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Information Acquisition and Crowding Out in Regulatory Hierarchies

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Abstract

This paper studies an externality common to all policy setting hierarchies in which information acquisition is essential to making policy decisions. The paper analyses the regulation of a monopolist with unknown cost both by a state and a federal agency with overlapping jurisdictions. When the objectives of the two tiers in the regulatory hierarchy differ, and the federal agency relies at least in part on the information acquired by the state regulator, the state agency has the incentive to acquire less than full information about the monopolist in order to prevent interference by the federal regulator. In this sense, the possibility of intervention by a higher level agency in the hierarchy crowds out information, and forces an apparently collusive (‘regulatory capture’) outcome in which the state regulator aligns itself with the interests of the monopolist. When crowding out is significant, policy decisions in a hierarchy may be worse than those made by single-tier organisations. The overlap of jurisdictions in hierarchies not only duplicates effort and wastes time and resources; more importantly, hierarchical decision-making wastes information.

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Non-Technical Summary

In an attempt to resolve conflicting decisions of policy setting and policy enforcement in federal-type structures, competencies are frequently assigned hierarchically. This is true for instance for the structure of competition and antitrust law enforcement in the European Union, where overlapping competencies are assigned hierarchically to member states' competition authorities and the European Commission. In this paper, I show that hierarchical organization of regulatory bodies results in a loss of information about the entity to be regulated (for instance, a monopoly firm), and that this leads to a loss of consumer welfare.

Although the idea is more general, in this paper I analyze the case of the regulation of a monopolist with privately known cost by two regulators with overlapping jurisdictions. In order to regulate price effectively, a regulator needs to acquire information about the firm's cost. Information acquisition is costly for the regulators. The paper is based on two observations: First, that lower-tier regulators (e.g. "state regulators") have better access to information than higher-tier regulators (e.g. "federal regulators"): federal regulators can request information through state regulators, but their power of autonomous investigation is limited. Second, that policy objectives of different tiers in the hierarchy frequently differ between the tiers of a hierarchy: state regulators, for instance, may be more interested in a firm's profit than the federal (higher-tier) regulator.

The idea of the paper is the following. When (as is natural) the two regulators' policy objectives differ, a high probability of intervention (in this case, price-setting) by the federal regulator creates incentives for the state regulator to corrupt the information that the federal regulator can acquire. Damaging the information at the disposal of the federal regulator ensures that the state regulator's decision is overturned less frequently. However, the simplest way to damage the federal regulator's information is for the state regulator not to acquire costly information in the first place, since the higher-tier agency has only limited power of autonomous investigation (we assume that the state regulator cannot selectively withhold information it has acquired, and that it cannot misrepresent information). A higher tier's intervention in the policy-setting process therefore "crowds out" information-acquisition by a lower tier, putting less information at the disposal of anyone in the hierarchy. In effect, the possibility of intervention of a higher tier in the policy hierarchy causes at least partial alignment of the lower tier regulator with the regulated firm. This is a variant of the old "regulatory capture" idea.

Furthermore, I show that the policy decisions made in hierarchies exposed to these incentives may result in prices that are higher when the monopoly firm is regulated by a hierarchy of regulators than if it were regulated by only one regulator. In this sense, hierarchical decision-making may be worse for consumers than if only one regulator acted independently.

1 Introduction

One frequently noted feature of federal structures is the overlap of the jurisdictions of state and federal agencies, and the resulting scope this creates for poor coordination of the actions of different agencies. To resolve conflicting decisions by different agencies, competencies are frequently assigned hierarchically, with state-level decisions subordinate to those made at the federal level. However, when information is not costlessly available to all tiers in a hierarchy, hierarchical structures introduce a new set of problems (e.g. Tirole (1986), Tirole (1992), Kofman and Lawarrée (1993)). Most of the literature on hierarchies has discussed the issue of collusion between different levels of a hierarchy when explicit side transfers are possible. This paper addresses a different set of inefficiencies that arise in hierarchical structures. In this paper, I analyze situations in which information is necessary to implement policy decisions, and I study the incentives for different tiers in a hierarchy, with different policy objectives, to acquire or hide that information. Hiding information leads to what appears like a collusive outcome, with undesirable welfare implications. Such collusion exists without the need for monetary side payments.

The problem this paper addresses is the following. When the policy objectives of state and federal agencies differ, the lower tier (state) agency may wish to damage or withhold information about the regulated entity (for instance, by not acquiring this information in the first place) from the higher tier (federal) agency. "Jamming" the signal (in a sense similar to Fudenberg and Tirole (1986)) which the federal agency receives reduces the probability that the state level agency's decision is overturned by a decision at federal level. The possibility of intervention by a higher tier agency therefore crowds out information acquisition by a lower level agency. The outcome is a collusive one in the sense that the lower level agency apparently aligns itself with the interests of the regulated entity. This is a novel twist on the old "regulatory capture" hypothesis. Since less information is available for policy decisions, hierarchical structures may produce outcomes inferior to those of single-tier structures. The overlap of jurisdictions in hierarchies not only duplicates effort and wastes time and resources; more importantly, hierarchical decision-making wastes information.

To study the problem, I develop a general model of informational crowding out in hierarchies. To motivate the results, I analyze price regulation of a monopolist with unknown cost by a hierarchy of federal and state regulatory agencies. Naturally, the idea of the paper straightforwardly applies to areas such as law enforcement, environmental regulation, educational curriculum design, and so on. One of the results of the paper is a counterintuitive conclusion: by introducing a higher-tier regulatory authority with a jurisdiction overlapping that of a state level agency, consumers may get a worse deal than if state regulators operated independently. The nature of this problem is a bilateral externality: When the federal authority obtains hard (verifiable) information about the monopolist's cost, it implements price regulation to maximize total surplus and consumers get the best possible deal (marginal cost pricing). But the possibility that the federal authority sets a price that is less preferred by the state regulator (for instance, because the state regulator places greater weight on its own constituency, i.e. firm profits) crowds out information acquisition by the state level authority. By acquiring less information, the state regulator can therefore damage the information ("jam" the signal) to the federal agency. The resulting lack of information acquired within the hierarchy increases the probability that the monopolist can set the monopoly price (no regulation). This apparent collusion between firm and state regulator of course is the worst possible outcome for consumers. As a result, the expected price of the monopolist's product may be higher than if the state regulator were to operate independently. More pertinently, expected consumer surplus may also be lower than if the state regulator were to work independently.

A concrete example of such a regulatory hierarchy is that within the European Union. Each member state to the European Union regulates industries directly. In addition, European antitrust law provides for a role for the European antitrust enforcement agency (the European Commission) in price regulation of a monopolist.[1] The system is hierarchical: decisions by the European Commission take precedence over decisions by the state regulator. While the European Commissions' price-setting objective is that of efficiency (the sum of consumer and producer surplus), a simple model of location choice by the monopolist implies that state regulators are likely to put greater relative weight on producer surplus (i.e. profits) than the European Commission.[2] However, the European Commission's powers of investigation are severely limited by comparison to those of state regulators.[3] For this reason, I simplify by modeling the information potentially available to the European Commission as a subset of that available to the state regulator.

One prediction of the model is that it is never in the federal authority's interest to crowd out state regulatory information acquisition fully.[4] More subtly, in a wide class of cases the federal authority will never investigate only tentatively: it will either not investigate at all, or invest substantial effort into its investigation of the monopolist's cost structure. I make this statement more precise by studying a special case of the more general model. Further, an important variable in the model is the size of the state regulator's jurisdiction relative to the size of the area of responsibility of the federal regulatory authority. This allows me to make state-specific predictions of state regulatory and federal authority behavior. In particular, in small states, neither the state regulator nor the federal authority will investigate fully. In large states, regulatory effort will always be maximal, regardless of the actions of the federal authority.

[1]cf. Art. 82 EC Treaty (Treaty of Rome); Judgement of the Court of November 13, 1975 (Case 26/75); Judgement of the Court of February 14, 1978 (Case 27/76)

[2]In the case of the European Commission it is not entirely clear what the objective is. Here we focus on efficiency (total surplus) as the objective. One might equally well argue for consumer protection (consumer surplus) as the objective. In fact, the objectives underlying European competition law are often inconsistent: although consumer protection is an important objective, the protection of "small and medium-sized undertakings" or the creation of a single market (for instance by allowing exclusive distributors in order to allow firms to penetrate a new market) are also important aims (cf. Whish (1993)). At any rate, all I need for my results is that the objective of the state industry regulator is between that of the federal (European) authority and that of the monopolist, so that there is some coincidence of interests between the federal authority and the state regulator.

[3]cf. Proposal for a Council Regulation COM(2000) 582 final (Sept. 27, 2000), 2.C.1.(c)

[4]A caveat is necessary: I show that regulatory effort will never be zero, unless it would be zero even in the absence of a federal authority. In this sense, the federal authority never *causes* complete crowding out.

The paper is structured as follows. Section 2 describes the model. Section 3 contains the main results of the paper. I solve the general model for optimal state regulator and federal regulatory agency effort and present results about these optimal choices. In particular, I derive the main proposition placing a bound on possible federal authority behavior. Finally, I engage in a comparative statics exercise to derive results about outcomes for states of different sizes. Section 4 sharpens some of these results for special cases of the more general model. This allows me to refine the bound derived in section 3. Section 5 discusses the informational modelling choice in greater depth and relates this model to the literature. The final section 6 concludes. All proofs are in the Appendix.

