CMPO Working Paper Series No. 04/110

Nonlinear Pricing and Multimarket Duopolists

Silvia Sonderegger

Department of Economics and CMPO, University of Bristol

October 2004

Abstract
This paper studies competition in price-quality menus within the context of a horizontally differentiated duopoly, where each firm also operates in a local, monopolistic market. It is assumed that the consumer's unobservable valuation for quality is determined by the nature of his preferences over brand product characteristics. I show that if competition between the two firms is sufficiently fierce, the equilibrium contract features overprovision of quality for sufficiently low types. Thus, with respect to the monopoly setting, competition may introduce new types of distortions, namely upward distortions. This suggests that the relationship between "toughness of competition" and welfare may not necessarily be monotonic.

Keywords: oligopoly, other forms of market imperfection

JEL Classification: D43, L13

Acknowledgements
I thank Roman Inderst, Bruno Jullien, Jean-Charles Rochet, Lars Stole, John Sutton and seminar participants at the University of Bristol, LSE, the conference of the European Association for Research in Industrial Economics 2002, and the European Economic Association Meeting, 2003. All errors are my own. I thank the Leverhulme Trust for funding this research.

Address for Correspondence
Department of Economics
University of Bristol
8 Woodland Road
Bristol
BS8 1TN
S.Sonderegger@bristol.ac.uk

CMPO is funded by the Leverhulme Trust.

1 Introduction

It is often argued that more competitive environments foster efficiency; however, the theoretical work in support of this proposition is fragmented. Although economists have gained a good understanding of the properties of both monopolistic and perfectly competitive markets, the welfare properties of what lies in between are still not entirely understood; except for the most stylised environments – such as the standard Cournot and Bertrand settings, where efficiency is (weakly) increasing in the number of market participants – the relationship between welfare and toughness of competition has not been generally characterized. A recent body of literature – such as Stole (1995), Armstrong and Vickers (2001) and Rochet and Stole (2002) – has worked towards filling this gap, by focussing on the properties of the equilibrium contracts offered by horizontally differentiated duopolists, who compete in price-quality menus to attract consumers whose preferences are unobservable. Their findings support the view that competition promotes efficiency: the duopoly outcome is generally qualitatively similar to the monopoly outcome, but distortions are reduced. Moreover, Rochet and Stole (2002) and Armstrong and Vickers (2001) find a remarkable result: conditional on sufficiently fierce competition, all distortions disappear, and the equilibrium quality allocations are efficient[1].

This work adds to this literature, by showing that, with respect to the monopoly scenario, duopolistic competition may introduce new types of distortions, namely upward distortions; the presence of a competitor induces each firm to engage in a sort of "arms' race", that results in the overprovision of quality for sufficiently low types. Hence, competition may result in a waste of resources.[2] This suggests that the relationship between "toughness of competition" and welfare may not necessarily be monotonic. Although in perfectly competitive environments the efficient allocation of resources always emerges, environments characterized by oligopolistic competition are not necessarily more efficient than those characterized by a monopoly.

This paper possesses two dinstinctive features: the first concerns the assumption that each firm operates both within a local market – where it is a monopolist – and a competitive market – where it competes against another firm. This division betwen markets may be interpreted either in a literal way, as modeling markets that are geographically separated, or as capturing

[1]Inderst (2001) finds a related result in a matching model of buyer-seller exchange.

[2]Indeed, the idea that competition may be socially wasteful is well known in the literature that studies competition in health care, and in particular competition among hospitals (see for instance Feldstein, 1971, Held and Pauly, 1983 and Robinson and Luft, 1985). In these markets, the presence of health insurance dampens the patients' sensitivity to cost and price differences among hospitals. In turn, insensitivity to price leads hospitals to compete through the provision of medically unnecessary services. As will become clear below, the intuition for the overprovision result obtained in the present paper is fundamentally different.

the fact that consumers may differ in their switching costs. Crucially, it is assumed that firms cannot discriminate between consumers located in different markets. This may be the case if firms cannot directly observe each consumer's market location, or if firms can observe market location, but are not allowed to discriminate according to it.[3] From the point of view of each firm, the presence of two markets is therefore equivalent to a situation where the consumer's reservation utility can be either equal to zero – when he is located in the local market – or taking a positive value – when he is located in the competitive market – with a positive probability.

The second distinctive feature of this paper concerns the assumption that the consumer's marginal valuation of quality is determined by his preferences over horizontal product characteristics: a consumer who prefers brand A over brand B derives more utility from an increase in the quality of good A rather than increase in the quality of good B. Moreover, the change in utility experienced by swiching from A to B is an increasing function of the consumer's valuation for quality when purchasing A. Hence, consumers who purchase goods of higher quality also have stronger brand preferences. From the point of view of each firm/brand, this implies that the competitive pressure generated by the presence of a rival firm is strongest for intermediate types. This is because high/low consumer types are strongly biased in favour of one brand, and are therefore reluctant to switch. Intermediate consumer types, on the other hand, are relatively brand-insensitive. Hence, competition between rival firms is concentrated on these intermediate types. The motivation for this way of modeling preferences is empirical, and comes from the observation that, in some markets, consumers who purchase higher qualities are more brand-loyal than those who purchase lower qualities.[4]

Together, these two features ensure that the mass of types with whom each firm contracts is strictly positive for all types[5], but experiences an upward jump as we move from low to high consumer types. This is because consumers with low valuations purchase the firm's product only when they have no viable alternative, i.e. when they are located in the firm's local market, while consumers with high valuations purchase the firm's product independently of their market location. Moreover, the price elasticities of demand possess the following feature: if the firm decreases its pricing schedule by a small amount[6], the units sold at high/low quality levels

[3] The following illustration is taken from Armstrong and Vickers (1993): prior to 1988, British Gas was free to set prices to its large consumers without any constraint on price discrimination. Customers without an alternative source of energy supply complained that they were charged more than the less captive customers. British Gas's freedom to discriminate was removed following the Monopolies and Mergers Commission Report that ensued.

[4] This is for instance well documented within the car market, as shown in Goldberg (1995), Berry, Lewinsohn and Pakes (1995), Feenstra and Lewinsohn (1995). Indeed, Verboven (1996) calls this feature a stilized fact of this market.

[5] In contrast, if the firms operated only in the competitive market, each firm would only contract with sufficiently high types.

[6] That is, if the price-quality schedule is shifted downward by a small constant.

remain unchanged, and the only increase occurs at intermediate quality levels. As will become clear below, these characteristics are the key for our results.

1.1 Relationship wih the existing literature

The literature that studies nonlinear pricing in duopoly settings mainly consists of Stole (1995), Rochet and Stole (2002) and Armstrong and Vickers (2001). All three papers focus on the behavior of firms that operate within only one market. Stole (1995) shares the assumption made in the present paper, that a consumer's valuation of quality is determined by the nature of his preferences over horizontal (brand) product characteristics, but assumes that the consumer's preferences over vertical product chracteristics are perfectly observable. Rochet and Stole (2002) and Armstrong and Vickers (2001) consider a duopoly setting in which the unobservable preferences over vertical and horizontal product characteristics are uncorrelated. This ensures smooth demand effects: if a firm decreases its price schedule, the units it sells at every quality level increase. In other words, competition occurs over all quality levels. In contrast, in the present setting, a correlation exists between a consumer's marginal valuation of quality and his preferences over horizontal product characteristics. As a consequence, competition between the two firms is concentrated on intermediate quality levels.

The paper to which the present work is most closely connected is Rochet and Stole (2002), that is therefore utilized as a benchmark.

The remainder of the paper is organized as follows: section 2 introduces the general model, while section 3 discusses the results. Section 4 concludes. All the proofs which are not in the main text can be found in the appendix.

2 The model

There are three players in the game: the consumer and the two producers/firms, denoted as l and r, standing for "left" and "right". The consumer may consume either zero or one unit of an indivisible good, which can be produced by either firm. More specifically, each firm can only produce a certain variety of the good: firm l can only produce variety (or brand) L, while firm r can only produce variety R. Also, each firm can produce the good at any quality level $q \in [0, Q]$, where Q is assumed to be finite but very large. Quality is an objectively measurable product characteristic.

