1 Supplementary Appendix

Proof of Lemma 1.

$$
\begin{aligned}
\frac{\partial \underline{\delta}^h}{\partial \mu} \overset{s}{=} \gamma\psi & \left(\eta^{\frac{1}{\gamma-1}} - \nu^{\frac{1}{\gamma-1}}\right) \left[\gamma \frac{(\theta_l-\theta_h)}{2\gamma}\nu^{\frac{1}{\gamma-1}} + \frac{1}{\gamma-1}\gamma\frac{(\theta_l-\theta_h)}{2\gamma}\nu\nu^{\frac{1}{\gamma-1}-1} \right. \\
& \left. -\frac{\gamma}{\gamma-1}\frac{(\theta_l-\theta_h)}{2\gamma}\nu^{\frac{\gamma}{\gamma-1}-1} - \gamma\frac{(\theta_l-\theta_h)}{2\gamma}\eta^{\frac{1}{\gamma-1}} \right] \\
& - \left(\gamma\nu\nu^{\frac{1}{\gamma-1}} - \nu^{\frac{\gamma}{\gamma-1}} - \gamma\nu\eta^{\frac{1}{\gamma-1}} + \eta^{\frac{\gamma}{\gamma-1}}\right) \left[-\gamma\frac{(\theta_l-\theta_h)}{2\gamma}\left(\eta^{\frac{1}{\gamma-1}} - \nu^{\frac{1}{\gamma-1}}\right) \right. \\
& \left. +\gamma\psi\left(-\frac{1}{\gamma-1}\frac{(\theta_l-\theta_h)}{2\gamma}\nu^{\frac{1}{\gamma-1}-1} \right) \right]
\end{aligned}
$$

$$
\begin{aligned}
= \; & \frac{(\theta_l-\theta_h)}{2\gamma}\psi\gamma\left(\eta^{\frac{1}{\gamma-1}} - \nu^{\frac{1}{\gamma-1}}\right)\left(\nu^{\frac{1}{\gamma-1}} - \eta^{\frac{1}{\gamma-1}}\right) \\
& + \frac{(\theta_l-\theta_h)}{2\gamma}\left(\gamma\nu\nu^{\frac{1}{\gamma-1}} - \nu^{\frac{\gamma}{\gamma-1}} - \gamma\nu\eta^{\frac{1}{\gamma-1}} + \eta^{\frac{\gamma}{\gamma-1}}\right)\left[\left(\eta^{\frac{1}{\gamma-1}} - \nu^{\frac{1}{\gamma-1}}\right) + \psi\frac{1}{\gamma-1}\nu^{\frac{1}{\gamma-1}-1}\right]
\end{aligned}
$$

$$
\begin{aligned}
= \; & \frac{1}{2}(\theta_l-\theta_h)\left\{\psi\gamma\left(\eta^{\frac{1}{\gamma-1}} - \nu^{\frac{1}{\gamma-1}}\right)\left(\nu^{\frac{1}{\gamma-1}} - \eta^{\frac{1}{\gamma-1}}\right)\right. \\
& \left. + \left(\gamma\nu\nu^{\frac{1}{\gamma-1}} - \nu^{\frac{\gamma}{\gamma-1}} - \gamma\nu\eta^{\frac{1}{\gamma-1}} + \eta^{\frac{\gamma}{\gamma-1}}\right)\left[\left(\eta^{\frac{1}{\gamma-1}} - \nu^{\frac{1}{\gamma-1}}\right) + \psi\frac{1}{\gamma-1}\nu^{\frac{1}{\gamma-1}-1}\right]\right\}
\end{aligned} \tag{1}
$$

Note that $\psi = (\eta - \nu)$. Then (1) is equivalent to:

