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Centre for Market and Public Organisation
Bristol Institute of Public Affairs
University of Bristol
2 Priory Road
Bristol BS8 1TX
http://www.bris.ac.uk/Depts/CMPO/

Tel: (0117) 33 10799
Fax: (0117) 33 10705
E-mail: cmpo-office@bristol.ac.uk

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Paul A. Grout, In–Uck Park and Silvia Sonderegger

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An Economic Theory of the Glass Ceiling

Paul A. Grout, In-Uck Park
and
Silvia Sonderegger[1]

[1] *Department of Economics and CMPO,
University of Bristol,
Bristol, BS8 1TN (UK)*

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Abstract

The glass ceiling is one of the most controversial and emotive aspects of employment in organisations. This paper provides a model of the glass ceiling that exhibits the following features that are frequently thought to characterise the problem: (i) there is a lower number of female employees in higher positions, (ii) women have to work harder than men to obtain equivalent jobs, (iii) women are then paid less than men when promoted, and (iv) some organisations are more female friendly than others. These features emerge as an equilibrium phenomenon, when identical firms compete in "Bertrand-like" fashion. Furthermore, they also occur even when offering women the same contract as men in higher positions would be sufficient to ensure that women in those positions would always prefer permanent career over non-market alternatives.

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Address for Correspondence
CMPO, Bristol Institute of Public Affairs
University of Bristol
2 Priory Road
Bristol
BS8 1TX
P.A.Grout@bristol.ac.uk
I.Park@bristol.ac.uk
www.bris.ac.uk/Depts/CMPO/

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1. Introduction

The glass ceiling is one of the most controversial and emotive aspects of employment in organisations. The term appears to have originated only in the mid 1980s but become so rapidly sealed in the lexicon that by 1991 the US had created a Federal Glass Ceiling Commission with the Secretary of Labor as its chair. When setting up the Glass Ceiling Commission in 1991 the US Department of Labor defined the concept as "those artificial barriers based on attitudinal or organizational bias that prevent qualified individuals from advancing upward in their organizations into management-level positions"; these barriers reflect "discrimination... a deep line of demarcation between those who prosper and those who are left behind." One only has to look at the casual empirical evidence to see why the issue remains topical and heated.

Women form a disproportionately small group in senior management positions. For example, Figure 1 provides the proportion of females in employment amongst US professions and the proportion of female within those employees working as officials and managers (US Equal Employment Opportunity Commission). Women constitute just over half of all professions but little more than a third of all officials and managers. In the US Fortune 500 women account for only 15.6% of all corporate officer positions of any type. The situation is even more extreme in Europe (16.5% of positions on all corporate governing bodies in the US are held by women compared to 7.6% in Europe).[2] Amongst the largest 100 companies in Europe in 2006 there were no female chief financial officers in any company in Germany or France, and one in the UK. In the US women fair better but even here there are only nine in the largest 100 US companies. As CFO magazine points out 'the paucity of female finance chiefs in Europe is puzzling given the steady flow of women into finance over the past few decades'. Indeed, women account for more than half all university graduates in the EU, and around 30% of MBAs and chartered accountants.

Although some of the disparity could be determined by occupational choice, women feel that they have to work harder to get promoted and that lower value is attached to their effort because of their gender. Again there is much anecdotal evidence. For example, in a recent survey 44% of current business owners women reported that the statement 'your contributions were not being recognized or valued' fitted their personal experience either well or very well, compared to 17% of men. Ferree and Purkayaastha report women's experience 'that you have to be twice as good to get half as far at higher levels of management' (Ferree and Purkayaastha (2000)).[3] Heilman (2001) suggests 'when women perform valuable work this may go

[2] Figures for 100 largest US corporations relative to 100 largest European companies.

[3] Women working on Wall Street as brokerage firm equity analysts are more likely to be highly rated by external assessments (e.g., designated as All-Stars by the Institutional Investor magazine) than their male counterparts yet on average within their firm they are allocated fewer stocks to cover

unacknowledged as their achievements are more likely to be attributed to external factors'. Interestingly, there also appear to be significant but complex differences between organisations within sector in terms of female employment. Obviously there is bound to be a variance but, more significantly, these differences appear to persist. For example, Cohen et al (1998) show that within the US savings and loans industry, savings and loans employing larger proportions of females are more likely to hire and promote women. Furthermore they show that this effect is statistically significantly stronger for promotion than for hiring. That it appears that, within the same sector, some firms are more female friendly than others.

Furthermore, not only do women form a minority of employees at senior levels, they also receive lower remuneration than men. This disparity is reflected throughout senior management. Figure 2 shows the relative salary of educated women (according to highest education attainment) to equivalent educated men in the US from 1990 to date. The ratio for both women with bachelors degree and those with an advanced qualification (i.e., higher than bachelors) are relatively constant and very similar, with mean values below 0.6 across the period.[4] Although over 15% of all corporate officers of Fortune 500 companies are female only 2.7% of their top earners are women.[5]

There is, of course, considerable theoretical and empirical debate as to what the glass ceiling actually is. The Federal Glass Ceiling Commission describes it as a barrier to obtaining management-level positions but many think of it as a barrier to obtaining the better management positions. Morrison et al (1987), one of the early authoritative sources on the topic, describe it as 'a transparent barrier that kept women from rising above a certain level in corporations ... It applies to women as a group who are kept from advancing higher because they are women'. Some authors argue that within organisations the glass ceiling is 'quite low'.[6] For example, Reskin and Padavic (1994) suggest that the glass ceiling and the 'sticky floor' are indistinguishable. We side-step this debate and assume that the glass ceiling refers to the situation where, amongst the group of employees that are hired to fill management slots within an organisation, a disproportionately small group of women get promoted to the upper part of the management chain and receive lower remuneration once there.

Although economists have not theorized on the phenomena, there is no shortage of explanations in other fields. One explanation, of course, is that there is pure gender

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than their male equivalent. Interestingly woman form a declining proportion of Wall Street brokerage firm equity analysts positions (falling from 16% in 1995 to 13% in 2005) but the employment evidence in overall appears to be more consistent with some form of self-selection than gender discrimination (Green et al (2007).

[4] Several papers confirm empirically the existence of a 'glass ceiling' in earnings (e.g., Albrecht et al (2001), de la Rica et al (2005) and Arulampalam et al (2004)).

[5] 2.7% of the 2,500 top earners (made up of the top five for each of the Fortune 500 companies).

[6] Britton and Williams (2000).

discrimination and there is evidence of pure discrimination in some arenas (see, for example, Goldin and Rouse (2000) and Neumark (1996)).

In contrast, strands of the evolutionary psychology literature argue a difference in genetic predisposition is responsible for the appearance of a ceiling on women's earnings and positions in management hierarchy. For example, Babcock and Laschever (2003) argue that women are poor negotiators and generally dislike the process of negotiating. Browne (1995, 1998) suggests that men are more interested in striving for status in hierarchies and 'engage in risk taking behaviour that is often necessary to reach the top of hierarchies'. Kanazawa (2005) uses General Social Survey data to show that men rank financial reward and power positions much higher in their preferences for employment, concluding that since men covet and strive for such positions they are the ones likely to succeed in achieving them, whereas "women have better things to do". Niederle and Vesterlund (2007) use experiments where subjects perform a task and are able to choose between non-competitive piece rate and a competitive tournament. There is a significant gender gap in choice with 35% of women and 73% of men selected the tournament. Gneezy et al (2003) also test performance in competitive environments and find within their sample that the bottom performance quintile is almost entirely composed of women. Indeed, some of the literature argues that this may not be true but the perception that it is limits the opportunities for women to be offered positions that provide equal opportunities for advancement (White (1992)).

Psychologists find gender effects in what they call the "romance of leadership". Essentially, the idea is that the outcome of good events is incorrectly attributed ex-post to good leadership rather than random draws. This phenomenon is referred to as the romance of leadership. In experiments where subjects are asked to allocate bonuses in hypothetical situations Kulich et al (2007)) show that, while the romance of leadership is present for both sexes, higher company performance does not lead to allocation of higher bonus for women. Indeed, women only receive bonuses based on perceptions of their charisma.

In contrast, to the voluminous theorizing in other social sciences, economists have not provided a formal model of the glass ceiling phenomenon as such that is consistent with the features outlined above. There is no economic model explaining whether, and if so how, a glass ceiling can arise as an equilibrium in the workplace if there is no 'discrimination' against women.[7] The gender specific features described above are inevitably the outcome of many factors but an interesting research exercise is to see if they can be explained as the outcome of a competitive process. One attraction

[7] There is, of course, a significant theoretical literature on statistical discrimination (stemming from the seminal papers of Phelps (1972) and Arrow (1973)) but this does not 'explain' several of the features of the glass ceiling.

of seeking an equilibrium explanation is that it helps to understand the role and limitations of "anti-discrimination" policies. The purpose of this paper is to provide such an equilibrium economic theory of the glass ceiling. We are able to show that, although it clearly disadvantages career focused women, we can characterise all these features in equilibrium as the natural outcome of a competitive process. To what extent these features, therefore constitute discrimination is not obvious. In our model there is a group of women who are clearly disadvantaged in many dimensions, relative to the equivalent male, but this is the consequence of employees engaging in Bertrand competition rather than exhibiting discrimination in the traditional manner.

