{aligned} \tag{26}$$

Using $w^* = W(w^*)$, this yields

$$W'(w^*) = \frac{4w^* - \left[\beta^2 + \left(4-\beta^2\right)c\right]}{2\beta(1-w^*)}. \tag{27}$$

We now have

$$\partial_{12}^2 \pi^P = \frac{2\beta}{\left(1-\beta^2\right)\left(4-\beta^2\right)} > 0,$$

so that the second-order conditions of the manufacturer's maximization program again imply $0 < W'(w^*) \leq 1$, which, combined with (27), defines the following interval for the equilibrium wholesale price:

$$\frac{\beta^2 + \left(4-\beta^2\right)c}{4} < w^* \leq \frac{\beta + (2-\beta)c}{2} = w^M. \tag{28}$$

The lower bound defined by (28) coincides again with that of the passive beliefs equilibrium. Thus, the equilibrium with wary beliefs is more competitive than the monopoly outcome but less competitive than the equilibrium with passive beliefs (and thus a fortiori less competitive than the standard Bertrand equilibrium).

- **The equilibrium retail price cannot be equal to the monopoly price**

To show that w^M cannot be the equilibrium retail price, we consider again

$$\delta(\varepsilon) = \pi_P(w^M + \varepsilon, w^M + \varepsilon) - \pi_P(w^M, w^M).$$

As for Cournot, if w^M is a symmetric equilibrium, $\delta'(0) = \delta''(0) = 0$, $W'\left(w^M\right) = 1$ and $\partial_{11}^2\pi_P\left(w^M, w^M\right) + \partial_{12}^2\pi_P\left(w^M, w^M\right) = 0$. Evaluating (27) and differentiating (26) at $w = w^M$ yields:

$$W''\left(w^M\right) = \frac{2(2+\beta)}{\beta(2-\beta)(1-c)}.$$

We have again $\partial_{112}^3\pi_P = 0$, and thus

$$\delta'''(0) = 2\partial_{12}^2\pi_P(w^M, w^M)W''\left(w^M\right) = \frac{8(2+\beta)^2}{\left(1-\beta^2\right)\left(4-\beta^2\right)^2(1-c)} > 0,$$

implying that a deviation $(w^M + \varepsilon, w^M + \varepsilon)$ is again profitable for small positive values of ε.

C Price competition and interim unobservability

We study here price competition with *interim* unobservability, and focus on beliefs $W_j(w_i)$ that depend only on the wholesale price (not on the franchise fee). R_i's best reply to the R_j's anticipated retail price $P_j(W_j(w_i))$, for $i \neq j = 1, 2$, is then given by:

$$P_i(w_i) = \arg\max_{p_i} (p_i - w_i) D(p_i, P_j(W_j(w_i))).$$

The first-order condition is necessary and sufficient; it writes as:

$$2P_i(w_i) - \beta P_j(W_j(w_i)) = 1 - \beta + w_i. \tag{P_i}$$

Wary beliefs satisfy:

$$\begin{aligned} W_j(w_i) = \arg\max_{w_j} \quad & [(w_i - c)D(P_i(w_i), P_j(w_j)) + (w_j - c)D(P_j(w_j), P_i(w_i)) \\ & (P_j(w_j) - w_j)D(P_j(w_j), P_i(W_i(w_j)))] . \end{aligned} \tag{29}$$

Moreover, using (P_j) we have:

$$D(P_j(w_j), P_i(W_i(w_j))) = \frac{P_j(w_j) - w_j}{1 - \beta^2}.$$

The following first-order condition (29) then implicitly defines the belief $W_j(w_i)$:

$$\begin{aligned} ((1 - \beta)c + \beta w_i - W_j(w_i)) P_j'(W_j(w_i)) + 1 - \beta - P_j(W_j(w_i)) + \beta P_i(w_i) \\ +2\left(P_j'(W_j(w_i)) - 1\right)(P_j(W_j(w_i)) - W_j(w_i)) = 0. \end{aligned} \tag{W_j}$$