$$
\begin{aligned}
& \frac{1}{2}(\theta_l-\theta_h)\left\{(\eta-\nu)\gamma\left(\eta^{\frac{1}{\gamma-1}} - \nu^{\frac{1}{\gamma-1}}\right)\left(\nu^{\frac{1}{\gamma-1}} - \eta^{\frac{1}{\gamma-1}}\right)\right. \\
& \left. + \left(\gamma\nu\nu^{\frac{1}{\gamma-1}} - \nu^{\frac{\gamma}{\gamma-1}} - \gamma\nu\eta^{\frac{1}{\gamma-1}} + \eta^{\frac{\gamma}{\gamma-1}}\right)\left[\left(\eta^{\frac{1}{\gamma-1}} - \nu^{\frac{1}{\gamma-1}}\right) + (\eta-\nu)\frac{1}{\gamma-1}\nu^{\frac{1}{\gamma-1}-1}\right]\right\}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{1}{2}\left(\theta_l-\theta_h\right)\Big\{\left(\eta-\nu\right)\gamma\left(\eta^{\frac{1}{\gamma-1}}-\nu^{\frac{1}{\gamma-1}}\right)\left(\nu^{\frac{1}{\gamma-1}}-\eta^{\frac{1}{\gamma-1}}\right)\\
&\quad+\left[\gamma\nu\left(\nu^{\frac{1}{\gamma-1}}-\eta^{\frac{1}{\gamma-1}}\right)-\left(\nu^{\frac{\gamma}{\gamma-1}}-\eta^{\frac{\gamma}{\gamma-1}}\right)\right]\left[\left(\eta^{\frac{1}{\gamma-1}}-\nu^{\frac{1}{\gamma-1}}\right)+\left(\eta-\nu\right)\frac{1}{\gamma-1}\nu^{\frac{1}{\gamma-1}-1}\right]\Big\}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{1}{2}\left(\theta_l-\theta_h\right)\Big[\left(\eta-\nu\right)\gamma\left(\eta^{\frac{1}{\gamma-1}}-\nu^{\frac{1}{\gamma-1}}\right)\left(\nu^{\frac{1}{\gamma-1}}-\eta^{\frac{1}{\gamma-1}}\right)\\
&\quad+\left(\eta-\nu\right)\frac{1}{\gamma-1}\nu^{\frac{1}{\gamma-1}}\gamma\left(\nu^{\frac{1}{\gamma-1}}-\eta^{\frac{1}{\gamma-1}}\right)-\left(\eta-\nu\right)\frac{1}{\gamma-1}\nu^{\frac{1}{\gamma-1}-1}\left(\nu^{\frac{\gamma}{\gamma-1}}-\eta^{\frac{\gamma}{\gamma-1}}\right)\\
&\quad+\gamma\nu\left(\nu^{\frac{1}{\gamma-1}}-\eta^{\frac{1}{\gamma-1}}\right)\left(\eta^{\frac{1}{\gamma-1}}-\nu^{\frac{1}{\gamma-1}}\right)-\left(\nu^{\frac{\gamma}{\gamma-1}}-\eta^{\frac{\gamma}{\gamma-1}}\right)\left(\eta^{\frac{1}{\gamma-1}}-\nu^{\frac{1}{\gamma-1}}\right)\Big]
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{1}{2}\left(\theta_l-\theta_h\right)\Big[\eta\gamma\left(\eta^{\frac{1}{\gamma-1}}-\nu^{\frac{1}{\gamma-1}}\right)\left(\nu^{\frac{1}{\gamma-1}}-\eta^{\frac{1}{\gamma-1}}\right)+\left(\eta-\nu\right)\frac{1}{\gamma-1}\nu^{\frac{1}{\gamma-1}}\gamma\left(\nu^{\frac{1}{\gamma-1}}-\eta^{\frac{1}{\gamma-1}}\right)\\
&\quad-\left(\eta-\nu\right)\frac{1}{\gamma-1}\nu^{\frac{1}{\gamma-1}-1}\left(\nu^{\frac{\gamma}{\gamma-1}}-\eta^{\frac{\gamma}{\gamma-1}}\right)-\left(\nu^{\frac{\gamma}{\gamma-1}}-\eta^{\frac{\gamma}{\gamma-1}}\right)\left(\eta^{\frac{1}{\gamma-1}}-\nu^{\frac{1}{\gamma-1}}\right)\Big]
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{1}{2}\left(\theta_l-\theta_h\right)\Big[\eta\gamma\left(\eta^{\frac{1}{\gamma-1}}-\nu^{\frac{1}{\gamma-1}}\right)\nu^{\frac{1}{\gamma-1}}-\eta\gamma\left(\eta^{\frac{1}{\gamma-1}}-\nu^{\frac{1}{\gamma-1}}\right)\eta^{\frac{1}{\gamma-1}}+\left(\eta-\nu\right)\frac{1}{\gamma-1}\nu^{\frac{1}{\gamma-1}}\gamma\nu^{\frac{1}{\gamma-1}}\\
&\quad-\left(\eta-\nu\right)\frac{1}{\gamma-1}\nu^{\frac{1}{\gamma-1}}\gamma\eta^{\frac{1}{\gamma-1}}-\left(\eta-\nu\right)\frac{1}{\gamma-1}\nu^{\frac{2}{\gamma-1}}+\left(\eta-\nu\right)\frac{1}{\gamma-1}\nu^{\frac{2-\gamma}{\gamma-1}}\eta^{\frac{\gamma}{\gamma-1}}\\
&\quad-\left(\nu^{\frac{\gamma}{\gamma-1}}-\eta^{\frac{\gamma}{\gamma-1}}\right)\eta^{\frac{1}{\gamma-1}}+\left(\nu^{\frac{\gamma}{\gamma-1}}-\eta^{\frac{\gamma}{\gamma-1}}\right)\nu^{\frac{1}{\gamma-1}}\Big]
\end{aligned}
$$