We suggest the factors that an attractive theoretical model of the glass ceiling should seek to explain are the following observations: (i) the lower number of female employees in higher positions, (ii) the fact that women feel that they have to work harder than men to obtain what appear to be equivalent jobs, (iii) on average women are then paid less than men when promoted, and (iv) some organisations appear to more female friendly than others (that otherwise appear to be identical).[8]

These four conditions are not trivial to reconcile. For example, if women have to work harder to achieve the same level of promotion then, if this has an effect on future productivity, it should increase it. Hence on average those women who make the higher level should be more attractive to employers than the typical male employee and are likely to receive higher not lower remuneration. Indeed this point has been made very clearly by Fryer (2006) who explores what happens to individuals who are successful within a statistical discrimination model. He identifies conditions for belief flipping, whereby groups that are statistically discriminated against at one level actually become positively favored at the upper level. Indeed in general if forces with similar effects to discrimination restrict promotion then those promoted should be more valuable and paid more. The model we present has no discrimination against women but displays all the four features described above in equilibrium.

The model depends on gender differences and, although having a different focus and outcome to the evolutionary psychology literature mentioned above, it has some similarities with some of the existing literature. The closest model to ours is that of Lazear and Rosen (1990). They assume that women have, on average, higher non-market options than men and are more likely to leave employment as a result. Consequently, in equilibrium the average ability of promoted women must be higher than men. The implication is that a smaller number of women are promoted but as suggested above, because of the difference in ability of those promoted relative to

[8]Note that we are not claiming that these four features are all obviously unambiguously empirically 'true'. The lower number of females in senior positions is an undisputable fact but whether the others are true depends on the conditioning factors. Our aim is to see if a plausible economic theory can explain these phenomena.

their male counterparts, women that are promoted earn more on average than men. In contrast, in our theory women may earn less than male counterparts. The primary similarity between our model and that of Lazear and Rosen is that we both assume that women have non-market options that men do not have. However, in our approach the relative value of non-market options is endogenous. The endogeneity plays a major role in the story since manipulation of the value of employment compared to the non-market options offers a pre-commitment strategy for those women who are less inclined to favor the non-market options.

A particular attraction of our model is that, although at times the results are not trivial to establish formally, the core forces that drive the equilibrium effects have a simple intuition. Here we provide a brief summary of the driving forces. Central to our story is the idea that employers believe (correctly) that on average a female employee is more inclined to leave or take a break in employment than an otherwise identical male employee.[9] At one extreme this can take the form of short employment breaks for each physical birth and the immediate consequent childcare (easily observable and contractible) and at the other extreme could take the form of reducing their "commitment" to the job and restricting the sacrifices they are willing to make as their career develops. The latter is far harder to measure and generally not contractible. Preferences over lifestyles differ, however, and so women themselves will differ significantly in the extent that the possibility of taking leave or a break in employment is a real issue for them. But these differences in preferences will be private information. Of course, this feature is not unique to differences rooted in gender. It can apply to a broad class of cases (we discuss others below) but we believe it is particularly relevant for a discussion of the glass ceiling phenomena.

A critical assumption is that there is more diversity in women with regard to this aspect of job commitment than men, who as a result have less potential for private information in this regard. Indeed, to keep things simple and to focus on the main issue, we assume in our model that men all display the same "commitment" to career but that women differ with regard to this. Women learn their preference but, as indicated, this is private information. One way of thinking about our model is that women are more productive than men in that they can do equivalent work in the workplace but also have alternative options that are not available for men. However, this very potential flexibility turns out to be harmful.

In this framework a woman whose personal commitment is, say, identical to the male group faces a clear problem that an otherwise equivalent male employee does not face. Such women 'lose' in two intertwined ways in our model.

[9] The CFO (Chief Finance Officer) Magazine (8th March 2007) claimed that... 'around 40% of women step off the career ladder at some point, most often driven by 'pull' factors such as having children or caring for a parent, rather than 'push' factors, such as career-related stress'.

First, since the (potential) alternative options have no value to her, she may value a pre-commitment strategy that reduces the attraction of the alternatives, providing this is observable. This is something that men do not have to do. For example, by working harder a woman may be able to signal her commitment. That is, working 'abnormally' hard may have a significant impact on promotion and financial returns if it is clear that working so hard changes the ex-post returns in such a way that leaving for alternative options is now sub-optimal. Such a strategy can sort employees by type.

The second way that women can lose is that firms will only offer as much as they need to meet competitor's offers. So there is no guarantee that these women obtain the equivalent return from this extra effort that a male employee would earn.

The formal model and the main results are loosely as follows. There are two technologically identical firms competing for employees.[10] Employees are hired at a low management level and each firm has need for one senior management slot for every two lower level management positions. The higher level managers are chosen from the intake at the lower level in the previous period. The amount of effort that employees put in at the lower level affects their productivity in a higher level but has no impact on their output in the second period if they are not promoted. It is this amount of work at the initial stage of the career that can act as a pre-commitment device. The firms compete (basically Bertrand-style) at an initial contract stage by offering contracts to the market. We analyze what happens when contracts can be made gender specific with a view to understanding the impact on the equilibrium when various possible legal anti-discrimination restrictions are placed on the market. Employees approach the firm that offers the best contract for them and then take up the other offer if they are not chosen by that firm. A contract offers promotion conditional on a specific amount of effort in the first period. This effort level may (explicitly or implicitly) be gender specific.

We assume that women differ in their preference for a career break (which is costly to employers). Whether this option is worthwhile to pursue, however, is endogenous to the model. Harder work increases productivity and in equilibrium affects the salary at the higher level, so this is an observable way that a woman can change ex-post commitment to the job. The equilibrium has two important features.

One is that firms differ in their proportion of female employees that are hired (even though firms make identical profit in equilibrium). The less "female friendly" firm is unattractive for the women who want good promotion opportunities. Because the 'female unfriendly' firm's offer is unattractive, it limits how much the "female friendly" firm needs to offer to attract female workers.

[10] We think of the two firm market being sustained by entry costs that limit the number of firms that can be accommodated in the market.

The second feature follows from the first. Since there is a limit on how much the "female friendly" firm needs to offer to attract female workers, a woman who is promoted has to work harder than an equivalent male (to ensure precommitment) and yet may receive a lower salary when promoted. Furthermore, this outcome is sustained even when the male promotion offer (i.e., a contract that offers guaranteed promotion for the male promotion-effort level) is also sufficient to provide full pre-commitment when offered to women. That is, if firms offered the male contract to women then all the women taking this contract would become identical in productivity terms to men (i.e., would never wish to leave the firm) but the equilibrium still provides a worse alternative for women.

What is the basic intuition why, even in the presence of Bertrand competition, women who work harder cannot receive full reward for this extra effort? Essentially a firm cannot employ only women since there will not be enough to fill the more senior jobs. That is, men are not completely 'priced' out of the firms hiring plans even when female workers have a 'lower price'. As a result of the need to employ some men, there will not be enough female jobs in the female friendly firm for all the women who would like them. Those that cannot find a job here end up in the female unfriendly firm, which offers a worse package. So the female friendly firm can itself extract a small surplus per female employee. However, both firms make the same profit. The female friendly firm has more women than the unfriendly one but pays them more. So the female friendly firm extracts a smaller surplus per female but has more of them whereas the female unfriendly firm has fewer female employees but extracts a higher surplus from each one. In equilibrium each firm has no incentive to change its offer.

Thus the model provides an explanation for the glass ceiling phenomenon that, although it disadvantages career focused women, is the natural outcome of a competitive process. We are able to show that this equilibrium does not depend on being able to offer gender specific contracts. However, if promotion rules (such as minimum promotion ratios for women) are imposed on the organization then the equilibrium can change to improve the position of those women who have a low preference for career breaks. A central feature of our approach that separates our conclusions from other discrimination models is that these restrictions cannot be temporary incursions into the labor market to shift the equilibrium to a more favorable one in a multi-equilibrium environment.[11] In our model the interventions would have to be "permanent" and in practice would require considerable information for the policy maker (although no more than is available to the agents in the model).

Before moving to the main body of the paper it is useful to discuss the potential

[11] Although many models have the feature that affirmative action only needs to be temporary, this is not always the case (see Coate and Loury (1993)).

applicability of the model. Although we have focused on gender we see the model as being relevant for many observationally distinguished groups. An immediate broad analogy is the treatment of immigrant employees (particularly where the culture of the country they have left is very different from the host country). If employers correctly believe that there is a higher probability the foreign employee may wish to return at some point then a similar pre-commitment problem arises for employees who have no preference for returning. An alternative anecdotal example (encountered by one of the authors) concerned a full time employee who enjoyed writing novels in his spare time. He claimed that he always felt disadvantaged relative to his colleagues since it was always implied that he was not as committed to the career as others because "surely he would really prefer to be a full time author" and was only waiting for the opportunity. He claimed this question arose to differing degrees of directness at every appraisal he ever had and that he thus felt obliged to display greater commitment to the cause than other employees.

In terms of the relevance of the model to the glass ceiling debate, the endogeneity/exogeneity of the gender differences needs some discussion. The paper relies on the primitive that women are more inclined than men to take some element of disruption in their career path. Where this is observable (e.g., formal breaks can be measured whereas lower willingness to make sacrifices may not be) the empirical evidence is clear (the extent differs between countries). However, although the empirical evidence vastly supports the disparity in short and long term disruption, one has to be careful in attributing all of this to an exogenous characteristic of genders. While disruption to physically have children and the immediate childcare can be treated as exogenous differences, one has to be careful when dealing with longer breaks. Some of this may be endogenous within the process, i.e., differences between genders in tendency to take breaks will be magnified by the equilibrium process, particularly if there are no legal restrictions on employer behavior.

The next section describes how we model the environment as a game. Section 3 provides a characterization of equilibrium of the game. Section 4 provides a discussion on the extent to which our analysis can be generalized and addresses policy implications. Section 5 contains some concluding remarks.