C.1 Polynomial beliefs

Let us now consider the polynomial solutions to the system consisting of equations $((W_i), (P_i))_{i=1,2}$. We denote by n_i and m_i the degrees of W_i and P_i, and by $\omega_{i,k}$ and $p_{i,k}$ the coefficients of their terms of degree k:

$$W_i(w) = \sum_{k=0}^{n_i} \omega_{i,k} w^k \text{ and } P_i(w) = \sum_{k=0}^{m_i} \pi_{i,k} w^k.$$

- **Any polynomial solution is affine ($0 \leq m_1, m_2, n_1, n_2 \leq 1$).**

  Consider (P_i):

$$\underbrace{2P_i(w_i)}_{\deg = m_i} - \underbrace{\beta P_j(W_j(w_i))}_{\deg = m_j n_j} = \underbrace{1 - \beta + w_i}_{\deg = 1}$$

Three cases can arise:

1. $m_i < m_j n_j$. This implies $m_i = 0$ and $m_j = n_j = 1$. Then (W_i) reduces to

$$1 - \beta - \pi_{i,0} + \beta P_j(w_j) - 2(\pi_{i,0} - W_i(w_j)) = 0,$$

and thus $n_i = 1$.

2. $m_i > m_j n_j$. This implies $m_i = 1$ and $m_j n_j = 0$. Thus, either $m_j = 0$ or $m_j > 0$ and $n_j = 0$.

   (a) The case $m_j = 0$ is similar to case 1 (reverting the roles of i and j).

   (b) If $m_j > 0$ then $n_j = 0$ and (P_j) is

$$\begin{aligned} 2P_j(w) &= \beta P_i(W_i(w)) + 1 - \beta + w \\ &= \beta(\pi_{i,0} + \pi_{i,1} W_i(w)) + 1 - \beta + w \end{aligned}$$

   and therefore $m_j = \max(n_i, 1)$. If $n_i \leq 1$, then no degree exceeds 1. The only remaining case is $m_j = n_j \geq 2$. Since $m_i = 1$ and $n_j = 0$, equation (P_i) leads to

$$P_i(w) = \frac{1}{2}(1 - \beta + \beta P_j(\omega_{j,0}) + w) \Rightarrow P_i'(w) = \frac{1}{2}. \tag{30}$$

   Differentiating (W_i) and (P_j) twice then yields respectively

$$\beta P_j''(w) = \frac{1}{2} W_i''(w) \text{ and } \beta W_i''(w) = 4P_j''(w),$$

   implying $2\beta^2 P_j''(w) = W_i''(w) = 4P_j''(w)$ ($\neq 0$ since $n_i = m_j \geq 2$), a contradiction.

3. $m_i = m_j n_j \geq 1$. In this case, either $m_j = m_i n_i \geq 1$ or all degrees are equal or lower than 1 (simply invert roles played by i and j in cases 1 and 2).

$m_i = m_j n_j \geq 1$ and $m_j = m_i n_i \geq 1$ imply $n_i = n_j = 1$ and $m_j = m_i = m \geq 1$. The only interesting case is when $m \geq 2$. Then (W_j) yields:

$$\underbrace{((1-\beta)c + \beta w_i - W_j(w_i)) P_j'(W_j(w_i)) + 1 - \beta - P_j(W_j(w_i)) + \beta P_i(w_i)}_{\deg \leq m} \\ \underbrace{+2\left(P_j'(W_j(w_i)) - 1\right)(P_j(W_j(w_i)) - W_j(w_i))}_{\deg = 2m-1 \geq 3} = 0,$$

which contradicts $m > 1$.

This concludes the proof and shows that polynomial solutions must be affine.

- **Any equilibrium with affine wary beliefs satisfies** $\pi_{1,1} = \pi_{2,1}$ **and** $\omega_{1,1} = \omega_{2,1}$.