2 The Model

The paper studies the bilateral externality in information acquisition between state and federal regulators with different policy objectives, when regulation can only be implemented based on verifiable information.[5] I model the regulated firm as a monopolist of privately known unit cost θ. Since I am not interested in market structure issues I assume, without loss of generality, the absence of fixed costs. The monopolist produces both for "home" (that is, in-state) consumption and for export (that is, out-of state consumption), and I assume that price discrimination is impossible. The monopolist faces a downward-sloping, differentiable, aggregate (i.e. home and export markets) demand curve $q(p)$, and in entirely conventional fashion is assumed to maximize profit $\pi(p, \theta)$. For ease of notation later, define a generalized objective function $f(p, \theta, \alpha, \beta) \equiv \alpha\pi(p, \theta) + \beta \int_p^\infty q(t)dt$. The parameters α and β reflect the relative importance of profit and consumer surplus in the generalized objective function. Furthermore, β has a straightforward interpretation as the fraction of output sold in the home market, which I assume is proportional to the size of the home market relative to the size of the export markets.[6] I will therefore sometimes refer to β as the size of the state. The monopolist seeks to maximize $f(p, \theta, 1, 0)$.

Both the state regulator and the federal regulatory authority can obtain information about the monopolist's cost parameter by investing in a stochastic information-revelation technology. An interesting feature of hierarchies is that of "local knowledge": higher level tiers in the hierarchy often have poorer access to information than lower tier agencies within the same hierarchy. I model the fact that state regulators have "better" information than the federal regulatory authority ("local knowledge") in the following way: while state regulators have powers to investigate firms directly, the federal authority may only request information from state regulators. The information potentially available to the federal authority is therefore a subset of the state regulator's information.

In general, the objectives of state regulator and federal authority will not coincide; furthermore, both the state regulator's and the federal authority's objective functions will differ from the monopolist's. This

[5] Information has to be verifiable ("hard") for instance because of the possibility of judicial review.

[6] Given a representative country i of size β_i ($\sum \beta_i = 1$), this assumption says that $\beta_i q(p) = q_i(p)$.

divergence of interests drives the "common agency" aspect of the problem: both authorities wish to obtain information about the monopolist's privately known cost parameter, and both wish to regulate the firm in different ways.[7] In this paper, I make strong simplifying assumptions about the state regulator's and the federal authority's objective function, but these can be viewed as reduced form results from, for instance, a model of interest group influence (career concerns by the state regulator), or a model of location choice by the firm. In particular, I assume that the monopolist is interested in profit maximization; the federal regulatory authority, in maximizing total (consumer and producer) surplus; and the state regulator in maximizing a weighted sum of producer and consumer surplus.[8]

The state regulator seeks to maximize $f(p, \theta, \alpha, \beta) = \alpha\pi(p, \theta) + \beta \int_p^\infty q(t)dt$. I assume that $\alpha < 1$ (for instance, when the state regulator cares about firm profit because profits are taxed, this reflects the shadow cost of taxation), and β denotes the relative size of the state (the fraction of production consumed in the "home" market, i.e. the extent to which consumer surplus figures in the objective function). This interpretation also requires that $\alpha > \beta$.[9] I model the state regulator as regulating price directly: I ignore any incentive issues that in practice drive a wedge between direct price regulation, rate-of-return regulation and price-cap regulation. Furthermore, price is the only policy variable for the regulator.[10]

The state regulator can invest (costly) effort $e \in [0, 1]$ in an investigation technology that yields a signal $\sigma \in \{\theta, \emptyset\}$ of the monopolist's cost parameter. The state regulator learns the monopolist's cost parameter with probability $\Pr\{\sigma = \theta | e\} = e$, and with probability $\Pr\{\sigma = \emptyset | e\} = 1 - e$ it learns nothing. The technology is available at cost $c_s(e)$, with standard assumptions on the cost function: $c_s(0) = 0$, $c_s(\cdot) > 0$ at all but a finite number of points (for instance, we will allow $c_s(0) = 0$), $c_s(\cdot) \geq 0$, and these derivatives exist everywhere.[11] The interpretation of effort as the probability of "success" (obtaining verifiable information

[7] cf. Martimort (1992) and Stole (1992) for a general model of common agency under adverse selection.

[8] The state regulator's greater concern for firm profits can be motivated in a number of ways: Any model of location choice by the monopolist would incorporate some concern of the state regulator for firm profits, to encourage the firm to locate within the state regulator's jurisdiction (e.g. Brander and Spencer (1985), Maggi (1999)), whereas the federal authority cares about the total surplus (location is not an issue for the federal agency). Alternatively, in the light of a theory of interest group influence one could justify this choice as arising from the fact that regulators have a concern for a future career in the regulated industry (that is, their objective is partly aligned with that of the firm), but need to signal their (privately known) ability by displaying competence in applying competition rules (so that part of their objective is welfare maximizing). Similarly, the state regulator may have an interest in a future career in state government and therefore aligns its objective with that of the government. Since state governments tax firms, but are elected by agents interested in their consumer surplus, this also justifies the regulator's objective as a weighted sum of consumer and producer surplus (cf. the literature on regulatory capture with direct money payments, e.g. Laffont and Tirole (1991), or career concerns e.g. Spiller (1990), Che (1995), and the seminal papers by Dewatripont, Jewitt, and Tirole (1999a), Dewatripont, Jewitt, and Tirole (1999b)). I make no assumption about which of these factors motivate regulators or federal authorities. At any rate, the aim of this paper is to study the externalities that arise from non-cooperative contracting, not the modelling of interest group pressure.

[9] In fact, this is also a technical assumption: it implies that if the regulator sets price, it will not choose a price at which the monopolist finds it unprofitable to produce any positive level of output.

[10] Issues such as quality monitoring and quality assurance open up a can of worms of their own. In this paper, I focus on price regulation and therefore ignore such issues, important as they are in practice.

[11] Throughout, subscript s denotes parameters or variables in the state regulator's utility function. Subscript m denotes the

about the monopolist's cost parameter) will be useful in the interpretation of the model's results.

The federal authority's objective, because it represents the interests of consumers in all states under its jurisdiction, is the maximization of total surplus; that is, if it intervenes it sets price so as to maximize $f(p, \theta, 1, 1)$. Similarly to the state regulator, it has the option to invest effort $i \in [0, 1]$ in a costly technology that reveals a signal $s \in \{\theta, \sigma\}$ such that $\Pr\{s = \sigma | i\} = i$, and with probability $\Pr\{s = \emptyset | i\} = 1 - i$. This is the previously argued assumption of the nestedness of information: the federal authority can only learn firm cost if the state regulator has received an informative signal. The information that the federal authority can buy is coarser than the state regulator's: and its coarseness varies with the state regulator's investigation effort. Effort level i costs the federal authority $c_f(i)$ (again assuming $c_f(0) = 0$, $c_f(\cdot) > 0$ at all but a finite number of points, $c_f(\cdot) \geq 0$, and that these derivatives exist everywhere).

Note at this point that none of the paper's results require the specific form for the payoff function $f(p, \theta, \alpha, \beta)$ that I have assumed. Any separable function of the form $\alpha g(p, \theta) + \beta h(p, \theta)$ (this includes $g(p, \theta)^{\alpha} h(p, \theta)^{\beta}$) that have a positive maximizer will do equally well. The restrictions that are necessary are that $\alpha g(p, \theta) + \beta h(p, \theta)$ be concave for all values of β (so that a maximum exists) and, for lemma 11, that $\frac{\partial h(p,\theta)}{\partial p} < 0$. This lends considerable generality to the model.

2.1 Timing of the Model

The timing of the game between monopolist, industry regulator, and federal authority is the following:

1. The monopolist learns its unit production cost θ.

2. The federal authority decides bindingly (and makes publicly known) its investigation probability, i. For instance, it commits to a certain staff size, budget, or sets public targets.

3. the state regulator decides its regulatory effort, e, and learns signal σ of firm cost. The federal authority learns its signal s, with the characteristics discussed above.

4. The monopolist privately decides its output price, $p_m(\theta)$.[12]

5. If the state regulator has received an informative signal ($\sigma = \theta$), it privately decides its preferred price for the monopolist's output, $p_s(\theta)$. Otherwise, it does nothing.

6. If the federal authority has received an informative signal ($s = \theta$), it privately decides its preferred price for the monopolist's output, $p_f(\theta)$. Otherwise, it does nothing.

7. If neither regulator nor federal authority have intervened, price p_m is published. If only the state regulator has intervened, price p_s is published. If both regulator and federal authority have intervened, price p_f is published. Monopolist, regulator, and federal authority receive their payoffs.

monopolist's and subscript f the federal regulatory authority's parameters or variables.

[12] For expositional purposes, I will occasionally suppress the dependence of prices on firm cost in notation.

3 Results

I solve for the subgame perfect equilibrium of this game by backward induction.

First, I find the Nash (actually, the dominant strategy) equilibrium in the price-setting game. When setting price, each agent's dominant strategy is to set price to maximize its objective.[13] Any other price, if it is published as the final price, gives the price-setter a lower payoff. But price-setting does not influence the probability of seeing one's preferred price published as the final price, so each agent's dominant strategy has to be to set the price that maximizes its payoff. Thus, the monopolist sets $p_m(\theta) = \arg\max_p f(p, \theta, 1, 0)$, the state regulator sets $p_s(\theta) = \arg\max_p f(p, \theta, \alpha, \beta)$, and the federal authority sets $p_f(\theta) = \arg\max_p f(p, \theta, 1, 1)$.

