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Is Commission-Based Financial Advice Always Bad Advice?

Andreas Bentz

Dartmouth College and CMPO, University of Bristol

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Abstract

For many financial products, quality is not obvious to the buyer on purchase (experience goods). Financial advisers who provide advice about the quality of financial products are often rewarded on a commission basis. This may create incentives for the mis-selling of financial products. This paper shows that, under certain plausible modelling assumptions on the behavior of financial (savings) products, commission payments to the retailer act as signals of quality to the purchaser: high commissions are attached to high-quality products. There is reason to believe that commission-based financial advice is not always bad advice.

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Address for Correspondence
Department of Economics
Dartmouth College
6106 Rockefeller Hall
Hanover
NH 03755
USA
Andreas.Bentz@dartmouth.edu

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Summary

For many financial products (for instance, insurance savings products), the product's quality is not immediately obvious to the buyer at the point of purchase. Financial advisers who retail these products are frequently rewarded with sales commissions that vary widely across apparently similar products. This leads to concerns about the mis-selling of financial products: The popular view is that financial advisers have an incentive to sell whatever product earns the highest sales commission, instead of giving unbiased buying advice.

In this paper, I point up a simple argument that breaks the connection between sales commission and biased advice. Even if financial advisers are interested purely in maximizing commission income, there is no problem of mis-selling of financial products if the best products earn the adviser the highest sales commission. In this case, even a purely commission-maximizing adviser gives the "right" advice.

The link between high-quality products and high sales commissions comes from a signaling model in which the producers of financial products have to attract initial buyers and in which owners of the product use any information they obtain about the quality of the product to make decisions about surrendering the financial product early, or continuing (or repeating) purchase. The argument is that owners of high-quality products will find out that they own a high-quality product, and will continue to purchase the product.. Owners of low-quality products will find out that they own a low-quality product, and will surrender the product early. Attracting an initial buyer is therefore worth more to the producer of a high-quality product than to the producer of a low-quality product. Since commission payments are just a way of attracting an initial purchaser, a high-quality producer is willing to pay higher sales commission to its retailers than a low-quality producer.

The crucial point in the argument is that owners need to be able to obtain information about the quality of the product that they have purchased. If a buyer will never know whether her financial adviser has provided good or bad advice, the signaling intuition no longer holds. This allows me to predict for different ways of modeling the behavior of returns to financial products over time whether commission payments contain information about the product's quality or not. Briefly, if today's returns contain information about yesterday's returns (for instance, because the returns follow a random walk), then the signaling intuition holds. The paper works through different plausible assumptions of the behavior of the returns of savings products.

1 Introduction

One of the cornerstones of modern economics is the competitive paradigm of price theory, in which a countable set of homogeneous goods, uniquely characterized by a vector of nonnegative prices, is traded amongst symmetrically informed, rational agents operating in a complete set of competitive markets. Under fairly general conditions the allocation that results from such trading has desirable welfare properties. Furthermore, the only information that agents need to possess in this framework is information about prices. In particular, the model implies that there is no role for intermediaries in trade (for instance, retailers).

However, when information is distributed asymmetrically among agents, many of the appealing properties of markets disappear. Furthermore, the product space consists of horizontally and vertically differentiated goods. Under these circumstances, trade in general will also depend on non-price characteristics of goods. In fact, in markets in which buyers possess no, or only imperfect, information about product characteristics, goods are often sold through intermediaries that apparently narrow the informational gap between the trading agents. In this case, we should expect informed sellers to compete in side payments (for instance, sales commissions) to intermediaries, rather than to compete purely in prices.

One way of understanding the economic role played by side payments such as sales commissions is as a way of conveying information about imperfectly known characteristics of goods. A simple intuition is the following: Suppose goods are vertically differentiated along a quality dimension, and that quality is unobservable by consumers at the time of purchase. In this case, one may conceptualize commission payments from a perfectly informed seller as a potential signal (in the sense of Spence (1974)) of product quality. If commission payments are observable by consumers, in a separating equilibrium these are a perfectly informative signal of product quality. This is the approach this paper explores.

Financial services, and in particular, insurance savings products, are commonly retailed through financial advisers who offer their customers advice on a portfolio of such policies. Usually, financial advisers receive commission payments from the producers of these savings products (e.g., insurers),[1]

[1] In the UK, two thirds of independent financial advisers retail life assurance savings products purely on commission terms (Personal Investment Authority (1998) *Life Assurance Disclosure: Three Years On* London: PIA). In the US, of the 36,000 financial planners licensed by the Certified Financial Planner Board of Standards, 25% are compensated purely by commission, and 41% by a combination of fee and commission payments (Certified Financial Planner Board of Standards (1999) *First Annual CFP Practitioner Survey* Denver: CFP Board).

and such commission payments differ widely across apparently similar products.[2] Despite the mandatory disclosure of commission payments to customers, these side payments have often been viewed as biasing the retailer's advice, and to lead consumers to buy inappropriate products. Indeed, commission payments to retailers of financial services have recently received much attention both in the press and from regulatory agencies.

In this paper, I interpret commission payments as a possible signal of unobserved product quality. The idea is that if high-quality products always earn higher commission payments than products of inferior quality, a retailer purely interested in maximizing commission revenue will always sell the appropriate policy. An interesting fact in this connection is that life assurance savings products sold through independent financial advisers (rather than through representatives of an insurer) exhibit greater "persistency." The persistency rate is the percentage of initial buyers of a policy that have not surrendered the policy before the end of its regular lifetime. Generally, early surrender (that is, low persistency) is more common for products sold through tied advisers than for those sold through independent financial advisers. In the UK, 87% of policies started in 1993 and sold through financial advisers were still in force in 1996, compared to 81% of policies sold through company (or other tied) representatives. This fact should raise doubts about whether commission payments indeed are distorting the quality of sales advice, at least relative to the advice given by tied advisers.

In this sense, commission payments are akin to uninformative, but observable, advertising expenditure. The link between advertising for experience goods and product quality has been studied before (e.g. Nelson (1970), Nelson (1974), Schmalensee (1978), Klein and Leffler (1981), Kihlstrom and Riordan (1984), Milgrom and Roberts (1986)).[3] We know from this literature that quality signaling for experience goods cannot arise in a one-period model; subsequent periods, in which buyers acquire some information about product quality are needed to achieve a separating equilibrium. A very rough intuition, of course, is that in order to achieve a separating equilibrium, the marginal return to advertising to a high-quality producer has to be greater than that to a

[2] In the UK in 1998, for instance, commission payments on a 25 year unit linked personal pension were distributed between £0 and £650 (based on gross premium of £60 per month). Similar variance in commission payments is true for other life assurance savings products.