2. Model

We consider a labor market with two populations of workers, male (m) and female (f), of the same measure 2. All male workers are ex ante identical. Female workers differ in their attitude towards career vs family, so they differ in their likelihood to leave their jobs for some non-market activities in the future. We model this heterogeneity as follows: each female worker has a private type θ drawn from a commonly known cdf G on $[0, 1]$, where θ is the probability that she receives a positive shock while on the job (the precise timing to be specified later). If hit by a shock, the value of her outside, non-market option jumps to $\hat{u}$ from 0, where $\hat{u} > 0.5$ is a fixed value. For simplicity, we assume G is uniform.

There are two firms, A and B, that are ex ante identical: each firm has a measure 2 of low-rank positions to fill in period 1 and a measure 1 of high-rank/managerial positions to fill in period 2. All workers (male and female) have the same productivity in low(-rank) posts, which we normalize to 0. However, they can make a human capital investment/effort, $e \in \Re_+$ in period 1, that would increase their productivity in the next period if they get promoted to a high-rank position. The worker's cost of making effort is quadratic, $c(e) = \frac{1}{2}e^2$, and is sunk at this stage. The exerted effort level is observable only by the firm that he/she works for. Female workers learn their private types, θ, early in period 1, in particular, before the effort decision.

In period 2, workers may get promoted to high-rank posts. A promoted worker generates a total revenue of $y_H(e) = 1 + e$ for the firm that realizes at the end of period 2, where e is the effort exerted in period 1. However, at the midpoint of period 2, female workers are hit by a shock with their private probability θ, in which case they choose whether to remain in the job and get the contracted wage or leave the job forfeiting the wage for the second half of period two[12] and get $\hat{u}$, the utility from the newly available outside option. We assume that if a worker leaves a high-rank post no revenue is generated by that post for that period unless it is replaced by another worker who has held a high rank job in either firm, in which case the revenue would be the same as when the lower-productivity worker of the two has kept the post for the entire period.

All workers that do not get promoted enter a competitive, unskilled job market for period 2, where they generate a constant flow revenue of $1 + \kappa$ after retraining which costs $\kappa > 0$ for the employer. Hence, competitive employers pay a flow wage that leaves them zero expected profit: $w_m = 1$ for men and $w_f < 1$ for women. Here, $w_f < 1$ reflects the market's expected loss in revenue due to the departure prospect of female workers for the non-market option of $\hat{u}$ in the middle of period 2. We assume that κ is very small, so that w_f is close to 1. For ease of exposition, we treat w_f as exogenous, although it can be determined endogenously in equilibrium taking into account what types of women enter the unskilled market, without affecting the main results.

Prior to period 1, the two firms compete in attracting workers by offering more favorable (for the workers) contracts than their rivals, i.e., in the spirit of Bertrand

[12]In principle, the contract may specify that she will forfeit the salary for the entire period 2. The firm would still suffer some damage from such departures, which is an alternative interpretation of one half of the wage paid to the departing worker. There also can be some "retiring" payment and/or moral hazard prior to departures.

competition. We model this process in two stages.

In the first stage, called the "campaign" stage, the two firms publicly announce their terms of employment/contract offers during an interval I of time. Specifically, a "contract" $C_i^g = (e, s)$ specifies a gender $g \in \{m, f\}$, an effort level $e \geq 0$ (to be qualified for promotion), and a flow salary level $s \geq 0$ in period 2 (to be paid if promoted). Note that a contract does not specify the salary in period 1, which we assume is equal to the productivity, 0. This assumption is for ease of exposition, and our main results extend to the case that a contract specifies period 1 salary as well.

There are various ways of modelling the specifics of the firms' announcement strategies, for instance, whether they may announce only once or multiple times, whether they may announce only at designated discrete points or anytime in I (discrete vs. continuous time model), etc, and the results are sensitive to such details. Here, we specify three rules to capture the Bertrand competition between firms and leave other details unspecified because, as it turns out, these are adequate for our equilibrium characterization:

(A1) Both firms are required to have announced one contract for each gender by the end of I.

(A2) Once a contract $C_i^g = (e, s)$ is announced, firm i may revise it during I but only for the better for the worker, that is, may reduce e and/or increase s.[13]

(A3) There is a final point in time when either firm may revise/announce contracts, i.e., I is right-closed.

We require (A3) because collusive outcomes are easily supported by punishment threats without it. A strategy in the campaign stage specifies a plan to announce and revise contracts according to (A1)-(A3).

After the campaign stage is concluded with the final contracts $\{C_i^g\}$, $i = A, B$, and $g = f, m$, the workers get matched with the firms in an "allocation" stage. A precise modeling of this process, such as initial application and selection procedures and the second matching process of unfilled posts and residual labor supply, would necessarily involve nontrivial ad hoc assumptions. Hence, we take an alternative approach of directly postulating workforce allocation outcomes based on the fundamental principle that the firm offering a more favorable contract gets the first pick in hiring decisions. Here an allocation outcome refers to a specification of measures μ_i^g of gender $g \in \{m, f\}$ workers hired by firm $i \in \{A, B\}$, such that $\mu_i^m + \mu_i^f = 2$ for $i = A, B$, and $\mu_A^g + \mu_B^g = 2$ for $g = f, m$.

[13] Alternatively, the firms are allowed to offer additional contracts during I and the workers choose one of the contracts offered for its gender after hired.

For each female contract C_i^f one can calculate the ex ante (face) value that a worker of gender g can derive from this contract, presuming that the specified terms of promotion will always be honored. This value, denoted by $v(C_i^f)$, is formally defined later.[14] The firm offering a female contract with a higher value has priority in hiring women as specified below.

Valuation of male contracts is different due to the homogeneity of male workers. Since they can guarantee a utility of 1 by exerting no effort and getting an unskilled job in period 2 (i.e., forgoing promotion), the value of any male contract is at least 1. On the other hand, even if firm i offers a contract (e, s) with $s - e^2/2 > 1$, as long as this firm hires more than measure 1 of male workers they would compete for promotion and as a result, they end up exerting an effort level, say $\tilde{e} > e$, that restores equivalence of pursuing promotion and not, i.e., $s - \tilde{e}^2/2 = 1$. Whichever male contract a firm offers, therefore, it is at least as attractive as the other firm's if the other firm hires more than measure 1 of male. This means that either firm is at least as well placed as the other firm in hiring additional male workers up to measure 1.

Based on these observations, we postulate the labor force allocation as below[15]:

(B1) The firm, say A, offering a female contract with a strictly higher value first hires as many women as it wants, and as many men as it wants up to measure 1. Then, firm B fills all its posts with the residual labor force. Finally, firm A hires any remaining workers.

(B2) If the two firms offer female contracts of the same value, they are allocated measure 1 of each gender. Then, a firm may propose an alternative allocation, which is implemented if accepted by the other firm.

A strategy of each firm in an allocation stage with final contracts $\{C_i^g\}$ is a hiring decision as per the rule (B1) if $v(C_A^f) \neq v(C_B^f)$, and a decision as to which alternative allocation to propose and/or to accept as per (B2) if $v(C_A^f) = v(C_B^f)$.

After the contracts $\{C_i^g\}$ are offered and workforce is allocated as $\{\mu_i^g\}$, a "promotion" subgame ensues, comprising periods 1 and 2. During these periods firms

[14] Since it is defined without reference to equilibrium, $v(C_i^f)$ is not necessarily the equilibrium utility of a female worker hired by firm i, because promotion may not be pursued in the exact terms specified in the contract. However, this potential discrepancy is not significant enough to invalidate using $v(C_i^f)$ to assess the relative attractiveness of the two firms' female contracts. Alternatively, one can use equilibrium values of contracts in this assessment, however this would complicate the exposition without additional insights.

[15] This allocation would ensue if the actual matching process is, for instance, as follows: All workers apply to both firms (at no cost) and the firms have one chance of offering positions and the workers choose among the offered positions (randomly if equivalent offers), given that the firms have to fill all positions to operate.

cannot fire workers, however workers may leave the firm at any time, forgoing any unpaid flow salary. At the beginning of period 1 all female workers learn their private types θ, and every worker in either firm exerts an effort level $e \in \Re_+$ observable only by the employer. In period 2, each firm decides who to promote based on gender and exerted effort level, and pays them the salaries specified in the relevant contracts.[16] All unpromoted workers leave the firm and get an unskilled job that pays a flow wage of w_g.

A worker may sue the firm if he/she was not promoted after exerting an effort level at least that specified in the contract, together with a claim that some high posts are either vacant or occupied by workers who failed to meet their contracted effort levels. If the claim is verified to be true, then the court finds in favor of the plaintiff and a hefty compensation payment is ordered from the firm to the plaintiff. We assume a sufficiently high verifiability of effort levels so that such breach of contract would never take place.

At the midpoint of period 2, female workers get hit by a positive shock with their private probabilities θ, in which case they may leave the job and get the non-market option of $\hat{u}$. In case a positive measure of high posts becomes vacant due to such departure, the firms may try to fill vacant positions by recruiting workers in the high posts of the other firm with a higher salary. This would inevitably launch a recruiting war between the two firms, pushing the salaries of all high-post workers up to their productivities, thereby depleting any positive profit of the firm.

In a promotion subgame of firm i with contracts C_i^g and allocation μ_i^g, $g = m, f$, each worker's strategy consists of an effort level, decisions as to whether to sue the firm or not in relevant contingencies, and for female workers, the decisions as to whether to leave the employment for the non-market option of $\hat{u}$ when hit by a positive shock; and firm i's strategy specifies who to promote based on gender and exerted effort level (contingent on the profile of efforts exerted by all workers), and the recruiting decisions when some high posts are vacated in the middle of period 2. Each worker maximizes the expected value of income stream, including that from non-market option and compensation from lawsuit, net of the effort cost. Each firm maximizes the expected profit, i.e., total revenue net of total salary and lawsuit-compensation payments.