There are three markets, denoted as m_l, m_r and m_c, where c stands for "competitive". Market m_l (respectively, m_r) is firm l' s (respectively, firm r's) local market. That is, firm l is market m_l's sole producer, and firm r is market m_r's sole producer. Market m_c, on the other hand, is supplied by both firms. Thus, each firm operates in two markets: its local market, and market m_c.

The consumer's private information is two-dimensional, and concerns both his preferences over product characteristics and his market location, which may be either m_l, m_r or m_c. Conditional on purchasing from firm i, $i = r, l$, the consumer is located in firm i's local market with probability s, and is located in market m_c with probability $1 - s$, for some $s \in]0, 1[$. Thus, s indicates the relative size of the local markets[7], with respect to market m_c.

Within each market, the consumer is located on an hypothetical segment of length $z \in]0, 1[$, measured between 0 and z; location over this segment is drawn according to a uniform[8] distribution. The intensity of the consumer's horizontal preferences with respect to a given brand is inversely proportional to the distance between the consumer's and the brand's location. It is assumed that brands L and R are located at the extremities of the segment; that is, we set brand L's position at zero, and brand R's position at z. This corresponds to a situation of maximal product differentiation[9]. The parameter z is therefore a measure of both the dispersion of the consumer's preferences and the degree of horizontal differentiation between the varieties sold by the two firms. A small z characterizes markets where consumer preferences are relatively homogeneous and the varieties sold by the two firms are close substitutes. Vice-versa, a large z characterizes markets where consumer preferences are strongly heterogeneous, and the varieties sold by the two firms are very dissimilar.

The consumer's preferences over vertical product characteristics are entirely determined by his brand preferences. That is, the consumer's marginal valuation of quality varies across brands according to the nature and intensity of his horizontal preferences. Denoting as $k_i(y)$, $i = l, r$ the marginal valuation of quality of a consumer located at y consuming the good produced by firm i we have

$$k_l(y) = 1 - y \text{ and}$$
$$k_r(y) = 1 - z + y$$

[7] In order to keep things as simple as possible, I only consider the case in which the two firms are perfectly symmetric, i.e. $s_l = s_r = s$.

[8] The assumption that the consumer's horizontal preferences are uniformly distributed ensures the perfect symmetry exists between the two producers, which in turn simplifies the analysis.

[9] D'Aspremont et al. (1979) show that, for quadratic trasportation costs, the equilibium of the two-stage game where (1) firms simultaneously choose their locations and (2) taking their locations as given, firms compete in prices, has the two firms locating at the extremities of the segment. The assumption of maximal differentiation can therefore be interpreted as hypothesizing the validity of this result when, in the second stage of the game, firms compete in price-quality menus.

Notice that there exists a one-to-one correspondence between $k_l(y)$ and $k_r(y)$, given by:

$$k_r(y) = 2 - z - k_l(y) \tag{1}$$

In what follows we refer to k_i as the consumer's type when purchasing from firm i. From the point of view of each firm i, k_i within each market is uniformly distributed on $[1 - z, 1]$.

The utility of a consumer of type $k_i = k$ purchasing quality q at price p from firm i is equal to

$$u_i(k, p, q) = kq - p$$

While his utility if he purchases quality q at price p from firm $-i$ is

$$u_{-i}(k, p, q) = (2 - z - k)\, q - p$$

This is the case because from (1) we have $k_{-i} = 2 - z - k_i$. Notice that the consumer's market location has no effect upon his utility from consumption. Instead, this is entirely determined by consumer's prefererence over horizaontal product characteristics. Finally, if the consumer does not consume the good at all, his utility is equal to 0.

The two firms are perfectly symmetric. If a firm sells a product of quality q at price p, its payoff is equal to $p - \frac{q^2}{2}$, while if it does not sell anything, its payoff is equal to zero. Hence, production does not entail any fixed cost.[10]

Competition between the firms takes the form of a simultaneous offer of price-quality menus. Specifically, each firm $i = l, r$ competes by offering a nonlinear price schedule $p_i(q_i)$ from which the consumer can chose a quality if he decides to purchase the product. We restrict attention to non-random pricing schedules.

Each firm's problem consists of designing a menu of contracts (or mechanism), from which the consumer may chose his preferred choice, conditional on purchase. At equilibrium each firm selects the optimal mechanism, taking the other firm's mechanism as given. From the revelation principle, we know that the search for the optimal menu of contracts may be confined to the set of direct revelation mechanisms, whereby the consumer is requested to report his type, and is offered a contract which is contingent upon his report. As mentioned above, the consumer's utility from consumption only depends upon his horizontal preferences, and is independent of the market

[10] Salop (1979) was the first to note how, in the presence of fixed costs of entry, more competition may result in lower total welfare. The private incentive for a firm to enter a market is higher than the social incentive, because the latter ignores the profits that are generated by the firm from "stealing the business" of incumbents. This fundamentally differs from the rationale for the results obtained in the present paper, where fixed costs are assumed to be irrelevant.

in which he is located. Hence, although the consumer's private information is two-dimensional, his type-space – defined as any private information that is relevant to the consumer's decision-making, conditional on purchasing from a given firm – is uni-dimensional. It is therefore without loss of generality that we consider direct revelation mechanisms of the form $\{q_i(k_i), p_i(k_i)\}$ in the analysis which follows. That is, we consider a direct revelation mechanism, taking the consumer's decision to purchase from the firm as given. For any given mechanism offered by firm i, we indicate the indirect utility of a consumer of type $k_i = k$ who truthfully reveals his type as $u_i(k)$. Throughout the analysis we concentrate on symmetric, pure strategy equilibria.

The consumer's reservation utility (or, equivalently, his outside option) when contracting with a given firm is defined as the highest utility which the consumer could obtain when not dealing with this firm. Thus, the consumer's reservation utility varies according to the market in which he is located. If the consumer is located in market m_i, $i = l, r$ his reservation utility when contracting with firm i is given by 0; this is because firm i is the sole active producer in market m_i. If the consumer is located in market m_c, on the other hand, his reservation utility is given by $\max\{0, u_{-i}(2 - z - k)\}$.

3 Implications

Consider firm $i = l, r$. The utility obtained by a consumer of type $k_i = k$ when contracting with firm i is equal to $u_i(k)$. The consumer's outside option when contracting with i depends upon his market location. If the consumer is located in market m_i, his outside option is zero. If the consumer is located in market m_c, his outside option is equal to

$$B_i(k) \equiv \max(0, u_{-i}(2 - z - k))$$

where $u_{-i}(2 - z - k)$ is the highest payoff which a consumer with marginal valuation $k_{-i} = 2 - z - k$ obtains when contracting with firm $-i$, firm i's rival. Notice that $u_{-i}(2 - z - k)$ is a function of k, the consumer's type when contracting with firm i. Hence, the consumer's outside option when located in the competitive market is type-dependent. This is in contrast with the standard monopoly situation, where all types of consumers typically have the same outside option. An additional departure from the standard model arises in the present setting from the inability of firms to observe the consumer's market location; this matters because, for any given type k, market location affects the consumer's outside option. From each firm's point of view, the situation is therefore equivalent to one where the outside option of a purchasing consumer is randomly drawn: with probability s, it is equal to 0, while with probability $1 - s$ it is equal to $B_i(k) \geq 0$.

The measure $M_i(u_i(k), k)$ of consumers of type k who contract with firm i is given by:

$$M_i(u_i(k), k) = \begin{cases} \frac{1}{z} \text{ if } u_i(k) > B_i(k) \\ \frac{s+1}{2z} \text{ if } u_i(k) = B_i(k) \\ \frac{s}{z} \text{ if } B_i(k) > u_i(k) \geq 0 \\ 0 \text{ if } u_i(k) < 0 \end{cases} \tag{2}$$

If $u_i(k) > B_i(k)$, the consumer purchases firm i's product with probability 1, independently of his market location. If $u_i(k) = B_i(k)$, the consumer purchases firm i' s product with probability 1 only if he is located in market m_i, while if he is located in market m_c he randomizaes, purchasing from each firm with probability 1/2. If $B_i(k) > u_i(k) \geq 0$ the consumer purchases firm i's product only if he is located in market m_i. Finally, if $u_i(k) < 0$ the consumer never purchases firm i's product.