[3] Nelson (1970) distinguishes between experience goods (for which quality is not directly observable on inspection) and search goods (for which quality is verifiable on inspection). Clearly, directly informative advertising can only arise in equilibrium for search goods; statements about the quality of an experience good (since unverifiable on inspection) will rationally be ignored by consumers.

low-quality producer. In this paper, I show conditions under which such signaling is informative of product quality for savings products such as life assurance savings products. In particular, I study how different assumptions on the stochastic nature of financial returns allow informativeness of commission payments about product quality.

This paper has two main implications. The first is a policy suggestion: if a separating equilibrium exists (in which commission payments signal product quality accurately), mandatory commission disclosure amounts to publishing product quality. Secondly, in this case, financial advisers (modeled here, admittedly simplistically, as maximizing only commission income) fulfil only the role of allowing insurers to signal to the consumer through retailer commission payments. Clearly, this implies that if insurers could find alternative ways of publicly throwing away money (for instance, running expensive advertising campaigns), there is no need for financial advisers. This, clearly, is the less robust conclusion of the paper, and should therefore not be overemphasized. In fact, financial advisers may serve different economic functions, which I do not address in this paper.[4]

If commission payments indeed are a way of conveying information about product quality to consumers, introduction of full commission disclosure rules (such as those introduced in the UK in 1995) should result in better, more informed, purchasing decisions by consumers of these products. As a result, one should expect to see the difference in persistency rates for policies retailed through tied advisers and those sold through independent financial advisers increase as a result of commission disclosure. This "difference in differences" should be observable in the data. Furthermore, when commission disclosure makes commission payments observable, one should expect the variance of commission payments to increase. Although I do not test these results in this paper, the model has clear implications for what we should see in the data.

[4] In this paper, I analyze a vertically differentiated product space, and therefore focus on the lack of information of buyers about product quality. A different way of viewing side payments to retailers is to focus on the informational asymmetry between sellers and retailer: the retailer has information about customers (or more generally, about market conditions) that sellers do not possess. Side payments (such as sales commissions) may be a way of eliciting that information. If a retailer acts on behalf of more than one seller, we have a typical common agency situation with adverse selection (cf. Martimort (1992), Stole (1992)). Such an approach naturally applies to a horizontally differentiated product space. Here, I study the case of vertically differentiated products, where the limited information of producers about their customers does not naturally force the modeling as one of common agency with asymmetrically informed principal. However, common agency models do not need to have conclusions orthogonal to those drawn in this paper. In general, when principals with substitutable products compete through a single agent (e.g. the retailer), for instance by effecting commission payments, common agency results in less inefficiency than when no such competition in commission payments can take place.

1.1 Literature

There are very few papers that model sales incentives for financial advisers within the framework of asymmetric information (although, of course, the general literature on optimal compensation schemes is enormous). Puelz and Snow (1991) derive the optimal incentive scheme for insurance sales agents under conditions of moral hazard where the agent's sales effort to new and renewal buyers is unobservable. They show that the optimal scheme (on which they exogenously impose linearity assumptions) involves commission payments that are high for an initial sale, and low for a repeat (renewal) sale if the agent's effort is more profitably directed at obtaining new (first-time) customers. My focus in the present paper however is on the adverse selection problem of asymmetrically distributed information about quality, and how commission payments may fill the information gap.

Gravelle (1993) studies the financial adviser's incentives for misrepresentation of product quality as a result of commission payments from the insurer. In his model, after initial purchase of a policy (based solely on the adviser's recommendation), consumers are "locked in" to the policy; in other words they cannot surrender their insurance savings products early. Unsurprisingly, in a framework in which consumers can never act on any information they may receive about their policy over its lifetime, Gravelle finds that commission payments cause additional inefficiency: When financial advisers can lie about the policies they sell, the resulting equilibrium is not even second-best efficient. As mentioned in the introduction, I find it more plausible to allow consumers to surrender insurance savings products early in response to information they obtain about the performance of the policy over the policy's lifetime. In the framework of the present paper, under plausible assumptions on the behavior of these savings products, commission payments cause no inefficiency.

The question of signaling of product quality has attracted relatively little empirical interest, and what evidence there is draws on different industries. In particular, the Milgrom-Roberts hypothesis of price and advertising signals of quality has attracted attention. For instance, Thomas, Shane, and Weigelt (1998) find that, in the US automobile market, prices and, importantly, advertising expenditures are consistent with a "signaling" story. They construct a measure of unobservable product quality (as the error in a regression of price on observable characteristics) and use this to regress advertising expenditure on observable and unobservable quality. They find that producers of high quality automobiles spend more on advertising than manufacturers of low-quality products. A similar conclusion should be true in the insurance context of the present paper.

2 The General Model

In the model, there are three time periods $t = 0, 1, 2$. I model the financial asset as a stochastic process with random per-period returns X_t, and denote realizations of each random variable by x_t.[5] In a very general formulation of the problem, some agents observe some $y_t = y_t(x_t)$, and $y_t(\cdot)$ is common knowledge. Write Y_t for $y_t(X_t)$. There is a seller of the financial asset and a representative consumer. Let the buyer's outside investment opportunities r (measured as rates of return) be distributed according to the distribution function $G(\cdot)$. If the buyer purchases the financial asset at the beginning of time t at price p, her payoff at the end of period t is $\frac{x_t}{p} - 1$. In this representative consumer model, per-period demand for an asset with known rate of return $\frac{x}{p} - 1$ is therefore $\Pr\{r \leq \frac{x}{p} - 1\} = G(\frac{x}{p} - 1)$. Generally, however, return of an asset is not known by the buyer. Payoffs are discounted at rate δ, $0 < \delta \leq 1$. The seller can signal (pay a commission) s_0 per unit sold in period $t = 0$. Denote the period t demand for the asset by q_t, and its price by p.[6] The seller's expected profit therefore is $(p - s_0)q_0 + \delta p E[q_1 | Y_0 = y_0]$. The retailer's objective function is common knowledge, so that for some specification of her objective (for instance, she maximizes commission income) or her behavior (say, she apportions the time spent explaining each policy according to the commission payments she receives on each) or by direct observation when commission is disclosed to the buyer, consumers can deduce the amount of commission payments. I can therefore ignore the role of the retailer in the following modeling. I refer to the producer of financial products as the "seller" and the consumer as the "buyer."