[16] That is, the firm commits to the salary specified in the contract for all promoted workers regardless of the effort exerted. This amounts to disallowing renegotiation at the point of promotion. Upward renegotiation would not happen even if allowed because the worker has no bargaining power given that the other firm does not know the exerted effort level. It is sensible that downward renegotiation is precluded for workers who met the contracted effort level because the contract would be upheld in the court. Even for workers who did not meet the contracted effort level, downward renegotiation would not be worth for the firm to pursue if verification of effort level is very costly and the worker is financially constrained.

A strategy profile in this promotion subgame, together with a belief profile on the type distribution of female workers contingent on the exerted effort level, constitutes a *(perfect Bayesian) continuation equilibrium* of this subgame if the strategies are mutual best-responses and the belief profile satisfies Bayes rule whenever possible.

A strategy profile of the two firms in the campaign stage, a strategy profile of the two firms in the allocation stage for each possible set of final contracts $\{C_i^g\}$, and a profile of continuation equilibria for every possible promotion subgame, constitute a *(subgame-perfect) equilibrium* of the grand game if, given the profile of continuation equilibria, i) the strategy profile in the allocation stage contingent on $\{C_i^g\}$ is an equilibrium in the continuation game, and ii) the strategies in the campaign stage are mutual best-responses given the rest of strategies.

An equilibrium outcome of the game consists of a (possibly stochastic) final contracts $\{C_i^g\}$, allocations $\{\mu_i^g\}$ contingent on $\{C_i^g\}$, and the ensuing effort profile of the workers and the promotion decisions, that arise in an equilibrium. We now characterize the equilibrium outcome.

3. Equilibrium Characterization

If a firm hires female workers for a contract (e, s) with $s < 2\hat{u}$, then the female worker in high posts would leave employment if a non-market option of value $\hat{u}$ becomes available at the midpoint of period 2, because the value of the income stream they forgo by leaving at that point is $s/2 < \hat{u}$. On the other hand, female workers promoted according to a contract (e, s) with $s \geq 2\hat{u}$ would never leave the employment. We describe the first kind of female promotion as "insecure," and the latter kind "secure".

Here, we take a view that the nature of positive shocks is such that when a non-market option becomes available after insecure promotion, departure is irreversible and renegotiation of salary is irrelevant at that point. Contract offers for secure promotion can be interpreted as preempting any such shocks by ensuring an income stream higher than any shock may bring.

If a firm hires female workers for insecure promotion, then it is certain that a positive measure of high posts will be vacated in the middle of period 2. As mentioned earlier, this would inevitably launch a recruiting war between two firms, wiping out all of firms' surplus from high posts. Foreseeing this, neither firm would hire for insecure promotion in equilibrium. We take this as granted and no longer consider insecure promotion,[17] i.e., we implicitly assume that all female contracts (e, s) considered below satisfy $s \geq 2\hat{u}$.

As will be verified shortly, the efficient male contract to be offered is $(e_m, s_m) = (1, 1.5)$, so that male workers get promoted by exerting an effort level of 1. If $\hat{u} =$

[17] The potential of recruiting war facilitates our analysis by providing a simple logic to eliminate insecure promotion as being a dominated strategy. However, it is not essential for our result because insecure promotion may be dominated without it.

3/4, the minimal female salary for secure promotion is also 1.5. By offering the efficient male contract to women as well, therefore, a firm can ensure that female workers up for promotion are identical to men: they generate the same revenue of 2 without any risk of departure and will be paid the same salary of 1.5. If the firms choose to offer a different female contract nonetheless, it cannot be explained by the general observation that male and female workers respond differently to the same incentives. For this reason, we are particularly interested in the cases that $\hat{u}$ is near 3/4. Therefore, we will present the analysis assuming that $\hat{u} = 3/4$, and discuss later that the main results and insights do not change in nearby cases, although the nominal high-cost compensation is lower for female workers if $\hat{u} < 3/4$ but is higher if $\hat{u} > 3/4$.

A. Employing male workers

Since men are guaranteed a utility of 1 by forgoing promotion, any male contract (e_m, s_m) that induces promotion should satisfy $s_m - e_m^2/2 \geq 1$. Subject to this constraint, the firm's surplus per promotion, $1 + e_m - s_m$, is uniquely maximized at $e_m = 1$ and $s_m = 1.5$, with the maximum surplus of 0.5.

Lemma 1: *The maximum surplus of a firm per male promotion is 0.5, which is uniquely obtained by the "efficient" male contract* $C_m^* = (1, 1.5)$.

B. Employing female workers

If a woman forgoes promotion, she would exert no effort, get an unskilled job for a wage of $w_f < 1$, and would leave the employment for a non-market option of $\hat{u}$ at midpoint with probability θ, which warrants an expected utility of

$$u_\ell(\theta) := w_f + \theta(\hat{u} - w_f/2). \tag{1}$$

On the other hand, if she pursues promotion as per a contract (e, s), she would get a utility of $s - e^2/2$ regardless of θ. Since the utility of non-promotion $u_\ell(\theta)$ increases in θ, there is a unique critical/threshold type that solves $u_\ell(\theta^c) = s - e^2/2$, so that female workers would pursue promotion if of a type lower than this threshold type, and not otherwise. Therefore, it proves useful to think of a secure promotion contract (e, s), i.e., with $e \geq 2\hat{u}$, in relation to the associated threshold type:

$$\theta^c(e, s) := u_\ell^{-1}(s - e^2/2).$$

Given a secure promotion contract (e, s) with a threshold type $\theta^c = \theta^c(e, s)$, a female worker would get a utility of $u_\ell(\theta^c)$ by pursuing promotion if of a type $\theta < \theta^c(e, s)$, or else would get a utility of $u_\ell(\theta)$ by not pursuing promotion. Hence, the ex ante value of this contract is defined as

$$v(e, s) := \theta^c u_\ell(\theta^c) + \int_{\theta^c}^{1} \frac{u_\ell(\theta)}{1 - \theta^c} d\theta \quad \text{where} \quad \theta^c = \theta^c(e, s). \tag{2}$$

Note that this values only depends on the threshold type and strictly increases in θ^c. This is the value of contracts used in allocation rules (B1) and (B2).

Given a threshold type $\theta \in (0,1)$, of all secure promotion contracts (e,s) such that $\theta^c(e,s) = \theta$, the contract that maximizes the per-promotion surplus of the firm, $1+e-s$, is

$$(e,s) = (\sqrt{2(1.5 - u_\ell(\theta))},\ 1.5) \quad \text{if} \quad u_\ell(\theta) \leq 1$$

and

$$(e,s) = (1,\ u_\ell(\theta) + 0.5) \quad \text{if} \quad u_\ell(\theta) \geq 1.$$

Consequently, the maximum surplus per-promotion is

$$\sqrt{2(1.5 - u_\ell(\theta))} - 0.5 \quad \text{if} \quad u_\ell(\theta) \leq 1, \quad \text{and} \quad 1.5 - u_\ell(\theta) \quad \text{if} \quad u_\ell(\theta) \geq 1.$$

Let $\bar{\theta}$ be such that $u_\ell(\bar{\theta}) = 1$, that is,

$$\bar{\theta} := \frac{2\hat{u} - 0.5 - w_f}{\hat{u} - 0.5 w_f} \in (0, 0.5).$$

Lemma 2: *Fix arbitrary $\theta \in (0, \bar{\theta})$. Conditional on inducing female promotion with a threshold type θ, the maximum per-promotion surplus is*

$$\pi_f(\theta) := \sqrt{2(1.5 - u_\ell(\theta))} - 0.5 > 0.5.$$

The unique contract that achieves this maximum is

$$(e_f(\theta), s_f) := (\sqrt{2(1.5 - u_\ell(\theta))},\ 1.5).$$

Note that $\pi_f(\theta)$ strictly decreases in θ down to $\pi_f(\bar{\theta}) = 0.5$.

As long as female promotion is induced with a threshold type $\theta \in (\bar{\theta}, 1)$, the per-promotion surplus is less than 0.5.

A graphical illustration might be useful. Consider $\theta \in (0, \bar{\theta})$ depicted in the first diagram of Figure 3. The part of the graph of $u_\ell(\theta) + c(e)$ in the second diagram above the horizontal line at level 1.5, is the set of all contracts (e,s) that would induce secure female promotion with the threshold type θ. Of these the one that maximizes the surplus, $1+e-s$, is depicted by the intersection point of this graph and the horizontal line at level 1.5. The horizontal coordinate of this point is $e_f(\theta)$ and the vertical distance from this point to the line $1+e$ is $\pi_f(\theta)$ which decreases as θ increases from 0 until $\pi_f(\bar{\theta}) = 0.5$.

[Figure 3 about here]

C. Optimal hiring with unconstrained workforce

Suppose there is unconstrained labor force for firm i, that is, firm i has enough labor force of either gender at its disposal so that it can fill its measure 2 of entry-level positions in any gender mix of its choice as long as the participation constraint is satisfied. This would be the case, for instance, if firm i is the only potential employer.

Recall from Lemma 1 that this firm can extract the maximum possible per-promotion surplus of 0.5 from men by offering $C_m^* = (1, 1.5)$. Since the firm can fill all upper positions with men by initially hiring measure 2 of male with this contract,[18] the firm may benefit by promoting women only if per-promotion surplus exceeds 0.5 for women. According to Lemma 2, this is feasible by inducing female promotion with a threshold type in $(0, \bar{\theta})$.