3.1 The State Regulator's Effort Choice

The state regulator chooses effort e as a function of the federal authority's choice of i. It chooses its effort to:[14]

$$\max_e E_\theta \left[(1-e) f(p_m, \theta, \alpha, \beta) + e \left(i \cdot f(p_f, \theta, \alpha, \beta) + (1-i) f(p_s, \theta, \alpha, \beta) \right) \right] - c_s(e).$$

(That is, with probability $(1-e)$ it will obtain no information about the monopolist's cost, and therefore the monopolist's price prevails. With probability e, either of two things happen: with probability i, the federal authority also obtains hard information and consequently sets p_f, or with probability $(1-i)$, the federal authority obtains no hard information about firm cost and therefore the state regulator's price prevails.)

The first-order condition defines the function $\hat{e}(i)$ such that

$$c_s'(\hat{e}(i)) \equiv E_\theta \left[f(p_s, \theta, \alpha, \beta) - f(p_m, \theta, \alpha, \beta) - i \left(f(p_s, \theta, \alpha, \beta) - f(p_f, \theta, \alpha, \beta) \right) \right]. \quad (1)$$

First note that, absent a federal authority, the state regulator would set effort such that

$$c_s'(\hat{e}) \equiv E_\theta \left[f(p_s, \theta, \alpha, \beta) - f(p_m, \theta, \alpha, \beta) \right].$$

Comparison of this first order condition and that in equation (1) reveals the nature of the externality the federal authority imposes on the state regulator: since $f(p_s, \theta, \alpha, \beta) - f(p_f, \theta, \alpha, \beta) \geq 0$ (p_s is the maximizer

[13] This is a feature specific to this model. For instance, if the timing were changed so that the regulator sets price before the international authority chooses effort and this price were observable, the regulator's price would likely act as a signal about the nature of its information. Similarly, the unobservability of the firm's price is important: since the firm's demand curve is known, cost could be calculated from the optimal price choice. This assumption is less controversial than it looks: the possibility of judicial review requires that national regulatory or international authority decisions are based on verifiable information about cost. In practice, the relationship between observed price, demand and underlying cost is made opaque by issues such as nonlinear pricing, allocation of cost to different products in a multiproduct firm, etc. Alternatively, and with purely cosmetic changes to the model, the nature of the monopolist's private information could be about the relation between production cost and demand.

[14] Since e is bounded between 0 and 1, the regulator in fact chooses $e(i) = \arg\max_e \max\{\min\{1, E_\theta[...] - c_r(e)\}, 0\}$. Because of the concavity of $E_\theta[...] - c_r(e)$, one can first maximize with respect to e and then take account of the boundaries later. This simplifies the exposition considerably.

of $f(p, \theta, \alpha, \beta)$), the existence of the externality from the federal authority generally tends to reduce regulatory effort by the state.[15] Bearing in mind the interpretation of e as a probability of successful investigation, this illustrates the hypothesis that, if there is a federal authority, consumers may, on occasion, get a worse pricing deal than without a federal authority.

From the first-order condition (1) comes the first proposition about the shape of the state regulator's optimal effort response function in the presence of a federal authority:

Proposition 1 *For $p_s \neq p_f$, the function $\hat{e}(i)$ is strictly decreasing if $c_s^{''}(\cdot) > 0$. If $p_s = p_f$, $\hat{e}(i)$ is constant if $c_s^{''}(\cdot) > 0$.*

Proposition 1 is a "crowding out" proposition: It says that the higher the federal authority's investigation effort is, the less effort the state regulator will invest in obtaining information about firm behavior. This proposition is intuitive: The higher the federal authority's effort, the greater the probability that it will set its own price. The state regulator can reduce this probability by reducing its own effort.

Note that the function $\hat{e}(i)$ is not compatible with the interpretation of e (or i) as probabilities: both points in its domain and in its range may lie outside the unit interval. The concavity of the state regulator's "unconstrained" objective however simplifies our task: we define a function $e(i)$ that takes on value 1 when $\hat{e}(i) > 1$, value $\hat{e}(i)$ when $0 \leq \hat{e}(i) \leq 1$, and value 0 when $\hat{e}(i) < 0$. By concavity of the state regulator's objective, this is the best "constrained" response. Since the function $e(i)$ is bounded between 0 and 1, and $\hat{e}(i)$ is monotone decreasing everywhere, there are at most three regions on function $e(i)$, as follows. For ease of presentation, define the boundary $b_0(e) \equiv \frac{E_\theta[f(p_s,\theta,\alpha,\beta) - f(p_m,\theta,\alpha,\beta)] - c_s^{'}(e)}{E_\theta[f(p_s,\theta,\alpha,\beta) - f(p_f,\theta,\alpha,\beta)]}$. The regions are:

1. $i \in [0, \max\{0, b_0(1)\})$ characterized by $e(i) \equiv 1$;

2. $i \in [\max\{0, b_0(1)\}, \min\{b(0), 1\}]$ characterized by $e'(i) < 0$, with $e(i) = \hat{e}(i)$ as in equation (1);

3. $i \in (\min\{b_0(0), 1\}, 1]$ characterized by $e(i) \equiv 0$.

Note that this function $e(i)$ is continuous (although not everywhere differentiable), and weakly monotone decreasing.

In region 1, optimal regulatory effort is high enough for increased federal authority activity not to discourage effort. In region 2, there is some crowding out of regulatory effort (as established in Proposition 1). Region 3 is a region of "total crowding out:" federal authority activity is so great that regulatory effort is fully discouraged.

[15] Note, however, that $\hat{e}$ is bounded between 0 and 1, so that this externality need not always reduce regulatory effort in fact.

3.2 The Federal Authority's Effort Choice

The federal authority's objective is to:

$$\begin{aligned}&\max_i E_\theta[i\left(e(i)f\left(p_f,\theta,1,1\right)+\left(1-e(i)\right)f\left(p_m,\theta,1,1\right)\right)+\\&+\left(1-i\right)\left(e(i)f\left(p_s,\theta,1,1\right)+\left(1-e(i)\right)f\left(p_m,\theta,1,1\right)\right)]-\\&-c_f(i).\end{aligned}$$

(That is, with probability $(1-e(i))$ the state regulator obtains no information, and the monopolist's price prevails. With probability $ie(i)$ both regulator and federal authority obtain hard information, in which case the federal authority sets its price. With probability $(1-i)e(i)$ the state regulator, but not the federal authority, obtains verifiable information about firm cost, and p_s is set.)

Let $u(i)$ denote this objective function. I now study this objective separately for all three regions, and investigate the federal authority's optimal effort choice.

3.2.1 Region 1:

For $i\in[0,b_0(1))$, we have $e(i)\equiv 1$. The federal authority's objective in this region is therefore to

$$\max_i E_\theta\left[i\cdot f\left(p_f,\theta,1,1\right)+\left(1-i\right)f\left(p_s,\theta,1,1\right)\right]-c_f(i),$$

with the first-order condition

$$c_f^{'}(i^*)=E_\theta\left[f\left(p_f,\theta,1,1\right)-f\left(p_s,\theta,1,1\right)\right].$$

This allows me to state the next result, which gives conditions for the federal authority's objective to be decreasing in region 1 (if region 1 exists):

Proposition 2 *For sufficiently large states (β sufficiently large), or for cost-of-effort functions that are sufficiently steep at the origin ($c_f^{'}(0)$ sufficiently large), the federal authority will not investigate at all.*

First, the following lemma is helpful (recall the restriction that $\beta<\alpha$):

Lemma 3 *As $\beta\to\alpha$, $p_s(\theta)\to p_f(\theta)$.*

The lemma is intuitive: it states the simple fact that if a state is large (in the limit, the state is the only state in the union), the state regulator's objective coincides with the federal authority's objective. In this limiting case, their pricing behavior is therefore identical.

Using this lemma, proposition 2 can be proven easily.

Under the conditions in proposition 2, the federal authority will never wish to investigate tentatively: it will either set $i^*=0$ (i.e. not investigate at all), or set i^* somewhere in region 2, which I will term "investigating rigorously."[16]

[16] I show below as a very general proposition that the federal authority will never choose to set i in (or even near) region 3.

The following proposition works out sufficient conditions under which the federal authority will choose to set i exactly at the right region 1 boundary $i^* \to b(1)$ (i.e. it investigates "rigorously").[17]

Proposition 4 *If region 1 exists, the federal authority will always investigate "rigorously" if it has*

(a) *a linear cost-of-effort function from the family characterized by* $c_f^{'}(i) < E_\theta[f(p_f, \theta, 1, 1) - f(p_s, \theta, 1, 1)]$ *for all* i *in region 1;*

(b) *a quadratic cost-of-effort function from the family characterized by* $c_f^{'}(i) = gi^h$ *for all* i *in region 1, with* $h > 1$, $0 < g < \frac{1}{h}\frac{E_\theta[f(p_f,\theta,1,1)-f(p_s,\theta,1,1)]}{(b_0(1))^{h-1}}$*;*

(c) *more generally a cost function with the following restriction on its concavity:* $\int_0^1 c_f''(t)dt \leq E_\theta[f(p_f, \theta, 1, 1) - f(p_s, \theta, 1, 1)]$.