The timing of the model is as follows:

- at $t = 0$, x_0 realizes; the seller observes y_0 and signals $s_0(\hat{y}_0)$; the buyer infers product quality $\tau^{-1}(s_0)$, where $\tau : \Upsilon \to \Xi$ (Υ is the set of possible values of y_0 and Ξ is the set of all possible values of x_0) and decides whether to buy the asset; if the buyer does not buy the asset, the game ends with zero payoffs for both players;

- at $t = 1$, x_1 realizes; the buyer observes y_1 and decides whether to repeat purchase of the asset; if she decides not to repeat purchase, the game ends and payoffs are realized; since I wish to model repeat purchase behavior, at $t = 1$, the asset can only be purchased by period

[5] Generally, I use capitals to denote random variables, and lower-case letters for their realizations.

[6] Note that in this model, price is not a choice variable. This is justified by the fact that, for life assurance savings products, the termly premium is typically chosen by the buyer. If sellers could choose prices as well as commission payments, one might expect signaling to occur through prices as well as sales commissions.

$t=0$ buyers, so that I effectively "track" repeat purchases by first-time buyers;

- at $t=2$, x_2 realizes; the game ends, and payoffs are realized.

2.1 The Buyer's Problem

At time $t=0$, the buyer purchases the asset if, and only if, the expected rate of return from holding the asset (taking into account that only buying the asset now gives the option of repeating purchase at $t=1$) is greater than the buyer's outside rate of return. Specifically, the buyer purchases the asset at $t=0$ if

$$(1+\delta)r \leq E_{X_1|Y_0}\left[\frac{X_1}{p} - 1 + \delta \max\left\{\frac{E_{X_2|Y_1,Y_0}\left[X_2|Y_1, s_0(Y_0)=s_0(\hat{y}_0)\right]}{p} - 1, r\right\} |s_0(Y_0)=s_0(\hat{y}_0)\right].$$

That is, demand at $t=0$ is

$$\begin{aligned} q_0 \;=\; & \Pr\left\{(1+\delta)r \leq E_{X_1|Y_0}\left[\frac{X_1}{p} - 1 \; + \right.\right. \\ & \left.\left. +\delta \max\left\{\frac{E_{X_2|Y_1,Y_0}\left[X_2|Y_1, s_0(Y_0)=s_0(\hat{y}_0)\right]}{p} - 1, r\right\} |s_0(Y_0)=s_0(\hat{y}_0)\right] |Y_0=y_0\right\}. \end{aligned} \tag{1}$$

Given that the buyer has purchased at $t=0$, she repeats purchase at $t=1$ if, and only if,

$$r \leq \frac{E_{X_2|Y_1,Y_0}\left[X_2|Y_1=y_1, s_0(Y_0)=s_0(\hat{y}_0)\right]}{p} - 1,$$

so that period 1 demand is

$$q_1 = \min\left\{q_0, \Pr\left\{r \leq \frac{E_{X_2|Y_1,Y_0}\left[X_2|Y_1=y_1, s_0(Y_0)=s_0(\hat{y}_0)\right]}{p} - 1 \,|Y_0=y_0\right\}\right\}. \tag{2}$$

2.2 The Seller's Profit

The seller's expected profit, after observing y_0, is consequently

$$\left(p - s_0(\hat{y}_0)\right) q_0 + \delta p E_{Y_1|Y_0}\left[q_1|Y_0=y_0\right],$$

where, of course, the conditional expectation in q_1 is conditional on $Y_1 = y_1|_{y_1=Y_1}$, i.e. evaluated at the random variable Y_1.

2.3 Three Models

To demonstrate the generality of the notation, consider the following three models. Model 1 and model 2 are extreme cases: in model 1, per-period returns are drawn from the same distribution; in model 2, per-period returns are independent. Model 3 is a more general Markov model. I will later restrict model 3 further to modeling per-period returns as following a martingale process.

2.3.1 Model 1

Let each X_t be distributed according to the same one-parameter distribution with parameter $\theta \in \Theta$, so that $X_t = X$. Let this parameter be observable to sellers at period 0 and buyers at period 1, so that $y_t = y_t(\cdot) = \theta$ (and denote $\hat{y}_t = \hat{\theta}$). Interpret θ as the quality of the asset (initially only observable to sellers). This is the simplest model of vertical product differentiation: the asset has a certain (fixed) quality, and realizations of the asset are drawn according to a distribution function with parameter θ.

2.3.2 Model 2

Let agents observe the realization of the X_t, so that $y_t = y_t(x_t) = x_t$ (and denote $\hat{y}_t = \hat{x}_t$). Furthermore, let the X_t be independently distributed. In this model, there is no fixed quality of the asset. Higher realizations are better than lower realizations, but each per-period rate of return on the asset is independently drawn.

2.3.3 Model 3

As in model 2, let agents observe the realization of the X_t, so that $y_t = y_t(x_t) = x_t$ (and denote $\hat{y}_t = \hat{x}_t$). However, let the X_t follow a stochastic process $\{X_t\}$ that is Markovian. In this model, asset quality evolves over time: this period's payoff is the mean to next period's payoff. If an asset starts off with a high return (x_0), then its quality (that is, its likely future returns) is high. Conversely, a low-quality asset is one with a low realization x_0.

The very general formulation of models 1–3 generally precludes a closed-form solution. When necessary, I will therefore introduce additional simplifying assumptions later.

3 Analysis of Models 1–3

In order to establish the existence of a separating equilibrium, it is sufficient to check whether the seller's expected profit obeys a Spence-Mirrlees single-crossing condition (cf., for instance, Milgrom and Shannon (1994)). The nonlinearity of the general model however precludes a simple solution. I therefore focus on the case where y_0 is drawn from a set Υ with $\#\Upsilon = 2$. In this case, I need to show that there exists an equilibrium in which no seller can gain from "lying" about her private information. (This of course is the two-type equivalent of checking for single-crossing.)