With affine beliefs, (P_i) reduces to

$$2\left(\pi_{i,0}+\pi_{i,1}w\right)-\beta\left(\pi_{j,0}+\pi_{j,1}\left(\omega_{j,0}+\omega_{j,1}w\right)\right)=1-\beta+w,$$

and since it holds for any w, it implies

$$\begin{aligned} 2\pi_{i,0}-\beta\pi_{j,0} &= 1-\beta+\beta\pi_{j,1}\omega_{j,0}, &(31)\\ 2\pi_{i,1}-\beta\pi_{j,1}\omega_{j,1} &= 1. &(32)\end{aligned}$$

(32_i) and (32_j) yield

$$\pi_{i,1}=\frac{2+\beta\omega_{j,1}}{4-\beta^2\omega_{1,1}\omega_{2,1}},$$

and thus:

$$\left(4-\beta^2\omega_{1,1}\omega_{2,1}\right)\left(\pi_{i,1}-\pi_{j,1}\right)=\beta\left(\omega_{j,1}-\omega_{i,1}\right). \quad (33)$$

Similarly, (W_j) implies:

$$\begin{aligned} 2\left(\pi_{j,1}^2-3\pi_{j,1}+1\right)\omega_{j,0} &= -1+\beta-(1-\beta)c\pi_{j,1}+\left(3-2\pi_{j,1}\right)\pi_{j,0}-\beta\pi_{i,0}, &(34)\\ 2\left(\pi_{j,1}^2-3\pi_{j,1}+1\right)\omega_{j,1} &= -\beta\left(\pi_{i,1}+\pi_{j,1}\right). &(35)\end{aligned}$$

Using (32) to replace $\pi_{j,1}\omega_{j,1}$ in (35) yields:

$$6+\beta^2\left(\pi_{i,1}+\pi_{j,1}\right)+4\pi_{i,1}\pi_{j,1}+2\beta\omega_{j,1}=12\pi_{i,1}-2\pi_{j,1}. \quad (36)$$

Substracting (36_j) to (36_i), we have:

$$5(\pi_{i,1}-\pi_{j,1})=\beta\left(\omega_{j,1}-\omega_{i,1}\right), \quad (37)$$

which, combined with (33), imposes:

$$\left(1+\beta^2\omega_{1,1}\omega_{2,1}\right)\left(\pi_{i,1}-\pi_{j,1}\right)=0. \quad (38)$$

But the second-order conditions of the manufacturer's program impose $0\leq\omega_{1,1}\omega_{2,1}\leq 1$.[13] Therefore, (38) imposes $\pi_{1,1}=\pi_{2,1}=\pi_1$ and thus $\omega_{1,1}=\omega_{2,1}=\omega_1$.

Given the symmetry, (32) and (35) simplify to

$$\begin{aligned} \beta\omega_1\pi_1 &= 1+2\pi_1, &(39)\\ \left(\pi_1^2-3\pi_1+1\right)\omega_1 &= -\beta\pi_1. &(40)\end{aligned}$$

[13] Beliefs satisfy $\partial_1\pi^P\left[W_1(w),w\right]=0$ and $\partial_2\pi^P\left[w,W_2(w)\right]=0$. Therefore, $\omega_{i,1}=-\partial^2_{12}\pi^P/\partial^2_{ii}\pi^P$ and the second-order conditions of the manufacturer's program impose

$$\omega_{1,1}\omega_{2,1}=\frac{\left(\partial^2_{12}\pi^P\right)^2}{\partial^2_{11}\pi^P\,\partial^2_{22}\pi^P}>0.$$

- **There exists a unique pair (π_1^*, ω_1^*) satisfying (39) and (40) as well as second-order conditions.**

Let us use (39) to eliminate ω_1 in (40):

$$\left(\pi_1^2 - 3\pi_1 + 1\right)(2\pi_1 - 1) = -\beta^2\pi_1^2 \tag{41}$$

$$\Leftrightarrow 2\pi_1^3 - \left(7 - \beta^2\right)\pi_1^2 + 5\pi_1 - 1 = 0. \tag{42}$$

The left-hand side is a polynomial φ of degree 3 such that:

$$\varphi(0) = -1 < 0 < \varphi\left(\frac{1}{2}\right) = \frac{\beta^2}{4} \text{ and } \varphi(1) = -(1-\beta^2) < 0 < \varphi(+\infty).$$

Therefore, φ has three roots: one in $\left]0, \frac{1}{2}\right[$, one in $\left]\frac{1}{2}, 1\right[$ and one in $]1, +\infty[$.