If this firm hires measure $\mu_f \in [0, 2]$ of women at a contract $(e_f(\theta), s_f)$ with a threshold type $\theta \in (0, \bar{\theta})$, then measure $\mu_f \theta$ of women would be promoted. Note $\mu_f \theta < 1$ because $\theta < \bar{\theta} < 0.5$. So, the firm would fill the remaining high posts with men at the efficient contract $C_m^* = (1, 1.5)$. If there are enough men to fill these positions, i.e., $1 - \mu_f \theta < 2 - \mu_f$, the firm's total profit, $\mu_f \theta \pi_f(\theta) + (1 - \mu_f \theta) 0.5$, increases in μ_f. Otherwise, i.e., if $1 - \mu_f \theta > 2 - \mu_f$, then all men pursue promotion yet some high posts are unfilled and the firm's total profit is $\mu_f \theta \pi_f(\theta) + (2 - \mu_f) 0.5$. Since $\sqrt{2(1.5 - w_f - \theta(\hat{u} - w_f/2))} < \sqrt{2}$ so long as $w_f \geq 0.5$, from the definition of $\pi_f(\theta)$ in Lemma 2 – and from $\theta < \bar{\theta} < 0.5$ – it follows that $\theta \pi_f(\theta) < 0.5$ and, therefore, the firm's profit decreases in μ_f in this contingency. Hence, the firm's total profit is maximized when measure μ_f of female workers are hired where μ_f solves $1 - \mu_f \theta = 2 - \mu_f$, as summarized in the next lemma.

Lemma 3: *Suppose there is unconstrained labor force for firm i. Conditional on hiring women with threshold type $\theta \in (0, \bar{\theta})$, firm i obtains its maximum total profit of*

$$\Pi(\theta) := \frac{\theta(\pi_f(\theta) - 0.5)}{1 - \theta} + 0.5 \tag{3}$$

by hiring measure $\mu_f(\theta) := 1/(1 - \theta) \in (1, 2)$ of women with the contract $(e_f(\theta), s_f)$ and measure $2 - \mu_f(\theta)$ of men with the contract C_m^*. $\Pi(\theta)$ is quasi-concave in $(0, \bar{\theta})$ with a unique interior maximum. The ex ante utility of a female worker from the contract is $v(e_f(\theta), s_f)$ as defined in (2), which is strictly increasing in θ.

Proof: It only remains to show that $\Pi(\theta)$ is quasi-concave and has a unique interior maximum. It is routine to show that $\pi_f'(\theta) = \frac{-\hat{u} + w_f/2}{\pi_f(\theta) + 0.5}$ and consequently,

$$\Pi'(\theta) = \frac{\pi_f(\theta) - 0.5 + \theta(1 - \theta)\pi_f'(\theta)}{(1 - \theta)^2} = \frac{\pi_f(\theta)^2 - 0.25 - \theta(1 - \theta)(\hat{u} - w_f/2)}{(\pi_f(\theta) + 0.5)(1 - \theta)^2}.$$

[18] Indifference may not guarantee that a half of men pursue promotion under this contract. However, the firm can get to arbitrarily close to this situation by offering $(e_m, s_m + \epsilon)$ for small $\epsilon > 0$: Then, in the equilibrium exactly measure 1 of men pursue promotion by exerting $e = \sqrt{2(0.5 + \epsilon)}$.

Differentiating the numerator wrt θ, we get $2\pi_f(\theta)\pi'_f(\theta) - (1-2\theta)(\hat{u} - w_f/2)$ which is negative for $\theta \in (0, \bar{\theta})$ because $\bar{\theta} < 0.5$, $\hat{u} - w_f/2 > 0$ and $\pi'_f(\theta) < 0$. Since the denominator is positive and $\Pi'(0) > 0$ and $\Pi'(\bar{\theta}) < 0$, it follows that $\Pi'(\theta) > 0$ for all θ less than some $\tilde{\theta} \in (0, \bar{\theta})$ and $\Pi'(\theta) < 0$ for all $\theta > \tilde{\theta}$, completing the proof. QED.

D. Optimal hiring with residual workforce

Consider an alternative situation in which labor force of total measure 2, consisting of measure $\mu_f^r < 1$ of female and measure $2 - \mu_f^r$ of male, remains available for firm i to hire. As before, since the firm can fill all upper positions with men hired with the efficient contract, the firm may benefit by promoting women only if per-promotion profit exceeds 0.5 for women, which is feasible with contracts the threshold types of which fall within $(0, \bar{\theta})$.

If this firm induces female promotion with a threshold type $\theta \in (0, \bar{\theta})$ by hiring the residual female workforce at a contract $(e_f(\theta), s_f)$, then it can obtain a total profit of

$$\Pi_r(\theta; \mu_f^r) := 0.5 + \mu_f^r \theta(\pi_f(\theta) - 0.5) \tag{4}$$

by filling all remaining high posts with men hired at the efficient contract. A routine calculation verifies that the second derivative of (4) wrt θ is

$$-\mu_f^r\left(\hat{u} - \frac{w_f}{2}\right)\frac{2(\pi_f(\theta) + 0.5)^2 + (\hat{u} - w_f/2)\theta}{(\pi_f(\theta) + 0.5)^3} < 0$$

for $\theta \in (0, \bar{\theta})$. Since (4) assumes 0.5 both at $\theta = 0$ and $\bar{\theta}$, it achieves a unique interior maximum. Thus, the optimal behavior of a firm with residual labor supply is characterized as below.

Lemma 4: *If firm i faces a residual labor force consisting of measure $\mu_f^r < 1$ of female and measure $2 - \mu_f^r$ of male, firm i obtains its maximum total profit of $\Pi_r(\theta^{**}; \mu_f^r) > 0.5$ by hiring all women with the contract $(e_f(\theta^{**}), s_f)$ and all men with the contract C_m^*, where*

$$\theta^{**} = \arg\max_{0<\theta<\bar{\theta}} (\pi_f(\theta) - 0.5)\theta. \tag{5}$$

Note that θ^{**} is independent of μ_f^r, hence so are the optimal contracts identified in Lemma 4. We say that a firm uses the "defensive (contracting) strategy" if it offers the female contract $(e_f(\theta^{**}), s_f)$ and the male contract C_m^* regardless the contracts offered by the other firm.

The maximum profits $\Pi(\theta)$ and $\Pi_r(\theta^{**}; \mu_f^r)$ are defined above by presuming that the appropriate measure of male workers choose to pursue promotion out of indifference given the efficient male contract C_m^*. Strictly speaking, therefore, those profits are not guaranteed if the firm offers C_m^*. Nevertheless, a firm can secure any profit level arbitrarily close to these levels from below, by offering a male contract that slightly sweetens C_m^*, as explained in the next lemma.

Lemma 5: *Suppose a firm hired measure μ_f of women at a contract (e, s) with $\theta^c(e, s) \in (0, \bar{\theta})$ and measure $2 - \mu_f$ of men at $C_m^\epsilon = (e_m, s_m + \epsilon)$ where $\epsilon > 0$ is small. If $2 - \mu_f \geq 1 - \mu_f \theta^c(e, s)$, the firm's profit in the ensuing promotion subgame exceeds $0.5 + \mu_f \theta^c(e, s)(0.5 + e - s) - \epsilon$.*

Proof: In this firm all women up to type $\theta^c(e, s)$ would exert e and pursue promotion, since their promotion is guaranteed because i) they generate higher surplus than any male would and ii) women of any other types find it a dominated strategy to pursue promotion by exerting at least as high effort level (the firm would never pay more than s for women promotion). If $2 - \mu_f = 1 - \mu_f \theta^c(e, s)$, i.e., there are exactly the same measure of men as the remaining high posts, all men would secure promotion by exerting e_m because, given ii), the firm would not deny promotion to them. If $2 - \mu_f > 1 - \mu_f \theta^c(e, s)$, then competition amongst men for promotion restores indifference, so that they would have to exert $e' > e_m$ such that $s_m - (e')^2/2 = 1$ to pursue promotion. Furthermore, exactly measure $1 - \mu_f \theta^c(e, s)$ of men would pursue promotion: if a smaller measure of men were to pursue promotion as such, other men would benefit by exerting e_m because the firm is obliged to promote them given ii). In either case, the firm's profit exceeds $0.5 + \mu_f \theta^c(e, s)(0.5 + e - s) - \epsilon$. QED.

Remark: From the FOC of (5), it is easy to show that $\Pi'(\theta^{**}) > 0$. This means that $\theta^{**} < \tilde{\theta}$ where $\tilde{\theta} \in (0, \bar{\theta})$ maximizes $\Pi(\theta)$ in (3). Because of this, there does not exist a mixed equilibrium analogous to that of capacity-constrained Bertrand duopoly in Kreps-Scheinkman (1983), Lemma 6. (Maybe, K-S assume "concave" demand function to ensure that the best-response to the other firm's "defensive" pricing" is just undercutting it, which is not the case in our model.) That is, if $\theta^{**} \geq \tilde{\theta}$, there would be a mixed equilibrium, in a simultaneous contract-offer version, in which the two firms atomlessly mix efficient female contracts corresponding to threshold types in $[\theta^{**}, \theta^{\dagger}]$ where $\Pi(\theta^{\dagger}) = \Pi_r(\theta^{**}; \mu_f(\theta^{**}))$.

E. Equilibrium

An ϵ-defensive strategy refers to the defensive strategy with C_m^* replaced with C_m^ϵ. Suppose a firm uses an ϵ-defensive strategy. Since this firm can guarantee itself a profit level arbitrarily close to 0.5 by Lemma 5,

[C] either firm can guarantee a profit of 0.5.

If a firm's final male contract specifies a salary $s < 1.5$, then no men would exert $e > \sqrt{2(s-1)}$ because the firm would never pay more than s and, therefore, the firm's surplus per male promotion, $1+e-s$, is less than 0.5. By a similar reason, the firm's surplus per male promotion is less than 0.5 as well if a firm's final male contract specifies a salary $s > 1.5$. If, in addition, the firm's final female contract (e, s) is such that $\theta^c(e, s) > \bar{\theta}$, then the firm's surplus per female promotion is less than 0.5: This is trivial if the contracted effort is exerted $(1+e-s < 0.5)$; if women exert $e' > e$ to get promoted, then they do so only up to type $\theta' < \theta^c(e, s)$ and all other high posts are filled by men who generate per promotion surplus less than 0.5 and, furthermore, the surplus per women promotion is no higher because otherwise women of type slightly above θ' would exert some effort in (e, e') and get promotion backed by lawsuit threat. In light of [C], therefore,

[D] neither firm offers a male salary other than 1.5 in conjunction with a female contract (e, s) such that $\theta^c(e, s) > \bar{\theta}$, because doing so is strictly dominated.