3.1 Model 1

In this model, each $X_t = X$ is distributed according to the same one-parameter distribution with parameter θ. This parameter is observable to sellers at period 0 and buyers at period 1, so that $y_t = y_t(\cdot) = \theta$. The model is thus one in which a financial product of a certain quality θ is sold, where $\theta \in \Theta$. Refer to the seller of this product as "type θ seller". Before θ is observed, refer to it as the random variable $\tilde{\theta}$. The type θ seller produces an asset with random per-period return X, distributed according to the cumulative distribution function $F(x|\theta)$ with conditional density $f(x|\theta) > 0$ on $[\underline{x}, \overline{x}]$. In period 0, θ is the seller's private information. Once first-period buyers have experienced the product, they know its quality perfectly; in period 1, therefore, θ is known to all players (in fact, it is sufficient to assume that it is known to all first-period buyers).

In this case (1) reduces to

$$q_0 = \Pr\left\{(1+\delta)r \leq E_{X|\tilde{\theta}}\left[\frac{X}{p} - 1 + \delta \max\left\{\frac{E_{X|\tilde{\theta}}\left[X|\tilde{\theta} = \hat{\theta}\right]}{p} - 1, r\right\}\middle|\tilde{\theta} = \hat{\theta}\right]\right\}.$$

Note that this further reduces to[7]

$$q_0 = \Pr\left\{r \leq \frac{E[X|\tilde{\theta} = \hat{\theta}]}{p} - 1\right\}.$$

Furthermore, (2) can be rewritten

$$q_1 = \min\left\{q_0, \Pr\left\{r \leq \frac{E_{X|\tilde{\theta}}\left[X|\tilde{\theta}\right]}{p} - 1 \middle| \tilde{\theta} = \theta\right\}\right\}.$$

[7]For assume that $r > \frac{E_{X|\tilde{\theta}}[X|\tilde{\theta}=\hat{\theta}]}{p} - 1$. Then we have $q_0 = \Pr\left\{(1+\delta)r \leq E_{X|\tilde{\theta}}\left[\frac{X}{p} - 1 + \delta r \middle| \tilde{\theta} = \hat{\theta}\right]\right\}$, or $q_0 = \Pr\left\{r \leq E_{X|\tilde{\theta}}\left[\frac{X}{p} - 1 \middle| \tilde{\theta} = \hat{\theta}\right]\right\}$ which is zero, and the case is not of analytical interest.

For concreteness, denote the asset's expected return conditional on θ by $E[X|\theta] = \int_{\underline{x}}^{\bar{x}} xdF(x|\theta)$.[8] I make the assumption that, for any $\theta' > \theta$, $F(x|\theta')$ stochastically dominates $F(x|\theta)$.[9] For generality, let the type θ seller's unit production cost be c_θ, and let the asset's price be exogenously fixed at p.[10] Further, assume that $c_\theta < p$ for all θ. (If, to the contrary, for some firm $c_\theta \geq p$, that seller would never make positive profits, and would therefore never enter the market.) Each type θ seller can pay the retailer a commission s_0 (which, in general, will be a function $s_0 : \Theta \to S$, where $S = \{s | s \in \Re_0^+\}$). Customers draw an inference $\hat{\theta} = \tau^{-1}(s_0)$ about seller type, where $\tau : \Theta \to S$. Note that in a separating equilibrium, $\tau(\cdot) = s_0(\cdot)$. (Here, I am intentionally more explicit about buyer inferences than in the general model.)

Let $G(\cdot)$, the distribution of buyer outside investment opportunities, be uniform on $[0, 1]$. Suppose that, in each period, any one customer has use for at most one policy. Accordingly, if θ were known, customers $r \leq \frac{E[x|\theta]}{p} - 1$ would want to buy the asset. By assumption, however, the asset can only be bought in period 1 (i.e. the policy is renewed) if it was bought in period 0.

3.1.1 Analysis of Model 1

It turns out to be convenient to rewrite demand for the asset slightly. In period 0, a customer will buy a policy she is offered if its expected return (which depends on her belief about the policy's quality) is greater than her outside investment opportunity. (Note that, in this model, buyers are willing to buy in period 0 only if they are (*ex ante*) also willing to buy in period 1, since expected per-period returns are identical.) With a uniform distribution of outside investment opportunities, therefore, period 0 demand for a product of (unknown) quality θ, when customers draw inference $\hat{\theta} = \tau^{-1}(s_0)$ about type, is therefore

$$q_0(s_0) = \max\left\{0, \frac{E[X|\tau^{-1}(s_0)]}{p} - 1\right\}.$$

[8] If customers are risk averse, with utility function $u(\cdot)$, $u' > 0$, $u'' < 0$, then the interest is in the expected utility; because of the assumption of stochastic dominance, which I will make shortly, this has no effect on our model. However, stochastic dominance may be felt to be too strong an assumption, so that an investigation of risk-aversion may hold independent interest.

[9] Recall that the definition of first-order stochastic dominance is that, for any increasing function h, $\int_{\underline{x}}^{\bar{x}} h(x)dF(x|\theta') > \int_{\underline{x}}^{\bar{x}} h(x)dF(x|\theta)$.

[10] Note that in models 2 and 3 I do not introduce a cost-parameter since it is no longer clear how cost varies with "quality".

Since only period-one buyers can renew their policies in period 2, the period 2 demand is

$$q_1(s_0,\theta) = \min\left\{\max\left\{0, \frac{E[X|\tau^{-1}(s_0)]}{p} - 1\right\}, \max\left\{0, \frac{E[X|\theta]}{p} - 1\right\}\right\}.$$

Profits of a type θ seller that signals s_0 are:

$$(p - c_\theta - s_0)q_0(s_0) + \delta(p - c_\theta)q_1(s_0,\theta).$$

We have:

$$\pi(s_0, q_0, \theta) = (p - c_\theta - s_0)\max\left\{0, \frac{E[X|\tau^{-1}(s_0)]}{p} - 1\right\} +$$

$$+\delta(p - c_\theta)\min\left\{\max\left\{0, \frac{E[X|\tau^{-1}(s_0)]}{p} - 1\right\}, \max\left\{0, \frac{E[X|\theta]}{p} - 1\right\}\right\}$$

3.1.2 The two-quality case

To simplify the analysis, let $\Theta \equiv \{L, H\}$, $L < H$. A separating equilibrium exists when the signal sent by the H type seller is large enough for the L type not to want to mimic the signal, yet low enough for the H type still to prefer to signal (rather than to be thought of as an L type). That is, in a separating equilibrium the signal s_0^* is bounded as follows: $s_0^* \in \left[\left(\frac{E[X|H]-E[X|L]}{E[X|H]-p}\right)(p - c_L); (1+\delta)\left(\frac{E[X|H]-E[X|L]}{E[X|H]-p}\right)(p - c_H)\right]$. Details are in the appendix.

