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An Economic Model of Learning Styles

Gervas Huxley and Mike Peacey

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Centre for Market and Public Organisation
University of Bristol
2 Priory Road
Bristol BS8 1TX
<http://www.bristol.ac.uk/cmipo/>

Tel: (0117) 33 10952

Fax: (0117) 33 10705

E-mail: cmipo-admin@bristol.ac.uk

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An Economic Model of Learning Styles

Gervas Huxley¹ & Mike Peacey²

¹ University of Bristol and CMPO

² University of Bristol and University of Bath

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Abstract

Much of the economic literature on education treats the actual process of learning as a 'black box'. While these 'black box' models have many interesting uses, they are of little use when a college seeks advice about reallocating resources from one input to another (e.g. from lecture hours to tutorials). Commenting on such questions requires us to 'open up' the black box. In this paper, we show what one such model would look like by explicitly modelling how students vary in their 'learning styles'. This model allows us to simulate how reforms to higher education would affect students with different learning styles. We consider alternative tuition fee structures and the technological change that has led to the introduction of massive open online courses (MOOCs).

Key words Human Capital, Education Production Function, Learning Style, Independent Learner, MOOC

JEL Classification: I20, I23

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Address for correspondence

CMPO

2 Priory Road, Bristol

BS8 1TX

www.bristol.ac.uk/cmipo/

cmipo-admin@bris.ac.uk

Tel +44(0) 117 33 10799

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¹ E-mail: gervas.huxley@bristol.ac.uk

² E-mail: mike.peacey@bristol.ac.uk

[In a production function with more than one input] “The i^{th} person has more ability if $f_i > f_j$... If sometimes $f_i > f_j$ and sometimes $f_j > f_i$, there is no unique ranking of their abilities.”

- Gary Becker (1975)¹

“In analyzing ability, society needs to recognize its multiple facets.”

- James Heckman (2010)

“Here is a bet and a hope that the next quarter century will see more change in higher education than the last three combined.”

- Lawrence Summers (2012)

1 Introduction

This paper is about how individuals invest in human capital and about the nature of the choices we face when we make these investments. Much of the economic literature on education treats the actual process of learning as a ‘black box’. The inputs provided by schools, colleges or parents (teachers, study materials, reading at home, etc.) are often lumped together into a single input (‘effort’, ‘investment’ or ‘expenditure’), which is assumed to affect student outcomes. Meanwhile the students themselves are often assumed to vary according to a single parameter (usually called ‘ability’) which affects their responsiveness to the learning input from educators.

¹Footnote 107, page 110

While these ‘black box’ models have many interesting uses, they are of little help when a school or college seeks advice about reallocating resources from one input to another (e.g. from lecture hours to tutorials, or from individual study time to group activities). Commenting on such questions requires us to ‘open up’ the black box capturing the variety of inputs available to educators, and also the heterogeneity of students in the ways that they respond to different inputs.

There is a large educational literature on pedagogy and a closely related literature on learning. This work explores the relationship between how teachers teach, how students learn and educational attainment. We follow this literature by emphasizing the distinction between teaching style, which refers to the mix of inputs used in the classroom, and learning style which refers how students respond to those inputs (see Bransford et al. 1999).

Differences in teaching style might focus on whether material is presented using visual, auditory or kinesthetic methods (VAK model, see Fleming (1995) and Bransford et al. (1999), part III). Differences in learning style refer to the responsiveness of students to these different teaching technologies. Both literatures emphasize that students are heterogeneous, not just in ability but also in learning style (see Gardner (1993) and Bransford et al. (1999), part II). The goal is to develop pedagogic strategies that achieve the best match between teacher and student.

The approach that comes closest to the one we adopt in this paper emphasizes the importance of ‘independent learning’, ‘interest-driven learning’, or ‘personalized learning’. The role of the teacher is to facilitate self directed learning (see Candy (1991)). These approaches all distinguish between

studying and being taught as mechanisms for acquiring education. We follow this approach by viewing investment in human capital as involving a choice between self study and tuition. As well as modeling the impact of each input separately, an economic approach emphasizes that the interactions between them will be crucial. The education literature usually ignores problems arising from scarcity. However, the choice of learning technology will have implications for, and be influenced by, the allocation and pricing of scarce educational inputs.

We apply our model to the current debate about the future of higher education. The full significance of massive open online courses (MOOCs) remains uncertain. Although millions of students have enrolled on MOOCs, for most of these students the education provided does not replace a traditional HE degree programme (Waldrop (2013)). It has been suggested that this will change, particularly if accreditation by MOOCs comes to be seen as comparable to that offered on a traditional degree program (Friedman (2012), Barber et al. (2013)). In this paper we assume that MOOCs offer an alternative to the traditional model. We investigate the consequences of this assumption on different kinds of learners.

As different pedagogical techniques are increasingly being subjected to rigorous, controlled trials, and the quantitative literature regarding different educational inputs (class sizes, teacher quality, etc.) continues to grow, we provide a framework for integrating these findings into a broader model of teaching and learning. Such a framework will hopefully lay the groundwork for increasingly fruitful interaction between developmental psychologists and educational economists, helping to translate findings from psychological and

pedagogical research more directly into concrete policy developments.

This paper has three key results. Firstly, for students of a given ability, the graduate premium will depend on learning style. Heterogeneity in learning styles results in different benefits from a given amount of tuition. Expenditure on tuition should be determined by how learners choose to invest in human capital, not simply their total expenditure on human capital. Secondly, the possibility that MOOCs will replace some traditional institutions depends on the distribution of learning styles within the population. Thirdly, the current ‘one size fits all’ model is inefficient and we characterize the welfare gains that would arise from unbundling.

1.1 Economic Literature

In Becker (1962) ability is a synonym for productivity. Since there is only one input, different means of investing in human capital are not considered. This ignores the possibility that, for example, students might make large investments in human capital without significant expenditure on education.

In Becker (1965) individuals allocate time between two goods: leisure and work. Grossman (1972) combined Becker’s work on human capital and allocation of time (Becker (1962), Becker (1965)). Grossman’s paper is about investment in health care seen as a form of human capital. In his model, time is allocated between three goods: leisure, work and time spent investing in health (e.g. exercise²). Biddle and Hamermesh (1990) also allocate time

²The comparative statics we are interested in are not explored by Grossman. For example, Grossman does not explore how a given health status can be achieved using different input combinations nor does he address the influence of price on the choice of inputs.

between three goods. In their model wages are affected by the time invested in sleep.

Since tuition is paid for through earnings, which involve an investment of time, the choice is ultimately one about the allocation of time between three goods: self study, tuition and earning.

The paper that comes closest to our work is Bratti and Staffolani (2002), which looks at the effect on exam performance of students' allocation of time between self study and lectures. In our model, students allocate time across the same three activities, however we also include price effects. Our paper departs from theirs by focusing on how various pricing structures affect students, and how students' decisions and attainment depends on different learning styles.

In addition to the literature on human capital, the work which comes closest to ours is on education production functions. We view this literature as motivated largely by econometric considerations. Early and more recent surveys are Hanushek (1979) and Machin and Vignoles (2005). This work measures the impact of a variety of inputs such as class size, peer effects and teacher quality. From our perspective these relate to teaching as opposed to learning (see section 4).

A more recent literature, which focuses on early childhood development, seeks to understand the role of parental characteristics and home environment in developing cognitive skills of young children (e.g. Todd and Wolpin (2003)). In our model these variables would be parameters not inputs. Using just tuition and study, we create a micro-foundation for different learning styles by modeling a learning production function.

‘The technology of skill formation’ provides a theoretical foundation for the early childhood literature (Cunha and Heckman (2007), Cunha et al. (2010)). In these production functions individuals must decide on investments at different stages of childhood. Although educational investments remain one dimensional, each intertemporal investment is an input in the education production function. Heckman is interested in how these inputs determine educational attainment and how the ratio used changes with respect to interest rates (prices).

We are interested in the degree of complementarity between study and tuition. Heckman is interested in the degree of complementarity between investments made at different stages of the life cycle. In this work, early childhood investments enhance non-cognitive ability and this increases the returns to investments made at later stages of the life cycle. The emphasis on non-cognitive ability has resulted in an interest in the psychology of personality formation and the relationship between personality, skill formation, and occupational choice. Almlund et al. (2011) use a modified version of the Roy model (Roy (1951)), here different endowments of personal characteristics determine occupational choice via comparative advantage. Heckman presents evidence that there are a relatively small number of personality types that once formed are hard to change. We are interested in how differences in personality might influence how students learn.