3.2.2 Region 2:

For $i \in [b_0(1), b_0(0)]$, $e(i)$ is defined through

$$c_s^{'}(e(i)) \equiv E_\theta[f(p_s, \theta, \alpha, \beta) - f(p_m, \theta, \alpha, \beta) - i(f(p_s, \theta, \alpha, \beta) - f(p_f, \theta, \alpha, \beta))].$$

In this region, the federal authority's objective has slope

$$\begin{aligned} &E_\theta[(ie'(i) + e(i))(f(p_f, \theta, 1, 1) - f(p_s, \theta, 1, 1)) + \\ &+e'(i)(f(p_s, \theta, 1, 1) - f(p_m, \theta, 1, 1))] - \\ &-c_f^{'}(i). \end{aligned}$$

This yields the following result:

Proposition 5 *Within region 2, if it exists, the federal authority will never set* $i^* = b_0(0)$*, i.e. it will never locate at the right boundary of region 2.*

The proposition establishes that in region 2, where $e(i)$ is negative monotone, the federal authority is always better off allowing at least some regulatory effort: it will locate to the left of the point where $e(i) = 0$. This proposition, again, is intuitive: moving away from the point where regulatory effort is fully crowded out gives the federal authority an increased probability of winning out over the state regulator in its price-setting. Furthermore, such a move to the left reduces costly effort, so it must be worthwhile.

[17] This nomenclature may be misleading: if region 1 does not exist, the proposition notwithstanding, the competition authority may not investigate at all if the objective in region 2 is decreasing.

3.2.3 Region 3:

For $i \in (b_0(0), 1]$, we have $e(i) \equiv 0$. The federal authority's objective in this region is therefore to

$$\max_i E_\theta \left[f\left(p_f, \theta, 1, 1\right) \right] - c_f(i).$$

This of course has a boundary solution, at $i^* \to b_0(0)$. The reason for this is entirely intuitive: where regulatory effort is zero, the information collected by the state regulator is never informative. By implication, and as a result of the informational structure of the model, the federal authority never receives hard information about firm cost. In this case, reducing federal authority effort does not decrease the information available to the federal authority (and therefore does not lead to a worse outcome for the authority), but it reduces effort cost. This argument establishes that it is never optimal for the federal authority to locate on the inside of region 3.

Together with the previous result about region 2, this results immediately in the following simple, and very general:

Proposition 6 *It is never in the federal authority's interest to discourage all regulatory effort.*

Proof. The proposition follows immediately from the discussion in the text, and from proposition 5. ■

3.3 Size Effects

The previous section has asked questions about the optimal federal authority effort. The maintained hypothesis has been that all three regions (region 1 of no crowding out, region 2 of partial crowding out, and region 3 of full crowding out) exist. The robust conclusion was that the federal authority will never crowd out regulatory effort fully. Furthermore, in a large number of cases, federal authority behavior is discontinuous: either the authority will not investigate at all, or it investigates rigorously. I study this last proposition in the context of a special case more rigorously below.

First, however, we need to understand the comparative statics of the boundaries between regions. In this section, I ask how state size interacts with our predictions. I focus on determining the relative size of regions 1, 2, and 3 along the $e(i)$ function, and study how these regions vary with state size.

The following limit result about the size of region 1 obtains:

Proposition 7 *For sufficiently (not only vanishingly) small states, region 1 does not exist (so that the federal authority's effort always crowds out some regulatory effort). For large states, if region 1 ever exists, it becomes large (so that no regulatory effort is crowded out).*

First, the following lemma that says that regulators in very small states set prices like monopolists is helpful:

Lemma 8 *As $\beta \to 0$, $p_s(\theta) \to p_m(\theta)$.*

The proof of proposition 7 is now simple.

Now that we know that region 1 becomes small for sufficiently small states, we may ask what fills the void. The following proposition is again intuitive: for very small states, regulatory and monopolist pricing behavior is identical, so that the state regulator (and therefore, by implication, the federal authority) will withhold all effort:

Proposition 9 *For small states, region 3 becomes large (neither regulator nor federal authority will intervene).*

It is worth noting that this last result applies for sufficiently (not only vanishingly) small states if $c_s^{'}(0)$ is sufficiently large.

3.4 Price Distribution

The state regulator's and the federal authority's effort choices induce a probability distribution over prices. Without a federal authority, price p_s will realize with probability $e(0)$, and price p_m with probability $1-e(0)$. With federal authority, the distribution is $\{(p_f, i^* e(i^*)), (p_s, (1-i^*)e(i^*)), (p_m, (1-e(i^*))\}$. Coming back to the question about whether consumers necessarily get a better deal when there is a federal authority, it is now possible to compare expected prices. In particular, price on average will be higher with a federal regulatory authority when

$$e(0) > e(i^*) \left(1 + i^* \frac{p_f - p_s}{p_s - p_m}\right). \tag{2}$$

Clearly, if the federal authority's effort does not crowd out any regulatory effort ($e(0) = e(i) = 1$), average price when there is a federal authority is lower than without federal authority. Equally obviously, at the point of total crowding out ($e(i) = 0$), the average price with federal authority (which is then just p_m) is higher than without (when it is a, possibly degenerate, mixture between p_m and p_s). When $e(i)$ is decreasing (in region 2), it is straightforward that the function on the right-hand side of inequality (2) tends to first increase with increasing i and then decrease. Whether the latter effect is sufficiently strong (and whether i^* can be sufficiently large) is an empirical question.

More pertinent, the effort choices of state regulatory and federal authorities induce a distribution over consumer surplus. Making use of the notation introduced earlier, consumer surplus from price p is $f(p, \theta, 0, 1)$. Without a federal authority, surplus $f(p_s, \theta, 0, 1)$ will realize with probability $e(0)$, and surplus $f(p_m, \theta, 0, 1)$ with probability $1-e(0)$. With a federal regulator, the distribution is $\{(f(p_f, \theta, 0, 1), i^* e(i^*)), (f(p_s, \theta, 0, 1), (1-i^*)e(i^*)), (f(p_m, \theta, 0, 1), (1-e(i^*))\}$. Average consumer surplus will be higher with a federal regulatory authority than without when

$$\begin{aligned} & e(0) f(p_s, \theta, 0, 1) + (1 - e(0)) f(p_m, \theta, 0, 1) \\ > \quad & i^* e(i^*) f(p_f, \theta, 0, 1) + (1 - i^*) e(i^*) f(p_s, \theta, 0, 1) + (1 - e(i^*) f(p_m, \theta, 0, 1). \end{aligned} \tag{3}$$

At $i^* = 0$, this inequality obviously holds as an equality. The right-hand side of inequality (3) is a function of i^*. As i^* increases, this function has slope

$$i^* e'(i^*)(f(p_f,\theta,0,1) - f(p_s,\theta,0,1)) - e'(i^*)(f(p_m,\theta,0,1) - f(p_s,\theta,0,1)) +$$
$$+ e(i^*)(f(p_f,\theta,0,1) - f(p_s,\theta,0,1)).$$

Since $f(p,\theta,0,1)$ is decreasing in p, it is obvious that the first two terms in this sum are nonpositive, and the last is nonnegative.

In region 1, where $e(\cdot) = 1$ (so that $e'(\cdot) = 0$), this slope is therefore $f(p_f,\theta,0,1) - f(p_s,\theta,0,1) \geq 0$. Proposition 7 states that, for large states, if region 1 ever exists, it becomes large. We also know that, for large states, $p_s \to p_f$, so that $f(p_f,\theta,0,1) - f(p_s,\theta,0,1) \to 0$. Therefore, if region 1 ever exists, for large states, expected consumer surplus is the same both with and without a federal authority.

If region 1 does not exist, then we have the result that, at least for sufficiently large states (where $p_s \to p_f$), this slope is negative, so that inequality (3) holds. In this illustrative case, expected consumer surplus is lower when a federal authority "regulates the (state) regulator."

3.5 Discussion

The nature of the externality between regulator and federal authority lies in the efficiency of the information flow from regulator to federal authority. In the model, greater federal regulatory authority effort leads to regulatory crowding out because it reduces the probability that the state regulator sees its preferred price imposed on the firm. At the same time, regulatory effort determines the quality of information available to the federal authority. I have chosen to model this informational assumption such that the information potentially available to the federal regulatory authority is a subset of the state regulator's information. The feature of differential quality of information is crucial to the model: as argued above, it opens up the possibility for potentially damaging external effects. Modelling the quality of information as two nested sets is clearly an extreme modelling choice. However, it is obvious that the main features of the model rely only on the relative coarseness of the federal regulatory authority's information and will therefore survive a less extreme modelling assumption.

4 Some Special Cases

Sharper predictions about state regulatory and federal authority behavior can be obtained by placing restrictions on the cost functions of state regulator and federal regulatory authority. First consider a regulator with constant unit cost of effort.

4.1 Case 1

Consider a regulator with constant unit cost of effort, $c_s(e) = ke$. This gives us an affine objective function which has a boundary solution. This regulator will therefore choose $e = 1$ if

$$E_\theta \left[i \cdot f \left(p_f, \theta, \alpha, \beta \right) + (1 - i) \, f \left(p_s, \theta, \alpha, \beta \right) \right] - k \geq E_\theta \left[f \left(p_m, \theta, \alpha, \beta \right) \right],$$

and $e = 0$ otherwise. This yields the following step function $e(i)$. Again, for presentational ease, define the boundary $b_1 \equiv \frac{E_\theta[f(p_s,\theta,\alpha,\beta) - f(p_m,\theta,\alpha,\beta)] - k}{E_\theta[f(p_s,\theta,\alpha,\beta) - f(p_f,\theta,\alpha,\beta)]}$.

$$e(i) = \begin{cases} 1 \text{ for } i \leq b_1 \\ 0 \text{ for } i > b_1 \end{cases}.$$

Note also that this result is a limit result for the general $e(i)$ function derived in section 3: for $c_s(e) = ke$, $c_s^{'}(0) = c_s^{'}(1) = k$, so that the boundaries between region 1 and region 2 and between region 2 and region 3 coincide. The linear cost function cuts out region 2.