Looking for existence of a separating equilibrium, we immediately have the following:[11]

Proposition 1 *There exists a separating equilibrium s_0^*, if and only if $p \leq E[X|H]$ and*

$$(1+\delta)c_H \leq c_L + \delta p. \tag{3}$$

Note that condition (3) can be rewritten, somewhat more intuitively, as

$$(1+\delta)p - (1+\delta)c_H \geq p - c_L$$

which is just the aforementioned inequality between the marginal return to signaling for high and low quality sellers: at the margin, a high quality seller sells an additional unit in both periods (and bears the production cost in both periods); similarly, at the margin, a low quality seller only manages to sell an additional unit in the first period (bearing the production cost in that period), and sells nothing in the second period. For a separating equilibrium to exist, the former has to be greater than the latter.

[11] All proofs are in the appendix.

When such a separating equilibrium exists, financial advice, even if based purely on commission payments, is good advice.

In fact, we can establish whether the type H seller would want to advertise (that is, pay commission to its sales agents) to distinguish itself away from a "pooling" equilibrium in which both types of sellers are not observably different. This gives us:

Corollary 2 *If, in addition to condition (3), we have the more stringent condition*

$$(1+\delta)c_H \leq c_L + \delta p + (c_L - p),$$

the type H seller prefers to advertise to distinguish itself from the type L seller.

Of course, the type L seller would never prefer signaling to not signaling.

Therefore, when signaling works in a separating equilibrium, the difference between payments to retailers from H and L type sellers is, as argued above, bounded between $\left(\frac{E[X|H]-E[X|L]}{E[X|H]-p}\right)(p-c_L)$ and $(1+\delta)\left(\frac{E[X|H]-E[X|L]}{E[X|H]-p}\right)(p-c_H)$.When signals cannot be observed by buyers (for instance, because there is no commission disclosure), sellers would rationally not engage in signaling, so that there is no difference in the payments to retailers from H and L type sellers. This "difference in differences" test should be observable in data.

3.2 Model 2

In this model, agents observe the realization of the X_t, so that $y_t = y_t(x_t) = x_t$ (and denote $\hat{y}_t = \hat{x}_t$). Furthermore, let the X_t be independently distributed. The model is therefore one in which the seller observes how well the asset has performed, and then signals regarding that information.

Note that if the X_t are independent, period-0 demand is

$$q_0 = \Pr\left\{(1+\delta)r \leq E\left[\frac{X_1}{p} - 1 + \delta \max\left\{\frac{E[X_2]}{p} - 1, r\right\}\right]\right\}.$$

Furthermore, if the X_t are independent, we have

$$q_1 = \min\left\{q_0, \Pr\left\{r \leq \frac{E[X_2]}{p} - 1\right\}\right\}.$$

Finally note that, if the X_t are independent, the seller's profit does not depend on x_0. We therefore have the following:

Proposition 3 *If per-period returns are independent, signaling cannot occur.*

Proof. The proposition follows from the derivation above. ■

The proposition is intuitive: if the buyer of an asset never learns whether the seller has lied to her about the asset's quality, there can be no separating equilibrium (there is no difference in marginal returns to signaling for sellers who observe different performance of the asset).

3.3 Model 3

I have just shown that if asset returns are independent, no signaling can occur. Signaling "works" in model 1 because per-period returns are identically distributed, and the shape of the distribution function is learned after the initial purchase. Model 1, however, is unsatisfactory for different reasons: it seems more natural to think of asset returns as following, for instance, a random walk. Model 3 therefore makes a "Markovian" assumption about $\{X_t\}$. As in model 2, agents observe the realization of the X_t, so that $y_t = y_t(x_t) = x_t$ (and denote $\hat{y}_t = \hat{x}_t$). However, let the X_t follow a stochastic process $\{X_t\}$ that is Markovian, i.e. we have $E[X_t|X_{t-1}, X_{t-2}, ...] = E[X_t|X_{t-1}]$.

When $\{X_t\}$ is Markovian, (1) reduces to:

$$q_0 = \Pr\left\{(1+\delta)r \leq E_{X_1|X_0}\left[\frac{X_1}{p} - 1 + \delta \max\left\{\frac{E_{X_2|X_1}[X_2|X_1]}{p} - 1, r\right\} |s_0(X_0) = s_0(\hat{x}_0)\right]\right\}$$

and (2) turns into

$$q_1 = \min\left\{q_0, \Pr\left\{r \leq \frac{E_{X_2|X_1}[X_2|X_1]}{p} - 1\,|X_0 = x_0\right\}\right\}.$$

In this case, q_0 depends only on $\hat{x}_0$; $E_{X_1|X_0}[q_1|X_0 = x_0]$, however, does depend on x_0. There is, therefore, scope for the existence of a separating equilibrium.

3.3.1 Analysis of Model 3

In fact, restrict further the nature of the stochastic process and assume that $\{X_t\}$ is a martingale. Below, I will make further assumptions on possible returns and discuss a discrete example similar to that used to study model 1. Note, for later use, that we have the following basic property from martingale theory: $E[X_n|X_0, ..., X_k] = X_k$, for $k < n$. Adopt similar simplifications as in model 1 (but note that now "quality" refers to observed performance in period 0, i.e. $x_0 \in [\underline{x}_0, \overline{x}_0]$). Use the notation introduced for model 1, but let $\tau : [\underline{x}_0, \overline{x}_0] \to S$. Then, in period 1, a buyer with outside investment opportunity r will buy (provided she has bought in period 0) if

$$\frac{E[X_2|X_1 = x_1, \tau^{-1}(s_0)]}{p} - 1 = \frac{x_1}{p} - 1 \geq r.$$

In period 0, this buyer will buy if: either

$$\frac{E[X_1|\tau^{-1}(s_0)]}{p} - 1 = \frac{\tau^{-1}(s_0)}{i} - 1 \geq r \tag{4}$$

or

$$\left(\frac{E[X_1|\tau^{-1}(s_0)]}{p} - 1\right) + \delta\left(\frac{E\left[E[X_2|X_1]\,|\tau^{-1}(s_0)\right]}{p} - 1\right) \geq (1+\delta)r$$

which, using the martingale property, can be rewritten as

$$\left(\frac{E[X_1|\tau^{-1}(s_0)]}{p} - 1\right) + \delta\left(\frac{E[X_1|\tau^{-1}(s_0)]}{p} - 1\right) \geq (1+\delta)r,$$

or

$$\left(\frac{\tau^{-1}(s_0)}{p} - 1\right) \geq r,$$

which is just the same as (4).