Suppose a firm's final female contract is such that $\theta^c(e, s) > \bar{\theta}$, whence the male contract is $(e', 1.5)$ by [D]. Due to [C], this firm's hiring strategy must be to secure at least measure 1 of male (which it can do as per (B1) and (B2)) because, regardless of what the other firm does, this is the only way of obtaining 0.5.

Now consider firm i that uses the ϵ-defensive strategy. Suppose that the other firm j's female contract (e, s) has a higher value than firm i's, i.e., $\theta^c(e, s) > \theta^{**}$. Then, firm j has priority in hiring and would hire no more than measure 1 of women if $\theta^c(e, s) > \bar{\theta}$ by the discussion just above; if $\theta^c(e, s) \leq \bar{\theta}$, on the other hand, it would hire a measure $\mu_f(\theta^c(e, s))$ of women[19] to maximize profit by the same logic behind the Lemma 3. Hence, firm i would have at least measure $2-\mu_f(\bar{\theta})$ of residual female labor force and, therefore, would secure a total profit arbitrarily close to $\Pi_r(\theta^{**}, 2-\mu_f(\bar{\theta}))$ be selecting ϵ arbitrarily close to 0. Since firm i would do at least as well if firm i's female contract is of no worse value than firm j's, by symmetry between firms, we deduce that

[E] either firm can guarantee a profit of $\Pi_d(\bar{\theta}) > 0.5$ by the defensive strategy,

where

$$\Pi_d(\theta) := \Pi_r(\theta^{**}; 2-\mu_f(\theta)) \tag{6}$$

is the maximum profit level of a firm facing a residual labor force after the other firm hired optimally with unconstrained labor force as described in Lemma 3, conditional on hiring women with threshold type $\theta \in (0, \bar{\theta})$. Recall that the latter firm's profit is $\Pi(\theta)$ as defined in (3).

[19] Note that the per promotion surplus must be at least 0.5 by [C], and the surplus per male promotion should be not much lower than 0.5 (this lower bound can be calculated), to ensure a total profit of at least 0.5 possible.

At this point it is instructive to examine the relationship between the $\Pi(\theta)$ and $\Pi_d(\theta)$ for $\theta^{**} \leq \theta \leq \bar{\theta}$. Note that $\Pi(\theta^{**}) > \Pi_d(\theta^{**})$ because $\mu_f(\theta^{**}) > 1$, whilst $\Pi(\bar{\theta}) = 0.5 < \Pi_d(\bar{\theta})$. Furthermore, Π_d is strictly decreasing in $\theta \in (\theta^{**}, \bar{\theta})$ because $\mu_f(\theta)$ is increasing in θ; and Π is single-peaked as shown in Lemma 4. These observations generate the graphs of $\Pi(\theta)$ and $\Pi_d(\theta)$ as below:

[Figure 4 about here]

There is a unique $\theta \in (\theta^{**}, \bar{\theta})$, denoted by θ^*, that satisfies $\Pi(\theta) = \Pi_d(\theta)$ in our specification of the model.[20]

Now, by an argument analogous to that leading to [E], we deduce that either firm can guarantee a profit of $\Pi_d(\theta')$ where $\theta' \in (\theta^*, \bar{\theta})$ such that $\Pi(\theta') = \Pi_d(\bar{\theta})$; then, analogously again, either firm can guarantee a profit of $\Pi_d(\theta'')$ where $\theta'' \in (\theta^*, \theta')$ such that $\Pi(\theta'') = \Pi_d(\theta')$; and so on ad infinitum. By this recursive process of elimination of dominated strategies,

Lemma 6: *Either firm can guarantee a profit of* $\Pi(\theta^*) = \Pi_d(\theta^*) > 0.5$ *by the defensive strategy.*

Finally, to show that no firm can get a profit higher than $\Pi(\theta^*)$, suppose to the contrary that a firm has a higher profit. The two firms' female contracts cannot be of the same value: if they were, the two firms would split labor force equally for both would want more than measure 1 of women (because female promotion must generate higher surplus than male for the total profit to exceed 0.5), therefore either firm would benefit by sweetening the female contract and hire more women. Hence, suppose firm A's female contract is of higher value than B's, i.e., $\theta_B < \theta_A \leq \theta^*$ where θ_i is the threshold types of firm $i = A, B$. If either firm's female contract is not of the form $(e_f(\theta_i), s_f)$ as specified in Lemma 3, then that firm can revise the female contract to $(e_f(\theta_i + \eta), s_f)$ at the final moment of campaign stage, where $\eta > 0$ is small, which would increase the surplus per female promotion by a discrete amount while maintaining $\theta_B < \theta_A$ and consequently, would increase its total profits. Hence, suppose that both female contracts are of the form $(e_f(\theta_i), s_f)$ specified in Lemma 3. Then, firm A would hire the unconstrained optimal measure $\mu_f(\theta_A)$ of female, and firm B would hire the residual workforce so that firm B's profit is

$$\Pi_r(\theta_B, \pi_B^m; \mu_f^r) := \mu_f^r \theta_B \pi_f(\theta_B) + (1 - \mu_f^r \theta_B) \pi_B^m$$

where $\mu_f^r = 2 - \mu_f(\theta_A) < 1$ is the residual female workforce and $\pi_B^m \leq 0.5$ is firm B's surplus per male promotion. If firm B deviates by just "undercutting" firm A's threshold type θ_A (which it can because $\theta_B < \theta_A$) it would get a profit level arbitrarily

[20] It is conceivable that there may be more intersection points in alternative specifications, e.g., different distribution of θ. Our main insights extends to these cases.

close to $\Pi(\theta_A) - \mu_f^r(0.5 - \pi_B^m)$, so the net benefit would be $\Pi(\theta_A) - \mu_f^r(0.5 - \pi_B^m) - \Pi_r(\theta_B, \pi_B^m; \mu_f^r)$, which is calculated as

$$(1 - \mu_f^r)\pi_f(\theta_A) - \mu_f^r\theta_B\pi_f(\theta_B) - (1 - \mu_f^r - \mu_f^r\theta_B)\pi_B^m. \tag{7}$$

Note that this is positive when $\pi_B^m = 0.5$ because then this is equal to $\Pi(\theta_A) - \Pi_r(\theta_B; \mu_f^r) > \Pi(\theta_A) - \Pi_r(\theta^{**}; \mu_f^r) > 0$. (Note that θ_A is on the downward slope of $\Pi(\theta)$, for otherwise firm A would benefit simply by sweetening its female contract.)

In addition, from $\Pi(\theta_A) > \Pi_r(\theta_B; \mu_f^r)$ we get $(1-\mu_f^r)(\pi_f(\theta_A)-0.5) > \mu_f^r\theta_B(\pi_f(\theta_B)-0.5)$. Since $\pi_f(\theta_B) > \pi_f(\theta_A)$, we further get $(1-\mu_f^r)(\pi_f(\theta_B) - 0.5) > \mu_f^r\theta_B(\pi_f(\theta_B) - 0.5)$, which implies that $1 - \mu_f^r - \mu_f^r\theta_B > 0$. Since (7) is positive when $\pi_B^m = 0.5$ as asserted above, it follows that the net benefit of deviation, $\Pi(\theta_A) - \mu_f^r(0.5 - \pi_B^m) - \Pi_r(\theta_B; \mu_f^r)$, is positive for all $\pi_B^m \leq 0.5$. This proves the next result.

Lemma 7: *In equilibrium neither firm's profits exceed* $\Pi(\theta^*) = \Pi_d(\theta^*) > 0.5$.

From Lemmas 6 and 7, both firms should have the same profits $\Pi(\theta^*)$ in equilibrium. Furthermore, as argued already, the female contracts of the two firms are of the form specified in Lemma 3, i.e., $(e_f(\theta_i), 1.5)$ where $\theta_i \geq \theta^*$ is the threshold type of firm $i = A, B$, and $\theta_A \neq \theta_B$ so that they may not be of identical values. Then, the firm with higher θ_i hires optimally with unconstrained labor force as in Lemma 3, thereby obtaining a profit of $\Pi(\theta_i)$, which implies that the higher θ_i must be θ^*. The other firm hires the residual labor force, thereby obtaining a profit of $\Pi_d(\theta^*)$, which implies that the lower θ_i must be θ^{**}. Given that the two firms offer these female contracts along with the efficient male contracts, it is straightforward to verify from Figure 4 that neither firm may benefit by unilaterally revising its own contract offers. Hence, we have a unique characterization of equilibrium as summarized below. The identity of the two firms, however, may be stochastic and symmetric: in a continuous-time campaign stage, for instance, the two firms may announce the contracts of the threshold type θ^* with a continuous hazard rate over time conditional on the other firm has not done so already, and once one firm has done so, the other firm would announce the contracts of the threshold type θ^{**}.