Note also that in this case, the federal authority's objective will be discontinuous at the point $\hat{\imath} = b_1$:

$$\begin{aligned} & \lim_{i \to \hat{\imath}^-} E_\theta [i \left(e(i) f \left(p_f, \theta, 1, 1 \right) + \left(1 - e(i) \right) f \left(p_m, \theta, 1, 1 \right) \right) + \\ & + (1 - i) \left(e(i) f \left(p_s, \theta, 1, 1 \right) + \left(1 - e(i) \right) f \left(p_m, \theta, 1, 1 \right) \right)] - c_f(i) \\ = \; & E_\theta [\hat{\imath} f \left(p_f, \theta, 1, 1 \right) + (1 - \hat{\imath}) \, f \left(p_s, \theta, 1, 1 \right)] - c_f(\hat{\imath}) \end{aligned}$$

and

$$\begin{aligned} & \lim_{i \to \hat{\imath}^+} E_\theta [i \left(e(i) f \left(p_f, \theta, 1, 1 \right) + \left(1 - e(i) \right) f \left(p_m, \theta, 1, 1 \right) \right) + \\ & + (1 - i) \left(e(i) f \left(p_s, \theta, 1, 1 \right) + \left(1 - e(i) \right) f \left(p_m, \theta, 1, 1 \right) \right)] - c_f(i) \\ = \; & E_\theta [f \left(p_m, \theta, 1, 1 \right)] - c_f(\hat{\imath}) \end{aligned}$$

Since p_f maximizes $f(p, \theta, 1, 1)$, we know that $f(p_f, \theta, 1, 1) \geq f(p_m, \theta, 1, 1)$, and from Lemma 11 we have $f(p_s, \theta, 1, 1) \geq f(p_m, \theta, 1, 1)$, so that the "step" at $\hat{\imath} = b_1$ is a "step down." Further, since the federal authority's objective is monotone decreasing for $i > b_1$ (by the familiar argument: regulatory effort is zero, so increasing federal authority effort brings no benefits, but is costly), we know that the objective for $i > b_1$ can never be greater than for $i \leq b_1$. Therefore, the behavior of the objective in region 1 determines entirely where (in region 1) the federal authority will locate. It will never set $i^* > b_1$.

If the federal authority's cost function is also linear, $c_f(i) = \kappa i$, we can say more about the federal authority's behavior. We already know that we need only study the objective function in region 1, where $e(i) \equiv 1$. With linear cost, we will obtain a boundary solution, so that the federal authority will choose $i^* = 0$ if $\kappa > E_\theta[f(p_f, \theta, 1, 1) - f(p_s, \theta, 1, 1)]$ and will choose $i^* = b_1$ otherwise.[18]

[18] This last conclusion is robust: Whenever the federal authority has a linear cost function, its behavior in region 1 will be at the boundary: either it will withhold all effort, or it will investigate with effort level of at least the level at the right boundary of region 1.

This case gives us a polarized result: Either the federal regulatory authority never intervenes, or it intervenes with a probability such that the state regulator is just indifferent between investing full and investing no effort. That is, we should observe the federal authority either to do nothing, or to "throw the book" at a regulated firm. Further, in equilibrium we would always expect to see the state regulator dedicate full effort to investigation of the monopolist's cost structure. Recalling the interpretation of effort as the probability of "success" (obtaining verifiable information), the testable implication of this result is that we should always see the state regulator (or the federal authority) trumping the monopolist's own price.

The state regulator's cost-of-effort function of course depends on the industry it regulates. For instance, a firm producing a large number of differentiated goods (so that cost allocation is an issue) is "harder" to regulate than a single-good monopolist; a cost-structure highly sensitive to random factors is "harder" to investigate than one that is not; etc. The case of linear cost-of-investigation, while clearly extreme, may be a pointer to one of the reasons why we would always expect certain industries to be regulated, while others are only subject to regulator-imposed pricing from time to time.

4.2 Case 2

We know that the federal authority will never want to locate in region 3. If it locates in region 2, can anything be said about where precisely? Given a set of assumptions on the state regulatory and federal regulatory authority cost-of-effort functions, we obtain a sharp result.

Consider the case where both regulator and federal authority have a quadratic cost-of-effort function, $c_s(x) = c_f(x) = \frac{1}{2}x^2$. About this case I find the following surprising result:[19]

Proposition 10 *With quadratic cost-of-effort, the upper bound on federal authority effort is half the right boundary of region 2.*

This result is surprising: it allows the sharp prediction that the federal authority will never wish to reduce the state regulator's effort below $e = \frac{1}{2}$.[20] This follows from the fact that, with quadratic cost-of-effort, the $\hat{e}(i)$ function is affine; since it decreases linearly from a value of one to a value of zero over the length of region 2, and we know that the federal authority will never wish to locate more than halfway into region 2 (depending on the size of region 1 possibly much less), it will never crowd out regulatory effort to a point below $e = \frac{1}{2}$.

Again, the testable implication is that we should expect the state regulator (or the federal authority) to "win out" over the firm in at least (and likely more than) half the number of cases in which the state regulator starts an investigation.

[19] One may ask whether the restriction of the cost-of-effort function to $c_s(e) = \frac{1}{2}e^2$, and $c_f(i) = \frac{1}{2}i^2$ is essential, rather than choosing ke^2 (κi^2, respectively). The answer is no. As can easily be ascertained, as long as $k > 0$, $\kappa > 0$, the result still holds. $c_s(e) = \frac{1}{2}e^2$, and $c_f(i) = \frac{1}{2}i^2$ are chosen for presentational ease.

[20] Unless, because of state size effects, the state regulator's effort in the absence of a feder authority would be below $e = \frac{1}{2}$.

5 Discussion and Related Literature

This section discusses briefly the informational structure in our model, and why I have chosen to model informational opportunities as a pair of nested sets. I then draw parallels between this model and the related literature.

5.1 Information Structure

I have chosen a very specific structure of the interaction of the timing of the model and the availability of information to regulator and federal authority. In particular, I have assumed that the federal regulatory authority can only observe the monopolist's cost when the state regulator has received "hard" information about firm cost. This created the nested information structure that this paper has exploited. Clearly, other modelling choices could have been made.

A limiting case is that of informational independence: Whether or not the state regulator observes firm cost (it observes cost with probability e and does not observe cost with probability $(1-e)$), the federal authority observes cost with probability i. The state regulator therefore seeks to

$$\begin{aligned} & \max_e E_\theta[(1-e)\left(i \cdot f\left(p_f, \theta, \alpha, \beta\right) + (1-i)\, f\left(p_m, \theta, \alpha, \beta\right)\right) + \\ & + e\left(i \cdot f\left(p_f, \theta, \alpha, \beta\right) + (1-i)\, f\left(p_s, \theta, \alpha, \beta\right)\right)] - c_s(e) \\ = \quad & \max_e E_\theta\left[i \cdot f(p_f, \theta, \alpha, \beta) + (1-e)(1-i)\, f\left(p_m, \theta, \alpha, \beta\right) + e\,(1-i)\, f\left(p_s, \theta, \alpha, \beta\right)\right] - c_s(e), \end{aligned}$$

with the first-order condition

$$c_s^{'}(\hat{e}(i)) = E_\theta\left[(1-i)\left(f\left(p_s, \theta, \alpha, \beta\right) - f\left(p_m, \theta, \alpha, \beta\right)\right)\right].$$

Again, one obtains an externality from federal authority to regulator: higher federal authority effort implies lower regulatory effort. By comparison with equation (1), we see that whether regulatory effort is crowded out more strongly in this model depends on the relative sizes of $E_\theta\left[f\left(p_s, \theta, \alpha, \beta\right) - f\left(p_m, \theta, \alpha, \beta\right)\right]$ and $E_\theta[(1-i)\left(f\left(p_s, \theta, \alpha, \beta\right) - f\left(p_f, \theta, \alpha, \beta\right)\right)]$. But there is no externality on the federal authority comparable to that examined in this paper: the federal authority's information set depends only on its own effort choice, not on that by the state regulator (although the externality through the pricing implication of the state regulator's lower effort persists). In general in a model like this, one should expect greater federal authority involvement. One would use this modelling approach if both the federal authority and the regulatory authorities had identical powers of investigation. In many interesting cases, for instance in the case of European competition law enforcement outlined above, this is not true: as argued above, states have greater powers of investigation than the European Commission. It is for this reason that my interest in this paper is in modelling externalities imposed by two competing principals on each other, and hence I have chosen the nested information structure above.

More generally, the information structure under which the federal authority operates is, of course, a choice variable for the designer of the hierarchical structure (the legislator; for instance, parliament). Independent investigation by the federal regulatory authority is, of course, costly: it loses information already acquired by state authorities. This opens up the larger question of how hierarchies should be structured optimally. While this paper has explored some of the implications of one specific structure (nested information), this larger question is beyond the scope of this paper.