Using the retailers's responses, the manufacturer's profit can be expressed as

$$\begin{aligned}\pi_P(w_1, w_2) = & \left[(w_1 - c)D(P_1(w_1), P_2(w_2)) + \frac{(P_1(w_1) - w_1)^2}{1-\beta^2}\right.\\ & \left. + (w_2 - c)D(P_2(w_2), P_1(w_1)) + \frac{(P_2(w_2) - w_2)^2}{1-\beta^2}\right].\end{aligned} \tag{43}$$

Therefore:

$$\begin{aligned}\partial_1\pi_P(w_1, w_2) &= \frac{\pi_1}{1-\beta^2}\left(-(w_1 - c) + \beta(w_2 - c)\right) + D(P_1(w_1), P_2(w_2))\\ & + \frac{2}{1-\beta^2}(\pi_1 - 1)(P_1(w_1) - w_1),\end{aligned}$$

and

$$\begin{aligned}\partial_{11}^2\pi_P &= \frac{2}{1-\beta^2}\left(\pi_1^2 - 3\pi_1 + 1\right),\\ \partial_{12}^2\pi_P &= \frac{2}{1-\beta^2}\beta\pi_1.\end{aligned}$$

A first necessary condition is $\partial_{11}^2\pi_P \leq 0$, that is $\pi_1^2 - 3\pi_1 + 1 \leq 0$. Together with (41), it implies

$$2\pi_1 - 1 > 0 \Leftrightarrow \pi_1 > \frac{1}{2}. \tag{44}$$

A second necessary condition is $\left(\partial_{11}^2\pi_P\right)^2 \geq \left(\partial_{12}^2\pi_P\right)^2$, which is equivalent to

$$\left(\pi_1^2 - 3\pi_1 + 1\right)^2 - \beta^2\pi_1^2 \geq 0$$

$$\begin{aligned}&\Leftrightarrow \underbrace{-\left(\pi_1^2 - 3\pi_1 + 1\right)(2\pi_1 - 1) - \beta^2\pi_1^2}_{=0 \text{ from } (41)} - \pi_1(1-\pi_1)\left(\pi_1^2 - 3\pi_1 + 1\right) \geq 0\\ &\Leftrightarrow \pi_1(1-\pi_1) \geq 0 \Leftrightarrow 0 \leq \pi_1 \leq 1.\end{aligned} \tag{45}$$

Together, (44) and (45) impose that the solution of (42) is the unique root of φ in $\left]\frac{1}{2},1\right[$. (39) then uniquely defines ω_1^*:

$$\omega_1^* = \frac{2\pi_1^* - 1}{\beta\pi_1^*} > 0.$$

- **The solution of the overall program, if it exists, is symmetric.**

Substracting (31_j) from (31_i) and (34_j) from (34_i) yields respectively:

$$\begin{aligned}(2+\beta)\left(\pi_{1,0}-\pi_{2,0}\right) &= \beta\pi_1\left(\omega_{1,0}-\omega_{2,0}\right), \\ 2\left(\pi_1^2-3\pi_1+1\right)\left(\omega_{1,0}-\omega_{2,0}\right) &= \left(3+\beta-2\pi_1\right)\left(\pi_{1,0}-\pi_{2,0}\right),\end{aligned}$$

thus implying

$$2\left(2+\beta\right)\left(\pi_1^2-3\pi_1+1\right)\left(\pi_{1,0}-\pi_{2,0}\right) = \beta\pi_1(3-\beta-2\pi_1)\left(\pi_{1,0}-\pi_{2,0}\right).$$

But then $\pi_1^2 - 3\pi_1 + 1 < 0$ and $\frac{1}{2} < \pi_1 < 1$ imply $\pi_{1,0} = \pi_{2,0}$ and thus $\omega_{1,0} = \omega_{2,0}$.