Writing down demands, we have

$$q_0 = \max\left\{0, \frac{\tau^{-1}(s_0)}{p} - 1\right\}, \tag{5}$$

and

$$q_1 = \min\left\{q_0, \max\left\{0, \frac{X_1}{p} - 1\right\}\right\}. \tag{6}$$

The seller's expected profit is

$$(p - s_0)\max\left\{0, \frac{\tau^{-1}(s_0)}{p} - 1\right\} + \delta p E_{X_1|X_0}\min\left\{q_0, \max\left\{0, \frac{X_1}{p} - 1\right\}\right\}.$$

At this point I need to simplify drastically in order to obtain a workable solution. Let all possible rates of return of the asset be contained within the unit interval. The seller's expected profit is then

$$(p - s_0)\left[\frac{\tau^{-1}(s_0)}{p} - 1\right] + \delta p E_{X_1|X_0}\min\left\{q_0, \left[\frac{X_1}{p} - 1\right]\right\}$$

Similarly to the two-quality case of model 1, let $x_0 \in \{l, h\}$, with $h > l$, and accordingly modify the domain of τ so that $\tau : \{l, h\} \to S$. Further, let $x_1 \in \{L, M, H\}$, with $H > h > M > l > L$, and with the "transition probabilities" $p_{x_0 x_1}$, such that $\sum_{x_1 \in \{L,M,H\}} p_{x_0 x_1} = 1$. The martingale property requires that $p_{hH}H + p_{hM}M + p_{hL}L = h$ and $p_{lL}H + p_{lM}M + p_{lL}L = l$. Further define

$\Delta_H \equiv H - h$, and $\Delta_L \equiv l - L$. In analogy to the analysis of model 1, we require, for a separating equilibrium to exist, the following version of the Spence-Mirrlees single-crossing property to hold:

$$\begin{aligned} & (p - s_0)\left(\frac{h}{p} - 1\right) + \delta p \left(\frac{p_{hH}h + p_{hM}M + p_{hL}L}{p} - 1\right) \\ \geq \; & p\left(\frac{l}{p} - 1\right) + \delta p \left(\frac{(p_{hH} + p_{hM})l + p_{hL}L}{p} - 1\right), \end{aligned}$$

that is

$$s_0 \leq \frac{(h - l) + \delta\left(p_{hH}h + p_{hM}M - (p_{hH} + p_{hM})l\right)}{\frac{h}{p} - 1}$$

and

$$\begin{aligned} & p\left(\frac{l}{p} - 1\right) + \delta p \left(\frac{(p_{lH} + p_{lM})l + p_{lL}L}{p} - 1\right) \\ \geq \; & (p - s_0)\left(\frac{h}{p} - 1\right) + \delta p \left(\frac{p_{lH}h + p_{lM}M + p_{lL}L}{p} - 1\right), \end{aligned}$$

that is

$$s_0 \geq \frac{(h - l) + \delta\left(p_{lH}h + p_{lM}M - (p_{lH} + p_{lM})l\right)}{\frac{h}{p} - 1}.$$

So a separating equilibrium exists if

$$p_{hH}h + p_{hM}M - (p_{hH} + p_{hM})l \geq p_{lH}h + p_{lM}M - (p_{lH} + p_{lM})l$$

Rewrite this using the definitions of Δ_H and Δ_L and the martingale property as

$$-p_{hH}\Delta_H + p_{hL}\Delta_L + h \geq -p_{lH}\Delta_H + p_{lL}\Delta_L + l.$$

I can now state the next result:

Proposition 4 *In the martingale case of model 3, a separating equilibrium exists if $h - l$ is sufficiently large, i.e. if $h - l \geq (p_{hH} - p_{lH})\Delta_H + (p_{lL} - p_{hL})\Delta_L$.*

Proof. The proposition follows self-evidently from the derivation above. ■

Once again, in a separating equilibrium, commission payments are a perfectly informative signal of product quality. Even if financial advice is based purely on commission payments, this does not compromise the quality of advice.

4 Welfare Properties of the Separating Equilibrium

4.1 Model 1

The attractive property of the separating equilibrium is that in every period, all customers who buy the policy they are offered are made weakly better off than they would be under their outside option. Those who do not buy the policy are better off investing in their outside option.

In a pooling equilibrium, the number of customers who buy in the first period is either (weakly) greater or (weakly) less than the number who optimally ought to buy in the first period, depending on whether the true quality of the policy they are offered is L or H. In the former case, some consumers who buy in the first period would have been better off investing in their outside option instead; in the latter case, some first-period non-buyers would have been better off buying the policy in the first period and furthermore, they cannot (under the rules of the game) buy in the second period either.

As discussed in section 1, the "renewal" of policies in the second period of our model is known in the insurance literature as persistency. The persistency rate is the ratio of investors who renew their policy. It is clear that in our two-quality model the persistency rate in a separating equilibrium is 100% for both high and low quality policies. In a pooling equilibrium (given that customers' expectation about policy performance is $0.5E[X|L] + 0.5E[X|H]$), the persistency rate for high quality policies is still 100%; for low quality policies, the persistency rate drops to $\frac{0.5E[X|H]-0.5E[X|L]}{0.5E[X|L]+0.5E[X|H]-p} < 1$.

Under the conditions identified in Corollary 2, the H-firm is also better off in a separating equilibrium. The L-firm, however, loses out: it will always be worse off in a separating than in a pooling equilibrium.