$$= \frac{1}{2}(\theta_l - \theta_h)\Big[-(\gamma-1)\,\eta^{\frac{1}{\gamma-1}}\eta^{\frac{\gamma}{\gamma-1}} - (\gamma-1)\,\eta\nu^{\frac{2}{\gamma-1}} + \frac{1}{(\gamma-1)}\nu^{\frac{\gamma}{\gamma-1}}\eta^{\frac{1}{\gamma-1}} + \frac{2\gamma(\gamma-2)}{(\gamma-1)}\nu^{\frac{1}{\gamma-1}}\eta^{\frac{\gamma}{\gamma-1}} + \frac{1}{\gamma-1}\eta\nu^{\frac{2-\gamma}{\gamma-1}}\eta^{\frac{\gamma}{\gamma-1}}\Big] \qquad (2)$$

Equation (2) shows that $\frac{\partial\delta^h}{\partial\mu} \overset{s}{=} \frac{1}{2}(\theta_l - \theta_h)\,\Xi$ where Ξ equals the expression in the square brackets. We now continue to work with Ξ.

$$\Xi = -(\gamma-1)\,\eta^{\frac{1}{\gamma-1}}\eta^{\frac{\gamma}{\gamma-1}} - (\gamma-1)\,\eta\nu^{\frac{2}{\gamma-1}} + \frac{1}{(\gamma-1)}\nu^{\frac{\gamma}{\gamma-1}}\eta^{\frac{1}{\gamma-1}} + \frac{2\gamma(\gamma-2)}{(\gamma-1)}\nu^{\frac{1}{\gamma-1}}\eta^{\frac{\gamma}{\gamma-1}} + \frac{1}{\gamma-1}\eta\nu^{\frac{2-\gamma}{\gamma-1}}\eta^{\frac{\gamma}{\gamma-1}}$$

$$= \frac{1}{(\gamma-1)}\nu^{-1}\Big[-(\gamma-1)^2\,\eta^{\frac{1}{\gamma-1}}\eta^{\frac{\gamma}{\gamma-1}}\nu - (\gamma-1)^2\,\eta\nu^{\frac{2}{\gamma-1}}\nu + \nu^{\frac{\gamma}{\gamma-1}}\eta^{\frac{1}{\gamma-1}}\nu + 2\gamma(\gamma-2)\,\nu^{\frac{1}{\gamma-1}}\eta^{\frac{\gamma}{\gamma-1}}\nu + \eta\nu^{\frac{2-\gamma}{\gamma-1}}\eta^{\frac{\gamma}{\gamma-1}}\nu\Big]$$

This proves that $\Xi = \frac{1}{(\gamma-1)}\nu^{-1}\Omega$ where Ω equals the expression in the square brackets. We continue to work with Ω.