Conditional on the consumer's truthfully declaring his type, the firm's profit when contracting with type k is given by $p_i(k) - \frac{q_i(k)^2}{2}$. Substituting for $p_i(k) = kq_i(k) - u_i(k)$ this becomes

$$kq_i(k) - u_i(k) - \frac{q_i(k)^2}{2} \tag{3}$$

Firm i's programme is to maximize

$$\int_{1-z}^{1} M_i(u_i(k), k) \left(kq_i(k) - u_i(k) - \frac{q_i(k)^2}{2} \right) dk \tag{P}$$

subject to incentive compatibility:

$$k = \arg\max_{\widehat{k}} \quad kq_i\left(\widehat{k}\right) - p_i\left(\widehat{k}\right) \tag{IC}$$

Lemma 1: *The following conditions are necessary and sufficient for incentive compatibility:*

IC.1 $u_i'(k) = q_i(k)$

IC.2 $q_i(k)$ *is non-decreasing in* k.

Condition IC.1 is the first order condition for local incentive compatibility, while condition IC.2 is the second-order condition. Together, these two conditions ensure global incentive compatibility. In what follows, we study the properties of the mechanism obtained when imposing condition IC.1 only. In the appendix, we show that the properties so derived also extend to the overall optimal mechanism.

Lemma 1 identifies the conditions which need to hold for both firms at equilibrium. This allows us to make some inferences concerning the consumer's reservation utility, when he is located in market m_c. From above, we know that

$$B_i(k) \equiv \max(0,\, u_{-i}(2-z-k))$$

Hence, $B_i'(k)$ is either equal to 0 or it is equal to $-u_{-i}'(2-z-k)$. From lemma 1, we know that

$$-u_{-i}'(2-z-k) = -q_{-i}(2-z-k) \rightarrow \quad \leq 0$$

where $q_{-i}(2-z-k)$ denotes the product quality which a consumer of type $k_{-i} = 2-z-k$ is offered when contracting with firm $-i$. Thus, $B_i'(k)$ is non-positive. Moreover, from lemma 1, the utility schedules offered by the two firms at equilibrium must be continuous. This brings us to the following lemma.

Lemma 2: *At equilibrium, the consumer's reservation utility when he is located in market m_c is continuous and non-increasing in his type.*

Lemma 2 characterizes the properties of the consumer's reservation utility in any symmetric equilibrium. Define firm i's marginal type k_i^M as the type for whom[11]

$$u_i\left(k_i^M\right) = B_i\left(k_i^M\right)$$

Because $B_i(k)$ is non-increasing in type, we can rewrite the measure $M_i(u_i(k), k)$ of con-

[11] We are implicitely assuming that the $u_i(k)$ and $B_i(k)$ schedules do not overlap for more than one type. This may however be the case if $u_i(k) = B_i(k) = 0$ for a whole set of types. In that case, the marginal type is defined as the highest type for whom $u_i(k) = B_i(k)$.

sumers of type k who contract with firm i as[12]:

$$M_i(u_i(k),k) = \begin{cases} \frac{s}{z} \text{ for } k \in \left[1-z, k_i^M\right[\\ \frac{s+1}{2z} \text{ for } k = k_i^M \\ \frac{1}{z} \text{ for } k \in \left]k_i^M, 1\right] \end{cases}$$

This is the case because the $u_i(k)$ and $B_i(k)$ schedules increase in opposite directions. Hence, they may cross only at one type. The implication is that $M_i(u_i(k),k)$ has a discontinuity at $k = k_i^M$. This is in contrast with Rochet-Stole (2002), where the firm's market share is a continuous function of the consumer's type. Moreover, we have

$$M_i'(u_i(k),k) = \begin{cases} 0 \text{ for } k \in \left[1-z, k_i^M\right[\\ \frac{1-s}{2z} \text{ for } k = k_i^M \\ 0 \text{ for } k \in \left]k_i^M, 1\right] \end{cases}$$

The increment in firm i's market share that results from a marginal rise in the consumer's utility is also discontinuous function of the consumer's type. Intuitively, from the definition of k_i^M, all types above k_i^M purchase firm i's product with probability one, independently of their market location. Hence, an increment in the utility offered to those types does not modify the probability with which they purchase firm i's product. Now consider those types located below k_i^M. For those types, $B_i(k) > u_i(k)$. Denoting the increment in the utility obtained when contracting with firm i as ε, we have $B_i(k) > u_i(k) + \varepsilon$ for ε sufficiently small. Hence, for types located below k_i^M a marginal increment is not sufficient to alter their purchasing behaviour when located in the competitive market. Finally, consider type k_i^M. For this type, $B_i\left(k_i^M\right) = u_i\left(k_i^M\right)$ and $B_i\left(k_i^M\right) < u_i\left(k_i^M\right) + \varepsilon$ for any $\varepsilon > 0$. By marginally increasing the utility that type k_i^M obtains, firm i is able to increase its market share; this is the case because the firm now trades with type k_i^M with probability 1, rather than $(1+s)/2$.

Hence, the price elasticities of demand possess the following feature: if firm i decreases its pricing schedule by a small amount, the units sold at quality level $q_i\left(k_i^M\right)$ increase, while those sold at quality levels above/below $q_i\left(k_i^M\right)$ remain unchanged. Again, this is in contrast with Rochet-Stole (2002), where the consumer's reservation utility is a smooth function of his type,

[12] If $u_i(k) = B_i(k) = 0$ for a whole set of types, the measure $M_i(u_i(k), k)$ of consumers of type k who contract with firm i is

$$M_i(u_i(k),k) = \begin{Bmatrix} \frac{s+1}{2z} \text{ for } k \in \left[1-z, k_i^M\right] \\ \frac{1}{z} \text{ for } k \in \left]k_i^M, 1\right] \end{Bmatrix}$$

and demand effects are accordingly smooth.

We now proceed to characterizing the equilibrium quality schedule. From condition IC.1 we have

$$u_i(k) = u_i(1-z) + \int_{1-z}^{k} q_i(x)\,dx \rightarrow \quad \geq u_i(1-z)$$

Hence, setting $u_i(1-z) \geq 0$ is sufficient to ensure the participation of all types located in the local market m_i. This is because these types have a null outside option, and will therefore accept any contract that gives them a non-negative utility. There is therefore no loss of generality in restricting attention to the case where each firm contracts will all types with a positive probability. In what follows, however, we will use the term "trade" as a synonym for "surplus-creating trade", i.e. to designate situations where the consumer and the firm trade a good that has stictly positive quality. Contracts that prescribe trade of a good of null quality will be designated as "null contracts", involving no trade.

We now explore the properties of the optimal mechanism. If we ignore monotonicity concerns, firm i's problem, $i = l, r$, can be written as

$$\max_{u_i(k), q_i(k)} \int_{1-z}^{1} M_i(u_i(k), k)\left(kq_i(k) - \frac{q_i(k)^2}{2} - u_i(k)\right) dk \tag{P}$$

subject to

$$u_i'(k_i) = q_i(k_i) \tag{IC1}$$

In the canonical setting, where the firm under consideration is a monopolist, the consumer's reservation utility corresponds to the utility the consumer derives if he foregoes consumption altogether, and is therefore equal to zero for all types. When this is the case, setting $u_i(1-z) \geq 0$ is sufficient to ensure the participation of *all* consumers. Here, in contrast, the consumer 's reservation utility when located in market m_c is equal to the utility he derives from purchasing the product sold by the firm's rival, and is therefore strictly positive for a non-empty set of types. Hence, although setting $u_i(1-z) \geq 0$ is sufficient to ensure the participation of all types located in the local market m_i, this is not necessarily the case for those consumers located in market m_c, the competitive market. The implication is that an upward shift in the consumer's utility schedule may allow firm i to expand its market share. This is in contrast with the standard setting, where an upward shift in the consumer's utility schedule has no impact upon the monopolist's sales volume.