2 The Model

An individual i lives for N periods³. In each period she will make choices which maximise her lifetime utility. Her lifetime utility, U , is increasing in consumption of a numéraire good, m_t :

$$U = \sum_{t=1}^N u(m_t) \quad (1)$$

She faces choices about investment in education. In any period, t , it is possible to obtain e_t education in a number of ways according to her education production function:

$$e_{i,t}(S_t, \tau(T_t)) \quad (2)$$

where S_t is time spent on study in period t , T_t is time spent receiving tuition in period t and τ is an intensity adjusted measure of tuition (see section 4). Education is strictly increasing in both S_t and T_t .

$$\frac{\partial e_{i,t}}{\partial S_t} > 0, \quad \frac{\partial e_{i,t}}{\partial T_t} > 0 \quad (3)$$

In period t it is possible to obtain a “level t diploma” subject to:

- (1) meeting the prerequisite (having a level $t-1$ diploma).
- (2) attaining a level of education of at least e_t^* in this period⁴.

Education is cumulative and as such i 's stock of education in period t is

³Discounting of utility, depreciation of education and the role of interest rates are not of interest in our model. Hence we do not include these in equations 1, 4 and 6.

⁴We assume that a single diploma cannot be obtained over multiple periods.

given by:

$$E_{i,t} = \sum_{s=1}^t \mathbb{1}(e_{i,s}^*) \quad (4)$$

where $\mathbb{1}(e_s^*)$ denotes if a diploma was acquired in period s .

The wage, w_t , is an increasing function of this education stock:

$$w_t = W(E_{t-1}) \quad (5)$$

The individual's decision problem is to choose a stream of education and work to maximize consumption. She is subject to an intertemporal money constraint (6), and N intratemporal time constraints (7). The first constraint implies that the lifetime value of the stream of expenditure (on tuition or consumption) cannot exceed the lifetime value of the stream of earned income:

$$\sum_{t=1}^N (w_t H_t - m_t - p T_t) = 0 \quad (6)$$

where H_t is hours worked in period t and p is the price of tuition. p is assumed an exogenous positive constant. The second set of constraints implies the amount of time devoted to each activity must add up to the endowment of time, Ω , in each period:

$$\Omega = S_t + T_t + H_t \quad t = 1 \dots N \quad (7)$$

We define the “graduate premium” as the net increase in consumption achieved through education: the difference between consumption available as a graduate minus the consumption that could have been achieved without education.

The utility maximization problem involves choosing the *level* of education to acquire (E_t) and *how* it is acquired (S_t, T_t).

2.1 Two Period Model

Rather than looking at how much to invest over the life cycle (Grossman (1972)), we focus on how investments are made during any one period. Focusing on one period of investment is equivalent to considering a two period model, since investment only occurs in the first period. In the appendix we present an N period model, but here we focus on two periods and after solving we remove the time period indexing.

$$E_1 = \begin{cases} 1 & \text{if } e_{i,1}(S_1, \tau(T_1)) \geq e_1^* \\ 0 & \text{otherwise} \end{cases} \quad (8)$$

We adopt the following notation for the wage function:

$$W(E_1) = \begin{cases} \bar{w} & \text{if } E_1 = 1 \\ \underline{w} & \text{if } E_1 = 0 \end{cases} \quad (9)$$

Since the individual only lives for 2 periods, it is clear that no investment will occur in period 2. If the individual chooses $e_1 = e_1^*$ she becomes a “graduate”, and her consumption is:

$$m^{E_1} = \Omega \bar{w} + \underline{w}(\Omega - S_1 - T_1) - pT_1 \quad (10)$$

If she chooses $e_1 = 0$ and becomes a “non-graduate” her consumption is:

$$m^{E0} = 2\Omega\underline{w} \quad (11)$$

The graduate premium is $G = (m^{E1} - m^{E0})$. The individual maximizes G subject to:

- (1) The education production function (equation 8)
- (2) The wage function (equation 9)
- (3) The time constraint: $\Omega \geq S_1 + T_1$

Clearly this will result in an optimal choice of $e_1 \in \{0, e_1^*\}$. If education is chosen, the optimal inputs can be found by minimizing $c_1(S_1, T_1) = \underline{w}(T_1 + S_1) + pT_1$, subject to the above constraints.

2.2 C.E.S. Education Production Function

We use a CES education production function to express and interpret the differences in learning styles set out in section 3:

$$e_i(S, \tau) = A(\alpha_i S_i^\rho + (1 - \alpha_i)\tau^{\rho_i})^{\frac{1}{\rho_i}} \quad (12)$$

where τ denotes the value of $\tau(T)$, $\rho \leq 1, 0 \leq \alpha \leq 1$ and $A = 1$.

We allow α and ρ to vary between individuals. Using this function, an individual’s learning style is stationary and independent of prices, education level and wealth⁵. Later in the paper we discuss how learning styles change

⁵The optimal ratio of inputs used will be independent of education, but will depend on price and learning styles.

during the education lifecycle.

In a two-input model it is natural to use total factor productivity (A) as a measure of what Becker called ability (see section 3.3). We are interested in how differences in learning style affect returns. Therefore in our simulations (section 5) we hold A constant across individuals. This gives no individual an unambiguous advantage in producing E .

The CES share parameter, α , is the *independence parameter* (see section 3.1). The *flexibility parameter*, ρ , represents the degree of complementarity between study and tuition in producing education (see section 3.2)⁶. Two special cases are $\rho = 1$ and $\rho \rightarrow \infty$. In the first case, study and tuition are perfect substitutes. When $\rho \rightarrow \infty$, study and tuition are perfect complements. In the CES production function, the parameters are interdependent and therefore must be carefully interpreted (see appendix)⁷.

2.3 The Wage Function

Human capital is the only variable in our wage function, we ignore the influence of all other variables that might determine income. This specification has two important implications:

- (A1) The wage function is independent of ability;
- (A2) The wage function is independent of the mix of inputs and the individual's learning style.

⁶The elasticity of substitution is related by $\sigma = \frac{1}{1-\rho}$.

⁷A more detailed interpretation of the CES share parameter can be found in Temple (2012).

A1 assumes that ability only affects the wage indirectly via its influence on educational attainment⁸. Becker (1962) does not make this assumption. At least in principal he allows the wage to depend on both ability and education⁹.

A2 becomes relevant in a two input model. Employers have no preferences about how human capital is acquired. For example how an individual learns a language does not matter, what matters is whether she can speak it. For this reason, we assume that study and tuition affect the wage only through education. Hence choices about study and tuition will depend on both the wage elasticity of education and an individual's learning style.

In neoclassical theories of production, consumers have no preferences over how goods are produced. There are good reasons to believe that this assumption may not hold in the case of education¹⁰. If employers want workers to replenish their human capital on the job they will seek out 'life long learners'. In this case employers might have preferences over both the level of education and the learning style.

3 Learning Styles and Education Production Functions

The educational literature emphasizes that students have different learning styles. In our model differences in learning style correspond to different ed-

⁸It is of course true that in equilibrium there will be a relationship between ability and wages.

⁹It turns out this assumption also has implications for the discussion of ability. See appendix.

¹⁰In this case the mix of inputs used affects the wage through both the indirect effect on education and the direct effect on productivity.

education production functions. We propose that the fundamental distinction can be modeled in terms of two variables: study and tuition. Although the resources suitable for different learning styles vary, we suggest that many of these differences can be captured in two dimensions: time and money. For example, a visual learner will respond better to an educational DVD than practical experiments (ideal for kinesthetic learners)¹¹. The former is likely to require less instruction than the latter.

A two-input model provides a rationale for one of the most important ideas in the education literature: the belief that one learning style is not necessarily better than another. Since the optimal mix of inputs is determined by both prices and learning style, ability is not independent of the vector of prices.

We define two aspects of learning style: *independence* and *flexibility*. Independence is measured by the ratio of tuition to study. The degree of flexibility refers to the extent to which learners can substitute the two inputs.

Since higher education typically bundles tuition and study, the relevance of these learning style parameters is usually ignored. The introduction of MOOCs represents a change in technology that is likely to unbundle these inputs, which has implications for demand.