$$\Omega = -(\gamma-1)^2\,\eta\nu\left(\eta^{\frac{2}{\gamma-1}} + \nu^{\frac{2}{\gamma-1}}\right) + \nu^{\frac{\gamma}{\gamma-1}}\eta^{\frac{1}{\gamma-1}}\nu + 2\gamma(\gamma-2)\,\nu\nu^{\frac{1}{\gamma-1}}\eta^{\frac{\gamma}{\gamma-1}} + \nu^{\frac{1}{\gamma-1}}\eta\eta^{\frac{\gamma}{\gamma-1}}$$

$$
\begin{aligned}
= \quad & -(\gamma-1)^2 \eta\nu \left(\eta^{\frac{2}{\gamma-1}} + \nu^{\frac{2}{\gamma-1}}\right) - 2(\gamma-1)^2 \eta\nu (\nu)^{\frac{1}{\gamma-1}} (\eta)^{\frac{1}{\gamma-1}} \qquad (3) \\
& +2(\gamma-1)^2 \eta\nu (\nu)^{\frac{1}{\gamma-1}} (\eta)^{\frac{1}{\gamma-1}} \\
& +\nu^{\frac{\gamma}{\gamma-1}} \eta^{\frac{1}{\gamma-1}} \nu + 2\gamma(\gamma-2)\nu\nu^{\frac{1}{\gamma-1}} \eta^{\frac{\gamma}{\gamma-1}} + \nu^{\frac{1}{\gamma-1}} \eta\eta^{\frac{\gamma}{\gamma-1}}
\end{aligned}
$$

Now take into account that

$$\left(\eta^{\frac{1}{\gamma-1}} - \nu^{\frac{1}{\gamma-1}}\right)^2 = \eta^{\frac{2}{\gamma-1}} - 2\nu^{\frac{1}{\gamma-1}} \eta^{\frac{1}{\gamma-1}} + \nu^{\frac{2}{\gamma-1}}$$

Equation (3) is then equivalent to:

$$
\begin{aligned}
& -2(\gamma-1)^2 \eta\nu (\nu)^{\frac{1}{\gamma-1}} (\eta)^{\frac{1}{\gamma-1}} \\
& +\nu^{\frac{\gamma}{\gamma-1}} \eta^{\frac{1}{\gamma-1}} \nu + 2\gamma(\gamma-2)\nu\nu^{\frac{1}{\gamma-1}} \eta^{\frac{\gamma}{\gamma-1}} \\
& +\nu^{\frac{1}{\gamma-1}} \eta\eta^{\frac{\gamma}{\gamma-1}} - (\gamma-1)^2 \eta\nu \left(\eta^{\frac{1}{\gamma-1}} - \nu^{\frac{1}{\gamma-1}}\right)^2
\end{aligned}
$$

$$
\begin{aligned}
= \quad & 2(\gamma-1)\eta\nu (\nu)^{\frac{1}{\gamma-1}} (\eta)^{\frac{1}{\gamma-1}} + \nu^{\frac{\gamma}{\gamma-1}} \eta^{\frac{1}{\gamma-1}} \nu \\
& -2\gamma\nu\nu^{\frac{1}{\gamma-1}} \eta^{\frac{\gamma}{\gamma-1}} + \nu^{\frac{1}{\gamma-1}} \eta\eta^{\frac{\gamma}{\gamma-1}} - (\gamma-1)^2 \eta\nu \left(\eta^{\frac{1}{\gamma-1}} - \nu^{\frac{1}{\gamma-1}}\right)^2
\end{aligned}
$$

$$= -2\eta\nu (\nu)^{\frac{1}{\gamma-1}} (\eta)^{\frac{1}{\gamma-1}} + \nu^{\frac{\gamma}{\gamma-1}} \eta^{\frac{1}{\gamma-1}} \nu + \nu^{\frac{1}{\gamma-1}} \eta\eta^{\frac{\gamma}{\gamma-1}} - (\gamma-1)^2 \eta\nu \left(\eta^{\frac{1}{\gamma-1}} - \nu^{\frac{1}{\gamma-1}}\right)^2$$

$$= \eta^{\frac{1}{\gamma-1}} \nu^{\frac{\gamma}{\gamma-1}} (\nu - \eta) + \eta^{\frac{\gamma}{\gamma-1}} \nu^{\frac{1}{\gamma-1}} (\eta - \nu) - (\gamma-1)^2 \eta\nu \left(\eta^{\frac{1}{\gamma-1}} - \nu^{\frac{1}{\gamma-1}}\right)^2$$

$$= \nu^{\frac{1}{\gamma-1}}\eta^{\frac{1}{\gamma-1}}\left(\eta-\nu\right)^2 - \left(\gamma-1\right)^2\eta\nu\left(\eta^{\frac{1}{\gamma-1}}-\nu^{\frac{1}{\gamma-1}}\right)^2 \tag{4}$$