A further modelling option would have been to give the federal regulatory authority a certain amount of autonomy in its investigation: While the state regulator's effort influences the probability of success of the federal authority's effort, the federal authority may be able to obtain hard information, even if the state regulator has happened not to observe firm cost. This could be modeled as follows: The signal $s \in \{\theta, \emptyset\}$ that the federal authority obtains could be such that $\Pr\{s = \theta | e, i\} = e \cdot i$, so that the probability distribution over pricing outcomes is $\{(p_f, e \cdot i), (p_s, e(1 - e \cdot i)), (p_m, (1 - e)(1 - e \cdot i))\}$. The state regulator therefore seeks to

$$\max_e E_\theta \left[(1-e)(1-ei) f(p_m, \theta, \alpha, \beta) + e(1-ei) f(p_s, \theta, \alpha, \beta) + eif(p_f, \theta, \alpha, \beta)\right] - c_s(e),$$

with the first-order condition

$$c_s^{'}(\hat{e}(i)) = (2\hat{e}(i)i - i - 1) f(p_m, \theta, \alpha, \beta) + (1 - 2\hat{e}(i)i) f(p_s, \theta, \alpha, \beta) + i \cdot f(p_f, \theta, \alpha, \beta).$$

The resulting function $\hat{e}(i)$ has derivative

$$\frac{d\hat{e}(i)}{di} = \frac{(f(p_f, \theta, \alpha, \beta) - f(p_m, \theta, \alpha, \beta)) - 2\hat{e}(i)(f(p_s, \theta, \alpha, \beta) - f(p_m, \theta, \alpha, \beta))}{c_s^{''}(\hat{e}(i)) + 2i(f(p_s, \theta, \alpha, \beta) - f(p_m, \theta, \alpha, \beta))}.$$

Here one cannot even prove in general that regulatory effort is crowded out—this depends on the size of $2\hat{e}(i)(f(p_s, \theta, \alpha, \beta) - f(p_m, \theta, \alpha, \beta))$ relative to $f(p_f, \theta, \alpha, \beta) - f(p_m, \theta, \alpha, \beta)$.[21] This also illustrates the increase in the model's computational complexity at, presumably, little informational benefit.

I have chosen to focus on an admittedly limiting case of nested information sets because I believe it brings out an important aspect of hierarchical structures: the relative poverty of means to acquire information of the federal agency relative to the state agencies. In this model, all the federal authority can do is request material previously acquired by antitrust enforcement agencies in federal states. While clearly extreme, the results from this model should still be informative of actual decision mechanisms.

5.2 Related Literature

The model in this paper combines elements from common agency under adverse selection (Martimort (1992), Stole (1992)) and hierarchical "principal-supervisor-agent" models (Tirole (1986)). The common agency

[21] For instance, with linear demand $q(p) = a - bp$, quadratic cost ($c(e) = \frac{1}{2}e^2$) and uniform distribution of cost, the function $\hat{e}(i)$ is monotone decreasing if, and only if,

$$\frac{a^2}{b} > \frac{-12(2\alpha - 3\beta)(-2\alpha + \beta)^2}{\beta^4}.$$

aspect enters through the competition for information about the monopolist's cost (which allows setting of the price preferred by each principal) between state regulator and federal regulatory authority. Both principals have different objectives, and each principal's objective differs from that of the regulated firm.

Spiller (1990) has a common agency model (in a hidden action setting) of competition between congress and an organized interest group ("industry") for favorable pricing decisions by the regulator. In Spiller's paper, congress and industry bid (through budget-setting and direct money transfers, respectively) for favorable price outcomes. The informational asymmetry is about unobservable regulator actions which induce a distribution over observable pricing outcomes. Spiller's paper differs from ours both in the informational structure of the problem it studies, and in emphasis: while Spiller explores the objective of the regulator, and how regulatory actions are influenced by political pressure, we emphasize the problems regulatory agencies with differing incentives create for each other in extracting information about a monopolist subject to regulation.[22]

In this sense, my model is closer to that of Martimort (1996). Martimort models two regulatory agencies that non-cooperatively choose subsidies for a firm with privately known cost of performing a project beneficial to the constituencies of both regulators. The firm chooses whether or not to invest in the project. Martimort shows that under non-cooperative contracting, the project is less likely to be performed than under co-operative or full information assumptions on the model. In Martimort's model, as in mine, regulators impose externalities on each other: one regulator's subsidy has external benefits (if the project is performed) for the other regulator's constituency. Both regulators, however, are identical: there is no sense of one regulator having greater discretion or power.

In many applications, there is no such symmetry of regulatory power; often one authority is subordinated (if sometimes only partially) to another. At the same time, these principals may maximize their own objectives, which depend on the agent's private information in opposite ways. This is the case for the regulatory example I address in this paper. I find this aspect of reality interesting and therefore choose to introduce elements of hierarchical decision-making into the model.

Tirole (1986) develops an hierarchical "principal-supervisor-agent" model, where the agent produces (observable) output with privately known effort, according to some stochastic technology. The principal obtains the proceeds from selling the agent's output and pays the agent a wage contingent on output and a report on the agent's performance from a supervisor. While the agent knows the size of the productivity shock in her production technology, the supervisor has coarser information. Specifically, the supervisor receives a signal that either reveals the productivity variable or else reveals nothing. The supervisor makes a report to the principal, in which she can either report the state of the productivity variable truthfully or report that she has observed nothing, but cannot "lie" directly. This is the extent of the supervisor's

[22] Spiller's paper is interesting because his empirical discussion finds career concerns (post-regulatory employment opportunities in regulated firms) as an important motivational factor for regulators. My model uses the idea of career concerns to justify a divergence between state and federal regulators.

discretion, and it drives the model's results. The supervisor is rewarded by the principal on the basis of her report and the productive output of the agent.

The focus of Tirole's paper is to describe the optimal collusion-proof contract in a world of moral hazard, that is the mechanism that prevents agent and supervisor from exchanging side transfers. In general, the possibility of side contracts (although, in equilibrium, no side contracts are actually made), implies that the supervisor will use her discretion to act as an advocate for the agent, i.e. she will sometimes hide information detrimental to the agent: the principal would be better off if side contracts could be prevented costlessly. Yet, there is a role for the supervisor: if the agent could produce verifiable information herself, she would only ever choose to reveal information that is not detrimental to her. Tirole argues that organizational design is partly a response to the threat of collusion; organizations are designed so as to minimize the possibility of side transfers (e.g. short-run relationships, or bureaucratic rules).[23]

In this paper, I use part of Tirole's hierarchical structure, but model a situation of adverse selection. In particular, I make use of the nested information structure between agent and supervisor. In my model, the state regulator can acquire (unlike in Tirole's paper, at a cost) an imperfect signal of the firm's cost parameter, and it can control the accuracy of the signal. I then duplicate this structure again and allow the federal agency to acquire, at a cost, an imperfect signal of the state regulator's signal. Unlike in Tirole's paper, where the supervisor determines what the principal knows directly, I allow the competition authority to determine what information it obtains—but the opportunities for this information acquisition are restricted by the regulator's choice of how much (and how precise) information it acquires.

An example of a hierarchical model in the field of regulation is the paper by Che (1995), which applies Tirole's framework to the study of regulatory capture by the firm. In a hierarchy of regulated firm (agent), regulator (supervisor), and government (principal), Che studies the question of whether the existence of a "revolving door" (post-government employment opportunities in the regulated industry for the regulator, and the opportunity for collusion between regulator and regulated firm) always needs to be harmful. In fact, Che finds that the revolving door can give the regulator the (*ex ante*) incentive to acquire industry-specific human capital, and to regulate the industry strictly (*ex post*) in order to signal her qualification for post-regulatory employment in the industry. Che's emphasis, unlike mine, however is not a description of the acquisition (or lack of acquisition) of regulation-relevant information.

6 Conclusion

This paper has examined the restrictions placed on the action space of a federal competition authority that interacts with a set of subsidiary state industry regulators. I have isolated an important limiting factor on the set of actions a federal competition authority (in its price regulating function) would wish to take. This limiting factor is the reciprocal externality the federal authority imposes by seeking to acquire information

[23] cf. also the more extensive discussion in Tirole (1992).

that can be used to implement price regulation: the more closely the federal agency monitors an industry (and therefore the more often it will overrule the state regulator's decision with its own), the lower the incentive for the state regulator to acquire the information necessary for regulation of the industry. Since the information at the disposal of the federal agency is correlated with the information acquired by the state regulator, the less information is at the disposal of the state regulator the more frequently the monopolist remains unregulated. In this sense, the presence of a higher level agency in a hierarchy with overlapping jurisdiction crowds out information and forces an outcome that is, apparently, collusive. The mechanism I have exploited in this paper is the observation that, in many hierarchies the higher (federal) tier's powers of investigation (for instance, such as under European Union antitrust law, the European Commission's) are clearly weaker than the state authority's powers (in the sense that the federal agency depends on the state agency for its information). The modelling tool I have used is to view the information available to the federal authority as a subset of the information obtained by the state regulator. This approach, while clearly extreme, allows a clearer focus on the nature of the bilateral externality between federal authority and state regulator.