3.1 Independent Learner

Independence measures the weight given to each input in the production function. For independent learners, the output elasticity for study will be

¹¹See Fleming (1995).

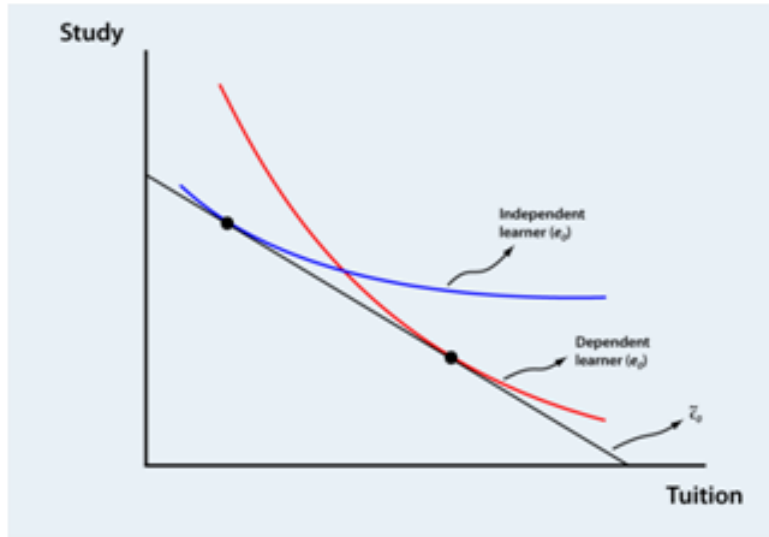


Figure 1: The independent learner has a flatter isoquant.

higher than for tuition.

The marginal rate of transformation measures the trade-off between tuition and study:

$$MRT_{(S,T)} = \frac{\frac{\partial e}{\partial T}}{\frac{\partial e}{\partial S}} \quad (13)$$

Independent learners will usually buy less tuition than directed learners (Figure 1). Although independent learners will choose to make most of their human capital investments in the form of study, these students need not have higher returns than directed learners. Total investments will not necessarily be greater for one type of learner than another. If low-return students are dependent learners and therefore require a tuition intensive education it would reverse the relationship between ability and expenditure set out in the literature.

3.2 Flexible Learner

A flexible learner is one who can adapt to different teaching styles. In the VAK model a flexible learner will be equally at home with visual, auditory, or kinesthetic teaching styles. In our model, flexibility is a measure of how a student can adapt to different combinations of the two inputs. The more easily the inputs can be substituted the more flexible the learner will be. A perfectly flexible learner can substitute teaching and study in a fixed ratio, whereas a perfectly inflexible learner must use the two inputs in a fixed ratio.

This implies that we are interested in whether tuition and study are substitutes or complements. In the standard model, it is implicit that tuition and study are perfect complements. We believe that this assumption is inconsistent with the pedagogic literature, much of which emphasizes the importance of how teachers and learners interact (Bransford et al. (1999)).

The elasticity of substitution (equation 14) measures the curvature of the education production function for a constant level of education: how much individuals can substitute inputs conditional on how much of each input they are already using.

$$\sigma_{(S,T)} = \frac{d(\frac{S}{T})}{(\frac{S}{T})} \frac{MRS_{(S,T)}}{dMRS_{(S,T)}} \quad (14)$$

If as seems likely investments in tuition and study are complementary then high investment in study will increase the return to investment in tuition and vice versa. Almost everybody would agree that if you do not study there is little point in investing in tuition. However this statement can be reversed; if you work hard it will be worthwhile investing in tuition.

Consider a parallel with sport: talented athletes make large investments

in exercise (study). However, precisely because they are so talented it is also worth their while to purchase large amounts of training. It is not uncommon for world class athletes to employ a full-time trainer.

In our model this translates into the statement that provided study and tuition are complementary the higher your ability *and* the more time you spend studying, the higher should be your expenditure on tuition.

3.3 Ability

Ability is defined by the ratio of output to input; when considering more than one input this definition is no longer straightforward. Inputs must be aggregated and this requires weighting by price. Because we allow marginal returns for each input to vary between individuals the focus shifts to relative price, and who can make better use of the cheaper input. In Figure 2 when the price of tuition increases, the cost of acquiring the same level of education increases more for the directed learner (DL) than the independent learner (IL).

This change is bound to raise objections however we can see no escape from the conclusion. This does not imply ability is meaningless - ability rankings can change when prices change. Becker understood this problem¹². We refer to ability rankings that are independent of prices as a “Becker ability ranking”. We aim to use this insight to develop a definition of ability that is intuitive and consistent with the pedagogical literature¹³.

¹²See Becker (1975), page 110 footnote 107.

¹³In general ability will also depend on e^* . Thus ability rankings can change through the education life cycle. The highest ability 10 year old may no longer be the highest by the time she enrolls at university. In a CES production function (section 2.2) this problem does not arise.

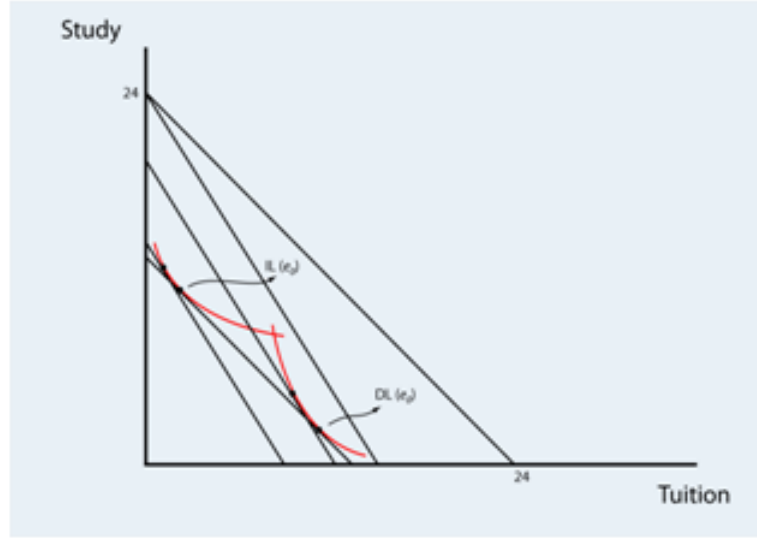


Figure 2: The impact of learning style on cost minimization

Definition (ability) If two individuals choose to produce e^* , if individual A has a lower cost than B, then A will have a higher level of utility and is said to be of higher ability.

$$e_A(T^{*A}, S^{*A}) = e_B(T^{*B}, S^{*B}) \quad (15)$$

and

$$c(T^{*A}, S^{*A}) < c(T^{*B}, S^{*B}) \quad (16)$$

This is equivalent to saying that if the cost of e^* is fixed for individuals A and B, then the individual with the highest ability will acquire more education.

Heckman's 2 period, 1 input model is similar to our 1 period, 2 input model. It follows that the same ambiguity must arise. The cost of investment I_0 is the opportunity cost of investment in the next period, $(1+r)I_1$. If the

interest rate changes, the relative price of the two investments change. The higher ability individual is now whoever makes better use of the cheaper investment¹⁴.

4 The Tuition Production Function

Tuition is dissemination of knowledge by a teacher and is an input in our education production function. The input T will always correspond to one hour of tuition, however this tuition may vary in quality: a student in a one hour class of five students is likely to obtain more from this hour than if they were in a larger class¹⁵. We use the function $\tau(T)$ to adjust tuition time, T , to account for quality. In addition to class size τ will depend on parameters such as content, classroom resources and teacher effects (see Vignoles et al. (2000) and Hanushek and Rivkin (2006)).

Decisions about quality will depend on how it varies with demand (which will depend on learning style - see section 3) and costs. We illustrate this idea with class size.

In our model we emphasize the importance of differences in content by focusing on material that is either ‘core’ or ‘discursive’. With core material class size has little impact on quality. In contrast we suggest that discursive material is harder to provide in large classes. A MOOC requires students to follow a syllabus, view videos and participate in online forums and thus delivers core material. However the delivery of discursive material is much

¹⁴In the appendix we show that this problem arises even in a one-input model.

¹⁵In principal almost all study requires at least some tuition. A learner who uses a text book is receiving a small amount of tuition.

less effective, and therefore the amount of this type of tuition provided by a MOOC is strictly limited (see Bowen (2012)).