- **There exists a unique solution.**

Given the symmetry, (31) and (34) reduce to:

$$(2-\beta)\pi_0 - \beta\pi_1\omega_0 = 1-\beta, \tag{46}$$

$$(3-\beta-2\pi_1)\pi_0 - 2\left(\pi_1^2-3\pi_1+1\right)\omega_0 = 1-\beta+(1-\beta)c\pi_1. \tag{47}$$

The determinant is

$$-2(2-\beta)\left(\pi_1^2-3\pi_1+1\right) + \beta\pi_1(3-\beta-2\pi_1) > 0$$

It is positive since $(\pi_1^2 - 3\pi_1 + 1) < 0$ and $\frac{1}{2} < \pi_1 < 1$. Therefore, (46) and (47) uniquely define π_0^* and ω_0^* as functions of π_1. The equilibrium retail price is then

$$p^* = \frac{1-\beta+w^*}{2-\beta},$$

where

$$w^* = W(w^*) = \omega_0^*\left(\pi_1^*\right) + \omega_1^*\left(\pi_1^*\right)w^* = \frac{\omega_0^*\left(\pi_1^*\right)}{1-\omega_1^*\left(\pi_1^*\right)}.$$

C.2 Any symmetric equilibrium price is below the monopoly level

Using symmetry, (P_i) and (W_i) simplify to

$$2P(w) - \beta P(W(w)) = 1 - \beta + w, \tag{48}$$

$$\begin{aligned} &1 - \beta + \beta P(w) - 3P(W(w)) + 2W(w) \\ &+ [(1-\beta)c + \beta w + 2P(W(w)) - 3W(w)] P'(W(w)) = 0. \end{aligned} \tag{49}$$

In a symmetric equilibrium,

$$p^* = p^B(w^*, w^*) = \frac{1 - \beta + w^*}{2 - \beta},$$

and thus (49) yields:

$$P' = \frac{1 - w^*}{2 + (2 - \beta)c - (4 - \beta)w^*}, \tag{50}$$

while (unless mentioned otherwise, all expressions are evaluated at the equilibrium w^*)

$$\partial_{12}^2 \pi^P = \frac{2\beta P'}{1 - \beta^2},$$

and thus:

$$W' = \frac{\partial_{12}^2 \pi^P}{-\partial_{11}^2 \pi^P} = \frac{2\beta P'}{-\left(1 - \beta^2\right) \partial_{11}^2 \pi^P}.$$

The second-order conditions of the manufacturer's maximization program $\partial_{11}^2 \pi^P < 0$ and $\partial_{11}^2 \pi^P + \partial_{12}^2 \pi^P < 0$.imply now $W' \leq 1$ and $P'W' \geq 0$ (and $P'W' > 0$ if $P' > 0$). Moreover, (48) imposes

$$2P' - \beta P'W' = 1, \tag{51}$$

and thus, in equilibrium:

$$2P' = 1 + \beta P'W' \geq 1,$$

which implies (since $P'W' > 0$ if $P' > 0$):

$$P' > \frac{1}{2}, W' > 0.$$

Together, (50) and (51) yield

$$P' = \frac{1 - w^*}{2 - (4 - \beta)w^* + (2 - \beta)c} = \frac{1}{2 - \beta W'}.$$

$0 < W' \leq 1$ then implies:

$$\frac{1}{2} < \frac{1 - w^*}{2 - (4 - \beta) w^* + (2 - \beta) c} \leq \frac{1}{2 - \beta}.$$

The second inequality implies

$$w^* \leq w^M = \frac{\beta + (2 - \beta) c}{2},$$

and thus $p^* \leq p^M$, while the first one implies

$$w^* > c,$$

and thus $p^* > p^B$.

To establish $p^* \neq p^M$, we consider again

$$\delta(\varepsilon) = \pi_P(w^M + \varepsilon, w^M + \varepsilon) - \pi_P(w^M, w^M).$$

If w^M is a symmetric equilibrium wholesale price, then by assumption $\delta'(0) = 0$ and $-\partial^2_{11}\pi_P = \partial^2_{12}\pi_P$, so that $W'(w^M) = 1$ and $\delta''(0) = 0$. It is thus necessary to compute

$$\delta'''(0) = 2\partial^3_{111}\pi_P(w^M, w^M) + 6\partial^3_{112}\pi_P(w^M, w^M).$$