Equation (4) shows that Ω is positive if and only if

$$\nu^{\frac{1}{\gamma-1}}\eta^{\frac{1}{\gamma-1}}\left(\eta-\nu\right)^2 > \left(\gamma-1\right)^2\eta\nu\left(\eta^{\frac{1}{\gamma-1}}-\nu^{\frac{1}{\gamma-1}}\right)^2$$

Taking logs of this equation we obtain:

$$\frac{1}{\gamma-1}\ln\nu+\frac{1}{\gamma-1}\ln\eta+2\ln\left(\eta-\nu\right) > 2\ln\left(\gamma-1\right)+\ln\eta+\ln\nu+2\ln\left(\eta^{\frac{1}{\gamma-1}}-\nu^{\frac{1}{\gamma-1}}\right)$$

$$\Leftrightarrow \frac{2-\gamma}{\gamma-1}\ln\nu+\frac{2-\gamma}{\gamma-1}\ln\eta+2\ln\nu > 2\ln\left(\gamma-1\right)+2\ln\left(\eta^{\frac{1}{\gamma-1}}-\nu^{\frac{1}{\gamma-1}}\right)$$

$$\Leftrightarrow \frac{2-\gamma}{\gamma-1}\ln\nu+\frac{2-\gamma}{\gamma-1}\ln\eta-2\ln\left(\gamma-1\right)+2\ln\left(\eta-\nu\right)-2\ln\left(\eta^{\frac{1}{\gamma-1}}-\nu^{\frac{1}{\gamma-1}}\right) > 0$$

$$\Leftrightarrow \frac{2-\gamma}{\gamma-1}\ln\nu+\frac{2-\gamma}{\gamma-1}\ln\eta-2\ln\left(\gamma-1\right)+2\ln\left(\frac{\left(\eta-\nu\right)}{\eta^{\frac{1}{\gamma-1}}-\nu^{\frac{1}{\gamma-1}}}\right) > 0$$

$$\Leftrightarrow \frac{2-\gamma}{\gamma-1}\ln\left(\nu\eta\right)+2\ln\left(\frac{\left(\eta-\nu\right)}{\left(\gamma-1\right)\left(\eta^{\frac{1}{\gamma-1}}-\nu^{\frac{1}{\gamma-1}}\right)}\right) > 0$$

$$\Leftrightarrow \frac{2-\gamma}{\gamma-1}\ln(\nu\eta) > -2\ln\frac{(\eta-\nu)}{(\gamma-1)\left(\eta^{\frac{1}{\gamma-1}}-\nu^{\frac{1}{\gamma-1}}\right)}$$

$$\Leftrightarrow (\nu\eta)^{-\frac{2-\gamma}{2(\gamma-1)}} < \frac{(\eta-\nu)}{(\gamma-1)\left(\eta^{\frac{1}{\gamma-1}}-\nu^{\frac{1}{\gamma-1}}\right)}$$

$$\Leftrightarrow \left(\eta^{\frac{1}{\gamma-1}}-\nu^{\frac{1}{\gamma-1}}\right)(\nu\eta)^{-\frac{2-\gamma}{2(\gamma-1)}} < \frac{1}{(\gamma-1)}\eta-\frac{1}{(\gamma-1)}\nu$$

$$\Leftrightarrow \nu\left(\frac{1}{(\gamma-1)}-\nu^{\frac{2-\gamma}{\gamma-1}}(\nu\eta)^{-\frac{2-\gamma}{2(\gamma-1)}}\right) < \eta\left(\frac{1}{(\gamma-1)}-\eta^{\frac{2-\gamma}{\gamma-1}}(\nu\eta)^{-\frac{2-\gamma}{2(\gamma-1)}}\right) \qquad (5)$$

$$\Leftrightarrow f(\nu) < f(\eta)$$

where $f(x) = x\left(\frac{1}{(\gamma-1)}-x^{\frac{2-\gamma}{\gamma-1}}(\nu\eta)^{-\frac{2-\gamma}{2(\gamma-1)}}\right)$. Therefore $\Omega > 0$ if and only if $f(\nu) < f(\eta)$ and

$$\frac{\partial\underline{\delta}^h}{\partial\mu} \overset{s}{=} \frac{1}{2}(\theta_l-\theta_h)\Xi = \frac{1}{2}(\theta_l-\theta_h)\frac{1}{(\gamma-1)}\nu^{-1}\Omega = \frac{1}{2}(\theta_l-\theta_h)\frac{1}{(\gamma-1)}\nu^{-1}[f(\eta)-f(\nu)]$$