The traditional university faces capacity constraints on lecture theaters. Adding students up to this constraint will not increase cost, and thus a university will be solely concerned with the reduction of intensity. For core material this quality reduction is close to zero and therefore universities will always try to fill lecture theaters.

We model traditional HE as maintaining quality when discursive material is taught ($\tau(T) = T$). This is possible because a traditional university provides this type of material in small group tutorials. Our model of traditional HE is one in which a generous amount of tuition is available for a fixed fee (see section 5.2).

We believe that with current technology, one-hour of online interaction with peers is lower quality than a traditional one hour small group tutorial with an experienced professor¹⁶. To simplify we assume that MOOCs can only provide core material, and they do so for free¹⁷. Therefore they can deliver \bar{T}^{core} with the same quality as a traditional HE institution:

$$\tau(T) = \begin{cases} T, & \text{if } T \leq \bar{T}^{core} \\ \bar{T}^{core}, & \text{if } T > \bar{T}^{core} \end{cases} \quad (17)$$

In general, cost minimization will result in universities teaching large groups for core material and small groups for discursive material¹⁸. Overall,

¹⁶This is disputed (see Barber et al. (2013)).

¹⁷In the future MOOCs may charge; provided pricing is much lower than traditional HE, our results will still hold.

¹⁸It may also involve providing different class sizes for different learner types (see section

a traditional university has constant returns to scale (Getz et al. (1991) and Cohn et al. (1989)). The technological change represented by MOOCs implies that core material may be provided, by natural monopolies, to ever larger audiences (Bowen (2012)).

5 Results

This section considers the delivery of higher education. We investigate how an individual's graduate premium is affected by her learning style and the pricing structure, holding 'ability' constant¹⁹. We use this framework to address two questions: firstly who would choose to acquire education through a MOOC compared to traditional HE; secondly how behavior changes when tuition is unbundled, and how these changes depend on learning style.

An algebraic solution to our model is given in the appendix. This gives the optimal inputs (S^*, T^*) which would be chosen under a *variable fee* (see section 5.3) and shows how this bundle changes with price and learning style. However, since the interactions between the learning style parameters are complicated, the results presented here are simulated. The appendix also contains more details on the level of the graduate premiums and the effect of compulsory attendance.

In these simulations 100,000 possible individuals are randomly generated, with the same level of Becker ability but heterogeneous learning styles. We maximize the individual's graduate premium subject to various constraints on (S, T) .

3).

¹⁹See section 3.3.

We make no attempt to quantify the distribution of learning styles present in a population²⁰. Instead we consider all learning styles, by allowing the two parameters in the CES production function to vary over their entire domains²¹.

5.1 Graduate Premium and Learning Style

In Figures 3, 5 and 7 different points represent individuals with specific learning styles, with independence and flexibility measured on the vertical and horizontal axes respectively. The diagram plots all individuals who obtain a positive graduate premium. These individuals are ranked by their graduate premium into four colored groups.

To illustrate the importance of learning styles in determining the graduate premium we consider the case where, apart from opportunity cost, education is free and unlimited. In this case, we might expect all individuals to obtain the same graduate premium since they have the same Becker ability²². However individuals with different learning styles use inputs differently and therefore have different opportunity costs²³.

In Figure 3, extreme values of the vertical axis correspond to either independent or directed learners. The high graduate premiums these individuals obtain illustrate the benefits that result from specialization in the use of one

²⁰Therefore we cannot address such questions as “What proportion of school leavers progress to HE?”.

²¹We generate 100,000 individuals with characteristics distributed in $\alpha \in (0, 1) \times \rho \in (-\infty, 1)$. The graphs are truncated at $\rho = -10$. Inputs are measured in units of 0.25 hours.

²²See section 3.3.

²³Individuals only obtain the same graduate premium if they are all forced to use bundles which are symmetric. This is the only case where graduate premium is independent of learning style.

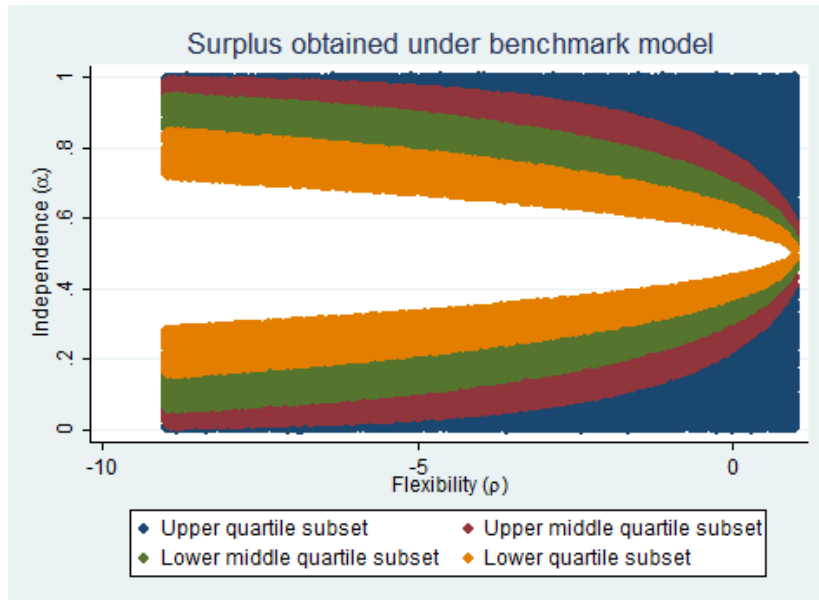


Figure 3: Graduate premium by learning style. Parameter values: $w=5$, $\exp=0$, $p=0$, $ed=12$

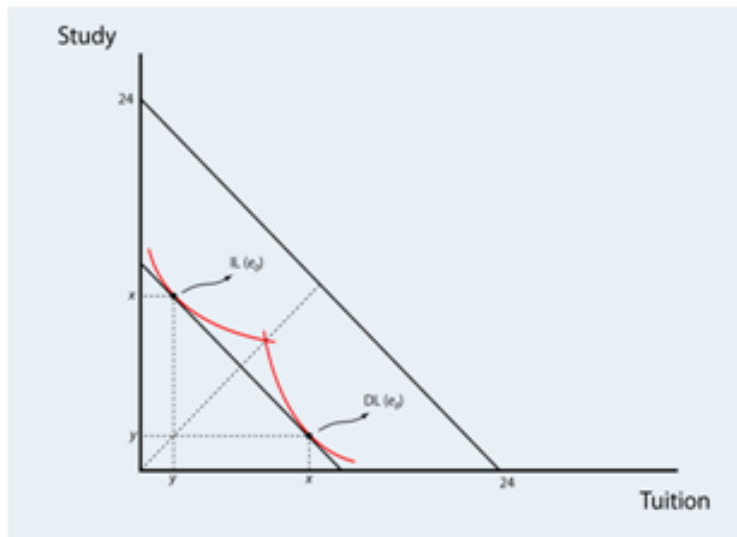


Figure 4: Optimal inputs under benchmark constraint

input.

However, specialization comes at a cost: low returns to the other input can matter. As students become less flexible, S and T become complements, and so they increasingly require a mix of inputs. As we move left along the horizontal axis, individuals with lower flexibility (ρ) generate a lower graduate premium.

These two effects mean that size of the graduate premium becomes increasingly responsive to changes in independence as we decrease flexibility. This can be seen by the thinning bands.

Figure 3 is symmetric about $\alpha = 0.5$. This symmetry arises because individuals who are symmetric with respect to the way they use inputs generate the same graduate premium when they face the same price. This can be shown in Figure 4. Two symmetric individuals (IL with $\alpha = 0.25$ and DL with $\alpha = 0.75$) produce the same level of education. The inputs used by one person ($S = x, T = y$) are the opposite of the inputs used by the other person ($S = y, T = x$). Each person consumes a different bundle of inputs, but the total amount of inputs used will be the same, and hence the cost will be the same.

When students must pay a price for tuition, as well as an opportunity cost, their choice of S and T will change and this has differential effects on the graduate premium.

The individuals who lose most when price increases are dependent learners, since they buy a lot of tuition. However, these losses are reduced by flexibility - as complementarity increases the cost of substituting study for teaching.

5.2 Traditional HE vs MOOCs

Following section 5.1, we compare the graduate premium under traditional HE and MOOCs. Individuals maximize their graduate premium by choosing the system which best suits their learning style (Figure 5).