The consequences of this extra effect upon the optimal mechanism can be be best illustrated in a finite-type setting. Suppose that the type-set is discrete, with N components, each of lenght $1/N$. Ignoring monotonicity concerns, the firm's problem is

$$\max_{\substack{u_i(k)\\q_i(k)}} \sum_{1-z}^{1} M_i\left(u_i(k), k\right)\left(kq_i\left(k\right) - \frac{q_i\left(k\right)^2}{2} - u_i(k)\right) \tag{P}$$

subject to the incentive compatibility constraint:

$$u_i\left(k\right) - u_i\left(k - \frac{1}{N}\right) = \frac{q_i\left(k\right)}{N} \tag{IC.1}$$

From IC.1, we have

$$u_i(k) = u_i(1-z) + \frac{1}{N}\sum_{1-z+\frac{1}{N}}^{k-\frac{1}{N}} q(x) \tag{4}$$

The problem can therefore be rewritten as

$$\max_{\substack{u_i(1-z)\\q_i(k)}} \sum_{1-z}^{1} M_i\left(u_i(1-z) + \frac{1}{N}\sum_{1-z+\frac{1}{N}}^{k-\frac{1}{N}} q_i(x),\ k\right)\left(kq_i\left(k\right) - \frac{q_i\left(k\right)^2}{2} - u_i(1-z) - \frac{1}{N}\sum_{1-z+\frac{1}{N}}^{k-\frac{1}{N}} q_i(x)\right) \tag{5}$$

The derivative of (5) with respect to $u_i\left(1-z\right)$ is

$$-\frac{1}{N}\sum_{k}^{1} M_i\left(u_i(x), x\right) + \frac{1}{N}\sum_{k}^{1} M_i'\left(u_i(x), x\right)\left(kq_i\left(x\right) - \frac{q_i\left(x\right)^2}{2} - u_i(x)\right) \tag{6}$$

A change in $u_i\left(1-z\right)$ generates a shift in the consumer's utility schedule. This has two effects: on one hand, it affects the rents allocated to the consumer, whenever he purchases from firm i; this effect is captured by the first term in (6). On the other hand, however, a shift in the consumer's utility schedule also alters the firm's market share. This effect is captured by the second term in (6). Figure 1 illustrates how an increment in $u_i\left(1-z\right)$ allows firm i to increase its market share. The upward shift in the consumer's utility schedule that arises from the increment in $u_i\left(1-z\right)$ results in the firm's marginal type moving from k_i^M to $k_i^{M'} < k_i^M$. The range of types that purchase exclusively from firm i is expanded, from $\left]k_i^M, 1\right]$ to $\left]k_i^{M'}, 1\right]$.

The presence of this market share effect of a change in $u_i\left(1-z\right)$ has important implications for the equilibrium contract. In particular, it may induce the firm to optimally set $u_i\left(1-z\right) > 0$. This is in contrast with the standard monopoly setting, where the participation constraint of the lowest type is always binding.

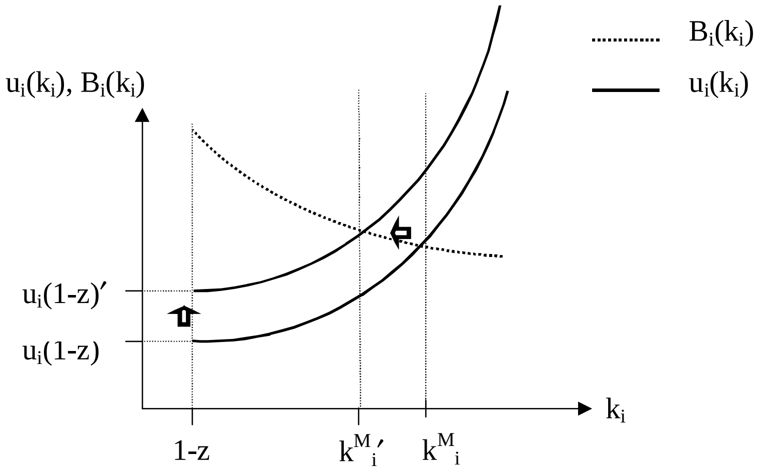

Figure 1:

The derivative of (5) with respect to $q_i(k)$ is

$$M_i(u_i(k),\ k)(k - q_i(k) - u_i(k)) - \frac{1}{N}\sum_{k}^{1} M_i(u_i(x), x) + \tag{7}$$
$$+\frac{1}{N}\sum_{k}^{1} M_i'(u_i(x), x)\left(kq_i(x) - \frac{q_i(x)^2}{2} - u_i(x)\right)$$

In addition to the standard efficiency/informational rents trade-off the firm is confronted with an extra effect, that arises from the fact that by increasing $q_i(k)$ the firm can enlarge the mass of types with whom it contracts. This effect – captured by the last term in (7) – emerges because a change in $q_i(k)$ generates a shift in the consumer's utility schedule, that initiates at $u_i(k+\frac{1}{N})$. In a similar manner to a movement in $u_i(1-z)$, a movement in $q_i(k)$ might therefore affect firm i's market share. Importantly, however, this maket share effect of a change in $q_i(k)$ arises only for types situated *below* k_i^M, the firm's marginal type. This can be seen by recalling that

$$M_i'(u_i(k), k) = \begin{cases} 0 \text{ for } k \in \left[1-z, k_i^M\right[\\ \frac{1-s}{2z} \text{ for } k = k_i^M \\ 0 \text{ for } k \in \left]k_i^M, 1\right] \end{cases}$$

Hence, in order for a marginal shift in the utility schedule to have an impact upon the firm's market share, it is necessary that the shift affect the utility offered to the marginal type k_i^M. In turn, this can only be generated by a movement in the quality allocation of types situated *below* k_i^M

A second important observation is that the market share effect of a marginal increment in $q_i(k)$ is the same for all $k < k_i^M$. Intuitively, for an arbitrarily small shift in the consumer's utility schedule, $k_i^{M'}$ tends to k_i^M. In that case, the expansion in firm i's market share consists in the firm trading with type k_i^M with probability 1, rather than $(1+s)/2$. The extra profit that this generates is independent of the precise location at which the shift is initiated. This is in contrast with Rochet-Stole (2002), where the market share effect of an upward shift in the consumer's utility schedule depends upon the location at which the shift begins.

The intuitions derived in the finite-type setting carry over to the continuous-type case. As shown in the appendix, at an interior optimum, $u_i(1-z)$ satisfies

$$\underbrace{-\int_{1-z}^{1} M_i(u_i(x),\ x)\,dx}_{\text{informational rents effect}} + \underbrace{\int_{1-z}^{1} M_i'(u_i(x),\ x)\left(xq_i(x) - \frac{q_i(x)}{2} - u_i(x)\right)dx}_{\text{market share effect}} = 0 \qquad \text{(E1)}$$

and the optimal quality allocation in the absence of bunching satisfies

$$\underbrace{M_i(u_i,\ k)(k - q_i(k)) - \int_{k}^{1} M_i(u_i(x),\ x)\,dx}_{\text{efficiency/informational rents trade-off}} + \underbrace{\int_{k}^{1} M_i'(u_i(x),\ x)\left(xq_i(x) - \frac{q_i(x)}{2} - u_i(x)\right)dx}_{\text{market share effect}} = 0 \qquad \text{(E2)}$$

The last term in (E2) captures the market share effect generated by a marginal upward shift in $u_i(k+\varepsilon)$, for some ε arbitrarily small. As discussed above, for types located above the marginal type this effect is null; as a result, for those types the optimal quality allocation is found by optimally balancing informational rents and efficiency – the same trade-off as in the canonical monopoly setting. Generally, therefore, for sufficiently high types the familiar Mussa and Rosen (1978) result of underprovision and "efficiency at the top" also persists in the duopoly setting. This brings us to the following proposition.

Proposition 1: *For sufficiently high types, the optimal quality allocation exhibits the same properties as that offered by a monopolist. In any equilibrium where the marginal type is the lowest type with whom firms trade, this is the case for all types.*

Although undeprovision and "efficiency at the top" persist for sufficiently high types, for lower types the competitive stimulus provided by the presence of a rival might induce firms to offer quality levels that are above those that would be offered by a monopolist. As proposition 2 indicates, this is always the case whenever z is sufficiently small.