Proof of Lemma 2.

$$\frac{\partial\underline{\delta}^h}{\partial\theta_h} \overset{s}{=} \psi\gamma\left[\eta^{\frac{1}{\gamma-1}}-\nu^{\frac{1}{\gamma-1}}\right]\left[-\nu\gamma\frac{1}{\gamma-1}\frac{1}{\gamma}\eta^{\frac{1}{\gamma-1}-1}+\frac{\gamma}{\gamma-1}\frac{1}{\gamma}\eta^{\frac{\gamma}{\gamma-1}-1}\right]$$
$$-\left[\nu\gamma\nu^{\frac{1}{\gamma-1}}-\nu^{\frac{\gamma}{\gamma-1}}-\nu\gamma\eta^{\frac{1}{\gamma-1}}+\eta^{\frac{\gamma}{\gamma-1}}\right]\left[\eta^{\frac{1}{\gamma-1}}-\nu^{\frac{1}{\gamma-1}}+\psi\gamma\frac{1}{\gamma-1}\frac{1}{\gamma}\eta^{\frac{1}{\gamma-1}-1}\right]$$

$$
\begin{aligned}
= \; & \frac{\gamma\psi}{\gamma-1}\left[\eta^{\frac{1}{\gamma-1}}-\nu^{\frac{1}{\gamma-1}}\right]\left[\eta^{\frac{1}{\gamma-1}}-\nu\eta^{\frac{2-\gamma}{\gamma-1}}\right] \\
& -\left[(\gamma-1)\,\nu^{\frac{\gamma}{\gamma-1}}-\gamma\nu\eta^{\frac{1}{\gamma-1}}+\eta^{\frac{\gamma}{\gamma-1}}\right]\left\{\eta^{\frac{1}{\gamma-1}}-\nu^{\frac{1}{\gamma-1}}+\psi\frac{1}{\gamma-1}\eta^{\frac{2-\gamma}{\gamma-1}}\right\}
\end{aligned}
$$

$$
\begin{aligned}
= \; & \left[\eta^{\frac{1}{\gamma-1}}-\nu^{\frac{1}{\gamma-1}}\right]\left\{\frac{\gamma\psi}{\gamma-1}\eta^{\frac{1}{\gamma-1}}-\frac{\gamma\psi}{\gamma-1}\nu\eta^{\frac{2-\gamma}{\gamma-1}}-(\gamma-1)\,\nu^{\frac{\gamma}{\gamma-1}}+\gamma\nu\eta^{\frac{1}{\gamma-1}}-\eta^{\frac{\gamma}{\gamma-1}}\right\} \\
& -\psi\frac{1}{\gamma-1}\eta^{\frac{2-\gamma}{\gamma-1}}\left[(\gamma-1)\,\nu^{\frac{\gamma}{\gamma-1}}-\gamma\nu\eta^{\frac{1}{\gamma-1}}+\eta^{\frac{\gamma}{\gamma-1}}\right]
\end{aligned}
$$

$$
\begin{aligned}
= \; & \left[\eta^{\frac{1}{\gamma-1}}-\nu^{\frac{1}{\gamma-1}}\right]\left\{\frac{\gamma\psi}{\gamma-1}\eta^{\frac{1}{\gamma-1}}-\frac{\gamma\psi}{\gamma-1}\nu\eta^{\frac{2-\gamma}{\gamma-1}}-(\gamma-1)\,\nu^{\frac{\gamma}{\gamma-1}}+\gamma\nu\eta^{\frac{1}{\gamma-1}}-\eta\eta^{\frac{1}{\gamma-1}}\right\} \\
& -\psi\frac{1}{\gamma-1}\eta^{\frac{2-\gamma}{\gamma-1}}\left[(\gamma-1)\,\nu^{\frac{\gamma}{\gamma-1}}-\gamma\nu\eta^{\frac{1}{\gamma-1}}+\eta\eta^{\frac{1}{\gamma-1}}\right]
\end{aligned}
$$