The most general result from the model is that federal authorities have to be wary of overregulation: the more the federal authority investigates, the more the state regulator aligns itself with the interests of the regulated monopolist, resulting in no regulation. This is a novel twist on the old regulatory capture hypothesis.[24]

More specifically, the results from the model are limiting results. I can derive bounds on the federal authority's actions that should observed. In particular, the model predicts that regulatory effort will never be completely crowded out by federal authority investigation. I obtain a surprisingly sharp limiting result in a special case, that predicts that regulatory effort is never crowded out by more than 50%. The paper also has a result that suggests that federal authorities should either do nothing or seek to acquire information with some degree of rigor: dabbling, the model suggests, under a wide range of circumstances, is not optimal.

The results from the present paper should give material for thought to policy designers and economists alike. To policy designers who set the budget for federal authorities, the advice is that more is not always better. To the economist, a note of caution: there is much that we still have to learn about the interaction of authorities in hierarchical structures that compete for policy outcomes.

[24] In this sense, my results are similar to those derived in a different context (congressional oversight of administrative agencies when lobby group influence is important), and from a different model, by Epstein and O'Halloran (1995). Epstein and O'Halloran provide a model where the administrative agency (bureaucracy) aligns itself with the interests of a lobby group in order to prevent the lobby group sounding a "fire alarm" to congress about bureaucracy behavior.

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7 Appendix: Proofs

Proof of Proposition 1. Equation (1) holds as an identity and can therefore be differentiated. Differentiation with respect to i yields

$$\hat{e}'(i)c_s^{''}(\hat{e}(i)) = -E_\theta\left[(f(p_s,\theta,\alpha,\beta) - f(p_f,\theta,\alpha,\beta))\right].$$

Since p_s maximizes $f(p_s,\theta,\alpha,\beta)$, we have $E_\theta\left[(f(p_s,\theta,\alpha,\beta) - f(p_f,\theta,\alpha,\beta))\right] \geq 0$. If $p_s \neq p_f$, $E_\theta\left[(f(p_s,\theta,\alpha,\beta) - f(p_f,\theta,\alpha,\beta))\right] > 0$, so that $\hat{e}'(i) < 0$ if $c_s^{''}(\cdot) > 0$. The result for $p_s = p_f$ follows immediately. ■

Proof of Lemma 3.

$$\begin{aligned}
\lim_{\beta\to\alpha} p_s(\theta) &= \\
&= \lim_{\beta\to\alpha} \arg\max_p \alpha\pi(p,\theta) + \beta\int_p^\infty q(t)dt \\
&= \lim_{\beta\to\alpha} \arg\max_p \pi(p,\theta) + \frac{\beta}{\alpha}\int_p^\infty q(t)dt \\
&= \arg\max_p \pi(p,\theta) + \int_p^\infty q(t)dt \\
&= p_f(\theta),
\end{aligned}$$

This proves the lemma. ■

Proof of Proposition 2. The federal authority's objective is monotone decreasing over the entirety of region 1, so that the optimal choice of i in this region is at the boundary $i^* = 0$, if

$$c_f^{'}(0) \geq E_\theta\left[f(p_f,\theta,1,1) - f(p_s,\theta,1,1)\right].$$

Since p_f is the maximizer of $f(p,\theta,1,1)$, we have $E_\theta\left[f(p_f,\theta,1,1) - f(p_s,\theta,1,1)\right] \geq 0$. By lemma 3, we have $\lim_{\beta\to\alpha} E_\theta\left[f(p_f,\theta,1,1) - f(p_s,\theta,1,1)\right] = 0$. In region 1, therefore, the federal authority will wish to set $i^* = 0$.

Next, we need to prove that if $c_f^{'}(0) \geq E_\theta\left[f(p_f,\theta,1,1) - f(p_s,\theta,1,1)\right]$, the federal authority's objective in region 2 is also monotone decreasing.

In region 2, the federal authority's objective has slope

$$\begin{aligned}
&E_\theta[(e(i) + ie'(i))(f(p_f,\theta,1,1) - f(p_s,\theta,1,1)) + \\
&+e'(i)(f(p_s,\theta,1,1) - f(p_m,\theta,1,1))] - c_f^{'}(i).
\end{aligned}$$

From $c_f^{'}(0) \geq E_\theta\left[f(p_f,\theta,1,1) - f(p_s,\theta,1,1)\right]$ and $c_f^{''}(\cdot) > 0$, we know that $c_f^{'}(i) \geq E_\theta[f(p_f,\theta,1,1) - f(p_s,\theta,1,1)]$. But this implies that the federal authority's objective has a slope less than

$$\begin{aligned}
&E_\theta[(e(i) - 1 + ie'(i))(f(p_f,\theta,1,1) - f(p_s,\theta,1,1)) + \\
&+e'(i)(f(p_s,\theta,1,1) - f(p_m,\theta,1,1))].
\end{aligned}$$

Since we know that $E_\theta[f(p_f,\theta,1,1)-f(p_s,\theta,1,1)] \geq 0$ and $E_\theta[f(p_s,\theta,1,1)-f(p_m,\theta,1,1)] \geq 0$, and $e(i) \leq 1$, by $e'(i) < 0$ (in region 2), it follows immediately that the federal authority's objective in region 2 is monotone decreasing. ■

Proof of Proposition 4. Part (a) of the proposition follows immediately. For part (b) note that continuity and concavity of the authority's objective guarantee that, if $c_f^{'}(0) < E_\theta[f(p_f,\theta,1,1)-f(p_s,\theta,1,1)]$ and $c_f^{'}(b_0(1)) \leq E_\theta[f(p_f,\theta,1,1)-f(p_s,\theta,1,1)]$, the objective will be monotone increasing over the entirety of region 1. All we require from a quadratic function is therefore that

$$ab(b_0(1))^{b-1} \leq E_\theta[f(p_f,\theta,1,1)-f(p_s,\theta,1,1)].$$

Since, by assumption $b_0(1) \geq 0$, the expression in the proposition follows. Part (c) of the proposition immediately follows from the requirement that $c_f^{'}(1) \leq E_\theta[f(p_f,\theta,1,1)-f(p_s,\theta,1,1)]$. ■

Proof of Proposition 5.

First we need a lemma.

Lemma 11 $E_\theta[f(p_s,\theta,1,1)-f(p_m,\theta,1,1)] \geq 0$.

Proof. The proof proceeds in two steps. Note first that p_m, p_s and p_f can be ordered, as follows: $p_m \geq p_s \geq p_f$. This follows from the maximization problem that p_m, p_s and p_f solve:

$$\begin{aligned} p_m &= \arg\max_p \pi(p,\theta) \\ p_s &= \arg\max_p \pi(p,\theta) + \frac{\beta}{\alpha}\int_p^\infty q(t)dt \\ p_f &= \arg\max_p \pi(p,\theta) + \int_p^\infty q(t)dt. \end{aligned}$$

Since $\int_p^\infty q(t)dt$ is a decreasing function of p, the ranking of p_m, p_s and p_f follows. Next, we need to show that this ranking implies a ranking over $f(p,\theta,1,1)$. But this is straightforward: since $f(p,\theta,1,1)$ is concave in p, and is maximized by p_f, the ranking of p_m, p_s and p_f implies that $f(p_s,\theta,1,1) \geq f(p_m,\theta,1,1)$. ■

Using Lemma 11, we can prove proposition 5 easily.

At $i = b_0(0)$, $e(i) = 0$. Since $e'(i) < 0$ in region 2, and since $E_\theta[f(p_f,\theta,1,1)-f(p_s,\theta,1,1)] \geq 0$ and (by lemma 11) $E_\theta[f(p_s,\theta,1,1)-f(p_m,\theta,1,1)] \geq 0$, and since $c_f^{'}(\cdot) > 0$, we know that at the right boundary of region 2, we have $\frac{du}{di}\left(\frac{E_\theta[f(p_s,\theta,\alpha,\beta)-f(p_m,\theta,\alpha,\beta)]-c_s^{'}(0)}{E_\theta[f(p_s,\theta,\alpha,\beta)-f(p_f,\theta,\alpha,\beta)]}\right) < 0$, so that moving ε to the left increases the federal authority's objective. ■

Proof of Lemma 8.

$$\begin{aligned}
\lim_{\beta\to 0} p_s(\theta) &= \\
&= \lim_{\beta\to 0} \arg\max_p \alpha\pi(p,\theta) + \beta\int_p^\infty q(t)dt \\
&= \lim_{\beta\to 0} \arg\max_p \pi(p,\theta) + \frac{\beta}{\alpha}\int_p^\infty q(t)dt \\
&= \arg\max_p \pi(p,\theta) \\
&= p_m(\theta).
\end{aligned}$$

This proves the lemma. ■

Proof of Proposition 7. The boundary between region 1 and region 2 is $i = b_0(1) \equiv \frac{E_\theta[f(p_s,\theta,\alpha,\beta)-f(p_m,\theta,\alpha,\beta)]-c_s^{'}(1)}{E_\theta[f(p_s,\theta,\alpha,\beta)-f(p_f,\theta,\alpha,\beta)]}$. Since, by the previous lemma (lemma 8), $\lim_{\beta\to 0} p_s(\theta) = p_m(\theta)$, for small states this boundary converges to $\lim_{\beta\to 0} \frac{E_\theta[f(p_s,\theta,\alpha,\beta)-f(p_m,\theta,\alpha,\beta)]-c_s^{'}(1)}{E_\theta[f(p_s,\theta,\alpha,\beta)-f(p_f,\theta,\alpha,\beta)]} = -\frac{c_s^{'}(1)}{E_\theta[f(p_m,\theta,\alpha,0)-f(p_f,\theta,\alpha,0)]}$. But since $f(p_m,\theta,\alpha,0) = \alpha f(p_m,\theta,1,0)$, and p_m is the maximizer of $f(p,\theta,1,0)$, we know that $E_\theta\left[f(p_m,\theta,\alpha,0) - f(p_f,\theta,\alpha,0)\right] \geq 0$. Furthermore, by $c_s^{'}(0) \geq 0$ and $c_s''(\cdot) > 0$ we know that $c_s^{'}(1) > 0$. Therefore, $-\frac{c_s^{'}(1)}{E_\theta[f(p_m,\theta,\alpha,0)-f(p_f,\theta,\alpha,0)]}$ is a negative number for all cost functions. By continuity of $\frac{E_\theta[f(p_s,\theta,\alpha,\beta)-f(p_m,\theta,\alpha,\beta)]-c_s^{'}(1)}{E_\theta[f(p_s,\theta,\alpha,\beta)-f(p_f,\theta,\alpha,\beta)]}$ in β (for $\beta < \alpha$), the first part of the proposition follows for sufficiently (not only vanishingly) small states.