4.2 Model 3

Purely for the purpose of illustration, consider the simplified (martingale) version of model 3. In a separating equilibrium, if x_0 is h, period 0 demand is $q_0 = \frac{h}{p} - 1$, and expected period 1 demand is $q_1 = \frac{p_{hH}h+p_{hM}M+p_{hL}L}{p} - 1$. Therefore, $\frac{(1-p_{hH})h-p_{hM}M-p_{hL}L}{p}$ customers do not renew their policy at the end of period 0.

If x_0 is l, period 0 demand is $q_0 = \frac{l}{p} - 1$, and expected period 1 demand is $q_1 = \frac{(p_{lH}+p_{lM})l+p_{lL}L}{p} - 1$. Therefore, $\frac{p_{lL}l-p_{lL}L}{p}$ customers do not renew their policy at the end of period 0.

In a pooling equilibrium, assuming that buyers' priors on x_0 are $\Pr\{x_0 = h\} = \Pr\{x_0 = l\} = 0.5$, period 0 demand is $q_0 = \frac{0.5l+0.5h}{p} - 1$.

If x_0 is h, expected period 1 demand is $Eq_1 = \frac{p_{hH}(0.5h+0.5l)+p_{hM}\min\{0.5h+0.5l,M\}+p_{hL}L}{p} - 1$. Therefore, $\frac{0.5h+0.5l-(p_{hH}(0.5h+0.5l)+p_{hM}\min\{0.5h+0.5l,M\}+p_{hL}L}{p}$ customers do not renew their policy at the end of period 0.

If x_0 is l, expected period 1 demand is $Eq_1 = \frac{p_{lH}(0.5h+0.5l)+p_{lM}\min\{0.5h+0.5l,M\}+p_{lL}L}{p} - 1$. Therefore, $\frac{0.5h+0.5l-(p_{lH}(0.5h+0.5l)+p_{lM}\min\{0.5h+0.5l,M\}+p_{lL}L}{p}$ customers do not renew their policy at the end of period 0.

Comparison of non-renewals yields our final result:

Corollary 5 *In the simple martingale example of model 3, persistency is higher in a separating equilibrium unless* $x_0 = h$ *and* $M > 0.5h + 0.5l$.

These results for both model 1 and model 3 suggest a further testable implication of the model. With the introduction of commission disclosure, the differences between persistency rates for products sold through independent financial advisers and those sold through tied advisers should have increased, in the way suggested above.

5 Conclusion

I began this discussion with the observation that, when sellers' marginal returns to advertising vary systematically with unobservable product quality, commission payments to retailers (uninformative advertising) may act as a signal of product quality. This paper has explored repeat purchases as the link between quality and returns to commission payments, and found that, under the assumptions of proposition 1 and proposition 4, a separating equilibrium in the quality signaling game exists. Furthermore, these equilibria have desirable welfare properties.

In viewing sales commissions as a quality signal, this paper has followed the lead of Milgrom and Roberts (1986) who explore advertising expenditures as signals of quality. In the present context, however, care must be taken not to overemphasize the importance of the signaling function of commission payments: signaling is just one aspect of a complex problem. In this paper, retailers serve no function other than that of conduit for commission payment signals of quality. However, as I have pointed out above, retailers may well acquire information about customers that producers want to extract. Commission payments may therefore be part of a mechanism designed by the producer in order to elicit this information. When the retailer is a common agent for more than one producer, the producers will typically compete in the mechanisms (the commission payment schedules) they design for the retailer. This, in essence, is the question addressed by the recent literature on common agency (cf. Martimort (1992), Stole (1992), and in particular, Mezzetti (1997)). For the reasons pointed out above, these concerns are largely orthogonal to the vertical differentiation issues of this paper. Nevertheless, making them explicit in the current context (with horizontal product differentiation) is a worthwhile future research project.

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6 Appendix: Proofs

Proof of Proposition 1. First, observe that if $p > E[X|H]$, demand is zero for both firms. Therefore, $p \leq E[X|H]$.

Next, note that, in a separating equilibrium, the low-quality (type L) seller's best choice of commission is 0. Therefore, in a separating equilibrium, we want $\pi(s_0^*, q_0(s_0^*); H) \geq \pi(0, q_0(0); H)$ and $\pi(0, q_1(0); L) \geq \pi(s_0^*, q(s_0^*); L)$, or:

$$\begin{aligned}
& \max\left\{0, \frac{E[X|H]}{p} - 1\right\}(p - c_H - s_0^*) + \\
& +\delta \min\left\{\max\left\{0, \frac{E[X|H]}{p} - 1\right\}, \max\left\{0, \frac{E[X|H]}{p} - 1\right\}\right\}(p - c_H) \\
\geq\ & \max\left\{0, \frac{E[X|L]}{p} - 1\right\}(p - c_H) + \\
& +\delta \min\left\{\max\left\{0, \frac{E[X|L]}{p} - 1\right\}, \max\left\{0, \frac{E[X|H]}{p} - 1\right\}\right\}(p - c_H)
\end{aligned}$$

and

$$\begin{aligned}
& \max\left\{0, \frac{E[X|L]}{p} - 1\right\}(p - c_L) + \\
& +\delta \min\left\{\max\left\{0, \frac{E[X|L]}{p} - 1\right\}, \max\left\{0, \frac{E[X|L]}{p} - 1\right\}\right\}(p - c_L) \\
\geq\ & \max\left\{0, \frac{E[X|H]}{p} - 1\right\}(p - c_L - s_0^*) + \\
& +\delta \min\left\{\max\left\{0, \frac{E[X|H]}{p} - 1\right\}, \max\left\{0, \frac{E[X|L]}{p} - 1\right\}\right\}(p - c_L).
\end{aligned}$$