$$
\begin{aligned}
= \; & \left[\eta^{\frac{1}{\gamma-1}}-\nu^{\frac{1}{\gamma-1}}\right]\left\{\frac{\psi}{(\gamma-1)}\eta^{\frac{1}{\gamma-1}}-\frac{\gamma\psi}{\gamma-1}\nu\eta^{\frac{2-\gamma}{\gamma-1}}-(\gamma-1)\,\nu\nu^{\frac{1}{\gamma-1}}+(\gamma-1)\,\nu\eta^{\frac{1}{\gamma-1}}\right\} \\
& -\psi\eta^{\frac{2-\gamma}{\gamma-1}}\nu\left[\nu^{\frac{1}{\gamma-1}}-\eta^{\frac{1}{\gamma-1}}\right]-\frac{1}{\gamma-1}\eta^{\frac{2-\gamma}{\gamma-1}}\psi^2\eta^{\frac{1}{\gamma-1}}
\end{aligned}
$$

$$
\begin{aligned}
= \; & \left[\eta^{\frac{1}{\gamma-1}}-\nu^{\frac{1}{\gamma-1}}\right]\left\{\frac{\psi}{(\gamma-1)}\eta^{\frac{1}{\gamma-1}}-\psi\eta^{\frac{2-\gamma}{\gamma-1}}\nu\frac{1}{(\gamma-1)}-(\gamma-1)\,\nu\nu^{\frac{1}{\gamma-1}}+(\gamma-1)\,\nu\eta^{\frac{1}{\gamma-1}}\right\} \\
& -\frac{1}{\gamma-1}\eta^{\frac{2-\gamma}{\gamma-1}}\psi^2\eta^{\frac{1}{\gamma-1}}
\end{aligned}
$$

$$
\begin{aligned}
&= \left[\eta^{\frac{1}{\gamma-1}} - \nu^{\frac{1}{\gamma-1}}\right] \left\{ \frac{\psi}{(\gamma-1)}\eta^{\frac{1}{\gamma-1}} - \psi\eta^{\frac{2-\gamma}{\gamma-1}}\nu\frac{1}{(\gamma-1)} + (\gamma-1)\,\nu \left[\eta^{\frac{1}{\gamma-1}} - \nu^{\frac{1}{\gamma-1}}\right] \right\} \\
&\quad -\frac{1}{\gamma-1}\eta^{\frac{2-\gamma}{\gamma-1}}\psi^2\eta^{\frac{1}{\gamma-1}}
\end{aligned}
$$

$$
\begin{aligned}
&= \left[\eta^{\frac{1}{\gamma-1}} - \nu^{\frac{1}{\gamma-1}}\right] \\
&\quad * \left\{ \frac{\psi}{(\gamma-1)}\eta^{\frac{1}{\gamma-1}} - \psi\eta\eta^{\frac{2-\gamma}{\gamma-1}}\frac{1}{(\gamma-1)} + \eta^{\frac{2-\gamma}{\gamma-1}}\psi^2\frac{1}{(\gamma-1)} + (\gamma-1)\,\nu \left[\eta^{\frac{1}{\gamma-1}} - \nu^{\frac{1}{\gamma-1}}\right] \right\} \\
&\quad -\frac{1}{\gamma-1}\eta^{\frac{2-\gamma}{\gamma-1}}\psi^2\eta^{\frac{1}{\gamma-1}}
\end{aligned}
$$

$$
\begin{aligned}
&= \left[\eta^{\frac{1}{\gamma-1}} - \nu^{\frac{1}{\gamma-1}}\right] \left\{ \frac{\psi}{(\gamma-1)}\eta^{\frac{1}{\gamma-1}} - \psi\eta^{\frac{1}{\gamma-1}}\frac{1}{(\gamma-1)} + (\gamma-1)\,\nu \left[\eta^{\frac{1}{\gamma-1}} - \nu^{\frac{1}{\gamma-1}}\right] \right\} \\
&\quad -\frac{1}{\gamma-1}\eta^{\frac{2-\gamma}{\gamma-1}}\psi^2\eta^{\frac{1}{\gamma-1}} + \eta^{\frac{2-\gamma}{\gamma-1}}\psi^2\frac{1}{(\gamma-1)}\left[\eta^{\frac{1}{\gamma-1}} - \nu^{\frac{1}{\gamma-1}}\right]
\end{aligned}
$$