Proposition 2: *When the marginal type is not the lowest type with whom firms trade, the quality level offered to sufficiently low types is above what would be offered by a monopolist. A sufficient condition for this to be the case at equilibrium is that* $z < \frac{\left(-\frac{1}{2}\sqrt{-0.37\,s-0.19\,s^2+1.56}+0.5\right)}{0.19s-0.19}$.

Intuitively, a smaller z corresponds to a higher the degree of substitutability between the brands sold by the two firms, and therefore to a more competitive environment. When competition is sufficiently strong, firms have an incentive to inflate the quality levels they offer to sufficiently low types. This is because, through incentive compatibility, higher quality offers to low types directly translate into higher utility offers to higher types. Hence, by offering higher quality levels to low types, each firm is able to increase its market share in market m_c.

Figure 2 depicts $z < \frac{1}{0.19s-0.19}\left(-\frac{1}{2}\sqrt{-0.37\,s-0.19\,s^2+1.56}+0.5\right)$, the threshold value of z below which competition starts to bite. This value is increasing in s, reaching 1 for $s \to 1$. This is because, as seen above, the market share effect is only present for sufficiently low types, namely those that are located below the marginal type; hence, the presence of competition may alter the equilibrium quality schedule only in equilibria where firms trade with types located below the marginal type, i.e. with types that contract with the firm only when they are located in the firm's local market. Larger local markets – that is, a larger s – make trade with these types more attractive, and therefore increase the likelyhood of obtaining quality levels that are above those that would be offered by a monopolist.

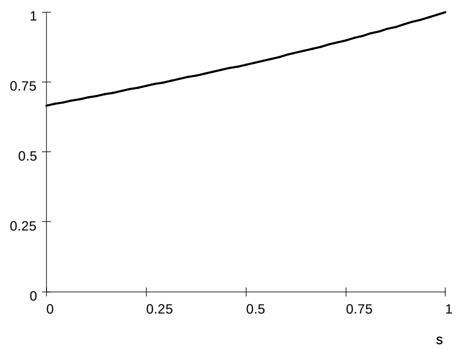

Figure 2

Proposition 2 tells us that, conditional on rivalry being sufficiently strong, the presence of competition reduces the distortions that are associated with monopoly power. This suggests that competition might be welfare-enhancing. The next proposition shows that this intuition might be misguided: when competition is very fierce, the presence of a rival induces firms to inefficiently inflate the quality offered to low types. Hence, with respect to a monopoly setting, competition might introduce new distortions, namely upward distortions.

Proposition 3: *When* $\frac{\left(\frac{1}{2}\sqrt{96s-48s^2+208}-8\right)}{3s-3}$ *any symmetric equilibrium exhibits overprovision for sufficiently low types.*

The proof of proposition 3 consists in two steps. First, we show that if at equilibrium we have $u_i\left(1-z\right)>0$, the optimal quality schedule always features overprovision for sufficiently low types. Hence, overprovision may *not* occur if and only if $u_i\left(1-z\right)=0$ at equilibrium. Second, we show when $z<\frac{1}{3s-3}\left(\frac{1}{2}\sqrt{96s-48s^2+208}-8\right)$, any equilibrium where $u_i\left(1-z\right)=0$ must also exhibit overprovision for sufficiently low types.

The intuition for the first part of the proof can be seen as follows: at an interior solution, the optimal $u_i\left(1-z\right)$ balances the firm's desire to enlarge his market share on one hand, and his desire to minimize the rents that have to be offered to the agent in order to ensure incentive compatibility on the other. Now consider the firm's choice of $q_i\left(1-z\right)$; this choice is determined by the interplay of three factors: the desire to expand the market share, the desire to minimize the informational rents that have to be offered to the consumer, and the the desire to maximize the total surplus obtained when contracting with type $1-z$. Because $u_i\left(1-z\right)$ has been chosen optimally, however, the first two factors annull each other; hence, in his contractual offer to type $1-z$, the principal has no incentive to deviate from the efficient quality level. Now consider a type k, located above $1-z$ but below the marginal type k_i^M. Evaluated at k, the market share effect has the *same* strenght as for type $1-z$, but the increment in informational rents that result from an increase in $q_i\left(k\right)$ is strictly *lower* than that arising from an increase in $q_i\left(1-z\right)$. This is because, in the present setting, the incentive compatibility constraint binds downwards; higher valuation types need to be offered rents in order to be dissuaded from understating their true valuations. It follows that the principal's incentive to increase quality must be higher for type $k\in\left]1-z,\,k_i^M\right]$ than for type $1-z$. Given that $q_i\left(1-z\right)$ is equal to the efficient level, we conclude that $q_i\left(k\right)$ must be *above* the efficient level.

The second part of the proof shows that, when z is sufficiently small, a situation where $u_{-i}(1-z)=0$ and overprovision does not occur for any type, the consumer's reservation utility would be sufficiently low to give firm i an incentive to capture a share of the competitive market that is above $1/2$. Clearly, this cannot be the case in any symmetric equilibrium.

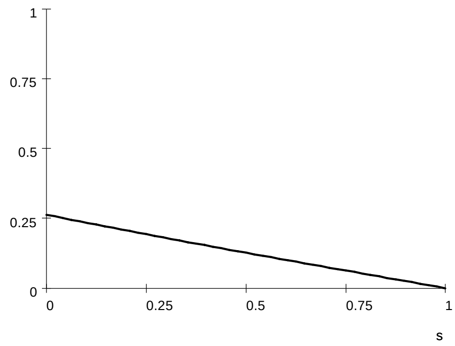

Figure 3

Figure 3 depicts $\frac{1}{3s-3}\left(\frac{1}{2}\sqrt{96s-48s^2+208}-8\right)$, the threshold value of z below which over-provision occurs for sufficiently low types. This value is a decreasing function of s, the size of the local market. Intuitively, firms will find it worthwhile to introduce distortions aimed at increasing their market share in the competitive market only if $1-s$, the density of consumers located in this market, is sufficiently high. Moreover, when s is small the expected cost of distorting the quality levels offered to sufficiently low types is also small. This is because low types only contract with firm i when they are located in firm i's local market. If the likelyhood of contracting with those types is small – that is, if s is small – the firm is less reluctant to introduce these distortions.

4 Concluding remarks

This paper contributes to the literature that studies nonlinear pricing within a duopoly setting in which products are spatially differentiated a la Hotelling (1929). Its novelty consists in combining two empirically sound features – namely the notion that firms may be serving monopolistic as well as competitive markets and the assumption that consumers who have a higher valuation for quality are also more brand loyal – and showing that these have important implications for the relationship between "toughness of competition" and welfare. In particular, we find that a strongly competitive environment will induce firms to inefficiently inflate the quality levels that they are offering to sufficiently low types. As a consequence, stronger competition may not necessarily result in higher efficiency. This suggests that the relationship between "toughness of competition" and welfare may not be monotonic.

5 Appendix

Proof of lemma 1:

The proof is standard and will be omitted.

Proof of lemma 2:

In text.