For the second part of the proposition, note that we know from lemma 3 that for large states, $p_s(\theta) \to p_f(\theta)$, and therefore $E_\theta\left[f(p_s,\theta,\alpha,\beta) - f(p_f,\theta,\alpha,\beta)\right] \to 0$. Therefore, if region 1 ever exists (i.e. $E_\theta\left[f(p_s,\theta,\alpha,\beta) - f(p_m,\theta,\alpha,\beta)\right] - c_s^{'}(1) > 0$), region 1 grows without bound for sufficiently large β. ■

Proof of Proposition 9. Recall that the left region 3 boundary is at $i = \frac{E_\theta[f(p_s,\theta,\alpha,\beta)-f(p_m,\theta,\alpha,\beta)]-c_s^{'}(0)}{E_\theta[f(p_s,\theta,\alpha,\beta)-f(p_f,\theta,\alpha,\beta)]}$. We know from lemma 8 that, as state size becomes small, $p_s(\theta) \to p_m(\theta)$, so that $\frac{E_\theta[f(p_s,\theta,\alpha,\beta)-f(p_m,\theta,\alpha,\beta)]-c_s^{'}(0)}{E_\theta[f(p_s,\theta,\alpha,\beta)-f(p_f,\theta,\alpha,\beta)]}$ becomes a nonpositive number (since $c_s^{'}(0) \geq 0$, and $E_\theta\left[f(p_s,\theta,\alpha,\beta) - f(p_f,\theta,\alpha,\beta)\right] \geq 0$). Therefore, region 3 becomes large. We know what this implies about regulatory behavior since, by definition, in region 3 $e(i) = 0$. The implication for federal authority behavior follows from the obvious fact that if $e(i) = 0$, reducing i increases the federal authority's objective. ■

Proof of Proposition 10. With quadratic cost-of-effort, the right boundary of region 2 is $\frac{E_\theta[f(p_s,\theta,\alpha,\beta)-f(p_m,\theta,\alpha,\beta)]}{E_\theta[f(p_s,\theta,\alpha,\beta)-f(p_f,\theta,\alpha,\beta)]}$. From the state regulator's maximization problem, given quadratic cost, we obtain immediately

$$\hat{e}(i) \equiv E_\theta\left[f(p_s,\theta,\alpha,\beta) - f(p_m,\theta,\alpha,\beta)\right] - iE_\theta\left[f(p_s,\theta,\alpha,\beta) - f(p_f,\theta,\alpha,\beta)\right],$$

with

$$\hat{e}'(i) \equiv -E_\theta\left[f(p_s,\theta,\alpha,\beta) - f(p_f,\theta,\alpha,\beta)\right].$$

Recall the federal authority's objective $u(i)$, with derivative:

$$\frac{du(i)}{di} = E_\theta[(ie'(i)+e(i))\left(f\left(p_f,\theta,1,1\right)-f\left(p_s,\theta,1,1\right)\right)+ \\ +e'(i)\left(f\left(p_s,\theta,1,1\right)-f\left(p_m,\theta,1,1\right)\right)]- \\ -c_f^{'}(i).$$

Substituting into this, we have as a first-order condition for maximization of $u(i)$ in region 2:

$$\begin{aligned} 0 &= \{E_\theta\left[f\left(p_s,\theta,\alpha,\beta\right)-f\left(p_m,\theta,\alpha,\beta\right)\right]-2i^*E_\theta\left[f\left(p_s,\theta,\alpha,\beta\right)-f\left(p_f,\theta,\alpha,\beta\right)\right]\}\cdot \\ &\quad E_\theta[f\left(p_f,\theta,1,1\right)-f\left(p_s,\theta,1,1\right)]- \\ &\quad -E_\theta\left[f\left(p_s,\theta,\alpha,\beta\right)-f\left(p_f,\theta,\alpha,\beta\right)\right]E_\theta[f\left(p_s,\theta,1,1\right)-f\left(p_m,\theta,1,1\right)]-i^*, \end{aligned}$$

or

$$\begin{aligned} i^* &= \frac{E_\theta\left[f\left(p_s,\theta,\alpha,\beta\right)-f\left(p_m,\theta,\alpha,\beta\right)\right]E_\theta[f\left(p_f,\theta,1,1\right)-f\left(p_s,\theta,1,1\right)]}{1+2E_\theta\left[f\left(p_s,\theta,\alpha,\beta\right)-f\left(p_f,\theta,\alpha,\beta\right)\right]E_\theta[f\left(p_f,\theta,1,1\right)-f\left(p_s,\theta,1,1\right)]}- \\ &\quad -\frac{E_\theta\left[f\left(p_s,\theta,\alpha,\beta\right)-f\left(p_f,\theta,\alpha,\beta\right)\right]E_\theta[f\left(p_s,\theta,1,1\right)-f\left(p_m,\theta,1,1\right)]}{1+2E_\theta\left[f\left(p_s,\theta,\alpha,\beta\right)-f\left(p_f,\theta,\alpha,\beta\right)\right]E_\theta[f\left(p_f,\theta,1,1\right)-f\left(p_s,\theta,1,1\right)]}. \end{aligned}$$

Consider the two summands separately. Since $E_\theta\left[f\left(p_s,\theta,\alpha,\beta\right)-f\left(p_m,\theta,\alpha,\beta\right)\right]>0$ and $E_\theta[f\left(p_f,\theta,1,1\right)-f\left(p_s,\theta,1,1\right)]>0$ (by the now familiar argument from maximization) we have $\frac{E_\theta[f(p_s,\theta,\alpha,\beta)-f(p_m,\theta,\alpha,\beta)]E_\theta[f(p_f,\theta,1,1)-f(p_s,\theta,1,1)]}{1+2E_\theta[f(p_s,\theta,\alpha,\beta)-f(p_f,\theta,\alpha,\beta)]E_\theta[f(p_f,\theta,1,1)-f(p_s,\theta,1,1)]} < \frac{1}{2}\frac{E_\theta[f(p_s,\theta,\alpha,\beta)-f(p_m,\theta,\alpha,\beta)]}{E_\theta[f(p_s,\theta,\alpha,\beta)-f(p_f,\theta,\alpha,\beta)]}$.[25] For essentially the same reason, and making use of lemma 11, we also know that $\frac{E_\theta[f(p_s,\theta,\alpha,\beta)-f(p_f,\theta,\alpha,\beta)]E_\theta[f(p_s,\theta,1,1)-f(p_m,\theta,1,1)]}{1+2E_\theta[f(p_s,\theta,\alpha,\beta)-f(p_f,\theta,\alpha,\beta)]E_\theta[f(p_f,\theta,1,1)-f(p_s,\theta,1,1)]} > 0$. The conclusion, therefore, that $i^* < \frac{1}{2}\frac{E_\theta[f(p_s,\theta,\alpha,\beta)-f(p_m,\theta,\alpha,\beta)]}{E_\theta[f(p_s,\theta,\alpha,\beta)-f(p_f,\theta,\alpha,\beta)]}$ follows directly.

That this is indeed the maximum, not a minimum, can be seen from the second-order condition, which holds:

$$\begin{aligned} \frac{d^2u(i)}{di^2} &= E_\theta[(ie''(i)+2e'(i))\left(f\left(p_f,\theta,1,1\right)-f\left(p_s,\theta,1,1\right)\right)+ \\ &\quad +e''(i)\left(f\left(p_s,\theta,1,1\right)-f\left(p_m,\theta,1,1\right)\right)]-c_f^{''}(i) \\ &= E_\theta[2e'(i)\left(f\left(p_f,\theta,1,1\right)-f\left(p_s,\theta,1,1\right)\right)-c_f^{''}(i)<0. \end{aligned}$$

(The equality follows because, with quadratic cost-of-effort, $e(i)$ is linear, so that $e''(i)=0$; the inequality follows since in region 2, $e'(i)<0$, $f\left(p_f,\theta,1,1\right)-f\left(p_s,\theta,1,1\right)>0$, and $c_f^{''}(i)>0$.) ■

[25] The expression is of the form $\frac{ac}{1+2bc}=\frac{1}{\frac{1}{ac}+2\frac{b}{a}}<\frac{1}{2\frac{b}{a}}=\frac{1}{2}\frac{a}{b}$. The inequality follows for $ac>0$.