Assuming that $p \leq E[X|L]$, these conditions reduce to:

$$s_0^* \leq (1+\delta)\left(\frac{E[X|H]-E[X|L]}{E[X|H]-p}\right)(p-c_H) \tag{7}$$

and

$$s_0^* \geq \left(\frac{E[X|H]-E[X|L]}{E[X|H]-p}\right)(p-c_L). \tag{8}$$

In this case, therefore, a separating equilibrium exists if and only if

$$(1+\delta)\left(\frac{E[X|H]-E[X|L]}{E[X|H]-p}\right)(p-c_H) \geq \left(\frac{E[X|H]-E[X|L]}{E[X|H]-p}\right)(p-c_L)$$

or

$$c_H \leq p + \frac{1}{1+\delta}(c_L - p)$$

or

$$(1+\delta)c_H \leq c_L + \delta p. \tag{9}$$

Assuming that $E[X|L] \leq p \leq E[X|H]$, the conditions reduce to:

$$s_0^* \leq (1+\delta)(p-c_H) \tag{10}$$

and

$$s_0^* \geq p - c_L. \tag{11}$$

In this case, a separating equilibrium exists if and only if

$$(1+\delta)c_H \leq c_L + \delta p \tag{12}$$

which is just the same condition as (9) above. ■

Proof of Corollary 2. Assume that, without advertising, customers' expectations about the policy's performance are $0.5E[X|L] + 0.5E[X|H]$. In the resulting pooling equilibrium, a type θ seller's profit is

$$\pi(0, q^P(0), \theta) = \max\left\{0, \frac{0.5E[X|L]+0.5E[X|H]}{p} - 1\right\}(p-c_\theta) +$$

$$+\delta \min\left\{\max\left\{0, \frac{0.5E[X|L]+0.5E[X|H]}{p} - 1\right\}, \max\left\{0, \frac{E[X|H]}{p} - 1\right\}\right\}(p-c_\theta) \tag{13}$$

A type H seller signaling at level s_0^* in a separating equilibrium makes profits

$$\pi(s_0^*, q(s_0^*); H) = (1+\delta)\left(\frac{E[X|H]}{p} - 1\right)(p - c_H) - \left(\frac{E[X|H]}{p} - 1\right)s_0^*. \tag{14}$$

Note first from (13) that if $p \geq 0.5E[X|L] + 0.5E[X|H]$, demand will be zero. In this case it is immediate from (14) that a type H seller will prefer to advertise if the advertising expenditure required to distinguish itself is

$$s_0^* \leq (1+\delta)(p - c_H),$$

which is just the same as condition (10), and weaker than (7). Therefore, assume in what follows that $p < 0.5E[X|L] + 0.5E[X|H]$.

In a (no-advertising) pooling equilibrium, in which customers' prior on qualities is $p_L = p_H = 0.5$, a type H seller's profit is

$$\pi(0, q^P(0); H) = (1+\delta)\left(\frac{E[X|H] + E[X|L]}{2p} - 1\right)(p - c_H). \tag{15}$$

For the type H seller to prefer to advertise, we need (14) > (15), so

$$s_0^* \leq \frac{1}{2}(1+\delta)(p - c_H)\frac{E[X|H] + E[X|L]}{E[X|H] - p}. \tag{16}$$

Suppose first that $p \leq E[X|L]$. For (16) to be possible in a separating equilibrium, we need (from condition (8))

$$\frac{1}{2}(1+\delta)(p - c_H)\frac{E[X|H] + E[X|L]}{E[X|H] - p} \geq \frac{E[X|H] - E[X|L]}{E[X|H] - p}(p - c_L)$$

or

$$c_H \leq p + 2\frac{1}{1+\delta}(c_L - p)$$

or

$$(1+\delta)c_H \leq c_L + \delta p + (c_L - p), \tag{17}$$

which is stronger than condition (3).

Now suppose $p > E[X|L]$. For (16) to be possible in a separating equilibrium, we need (from condition (11))

$$0.5(1+\delta)(p - c_H)\frac{E[X|H] + E[X|L]}{E[X|H] - p} \geq p - c_L.$$

or

$$c_H \leq p - \frac{2(p - c_L)}{(1+\delta)\frac{E[X|H]+E[X|L]}{E[X|H]-p}}$$

However, we know (from $p > E[X|L]$) that

$$\frac{E[X|H]+E[X|L]}{E[X|H]-p} > \frac{E[X|H]+E[X|L]}{E[X|H]-E[X|L]},$$

so that we have as a necessary condition for (16)

$$c_H \leq p + 2\frac{E[X|H]-E[X|L]}{E[X|H]+E[X|L]}\frac{1}{(1+\delta)}(c_L - p),$$

which is stronger than condition (3) if $\frac{E[X|H]-E[X|L]}{E[X|H]+E[X|L]} > 0.5$, but always weaker than (17). ■

Proof of Corollary 5. Compare the expected number of non-renewals in separating and pooling equilibrium.

For the case where $x_0 = l$, suppose first that $0.5h + 0.5l < M$. Then, a separating equilibrium exhibits higher persistency if $0.5(1 - p_{lH} - p_{lM})h + 0.5(1 - p_{lH} - p_{lM})l - (1 - p_{lH} - p_{lM})l \geq 0$, or $h - l \geq 0$, which is true by assumption. Suppose now that $0.5h + 0.5l > M$. Then, a separating equilibrium exhibits higher persistency if $0.5(1-p_{lH})h+0.5(1-p_{lH})l-p_{lM}M-(1-p_{lH}-p_{lM})l \geq 0$. However, from $0.5h + 0.5l > M$ (which we have just assumed), we know that

$$0.5(1 - p_{lH})h + 0.5(1 - p_{lH})l - p_{lM}M - (1 - p_{lH} - p_{lM}) >$$

$$> 0.5(1 - p_{lH})h + 0.5(1 - p_{lH})l - (0.5h + 0.5l)p_{lM} - (1 - p_{lH} - p_{lM})l =$$

$$= 0.5p_{lL}h - 0.5p_{lL}l > 0.$$

For the case where $x_0 = h$, suppose first that $0.5h + 0.5l > M$. Then, a separating equilibrium exhibits higher persistency if $0.5(l - h) + 0.5p_{hH}(h - l) \geq 0$, or $1 - p_{hH} \geq 0$, which is true by assumption. Suppose now that $0.5h + 0.5l < M$. Then, a separating equilibrium exhibits higher persistency if $0.5(1 - p_{hH} - p_{hM})h + 0.5(1 - p_{hH} - p_{hM})l - h + p_{hH}h + p_{hM}M \geq 0$, or $0.5(h + l)p_{hL} - (p_{hL} + p_{hM})h + p_{hM}M \geq 0$. However, since $0.5(h + l) < M$, and $h > M$, in this case only, persistency is higher in a pooling than in a separating equilibrium. ■