Since

$$\left(1 - \beta^2\right) \partial^2_{12}\pi_P(w_1, w_2) = \beta P'(w_1) + \beta P'(w_2),$$

all third-order cross derivatives of π_P are equal to $\beta P'' / \left(1 - \beta^2\right)$. In addition, differentiating $\partial_1\pi_P(W(w), w)$ twice at $w^M = W(w^M)$ leads to (using symmetry and $W'(w^M) = 1$)

$$3\partial^3_{112}\pi_P\left(w^M, w^M\right) + \partial^3_{111}\pi_P\left(w^M, w^M\right) + \partial^2_{11}\pi_P\left(w^M, w^M\right) W''\left(w^M\right) = 0.$$

Therefore,

$$\delta'''(0) = -2\partial^2_{11}\pi_P(w^M, w^M)W''\left(w^M\right) = 2\partial^2_{12}\pi_P(w^M, w^M)W''\left(w^M\right) = 4\beta P'\left(w^M\right) W''\left(w^M\right).$$

Evaluating (50) and differentiating (49) at w^M yields respectively:

$$P'(w^M) = \frac{1}{2 - \beta} \text{ and } P''(w^M) = \frac{4}{(2 - \beta)^3 (1 - c)}.$$

Differentiating (48) twice at w leads to

$$2P''(w) - \beta P''(W(w)) (W'(w))^2 = \beta P'(W(w)) W''(w),$$

and thus

$$W''(w^M) = \frac{4}{\beta(2 - \beta)(1 - c)}.$$

Therefore

$$\delta'''(\varepsilon) = \frac{16}{(2 - \beta)^2 (1 - c)} > 0.$$

C.3 Price comparisons

We have already established that any symmetric wary beliefs equilibrium outcome is more competitive than the monopoly outcome and less competitive than the unique candidate equilibrium with passive beliefs (that is, the unique "contract equilibrium" that resist pairwise deviations). The remaining conditions have been checked using Mathematica.[14]

D Resale Price Maintenance: proof of Proposition 8

We show here that with RPM there exists a symmetric equilibrium, with affine wary beliefs based on offered prices, that sustains the monopoly price p^M. Note that with RPM, *interim* observability does not matter, since retail prices are contractually set ex ante.

Receiving an offer $(f_i, w_i; p_i)$, R_i anticipates that R_j has accepted the contract $(W(w_i, p_i),$ $F(w_i, p_i); P(w_i, p_i))$ and accepts the offer if and only if

$$f_i \leq (p_i - w_i) D_i(p_i, P(w_i, p_i)).$$

The beliefs W, F and P must therefore satisfy

$$F(w, p) = (P(w, p) - W(w, p)) D(P(w, p), P(W(w, p), P(w, p))).$$

and

$$(W(w, p), P(w, p)) = \underset{(w_2, p_2)}{\arg\max} \left[(w - c) D(p, p_2) + (w_2 - c) D(p_2, p) + (p_2 - w_2) D(p_2, P(w_2, p_2)) \right].$$

The first-order conditions are:

$$\begin{aligned} \beta(p - P(W, P)) + (P(w, p) - W(w, p)) \frac{\partial P}{\partial w}(W, P) &= 0, \\ (1 - \beta)(1 - c) + \beta w - 2P(w, p) + \beta P(W, P) - \beta(P(w, p) - W(w, p)) \frac{\partial P}{\partial p}(W, P) &= 0. \end{aligned}$$

In equilibrium, we must also have

$$P(w^*, p^*) = p^* \text{ and } W(w^*, p^*) = w^*. \tag{52}$$

- **Affine beliefs leading to the monopoly price**

[14] Mathematica file available upon request.

We consider beliefs of the form

$$
\begin{aligned}
P(w,p) &= \alpha_P + \omega_P w + \pi_P p, \\
W(w,p) &= \alpha_W + \omega_W w + \pi_W p,
\end{aligned}
$$

and look for an equilibrium where $w^* = p^* = p^M = \frac{1+c}{2}$. (52) imposes

$$
\alpha_P = (1 - \pi_P - \omega_P)\, p^M = \frac{(1 - \pi_P - \omega_P)(1 + c)}{2}, \tag{53}
$$

$$
\alpha_W = (1 - \pi_W - \omega_W)\, p^M = \frac{(1 - \pi_W - \omega_W)(1 + c)}{2}. \tag{54}
$$