Figure 5 possesses some of the same symmetry as Figure 3. This is because for both traditional HE and a MOOC, the marginal cost of each input is the same (forgone wage). The fixed fee charged by a traditional university reduces the graduate premium for all students who choose to attend (compared to 5.1), this means that for some students it is no longer worth attending university (represented by the white area).

The MOOC allows some students who would attend a traditional university to complete their education for a lower cost and hence receive a larger graduate premium. In addition some students, discouraged by the high cost of traditional university, now find acquiring education worthwhile. The students who choose to attend a MOOC tend to be both independent and flexible (Figure 5, top right). These learners receive little benefit from the face to face contact provided in small groups by traditional universities. However, there will also be some directed learners who are sufficiently flexible that find the price reduction justifies the extra study required by a MOOC (shown in Figure 5 by the blue area continuing below $\alpha = 0.5$).

Independent inflexible learners will choose traditional universities (Figure 5, top left). For these students, tuition and study are complementary. Therefore these students benefit from the more intensive contact offered by a traditional university. Finally, directed learners choose traditional univer-

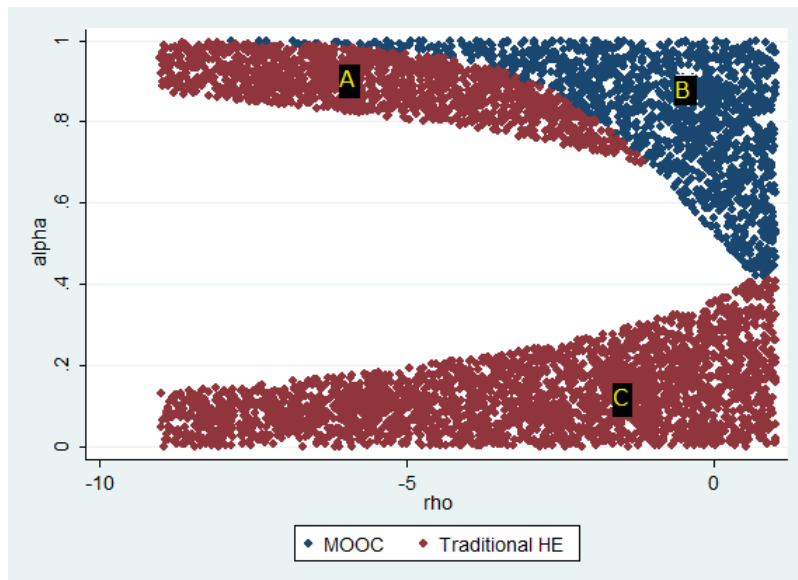


Figure 5: MOOC vs Traditional HE

sities (Figure 5, bottom). These learners require more tuition than can be provided by a MOOC because the returns to study are too low.

The recent development of MOOCs represents a radical change in the proportions in which S and T are bundled. We believe that the potential for MOOCs to transform HE depends in part on the importance of learning style, and the possibility that the current arrangement provides an inefficient bundle.

5.3 Unbundling (Fixed vs Variable Fees)

The HE model we have adopted is one in which the university charges a fixed fee and provides optional lectures and classes²⁴. This means study and tuition are implicitly bundled (Adams and Yellen (1976), Wang (1975), Norton

²⁴In the appendix we investigate the implications of compulsory class attendance.

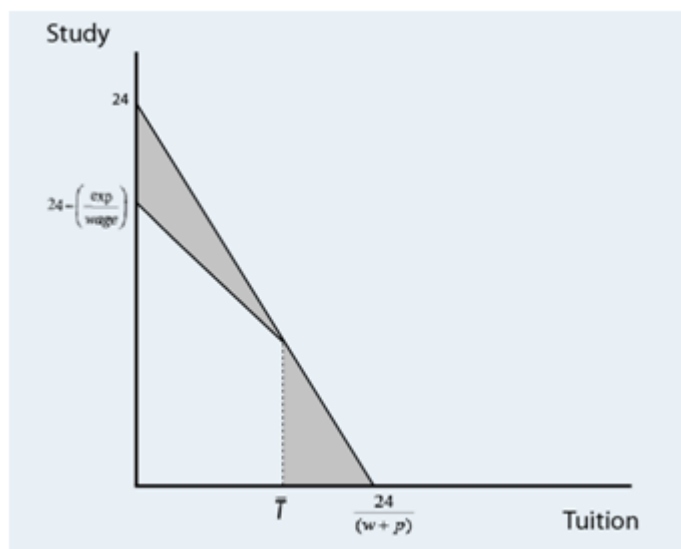


Figure 6: Unbundling relaxes the constraint

(2013)). In this section we attempt to investigate the welfare implications of unbundling on students with different learning styles.

In practice students are almost never offered a variable fee structure, rather they must choose S given T . If learning styles are heterogeneous at least some students must be purchasing a sub-optimal bundle. Unbundling would allow students to only pay for their desired quantity of T , resulting in hourly paid tuition.

To compare the welfare effects of variable fees relative to fixed fees, we set the price of tuition equal to $p = \frac{\text{fixed fee}}{T}$. This means any bundle available under the fixed fee is available under the variable fee²⁵. Relaxing the constraint in this way ensures that all learners will gain from unbundling (see Figure 6).

²⁵For a university this change is unlikely to be resource neutral. Unbundling will change the demand for tuition, and therefore affect revenue.

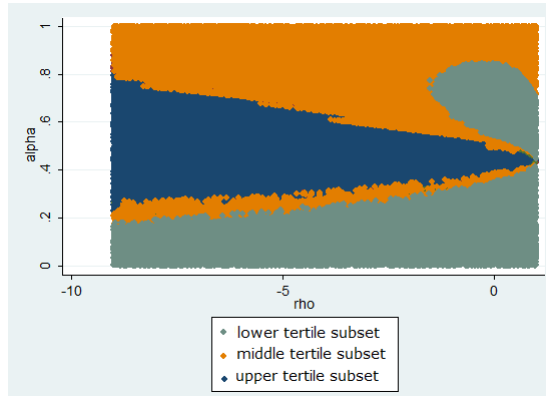


Figure 7: The gains from unbundling according to learning style.

Unbundling gives rise to both price and wealth effects. These depend on the student's initial bundle, how distorted they were and their learning style.

Independent, flexible learners choose zero tuition under the fixed fee because the opportunity cost of tuition is already too high. Thus their behavior will not change when an additional price is included. These students only benefit from the wealth effect and are shown in Figure 7, top left. Analogously, directed, flexible learners (Figure 7, bottom right) continue to choose zero study and therefore only benefit through the wealth effect²⁶.

Students who choose an interior solution with a fixed fee will tend to be more independent and inflexible (Figure 7, top left). The increase in price results in learners substituting study for tuition.

The most distorted learners will change their behavior the most, they will be flexible learners initially located at the kink. These learners will experience large gains. Directed learners will purchase more tuition after unbundling (Figure 7, bottom). Replacing x hours of study (opportunity

²⁶Note, unlike their independent, flexible counterparts some of these students may substitute away from tuition when its price increases.

cost wx) with y hours of tuition (cost $y(w + p)$), will generate a benefit of $y(w + p)wx$. Independent learners value the tuition they currently receive $\in (w, w + p)$. Thus when the price rises, they substitute away from tuition. How much study is required for this substitution will determine the size of the gain (Figure 7, grey area on right).

We can summarize these three results by saying that the graduate premium will depend on how higher education is delivered and will vary across heterogeneous learning styles (5.1).

MOOCs have the potential to provide large gains (5.2). These gains depend on learning style, with the independent, flexible learners gaining most. More generally, unbundling the inputs used by learners will benefit everyone (5.3), with largest gains going to the most distorted learners.

Combining MOOCs with unbundling would provide the most efficient outcome: offer core material via a MOOC and discursive material via a more flexible HE sector. Thus different kinds of learners can choose the bundle of tuition and study that best suits their individual learning style, whilst benefiting from the economies of scale provided by a MOOC.

6 Extensions

6.1 Returns to Scale and Technical Change

The education production function may change as the level of education increases, and therefore during the education lifecycle. The relevant measure in this case is how the cost changes as education increases, which is similar

to Arrow's learning by doing progress ratio²⁷.