Theorem 1: *In equilibrium, one firm offers the female contract* $(e_f(\theta^*), 1.5)$ *and the efficient male contract* $C_m^* = (1, 1.5)$*, and the other firm offers the defensive contracts,* $(e_f(\theta^{**}), 1.5)$ *and* $C_m^* = (1, 1.5)$ *for female and male workers, respectively. The former firm hires measure* $\mu_f(\theta^*) \in (1, 2)$ *of women and measure* $2 - \mu_f(\theta^*)$ *of men and the latter firm hires the residual labor force. The two firms have identical profits* $\Pi(\theta^*) = \Pi_d(\theta^*) > 0.5$*, however, the ex ante utility of women is higher when hired by former firm than by the latter. Furthermore, every female worker promoted in either firm generates a higher surplus to the firm but obtains a lower net utility than any promoted male worker.*

Note that in equilibrium workers of either gender get paid the same salary in high posts, although female workers have to work harder to get promoted ($e_f(\theta^*) > 1$). The equality of high-post salaries results from the assumption that the minimum female salary to preclude their departure is equal to the men's efficient high-post salary, i.e., $2\hat{u} = s_m = 1.5$. As mentioned earlier, we are interested in the cases in which offering the efficient male wage would render female promotions identical to male promotions by precluding any departure risk, which is the case if $2\hat{u} < 1.5$. In these cases (as long as $2\hat{u} > 1$), the equilibrium remains the same with one change: female workers have to work harder than male workers to get promoted, yet they get paid less after promotion. In the opposite cases, i.e., $2\hat{u} > 1.5$, the equilibrium also remains the same, although now the promoted female workers get paid more than their male counterparts, however this extra salary falls short of compensating the extra effort they have to exert. In all these cases, therefore, female workers in high posts generate a higher surplus to the firm per post but obtain a lower net utility than their male counterparts.

4. Discussion, Extensions and Policy Analysis

A. Discussion: differences with signaling and 'standard' screening models

In our framework, we rule out signaling, and instead adopt a screening approach. What would happen if instead we allowed for signaling? Not much. The model we consider is one where the *only* differences between women concern the probability with which they will be hit by a shock that gives them a positive (and of fixed value) utility from leaving the workforce (denoted as $\widehat{u}$). Since we concentrate on the case where promotions are secure, this implies that, once promoted, all women become *identical*. So, in our framework, the agent's type does not affect the principal's utility. In that sense, there is *no value* for a woman to signal her type. This is in contrast with more standard models (such as Spence 1973), where workers have an incentive to signal that they are of a high type, as this makes them more valuable for the employer (and therefore allows them to earn a higher wage).

Similar to what said above for signaling, the key distinction between ours and 'standard' screening models is that here the instruments available to the principal (wage in case of promotion, and effort necessary for promotion) affect all women *in the same way*. The only difference between types is their reservation utility. The utility obtained by a woman who decides to go for promotion is *independent* of her type.

In standard screening models, distortions in effort are introduced in order to reduce the agents' informational rents. Types with a lower disutility from effort have an incentive to mimic those who suffer a greater disutility, in order to obtain a larger compensation. To reduce this effect, the principal introduces distortions, namely inefficiently low effort for all but the lowest type. In our model this type of effect is absent, since greater effort affects all women in the same way. So the rationale for the distortion in the effort level required from promoted women is fundamentally different from that we would normally encounter with signaling (and, as seen above, signaling).

B. Extensions

Effort not verifiable. We carried out our analysis presuming that effort level is sufficiently verifiable so that the workers are protected by the court if they do not get promoted after meeting the contractual requirement. In some circumstances, though, it may be too costly to verify the effort level. In the extreme case that it is unverifiable at all, the effort component of a contract becomes cheap-talk. It is natural, therefore, that there exist multiple equilibria supported by various self-confirming belief profiles, and such beliefs may be the reputation of firms that they acquired in previous interactions for future benefits. In this context, the equilibrium described in Theorem 1 continues to be an equilibrium, albeit no longer the unique one, supported by the beliefs that the effort levels specified there are expected for promotion and a sufficient number of workers will full the expectation.

Contracts also specify wage in period 1. In the main analysis, for ease of exposition we disallowed a contract to specify a wage for period 1. Allowing this does not change anything strategically for male contracts because salaries in period 1 and 2 are perfect substitutes. For female contracts, however, this would allow the firms to reduce the threshold type of a contract and enhance its value at the same time. This additional scope may generate some benefits that might be shared between firms and workers, yet the essential features of equilibrium in Theorem 1 remain valid: One firm hires the residual labor force with the defensive contracts, and the other firm hires as if facing unconstrained workforce but with a sufficiently favorable female contract so that the former firm would not undercut it, and consequently, the two firms get identical profit levels.

More than two firms. Although our analysis dealt with competition of two symmetric firms in a labor market, it can be extended to two firms of different sizes and to more than two firms, generating qualitatively similar results. It seems possible that it can be extended further to a game that incudes an initial stage in which potentials firms make entry and size decisions.

C. Policy Analysis

Our model identifies a glass ceiling phenomenon for high-rank positions. In contrast to previous literature (such as Lazear and Rosen 1990) this glass ceiling is unjustified; offering women the same promotion offer as men would be sufficient to erase any difference between men and women in high-rank jobs (since all promoted women would remain in the job with certainty).

The disadvantage suffered by women in high-rank jobs is essentially a ‘spillover’ of the disadvantage they suffer when remaining in low-rank positions. In low-rank positions, women will always leave if hit by the shock (since in this case their productivity outside the job market is strictly higher than their productivity on the job). Hence, a woman’s expected productivity in the low-rank job is strictly below that of a man. This results in women earning strictly lower wages than men in low-rank jobs. Note that, although on average this lower wage is justified, some women (i.e., those with a low or zero probability of being hit by the shock) are earning less than their productivity. For those women, the existence of more ‘family-friendly’ fellow women imposes a negative externality.

The fact that women do have a lower expected productivity than men in low-rank jobs spills over to higher-rank jobs, where women are disadvantaged in spite of being potentially identical to men.

One obvious way to eliminate the problem would be that of imposing a policy that ensures that men and women earn the same wage in low-rank jobs. (Note: this would result in firms offering the same terms of promotion to men and women. However, it would also result in a measure zero of women being promoted, i.e., only those women who have exactly zero probability of being hit by the shock).

It is not clear however that such a policy would be feasible. Essentially, women in low rank jobs *are* on average less productive than men. So a policy that obliges firms to pay their low-rank male and female employees exactly the same wage would simply result in two types of firms emerging, those employing only female workers,and those employing only male workers in low-rank positions. Under perfect competition, in each type of firm employees would be paid their expected productivity, but, crucially, wages in firms specialized in female workers would be lower than those in firms specialized in male workers.

In what follows, we review a few simple policy measures that may be imposed on the firms' contractual offers for high-rank jobs, and discuss the extent to which they may affect the competitive equilibrium described in the previous section.

Gender-free contracts. First, consider a policy requiring that all contracts offered must be gender-free, i.e., the same set of contracts are available for workers of either gender to choose one from and apply for. So long as the promotion decisions among qualified workers are left to the firm's discretion, the competitive equilibrium continues to be an equilibrium under this policy, supported by the belief that the firm may promote women only if they select the contract intended for them. Therefore, although there are other equilibria, including one in which all workers get promoted in the same terms, putting this policy in place does not warrant transition to a more equitable promotion outcome. This is especially evident when the effort level is unverifiable: the effort component of a contract being cheap-talk, the contracts in the competitive equilibrium can be interpreted as being identical contracts with different implicit understanding of the requirements.

A certain fraction of women must be promoted. Consider an alternative policy that requires each firm to promote at least a fixed fraction, say θ^R, of its female employees, i.e., it imposes a restriction on the threshold female type that it may implement. If $\theta^R > \theta^*$, then this policy will improve the welfare of promoted women; So long as $\theta^R < \bar{\theta}$, however, the main features of the competitive equilibrium (albeit of a lesser degree) will remain in the ensuing equilibrium, in which one firm hires the residual labor force with the efficient male contract and a female contract of threshold type θ^R, and the other firm hires as if facing unconstrained workforce but with female contract sufficiently favorable so that the former firm would not undercut, and the two firms get identical profits. If $\theta^R > \bar{\theta}$, then a reversed glass ceiling phenomenon arises: men have to work harder to get promoted, yet may get paid less than their female counterparts. Only when $\theta^R = \bar{\theta}$, such a policy would correct the glass ceiling phenomenon. However, such remedy would be temporary in nature unless the policy were to remain in place permanently, for the competitive equilibrium would reemerge when the policy is lifted.

5. Conclusions

In this paper we have provided an economic equilibrium theory of the glass ceiling phenomena The model provides a theoretical equilibrium where there is a lower number of female employees in higher positions, women have to work harder than men to obtain what are equivalent jobs, on average women are then paid less than men when promoted and where the equilibrium implies that there will be female friendly and female unfriendly organisations, in a model where women are not formally discriminated against.

Women who wish to be career committed are worse off than their male since they have to make an observable commitment, which entails working harder than the equivalent male would do to obtain the same promotion. Firms compete Bertrand style for female employees but even so are able to hire women at remuneration levels that do not reflect the higher productivity that is generated by this extra effort (relative to the male remuneration contract). Some firms are 'female friendly' and hire larger numbers of women at remuneration that provides a small surplus per female. Other firms are 'female unfriendly' and hire less women and make a greater surplus per female employee. The net effect is that both make the same profit. As indicated women are not formally discriminated against by employees but it is the case that women who wish to be promoted have to work harder for less pay simply because, and only because, they are female. In this sense career women are clearly disadvantaged in the workplace.

Unfortunately, the model is somewhat depressing in terms of what can be done to improve things for career women. Insisting that contracts cannot be gender specific is not sufficient to improve things. Restrictions can be set on the minimum percentage of women that constitute the senior positions. There are two problems here. One is that the planner needs a great deal of information and is unlikely to get the proportion right. More of an issue is that a short-lived intervention will not help once it is lifted, unlike most statistical discrimination models. The gender disparity will go away if on average male and female employees are equally likely to wish to take an employment break or reduce their commitment to the job.