Proof of proposition 1:

Firm i solves

$$\max_{u_i(1-z),q_i(k)} \int_{1-z}^{1} M_i\left(u_i(k),k\right)\left(kq_i\left(k\right)-\frac{q_i\left(k\right)^2}{2}-u_i(k)\right)dk \tag{P}$$

subject to

$$u_i'(k)=q_i(k)$$

and

$$q_i\left(k\right) \text{ non decreasing in } k$$

The structure of the problem is identical to that analyzed by Rochet-Stole (2002). The Lagrangean for the problem is

$$L=\int_{1-z}^{1}\left(M_i\left(u_i,\ k\right)\left(kq_i(k)-\frac{q_i(k)}{2}-u_i(k)\right)-\lambda(k)\left(u'(k)-q_i(k)\right)\right)dk \tag{L}$$

where the multiplier $\lambda(k)$ is the costate variable for the problem. After integration by parts, we obtain

$$L=\int_{1-z}^{1} M_i\left(u_i,\ k\right)\left(kq_i(k)-\frac{q_i(k)}{2}-u_i(k)+\lambda q_i(k)+\lambda'(k)u_i(k)\right)dk-\lambda(1)u(1)+\lambda(1-z)u(1-z)$$

The Hamiltonian for the problem is:

$$H(q_i,u_i,k,\lambda)\equiv M_i\left(u_i,\ k\right)\left(kq_i-\frac{q_i}{2}-u_i\right)+\lambda q_i \tag{H}$$

Thus,

$$L=\int_{1-z}^{1}\left(H(q_i,u_i,k,\lambda)+\lambda' u_i\right)dk-\lambda(1)u(1)+\lambda(1-z)u(1-z)$$

The necessary and sufficient conditions[13] for optimality are[14]:

$$\lambda(1-z) = \lambda(1) = 0 \tag{i}$$

$$\lambda'(k) = M_i(u_i(k),\ k) - M_i'(u_i(k),\ k)\left(kq_i(k) - \frac{q_i(k)}{2} - u_i(k)\right) \tag{ii}$$

$$M_i(u_i(k), k)(k - q_i(k)) = -\lambda(k) \tag{iii}$$

From (i) and (ii) we can write

$$\lambda(k) = -\int_k^1 \lambda'(x)dx = -\int_k^1 \left\{M_i(u_i(x),\ x) - M_i'(u_i(x),\ x)\left(xq_i(x) - \frac{q_i(x)}{2} - u_i(x)\right)\right\} dx \tag{8}$$

From (i) we know that: $\int_{1-z}^1 \lambda'(x)dx = 0$. From (8), this implies that

$$-\int_{1-z}^1 M_i(u_i(x),\ x)\, dx + \int_{1-z}^1 M_i'(u_i(x),\ x)\left(xq_i(x) - \frac{q_i(x)}{2} - u_i(x)\right) dx = 0 \tag{E1}$$

Moreover, substituting for $\lambda(k)$ in (iii), we obtain

$$M_i(u_i,\ k)(k - q_i(k)) - \int_k^1 M_i(u_i(x),\ x)\, dx + \int_k^1 M_i'(u_i(x),\ x)\left(xq_i(x) - \frac{q_i(x)}{2} - u_i(x)\right) dx = 0 \tag{E2}$$

Condition (E1) is the first order condition with respect to $u_i(1-z)$. The first term on the lefthandside of (E1) captures the effect that a marginal change in $u_i(1-z)$ has upon the rents that have to be offered to all types. The second term on the lefthandside of (E1) captures the market share effect of a marginal change in $u_i(1-z)$. If the lefthandside of (E1) is negative for all $u_i(1-z) \geq 0$, we have a corner solution, and the optimal $u_i(1-z)$ is equal to zero. Condition (E2) is the first order condition with respect to $q_i(k)$. The first term on the lefthadside of (E2) captures the effect that a marginal change in $q_i(k)$ has upon the surplus created from trade between the firm and the consumer. The second term on the lefthadside of (E2) captures the effect that a marginal change in $q_i(k)$ has upon the informational rents that have to be offered to all types above k, in order to ensure incentive compatibility. The last term on the lefthadside

[13] See Rochet-Stole (2002), p.309.

[14] More precisely, condition (i) requires

$$\begin{aligned} \lambda(1) &\geq 0,\ \lambda(1)\, u(1) = 0 \\ \lambda(1-z) &\geq 0,\ \lambda(1-z)\, u(1-z) = 0 \end{aligned}$$

Conditional on trade occurring, $u(1) = 0 \to \lambda(1) = 0$. Moreover, at an interior solution, $u(1-z) \to \lambda(1-z) = 0$.

of (E2) captures the market share effect of a marginal change in $q_i(k)$.

We now show that

$$\int_k^1 M_i'(u_i(x),\ x)\left(xq_i(x)-\frac{q_i(x)}{2}-u_i(x)\right)dx=\begin{cases}0 & \text{for } k>k_i^M\\ \frac{1-s}{2z}\left(k_i^M q_i\left(k_i^M\right)-\frac{q_i\left(k_i^M\right)}{2}-u_i\left(k_i^M\right)\right) & \text{for } k\le k_i^M\end{cases}$$

As mentioned in the main text, when $k>k_i^M$ the consumer is already purchasing firm i's product with probability one. For those types, therefore, an increment in the consumer's utility does not alter $M_i(u_i(k),k)$, the mass of consumers that trade with firm i. Hence, any upward shift in the consumer's utility schedule that starts at $k>k_i^M$ has no effect upon the firm's expected profit. Now consider an upward shift in the consumer's utility schedule that starts at some type $k\le k_i^M$. Denote the size of this shift by ε. After the shift, firm i's new marginal type $k_i^{M'}$ is equal to $k_i^{M'}=\max\left(k,\widetilde{k}\right)$, where $\widetilde{k}$ is defined by $u_i(\widetilde{k})+\varepsilon=B_i\left(\widetilde{k}\right)$. Hence, we have

$$\begin{aligned}\int_k^1 M_i'(u_i(x),\ x)\left(xq_i(x)-\frac{q_i(x)}{2}-u_i(x)\right)dx &= \int_{k_i^{M'}}^{k_i^M}\frac{1-s}{z}\left(xq_i(x)-\frac{q_i(x)}{2}-u_i(x)\right)dx+\\ &\quad \frac{1-s}{2z}\left(k_i^M q_i\left(k_i^M\right)-\frac{q_i\left(k_i^M\right)}{2}-u_i\left(k_i^M\right)\right)\end{aligned}$$

Notice that $\lim_{\varepsilon\to 0}\widetilde{k}=k_i^M$; that is, when considering an infinitesimally small shift, we can approximate $\widetilde{k}=k_i^M$. Hence, the extra profit the firm earns after a marginal shift in the consumer's utility schedule that starts at $k\le k_i^M$ is equal to the profit that the firm gains from trading with k_i^M with probability 1, rather than $(1+s)/2$.

We now proceed to prove proposition 1. First, we show that in equilibria where the lowest type with whom firms trade is the marginal type, the optimal quality schedule exhibits the same properties as that offered by a monopolist. We then show that for sufficiently high types, this is the case in all equilibria.

Consider an equilibrium where the lowest type with whom each firm trades is the marginal type. At equilibrium, all $k<k_i^M$ are offered the null contract: $q_i(k)=0$. For $k>k_i^M$, the optimal quality allocation is given by

$$M_i(u_i,\ k)(k-q_i(k))=\int_k^1 M_i(u_i(x),\ x)\,dx \tag{9}$$

i.e.

$$\frac{1}{z}\left(k - q_i\left(k\right)\right) = \frac{1-k}{z} \rightarrow q\left(k\right) = 2k - 1 \tag{10}$$

For $k = k_i^M$, the optimal quality allocation is given by

$$\frac{1}{2z}\left(k_i^M - q\left(k_i^M\right)\right) = \frac{1-k_i^M}{z} \rightarrow q\left(k_i^M\right) = 3k_i^M - 2 \tag{11}$$

As discussed in the main text, for $k \geq k_i^M$ the market share effect of a marginal change in $q_i\left(k\right)$ is null; this is because all types $> k_i^M$ are already purchasing from firm i with probability one, independently of their market location. Hence, an increment in the utility level offered to those types does not alter the firm's market share. We conclude that for $k \geq k_i^M$ the trade-off faced by the firm when deciding upon quality allocations is exactly the same as that faced by a monopolist. By decreasing $q_i\left(k\right)$ below its efficient level, the firm reduces the informational rents that have to be allocated to all types above k; evaluated at $q_i\left(k\right) = q_i^{FB}\left(k\right)$ – where $q_i^{FB}\left(k\right)$ indicates the surplus-maximizing quality allocation for type k – this effect dominates the second-order efficiency loss generated by a marginal reduction in $q_i\left(k\right)$. Hence, the optimal quality allocation prescribes underprovision for all types, except the highest. In the present setting, $q_i^{FB}\left(k\right)$ maximizes $kq_i - \frac{q_i^2}{2}$, and is therefore equal to k. It can be easily verified that the optimal quality allocations for all $k \in \left[k_i^M, 1\right[$ are below their surplus-maximizing value, while the optimal quality allocation for the highest type is at the efficient level.