Technological change can affect the cost of existing pedagogy²⁸ or it can sometimes result in novel pedagogies. For example, the introduction of Powerpoint might have increased lecture quality and therefore raised the productivity of one hour of tuition. Whilst e-learning which promotes user generated content (e.g, blogs or wikis) might increase the productivity of study. These changes will affect the demand for each input.

It is perhaps worth emphasizing that new technology can change the productivity of both tuition and study. The introduction of dictaphones in the 1980s (and more recently podcasts), which permitted students to review lectures easily increases the direct productivity of both tuition and study, as well as increasing complementarity between the two. However, the productivity impact is unlikely to be the same for each input.

In our model, technology can change learning styles and these changes are unlikely to be uniform across learning styles. Much of the recent discussion about the role of e-learning suggests that learners are becoming increasingly independent. However technological change can increase the productivity of both tuition and study so the impact on learning styles is uncertain.

6.2 Education Lifecycle

Obtaining education is a cumulative process; to produce any education greater than e_0 , the individual must first obtain e_0 . We are interested in two mechanisms through which learning style might change:

²⁷See Arrow (1962).

²⁸See section 4.

- (1) Exogenously with education level; and
- (2) Endogenously via path dependencies.

By the latter we mean the possibility that current learning style depends how education at earlier stages of the life cycle was produced.

To address these questions we propose combing Heckman's two period model with our two input model. This would result in a two period, two input education production function that contains four inputs and four parameters;

$$e(s_0, t_0, s_1, t_1; \alpha_0, \alpha_1, \rho_0, \rho_1) \tag{18}$$

Exogenous variation in the learning style parameters means that investment in period 2 depends on the education level in period 1. Endogenous variation in the learning style parameters implies that the learning style in the second period depends on the inputs used in the first period. This means that, like Heckman, we are interested in the inputs' second order cross-partial derivatives. Heckman looks at how the size of an investment made at $t = 0$ has a direct effect on productivity of later investments. Heckman is therefore interested in the degree of substitutability between investments at different stages on the education life cycle.

For example a high proportion of study, relative to tuition in early childhood, might increase the independence later in life. Thus individuals can 'learn' to become independent learners. If the mix of inputs used in early childhood influences the learning style adopted later in life there may be a trade off between static and dynamic efficiency.

A production function that captures some of these properties would be one in which the period 1 independence parameter is a function of the proportion of inputs used in period 0; $\alpha_1(\alpha_0, \rho_0)$. Using a simple linear relationship we can express total education attainment as:

$$e_1(s_0, t_0, s_1, t_1) = ((\alpha P_0)t_1^\rho + \frac{(1 - \alpha)}{P_0}s_1^\rho)^{\frac{1}{\rho}} \quad (19)$$

where $P_0 = \frac{s_0}{t_0 + s_0}$ s.t. $E(s_0, t_0) = e_0$.

If resources cannot be shifted across periods, we can simply solve using backwards induction. If resources can be shifted across periods we will have to equate the marginal products, including externalities, of each input.

6.3 Signaling²⁹

The most important alternative to human capital as an explanation for education is the signaling model (Spence (1973)). In the Spence model education is used to signal unobserved ability. We believe employers may also be interested in learning styles, and so individuals may wish to signal these unobservables³⁰.

In this case employers will want to observe how individuals learn. We believe that it is more likely that tuition can be observed than study. How-

²⁹ S, T are inputs into production function - their observability addresses similar questions to those of effort in a moral hazard problem. (α, ρ) are parameters of the production function - their observability addresses similar questions to types in an adverse selection model.

³⁰If an employer cares about learning styles, A2 must be relaxed. This is discussed in section 2.3.

ever we think it is unlikely that either of the parameters in the education production function are directly observable. We can observe tuition because it may be common knowledge that different universities (providing the same E) offer different T ³¹. In this case employers might be able to infer learning style from the choice of university.

For example if employers are willing to pay an independent learner a higher wage but α is not observable, employees will have to signal their learning styles using the observables E and T .

The separating equilibrium is one where independent learners choose a university which gives very little tuition, and directed learners choose a university where there is more tuition. While both universities differ in the amount of tuition offered, the content of the education received is identical³².

An employer would offer a wage which is increasing in E (education attainment), but decreasing in tuition. The various costs and benefits would be captured in the relevant incentive compatibility constraints.

The need to signal learning style in addition to ability will increase the total cost of the signal. This increases the welfare loss relative to perfect information. In order to signal independence students will purchase less than their cost minimizing bundle of tuition and thus education is obtained inefficiently. This is true whether or not education itself is productive.

³¹In order to infer S from E and T , one would have to make several strong assumptions about the returns to scale exhibited by the production function.

³²For example $E = 10$ obtained at Harvard is identical to $E = 10$ obtained at William & Mary.

6.4 Empirical Testing

Because data on S is hard to observe, α and ρ are hard to identify. However, the CES production function we use has been identified in many contexts and we believe that with a rich enough dataset our model can be tested (see Klump et al. (2012)). Moreover, there is a small but growing literature that addresses many of the issues raised by our model.

Here we discuss question of whether attendance in class should be compulsory. In the appendix we show that in our model compulsory attendance reduces welfare for all students. Romer (1993) asks “Do students attend class? Should they?”. He finds that in three elite universities “absenteeism is rampant”. Attendance is effectively voluntary, which he states was not the case a generation ago. He finds that after controlling for motivation and ability attendance in class is strongly correlated with performance. It is also correlated with class size and teacher quality.

The paper generated an unusually large correspondence (JEP 1994). Many of these disagreed with Romer’s conclusion; firstly because of doubts about his identification strategy, and secondly because he ignores the opportunity cost associated with attending class, which is at the core of this paper.

If the correlation is causal Romer’s result implies that students are not optimizing. A behavioral explanation seems to be the most likely and suggests that a policy based on paternalism might be justified (Thaler and Sunstein (2003)).

If the students are maximizing, and therefore are substituting study for tuition, our model implies a negative correlation between ability and inde-

pendence. Perhaps the most natural interpretation in terms of our model is that students have not understood the complementarity between study and tuition.

Since Romer (1993), the case for compulsory attendance has probably been weakened since changes in technology mean the delivery of core material no longer requires face to face interaction (see section 4).

7 Conclusion

How teachers teach and students learn have consequences for the efficient allocation of scarce resources. A two input model with tuition and study captures critical differences in teaching and learning discussed in the education literature.

Because the current literature has yet to explore this distinction important efficiency considerations are ignored. We have shown that, even if students have the same Becker ability, the graduate premium depends on learning styles and mix of inputs matters for efficiency (Section 5.1). For a given cost structure tuition may be over or under supplied. With heterogeneous learning styles it becomes increasingly important to ensure that resources are structured to individual needs. In practice education markets offer students very little choice about the amount of tuition to purchase. They must choose how much to study given a fixed amount of tuition.

These results hold because we specify a price for each type of investment. The price of study comes in the form of an opportunity cost whereas the price students pay for tuition is a market price. Independent learners should pay

less for their education even if they are high ability and make large overall investments in human capital.

If high ability students are more independent than lower ability students it is possible that those universities that recruit the highest ability students should charge lower fees than universities that recruit lower ability students, at least to achieve a given level of education. If tuition and study are complementary the paradox that prestigious universities should charge less than lower ranked universities is reversed. We believe that the evidence suggests that there is considerable complementarity between study and tuition³³.

We have used our model to investigate how changes in technology and the introduction of MOOCs will change the way higher education is delivered, the types of learners who participate, and the graduate premium (Section 5.2).

We have shown that the importance of these changes depend on how much students vary in their learning styles. We propose a distinction between core and discursive material, and predict that the future of traditional HE institutions will depend on their ability to deliver discursive material.

We have used our model to show how unbundling the core and discursive dimensions of tuition increases the potential for exploiting the scale economies and cost reduction potential of MOOCs (Section 5.3). If this is combined with ‘private tuition’ different kinds of learners can choose the bundle of tuition and study that best suits their individual learning style.

³³Recent evidence (Bandiera et al. (2010)) suggests in higher education class size is important particularly for high ability students. This is important because the rapid expansion in HE participation in developed countries has resulted in students receiving less tuition.