Then, the first-order conditions lead to:

$$
1 - \pi_P^2 + \omega_P^2 - 2\omega_P(\pi_W + \omega_W) = 0, \tag{55}
$$

$$
\omega_P(\omega_P - \pi_P - 2\omega_W) = 0, \tag{56}
$$

$$
1 - \pi_P^2 + \omega_P(\pi_P - 2\pi_W) = 0, \tag{57}
$$

$$
\beta(\pi_P - \omega_P)(\pi_W + \omega_W) - \beta - 2(\pi_P + \omega_P)(1 - \beta\pi_P) = 0, \tag{58}
$$

$$
\beta(\pi_P - \omega_P)\,\omega_W - \beta + 2\omega_P(1 - \beta\pi_P) = 0, \tag{59}
$$

$$
\beta(\pi_P - \omega_P)\,\pi_W + 2\pi_P(1 - \beta\pi_P) = 0. \tag{60}
$$

It is easy to check that (55) = (56) + (57) and (58) = (59) + (60). Equations (59) and (60) lead to:

$$
\omega_W = \frac{\beta - 2\omega_P(1 - \beta\pi_P)}{\beta(\pi_P - \omega_P)} \text{ and } \pi_W = \frac{2\pi_P(1 - \beta\pi_P)}{\beta(\pi_P - \omega_P)}. \tag{61}
$$

Assuming that $\omega_P \neq 0$, compute $\frac{\pi_P}{\omega_P}(56) - (57)$:

$$
\frac{3\pi_P - \omega_P}{\pi_P - \omega_P} \Leftrightarrow \omega_P = 3\pi_P. \tag{62}
$$

Then, rewriting (57) using (61) and (62), it comes:

$$
8\beta\pi_P^2 - 6\pi_P + \beta = 0 \Leftrightarrow \pi_P = \frac{3 \pm \sqrt{9 - 8\beta^2}}{8\beta}. \tag{63}
$$

- **Second-order conditions**

We now show that one of the two above solutions satisfies the second-order conditions of the overall program. The manufacturer's maximization program is:

$$
\begin{aligned}
(w_1^*, p_1^*, w_2^*, p_2^*) = \underset{(w_1,p_1,w_2,p_2)}{\arg\max} \quad & [(w_1 - c)D(p_1,p_2) + (p_1 - w_1)D(p_1, P(w_1,p_1)) \\
& + (w_2 - c)D(p_2,p_1) + (p_2 - w_2)D(P(w_2,p_2),p_2)]
\end{aligned}
$$

Using the linear form of function P, the Hessian matrix only depends on π_P and ω_P and is equal to

$$H(\omega_P,\pi_P) = \begin{pmatrix} -2\beta\pi_P & \beta(\omega_P-\pi_P) & 0 & \beta \\ \beta(\omega_P-\pi_P) & -2(1-\beta\pi_P) & \beta & 0 \\ 0 & \beta & -2\beta\pi_P & \beta(\omega_P-\pi_P) \\ \beta & 0 & \beta(\omega_P-\pi_P) & -2(1-\beta\pi_P) \end{pmatrix}.$$

Using equation (62) the matrix can be simplified into:

$$H(\pi_P) = \begin{pmatrix} -2\beta\pi_P & 2\beta\pi_P & 0 & \beta \\ 2\beta\pi_P & -2(1-\beta\pi_P) & \beta & 0 \\ 0 & \beta & -2\beta\pi_P & 2\beta\pi_P \\ \beta & 0 & 2\beta\pi_P & -2(1-\beta\pi_P) \end{pmatrix}.$$

The four eigenvalues of the matrix $H(\pi_P)$ are given by:

$$\lambda(\pi_P) = -1 - 2\beta\pi_P \pm \sqrt{1 - 8\beta\pi_P + \beta^2(1 \pm 4\pi_P + 20\pi_P^2)}.$$

It can then be checked that the matrix $H\left(\frac{3-\sqrt{9-8\beta^2}}{8\beta}\right)$ is definite negative (the four eigenvalues are strictly negative). This ensures that there exists a symmetric equilibrium with affine wary beliefs leading to the monopoly prices and profit.