Now consider the optimal $u_i\left(1-z\right)$. From incentive compatibility, we have

$$u_i\left(k\right) = u_i\left(1-z\right) + \int_{1-z}^{k} q_i\left(k\right)dk$$

In the equilibrium under consideration, all $k < k_i^M$ are offered the null contract: $q_i\left(k\right) = 0$. Hence, $u_i\left(k\right) = u_i\left(1-z\right)$ for all $k \leq k_i^M$. When trading with $k < k_i^M$, firm i's profit is equal to $-u_i\left(1-z\right)$. We now show that in any equilibrium where the lowest type with whom each firm trades is the marginal type, we must have $u_i\left(1-z\right) = 0$. Suppose that this was not the case; consider type $k_i^M - \varepsilon$, for some very small ε. Suppose that instead of offering $q_i\left(k_i^M - \varepsilon\right) = 0$, $u_i\left(k_i^M - \varepsilon\right) = u_i\left(1-z\right)$ firm i was to offer $q_i\left(k_i^M - \varepsilon\right) = q_i(k_i^M) - \varepsilon$, $u_i\left(k_i^M - \varepsilon\right) = u_i\left(1-z\right) - \varepsilon q_i\left(k_i^M - \varepsilon\right)$. Under this new contract, the profit firm i would earn when trading with types $\geq k_i^M$ would remain unchanged, while that earned when trading with type $k_i^M - \varepsilon$ would be given by $\left(k_i^M - \varepsilon\right)q_i\left(k_i^M - \varepsilon\right) - \frac{q_i\left(k_i^M-\varepsilon\right)^2}{2} - u_i\left(k_i^M - \varepsilon\right)$, instead of $-u_i\left(1-z\right)$. In the limit, as ε approaches 0, $\left(k_i^M - \varepsilon\right)q_i\left(k_i^M - \varepsilon\right) - \frac{q_i\left(k_i^M-\varepsilon\right)^2}{2} - u_i\left(k_i^M - \varepsilon\right)$ approaches $k_i^M q_i(k_i^M) - \frac{q_i(k_i^M)^2}{2} - u_i\left(1-z\right)$. This is strictly higher than $-u_i\left(1-z\right)$ if $k_i^M q_i(k_i^M) - \frac{q_i(k_i^M)^2}{2} > 0$. Notice that in the equilibrium under consideration, this inequality must necessarily hold. If this was not the case, then surely the lefthandside of (E1) would be negative. This would contradict the initial assumption that the optimal $u_i(1-z)$ is strictly positive. Hence, when the marginal type is the lowest type with whom both firms trade, a situation where $u_i(1-z) > 0$ is necessarily dominated. We conclude that when the marginal type is the also lowest type with whom both firms trade, the optimal $u_i(1-z)$ must be equal to zero.

Ignoring monotonicity concerns, any equilibrium where the marginal type is also the lowest type with whom firms trade is therefore characterized by

$$u_i(1-z) = 0,\ q_i(k) = \begin{cases} 0 \text{ for } k < k_i^M \\ 3k-2 \text{ for } k = k_i^M \\ 2k-1 \text{ for } k > k_i^M \end{cases}$$

It can be easily verified that this mechanism also satisfies constraint IC.2.

Now consider an equilibrium where firms also trade with types below the marginal type. For $k \geq k_i^M$, the unconstrained optimal quality is the same as above:

$$q_i(k) = \begin{cases} 3k-2 \text{ for } k = k_i^M \\ 2k-1 \text{ for } k > k_i^M \end{cases}$$

However, in this case there is the possibility that the unconstrained quality schedule might violate the monotonicity constraint, IC.2, in which case the overall optimal quality schedule will exhibit pooling over some interval $[k_0, k_1]$.

Denote as $q_i(k)$ the unconstrained optimal quality schedule, and as $\overline{q}_i$ the pooling quality level offered to $k \in [k_0, k_1]$. At an optimum we have[15]:

$$\overline{q}_i = q_i(k_0) = q_i(k_1)$$

and

$$\int_{k_0}^{k_1} \left\{ M_i(u_i,\ k)(k - q_i(k)) - \int_k^1 M_i(u_i(x),\ x)\,dx + M_i'(u_i(x),\ x)\left(xq_i(x) - \frac{q_i(x)}{2} - u_i(x)\right)dx \right\} dk = 0 \qquad (12)$$

Notice that for $k \leq k_0$ and $k \geq k_1$ the overall optimal quality allocation is equal to the unconstrained quality allocation. Hence, for "sufficiently low" and "sufficiently high" types the

[15] See for instance Laffont-Martimort (2002).

properties of the unconstrained quality schedule extend to the overall optimal mechanism. For what follows, we therefore concentrate on the properties of the quality schedule obtained when imposing condition IC.1 only. ■

Proof of proposition 2:

The proof proceeds in two steps. We first show that when $z < \frac{1}{0.19s-0.19}\left(-\frac{1}{2}\sqrt{-0.37\,s-0.19\,s^2+1.\,56}+0.5\right)$, a situation where marginal type is also the lowest type with whom firms trade cannot be an equilibrium. Hence, any equilibrium must have the firms trading also with types located below the marginal type. We then show that any equilibrium where this is the case prescribes that the quality allocations to sufficiently low types are above those that would be offered by a monopolist.

First step:

Consider an equilibrium where the marginal type is the lowest type with whom firms trade. Take type $k_i^M-\varepsilon$, where ε is arbitrarily small; the equilibrium contractual offer prescribes $q_i\left(k_i^M-\varepsilon\right)=0$. We now prove that when z is sufficiently small, setting $q_i\left(k_i^M-\varepsilon\right)=0$ is suboptimal.

Recall the first order condition with respect to quality allocation:

$$M_i\left(u_i,\ k\right)\left(k-q_i\left(k\right)\right)-\int_k^1 M_i\left(u_i\left(x\right),\ x\right)dx+\int_k^1 M_i'\left(u_i\left(x\right),\ x\right)\left(xq_i\left(x\right)-\frac{q_i\left(x\right)}{2}-u_i\left(x\right)\right)dx=0 \tag{E2}$$

From the proof of proposition 1, we know that $\int_k^1 M_i'\left(u_i\left(x\right),\ x\right)\left(xq_i\left(x\right)-\frac{q_i(x)}{2}-u_i\left(x\right)\right)=\frac{1-s}{2z}\left(k_i^Mq_i\left(k_i^M\right)-\frac{q_i\left(k_i^M\right)}{2}-u_i\left(k_i^M\right)\right)$. Hence, condition (E2) can be rewritten as

$$M_i\left(u_i,\ k\right)\left(k-q_i\left(k\right)\right)-\int_k^1 M_i\left(u_i\left(x\right),\ x\right)dx+\frac{1-s}{2z}\left(k_i^Mq_i\left(k_i^M\right)-\frac{q_i\left(k_i^M\right)}{2}-u_i\left(k_i^M\right)\right)=0$$

In the equilibrium we are considering, the types located in the interval $\left[2-z-k_i^M,k_i^M\right]$[16] are offered the null contract $q\left(k\right)=u\left(k\right)=0$ by both firms. Whenever they are located in the competitive market, these types randomize, and contract with each firm with probability 0.5.

[16] This can be seen as follows: the consumer is offered the null contract by both firms whenever both k_i and k_{-i} are $\leq k_i^M=k_{-i}^M$, the symmetric marginal type. Recall that $k_{-i}=2-z-k_i$. Hence, $k_{-i}\leq k_i^M$ can also be written as $2-z-k_i\leq k_i^M$, i.e. $2-z-k_i^M\leq k_i$.

Hence, for $k \in \left[2 - z - k_i^M, k_i^M\right]$, $M_i\left(u_i,\ k\right) = \frac{1+s}{2z}$, and $\int\limits_k^1 M_i\left(u_i\left(x\right),\ x\right)dx = \int\limits_k^{k_i^M} \frac{1+s}{2z}dx + \int\limits_{k_i^M}^1 \frac{1}{z}dx$.