8 Appendix

8.1 N Period Model

Since U is increasing in m , maximizing utility is achieved through maximizing consumption. Assuming the individual chooses to graduate with a k -*diploma*, her graduate consumption is given by:

$$m^{Ek} = (N - k)\Omega w_k + \sum_{j=1}^k ((\Omega - T_j - S_j)w_j - pT_j) \quad (20)$$

If she chooses no education and becomes a non-graduate her consumption is:

$$m^{E0} = N\Omega w_0 \quad (21)$$

The graduate premium is $G = (m^{Ek} - m^{E0})$. The individual maximizes G subject to:

- (1) The education production function (equations 2 and 4)
- (2) The wage function (equation 5)
- (3) Time constraints (equation 7)

We note there is an equivalent cost minimization problem: Minimize $C^k = \sum_{j=1}^k c_j(T_j, S_j) = \sum_{j=1}^k ((T_j + S_j)w_j - pT_j)$, subject to the same constraints.

Lemma 8.1 *The individual would always minimize the cost of whatever education is undertaken in each period.*

Proof ³⁴

³⁴We are grateful to Alasdair Smith for suggesting this approach.

It is clear that the education levels chosen in each period will either be 0 or e_i^* . This is because, for $0 < e_i < e_i^*$, the marginal benefit from education is zero and the individual would be wasting resources (which could be used to increase her graduate premium) by not choosing $e_i = 0$. If $e_i > e_i^*$, then the gain would come from decreasing education to e_i^* .

Moreover, if it is optimal to choose no education in period i , then for $j = i + 1$, it will also be optimal to choose no education. In both periods the education production function and forgone wage are identical and the individual faces a similar decision. However, in period j the benefit of the increased wage would last for one less period.

Suppose that $S_1^*, S_2^*, \dots, S_N^*$ and $T_1^*, T_2^*, \dots, T_N^*$ are the amounts of study and tuition undertaken in each period to maximize the graduate premium. Since $\frac{\partial e_i}{\partial S_i} > 0$ and $\frac{\partial e_i}{\partial T_i} > 0$, it is clear that there is only one way to produce $e_i = 0$, which is to use $S_i = T_i = 0$. Hence S_i^* and T_i^* are also the cost minimizing inputs.

If $e_i = e_i^*$, we use proof by contradiction to show that S_i^* and T_i^* satisfy this period's cost minimization problem. Suppose using $(S'_i, T'_i) \neq (S_i^*, T_i^*)$ is the least costly way for an individual to produce e_i^* in period i .

By changing to (S'_i, T'_i) but keeping all other variables unchanged in all other periods would only affect the graduate premium through the payoff in this period (since e_i^* is still produced in period i). This switch to (S'_i, T'_i) would decrease her cost by $(p + w_i)(T_i^* - T'_i) + w_i(S_i^* - S'_i) > 0$, and hence increase her graduate premium by this amount. ■

8.2 Ability³⁵

This appendix follows from section 3.3. It turns out that this problem arises even in a one input model measured in different units (money and time). In this case the problem can be avoided with a homogeneous forgone wage. With heterogeneous foregone wages and one input the same ambiguity arises.

Consider the following case. In the standard model if two individuals, i and j , have the same number of years schooling and i attains a higher mark she is said to be higher ability than j .

$$\frac{y}{a_i} < \frac{y}{a_j} \quad (22)$$

Correspondingly, if two individuals attain the same mark, and if one has had fewer years of schooling, she is said to be higher ability.

In the standard model of human capital there is a role for opportunity cost. This is because the forgone wage is included in calculations of the optimal level of investment in education. However, opportunity cost does not enter into the standard definition of ability.

In a model where there are only two wages: \underline{w} (a non-graduate wage) and \bar{w} (a graduate wage) the inclusion of forgone costs is not problematic. This is because when ‘years of schooling’ are considered then the opportunity cost for all individuals is the same (everyone needs to give up $n\underline{w}$).

Thus a ratio becomes:

$$\frac{y + n\underline{w}}{a_i} < \frac{y + n\underline{w}}{a_j} \quad (23)$$

³⁵We are grateful to Paul Grout for suggesting this approach.

This can also be extended away from a binary case, so long as wages are only a function of education (and not ‘ability’).

There is slightly more to consider when effort is used, if this translates to time. But essentially, the problem is made insignificant by fixing wages.

However in reality individuals face different opportunity costs. In particular, opportunity cost is likely to increase with ability.

In this instance, the introduction of heterogeneous forgone wages causes ambiguity in the definition of ability. If the individual can obtain a forgone wage of $\underline{w}_i > \underline{w}_j$, then the input/output ratio becomes:

$$\frac{y + n\underline{w}_i}{a_i} <? > \frac{y + n\underline{w}_j}{a_j} \quad (24)$$

Which of these ratios is larger is ambiguous, since $a_i > a_j$ but $\underline{w}_i > \underline{w}_j$.

8.3 Optional vs Compulsory Class Attendance

In this section we expand on our tradition HE model by investigating the implications of compulsory class attendance. Thus rather than students choosing $T \leq \bar{T}$ we set $T = \bar{T}$, with the same fixed fee paid. In practice attendance is voluntary with “absenteeism rampant” (Romer (1993)).

Compulsory attendance will not increase the graduate premium for any student, because students are now more constrained (Figure 9)³⁶. The graduate premium is now determined by the minimum number of hours (of S) needed to reach the required education level. In this case what matters is the opportunity cost of the compulsory class, and this depends on learning

³⁶Students who previously chose to attend all the classes will not be affected.

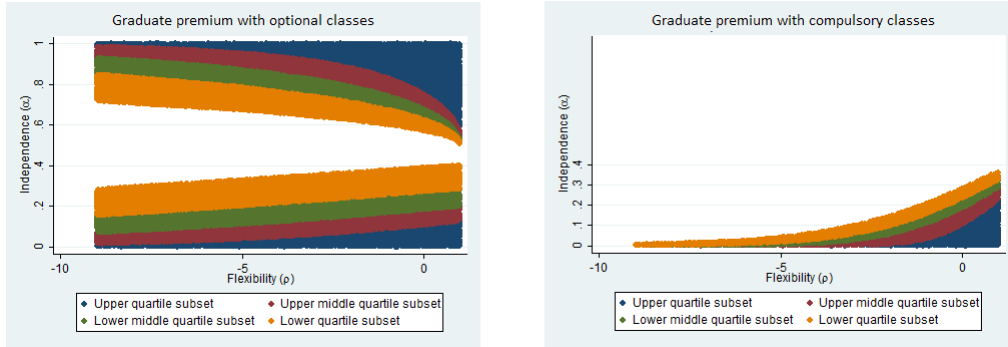


Figure 8: Optional vs compulsory classes. Parameter values: $w=5$, $\exp=10$, $ed=12$, $T=15$.

style.

The comparison in Figure 8 shows the reduction in the graduate premium is greatest for independent learners. For these students the ‘unnecessary’ tuition imposes the greatest cost. Figure 9 illustrates these asymmetric effects. With optional classes IL has a higher graduate premium than DL. Compulsory classes increase the cost of achieving e^* for both learners. However, the increase in cost to IL exceeds the increase to DL and DL now has a higher graduate premium.

These effects are ameliorated by flexibility (Figure 8). When tuition is compulsory the benefits of being flexible outweigh the advantages that can accrue from complementarity. Inflexible learners cannot use the ‘extra’ tuition to substitute for study, whilst flexible learners can do so.

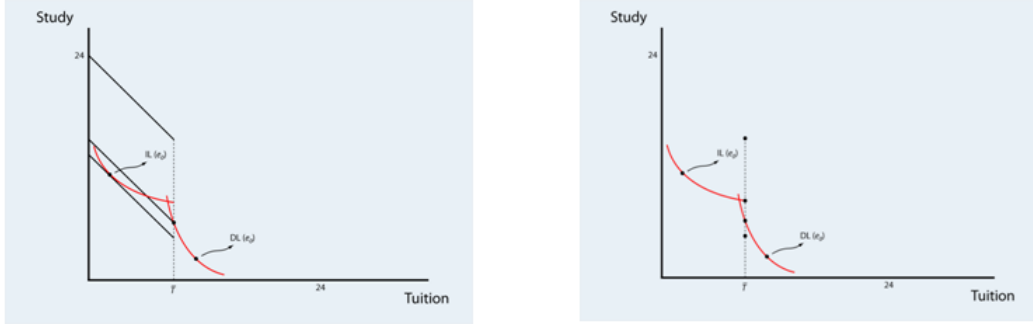


Figure 9: Optimal inputs under fixed fee with (L) optional classes and (R) compulsory classes

8.4 Marginal Products in the CES Production Function

This appendix follows from section 2.2. Temple (2012) notes that if one of the parameters in the CES function changes while the other is held constant, the economic interpretation of the constant parameter may change. For example if flexibility is varied, the interpretation of a given level of independence may change.