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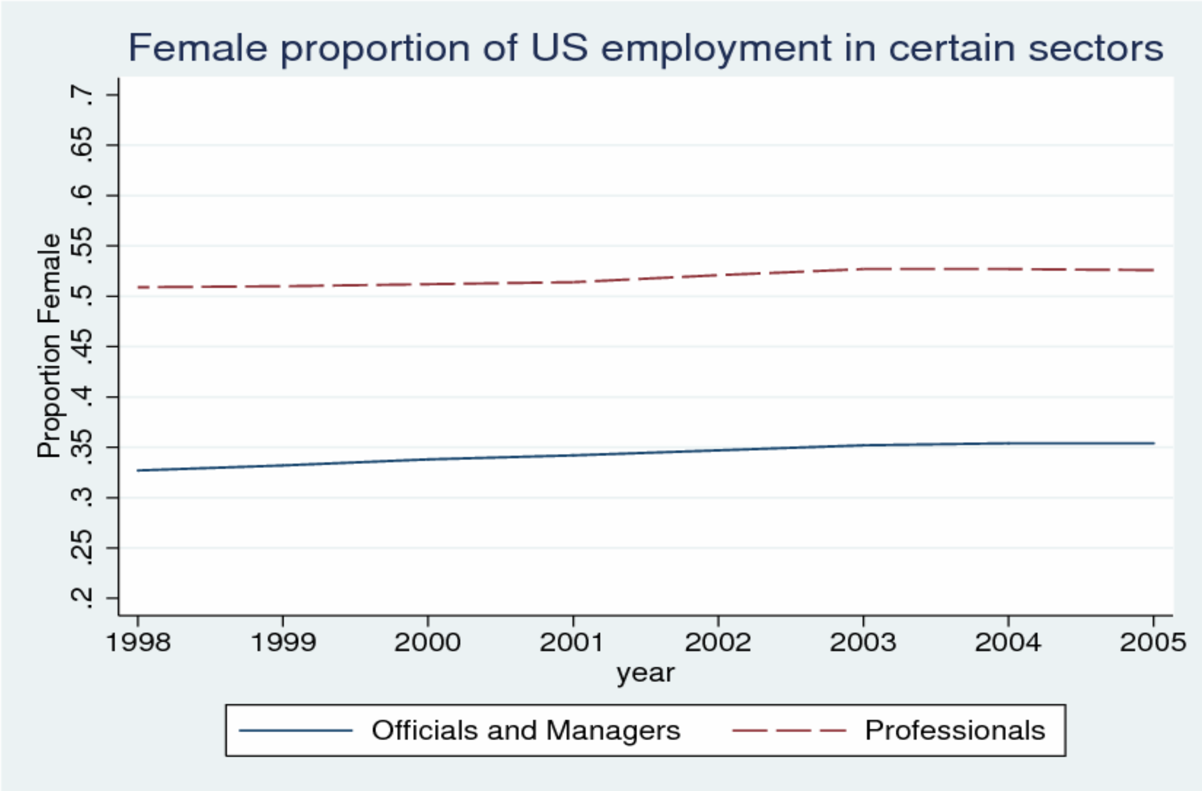
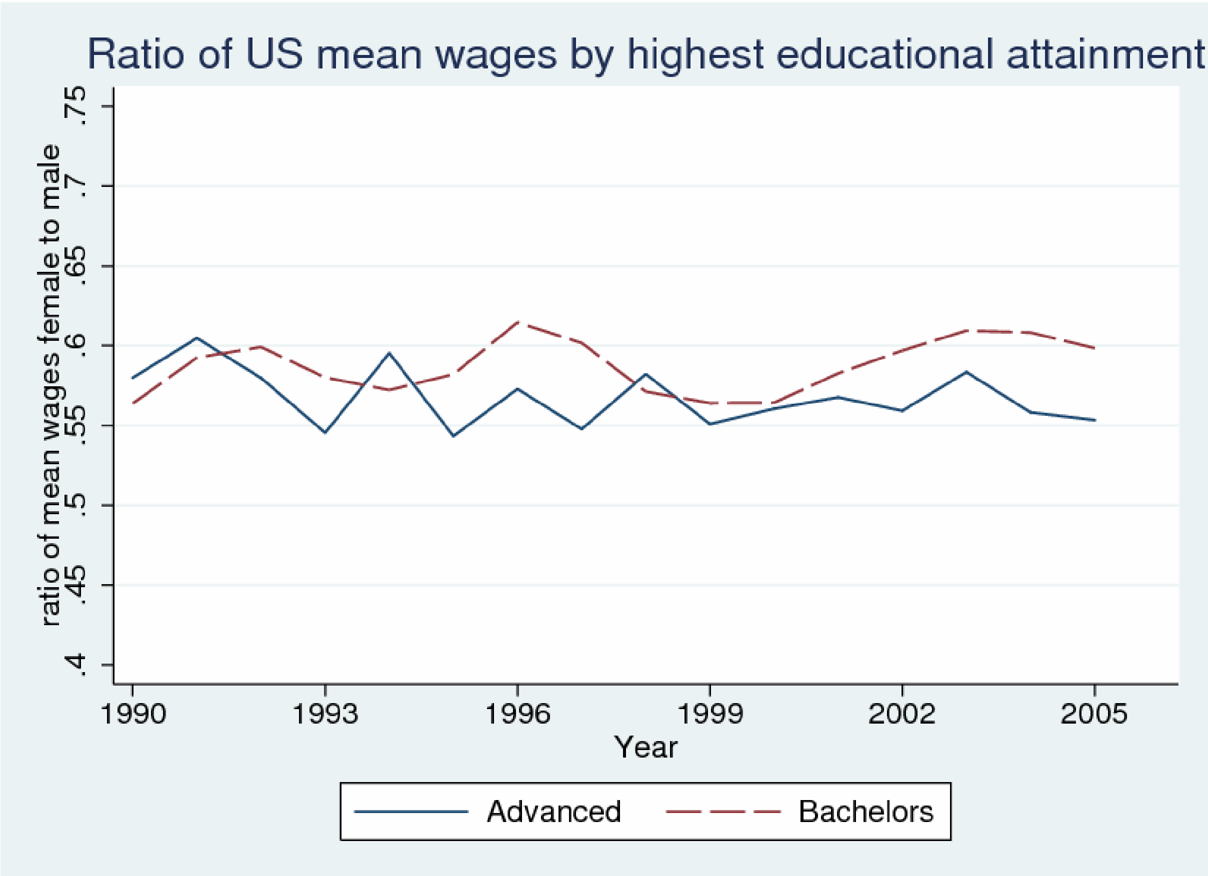
Figure 1.



Source: U.S. Equal Employment Opportunity Commission (EEOC).

Figure 2.



Source: US Census Bureau.

Figure 3

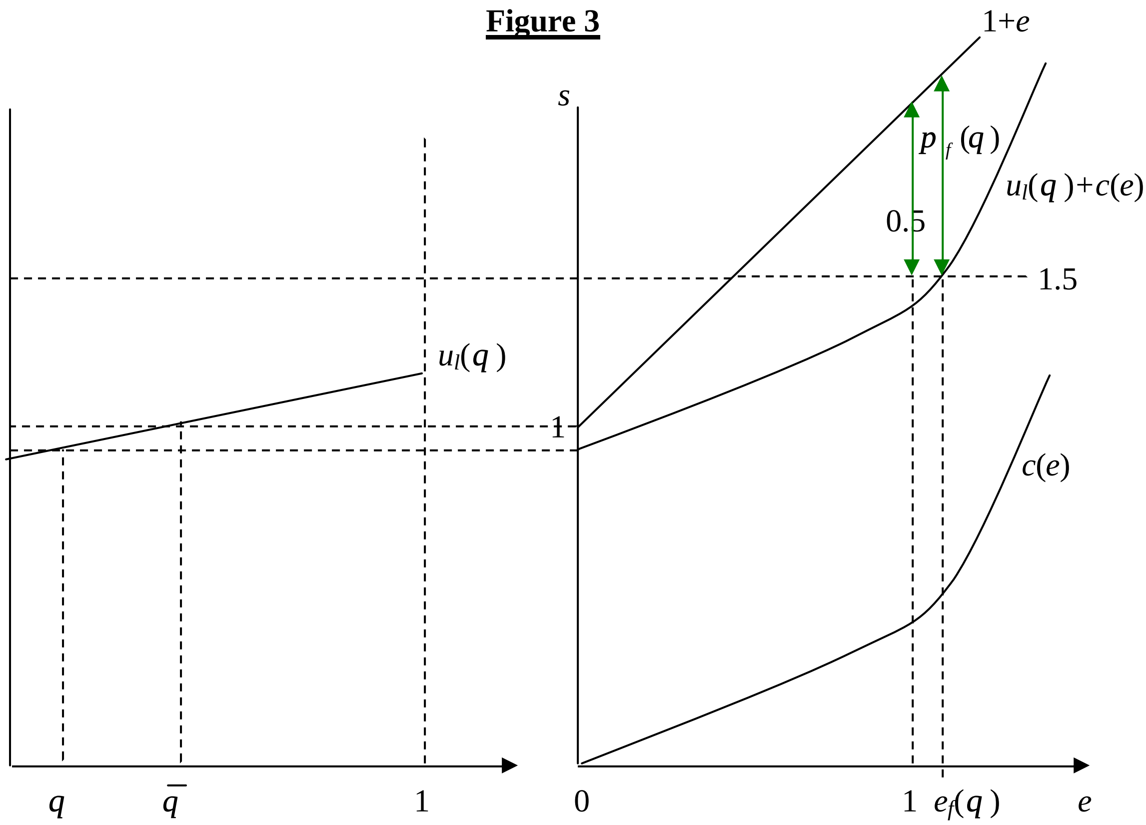


Figure 4