$$
\begin{aligned}
&= (\gamma-1)\,\nu \left[\eta^{\frac{1}{\gamma-1}} - \nu^{\frac{1}{\gamma-1}}\right]^2 \\
&\quad -\frac{1}{\gamma-1}\eta^{\frac{2-\gamma}{\gamma-1}}\psi^2\eta^{\frac{1}{\gamma-1}} + \eta^{\frac{2-\gamma}{\gamma-1}}\psi^2\frac{1}{(\gamma-1)}\left[\eta^{\frac{1}{\gamma-1}} - \nu^{\frac{1}{\gamma-1}}\right]
\end{aligned}
$$

$$
= (\gamma-1)\,\nu \left[\eta^{\frac{1}{\gamma-1}} - \nu^{\frac{1}{\gamma-1}}\right]^2 - \frac{1}{\gamma-1}\psi^2\eta^{\frac{2-\gamma}{\gamma-1}}\nu^{\frac{1}{\gamma-1}} \tag{6}
$$

Substitute in the values of η, ν, ψ taking into account that $\mu = 1$. Furthermore substitute in $\theta_l = 1$ (Assumption 1). Equation (6) then implies that $\frac{\partial \delta^h}{\partial \theta_h} < 0$ if and only if

$$\frac{(\gamma-1)}{\gamma}\left[\left(\frac{1+\theta_h}{\gamma}\right)^{\frac{1}{\gamma-1}}-\left(\frac{1}{\gamma}\right)^{\frac{1}{\gamma-1}}\right]^2 < \frac{1}{\gamma-1}\left(\frac{\theta_h}{\gamma}\right)^2\left(\frac{1+\theta_h}{\gamma}\right)^{\frac{2-\gamma}{\gamma-1}}\left(\frac{1}{\gamma}\right)^{\frac{1}{\gamma-1}}$$

$$(\gamma-1)^2\left[\left(\frac{1+\theta_h}{\gamma}\right)^{\frac{1}{\gamma-1}}-\left(\frac{1}{\gamma}\right)^{\frac{1}{\gamma-1}}\right]^2 < (\theta_h)^2\left(\frac{1+\theta_h}{\gamma}\right)^{\frac{2-\gamma}{\gamma-1}}\left(\frac{1}{\gamma}\right)^{\frac{\gamma}{\gamma-1}}$$

$$(\gamma-1)^2\left\{\left(\frac{1}{\gamma}\right)^{\frac{1}{\gamma-1}}\left[(1+\theta_h)^{\frac{1}{\gamma-1}}-1\right]\right\}^2 < (\theta_h)^2\left(\frac{1+\theta_h}{\gamma}\right)^{\frac{2-\gamma}{\gamma-1}}\left(\frac{1}{\gamma}\right)^{\frac{\gamma}{\gamma-1}}$$

$$(\gamma-1)^2\left(\frac{1}{\gamma}\right)^{\frac{2}{\gamma-1}}\left[(1+\theta_h)^{\frac{1}{\gamma-1}}-1\right]^2 < (\theta_h)^2(1+\theta_h)^{\frac{2-\gamma}{\gamma-1}}\left(\frac{1}{\gamma}\right)^{\frac{2}{\gamma-1}}$$

$$(\gamma-1)^2\left[(1+\theta_h)^{\frac{1}{\gamma-1}}-1\right]^2 < (\theta_h)^2(1+\theta_h)^{\frac{2-\gamma}{\gamma-1}}$$

$$(\gamma-1)\left[(1+\theta_h)^{\frac{1}{\gamma-1}}-1\right] < (\theta_h)(1+\theta_h)^{\frac{2-\gamma}{2(\gamma-1)}}$$

$$(1+\theta_h)^{\frac{1}{\gamma-1}}-1 < \frac{\theta_h}{(\gamma-1)}(1+\theta_h)^{\frac{2-\gamma}{2(\gamma-1)}}$$

$$\frac{(1+\theta_h)^{\frac{1}{\gamma-1}}}{(1+\theta_h)^{\frac{2-\gamma}{2(\gamma-1)}}}-\frac{1}{(1+\theta_h)^{\frac{2-\gamma}{2(\gamma-1)}}} < \frac{\theta_h}{(\gamma-1)}$$

$$(1+\theta_h)^{\frac{\gamma}{2(\gamma-1)}} - (1+\theta_h)^{\frac{\gamma-2}{2(\gamma-1)}} < \frac{\theta_h}{(\gamma-1)}$$