All the proposed solutions all involve some form of normalization. As Temple points out none of these solutions is entirely satisfactory.

The variations in these interactions have different impacts on the functional form of the CES, which directly determines the marginal productivity of each input. The result is that we must explain how marginal products relate to learning styles. The marginal product of an input (e.g, S) is given by:

$$\frac{\partial E_i}{\partial S} = \alpha \rho S^{\rho-1} (\alpha S^\rho + (1 - \alpha) T^\rho)^{\frac{1-\rho}{\rho}} \quad (25)$$

When $\rho = 1$, the marginal product of study is just the independence parameter. For all other values of ρ , the marginal product depends on the parameters in the model (α, ρ) and the bundle of inputs (S, T) . We are interested in how the marginal product of S and T depends on α and ρ .

8.4.1 Marginal product depends on α

Since α is a weighting parameter, the marginal product of study (tuition) is increasing in α (or $1-\alpha$). Specialization is beneficial because the marginal product of the input chosen will be higher than that of an ‘equivalent’ generalist.

8.4.2 Marginal product depends on ρ

The exponent of each input is increasing in ρ (see Ramsey (1968)). Therefore for a given level of independence, the marginal product of study (tuition) is increasing in ρ . As learners become less flexible the isoquant becomes increasingly convex and this demonstrates the reduction in the marginal product of each input. Flexibility reduces the importance of bundling because a flexible learner can make better use of sub-optimal bundles than an inflexible learner.

8.5 Algebraic Results

8.5.1 FOCs

The results of the model are found by solving the following Lagrangian equation:

$$L = wS + (w + P_t)T + \lambda((\alpha S^\rho + (1 - \alpha)T^\rho)^{\frac{1}{\rho}} - e^*) \quad (26)$$

Assuming the individual chooses a skilled job the following first order conditions hold:

$$\frac{\partial L}{\partial S} = w + \lambda \alpha S^{1-\rho} (\alpha S^\rho + (1-\alpha)T^\rho)^{\frac{1-\rho}{\rho}} \quad (27)$$

$$\frac{\partial L}{\partial T} = (w+p) + \lambda (1-\alpha) T^{\rho-1} (\alpha S^\rho + (1-\alpha)T^\rho)^{\frac{1-\rho}{\rho}} \quad (28)$$

$$\frac{\partial L}{\partial \lambda} = (\alpha S^\rho + (1-\alpha)T^\rho)^{\frac{1}{\rho}} - e^* \quad (29)$$

Solving:

$$S^* = \frac{e^*}{(\alpha + (1-\alpha) \left(\frac{\alpha(w+p)}{(1-\alpha)w}\right)^{\frac{\rho}{\rho-1}})^{\frac{1}{\rho}}} \quad (30)$$

$$T^* = \frac{e^* \left(\frac{\alpha(w+p)}{(1-\alpha)w}\right)^{\frac{\rho}{\rho-1}}}{(\alpha + (1-\alpha) \left(\frac{\alpha(w+p)}{(1-\alpha)w}\right)^{\frac{\rho}{\rho-1}})^{\frac{1}{\rho}}} \quad (31)$$

It is convenient to discuss these results in terms of a ratio, $\frac{S^*}{T^*}$:

$$R^* = \frac{S^*}{T^*} = \left(\frac{\alpha(w+p)}{(1-\alpha)w}\right)^{\frac{1}{1-\rho}} \quad (32)$$

8.5.2 Comparative Statics

Prices of tuition

Consider two individuals who have identical education production functions, but face different prices, $p_1 > p_2$:

$$R_1 = \frac{S^*}{T^*} = \left(\frac{\alpha(w+p_1)}{(1-\alpha)w}\right)^{\frac{1}{1-\rho}} \quad (33)$$

$$R_2 = \frac{S^*}{T^*} = \left(\frac{\alpha(w+p_2)}{(1-\alpha)w}\right)^{\frac{1}{1-\rho}} \quad (34)$$

$$\frac{R_1}{R_2} = \left(\frac{w+p_1}{w+p_2}\right)^{\frac{1}{1-\rho}} \quad (35)$$

Interpretation: When s and t are perfect complements, the change in price does not affect the consumption ratio. When s and t are perfect substitutes $\frac{R_1}{R_2} = \frac{(w+p_1)}{(w+p_2)} > 1$, which shows individual 1 (who faces a higher cost of tuition) will study more. This elasticity is clearly larger when the wage rate is low relative to the price of tuition.

Flexibility

Consider two individuals who face the same prices and have almost identical education production functions, except one is more flexible than the other, $\rho_1 > \rho_2$;

$$R_1 = \frac{S^*}{T^*} = \left(\frac{\alpha(w+p)}{(1-\alpha)w} \right)^{\frac{1}{1-\rho_1}} \quad (36)$$

$$R_2 = \frac{S^*}{T^*} = \left(\frac{\alpha(w+p)}{(1-\alpha)w} \right)^{\frac{1}{1-\rho_2}} \quad (37)$$

Clearly $\frac{\alpha(w+p)}{(1-\alpha)w}$ is important in the effect of ρ . This is a measure of the marginal products on inputs relative to their prices. Thus we consider three cases:

1. $\frac{\alpha(w+p)}{(1-\alpha)w} \in (0, 1)$. In this case, as ρ decreases $\frac{S^*}{T^*}$ increases. When tuition is cheap relative to its marginal product, then being less flexible results in the consumption of more study.
2. $\frac{\alpha(w+p)}{(1-\alpha)w} = 1$. In this case, ρ does not change $\frac{S^*}{T^*}$.
3. $\frac{\alpha(w+p)}{(1-\alpha)w} > 1$. In this case, as ρ decreases $\frac{S^*}{T^*}$ decreases. When tuition is expensive relative to its marginal product, then being less flexible results in the consumption of more tuition.

These results suggest that, *ceteris paribus*, being a flexible learner is a good thing. As a learner becomes more flexible, she is able to take advantage of the cheaper input. If the price of tuition is high, she will now be more able to substitute away from tuition (i.e, $\frac{\partial S^*}{\partial \rho} < 0$). If the price of tuition is low, she will now be more able to substitute towards tuition (i.e, $\frac{\partial S^*}{\partial \rho} > 0$). The same analysis applies for $\frac{\partial T^*}{\partial \rho}$.

Independence

Consider two individuals who face the same prices and have the same flexibility in their learning style, but one is more independent than the other, $\alpha_1 > \alpha_2$;

$$R_1 = \frac{T_s^*}{T_t^*} = \left(\frac{\alpha_1(w+p)}{(1-\alpha_1)w} \right)^{\frac{1}{1-\rho}} \quad (38)$$

$$R_2 = \frac{T_s^*}{T_t^*} = \left(\frac{\alpha_2(w+p)}{(1-\alpha_2)w} \right)^{\frac{1}{1-\rho}} \quad (39)$$

We consider $\frac{R_1}{R_2}$,

$$\left(\frac{\alpha_1(1-\alpha_2)}{(1-\alpha_1)\alpha_2} \right)^{\frac{1}{1-\rho}} \quad (40)$$

Since $\frac{\alpha_1(1-\alpha_2)}{(1-\alpha_1)\alpha_2} > 1$, $\frac{R_1}{R_2} > 1$. As ρ decreases, $\frac{R_1}{R_2}$ increases. Thus, individuals who are more independent will consume a larger proportion of study than their less independent peers.

8.5.3 Other Results

Result ECP (education, cost and price) In our model the choice about education is binary. If the cost of education is pushed up too high, individuals will choose not to purchase any education. By evaluating $c(e^*) = wS^* + (w +$

$p)T^*$,

$$\frac{\partial c(e^*)}{\partial p} = w \frac{\partial S^*}{\partial p} + T^* + (w + p) \frac{T^*}{\partial p} \quad (41)$$

It is clear that, for all individuals, the cost is an increasing function of price. Those whose costs rise the most are directed learners and individuals whose learning style is not particularly flexible. These individuals are particularly susceptible to deciding not to partake in education as the cost of tuition increases.

Corollary 8.2 (Ability) *Ability depends on price.*

This follows from our definition of ability and result ECP.

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