Evaluated at $k_i^M - \varepsilon$, condition (E2) therefore becomes

$$\frac{1+s}{2z}\left(k_i^M - \varepsilon\right) - \int\limits_{k_i^M-\varepsilon}^{k_i^M} \frac{1+s}{2z}dx - \int\limits_{k_i^M}^{1} \frac{1}{z}dx + \frac{1-s}{2z}\left(k_i^M q_i\left(k_i^M\right) - \frac{q_i\left(k_i^M\right)^2}{2} - u_i\left(k_i^M\right)\right) = 0 \quad (13)$$

In the limit, as $\varepsilon \to 0$ the lefthandside of (13) becomes

$$\frac{1+s}{2z}k_i^M - \frac{1-k_i^M}{z} + \frac{1-s}{2z}\left(k_i^M q_i\left(k_i^M\right) - \frac{q_i\left(k_i^M\right)^2}{2} - u_i\left(k_i^M\right)\right) \quad (14)$$

From the proof of proposition 1, we know that in any equilibrium where the marginal type is also the lowest type with whom firms trade we have $q_i\left(k_i^M\right) = 3k_i^M - 2$, $u_i\left(k_i^M\right) = 0$. Substituting for these in expression (14) and rearranging we obtain:

$$-0.75{k_i^M}^2\left(1-s\right) + k_i^M\left(3.5 - 1.5s\right) + s - 2 \quad (15)$$

Expression (15) is increasing in k_i^M[17]. The lowest admissible symmetric k_i^M is equal to $1 - 0.5z$[18]. Hence, to prove our claim it is sufficient that (15) be positive when $k_i^M = 1 - 0.5z$. Substituting for $k = 1 - 0.5z$ in (15) we see that this is the case whenever

$$z < \frac{1}{0.19s - 0.19}\left(-\frac{1}{2}\sqrt{-0.37\,s - 0.19\,s^2 + 1.56} + 0.5\right)$$

Second step.

Consider a symmetric equilibrium where the lowest type with whom each firm trades is below the marginal type. In that case, the mass of types that contract with firm i is equal to

$$M_i\left(u_i\left(k\right), k\right) = \begin{cases} \frac{s}{z} \text{ for } k \in \left[1-z, k_i^M\right[\\ \frac{1+s}{z} \text{ for } k = k_i^M \\ \frac{1}{z} \text{ for } k > k_i^M \end{cases}$$

[17]The derivative of this expression with respect to k is equal to $-1.5k(1-s) + 3.5 - 1.5s$. This expression is positive for all $k \leq 1$.

[18]$k_l^M = k_r^M = 1 - 0.5z$ corresponds to a situation where in the competitive market all consumers located in the interval $[0, 0.5z[$ contract exclusively with firm L, all consumers located in $]0.5z, z]$ contract exclusively with firm R, and consumers located at $0.5z$ randomizes between the two firms.

For $k < k_i^M$, the optimal quality allocation (ignoring monotonicity concerns) satisfies

$$\frac{s}{z}\left(k - q_i\left(k\right)\right) - \frac{1 - k_i^M}{z} - \frac{s\left(k_i^M - k\right)}{z} + \phi(k) = 0 \tag{16}$$

where $\phi(k) \equiv \int\limits_k^1 M_i'\left(u_i\left(x\right),\ x\right)\left(xq_i\left(x\right) - \frac{q_i(x)}{2} - u_i\left(x\right)\right)dx$. Now suppose that firm i is a monopolist over both markets m_i and m_c. In that case, the optimal quality allocation $q_i^m\left(k\right)$ satisfies

$$\frac{1}{z}\left(k - q_i^m\left(k\right)\right) - \frac{1-k}{z} = 0 \rightarrow q_i^m\left(k\right) = 2k - 1$$

Evaluated at $q_i\left(k\right) = q_i^m\left(k\right)$, the lefthandside of (16) becomes:

$$\frac{(1-s)(1-k_i^M)}{z} + \phi(k) \tag{17}$$

Hence, $\phi(k) > 0$ for all $k < k_i^M$ is a sufficient condition to prove that, in equilibria where the lowest type with whom firms trade is below the marginal type, the quality allocation for sufficiently low types is above what would be offered by a monopolist. We now show that at equilibrium we must necessarily have $\phi(k) > 0$ for all all $k \leq k_i^M$.

From the proof of proposition 1, we know that for all $k \leq k_i^M$ we can write

$$\phi(k) = \frac{1-s}{2z}\left(k_i^M q_i\left(k_i^M\right) - \frac{q_i\left(k_i^M\right)^2}{2} - u_i\left(k_i^M\right)\right)$$

The market share effect of an upward shift in the utility schedule that initiates at some $k \leq k_i^M$ is equal to the extra profit that the firm gains from trading with the marginal type with probability 1, rather than $(1+s)/2$. Notice that this is independent of k. In what follows, we therefore denote $\frac{1-s}{2z}\left(k_i^M q_i\left(k_i^M\right) - \frac{q_i\left(k_i^M\right)^2}{2} - u_i\left(k_i^M\right)\right)$ simply as ϕ. We now show by contradiction that at equilibrium it must be the case that $\phi > 0$.

Suppose that $\phi \leq 0$. Notice that in order for this to be the case we must have $k_i^M q_i\left(k_i^M\right) - \frac{q_i\left(k_i^M\right)^2}{2} - u_i\left(k_i^M\right) \leq 0$. The optimal quality allocation (ignoring monotonicity concerns) for all $k < k_i^M$ satisfies

$$\frac{s}{z}\left(k - q_i\left(k\right)\right) - \frac{1 - k_i^M}{z} - \frac{s\left(k_i^M - k\right)}{z} + \phi = 0 \tag{18}$$

If $\phi \leq 0$ then necessarily $q_i\left(k\right) < k = q_i^{FB}\left(k\right)$ for all $k < k_i^M$. Moreover, from the proof of proposition 1, we know that the unconstrained quality schedule for $k \geq k_i^M$ also satisfies

$q_i(k) \leq k = q_i^{FB}(k)$, with strict inequality for all $k < 1$. Hence, the overall optimal quality schedule – that is, the optimal quality schedule when taking conditions IC.1 *and* IC.2 into consideration – must satisfy $q_i(k) < k = q_i^{FB}(k)$ for all $k < 1$. Now consider $kq_i(k) - \frac{q_i(k)}{2} - u_i(k)$, the profit that firm i earns when trading with type k; the derivative of this expression with respect to k gives $(k - q_i(k))q_i'(k) \rightarrow > 0$ for all $k < 1$. Hence, for $k < k_i^M$ we must have

$$kq_i(k) - \frac{q_i(k)}{2} - u_i(k) < k_i^M q_i\left(k_i^M\right) - \frac{q_i\left(k_i^M\right)^2}{2} - u_i\left(k_i^M\right) \leq 0$$

But if this is the case, the equilibrium is dominated by one where the firm offers $u_i(k)' = u_i(k) - \varepsilon$, for some small $\varepsilon > 0$, and the marginal type $k_i^{M'}$ is above k_i^M. This brings a contradiction. Hence, in all symmetric equilibria where the lowest type with whom firms trade is below the marginal type, the quality offered to sufficiently low types is above what would be offered by a monopolist. ■

Proof of proposition 3:

The proof consists in two steps. We first prove that if at equilibrium we have $u_i(1-z) > 0$, than necessarily the optimal mechanism must prescrive overprovision for sufficiently low types. Second, we prove that whenever $z < \frac{1}{3s-3}\left(\frac{1}{2}\sqrt{96s - 48s^2 + 208} - 8\right)$, a situation where $u_i(1-z) = 0$ and $q_i(k) \leq q_i^{FB}(k)$ for all $k \leq 1 - \frac{z}{2}$ cannot be an equilibrium.

First step.

From the proof of proposition 1, we know that in any equilibrium where the marginal type is the lowest type with whom firms trade, we must have $u_i(1-z) = 0$. Hence, $u_i(1-z) > 0$ at equilibrium can only occur when the marginal type is not the lowest type with whom firms trade. Recall the first order condition for $u_i(1-z)$:

$$-\int_{1-z}^{1} M_i(u_i(x),\ x)\,dx + \int_{1-z}^{1} M_i'(u_i(x),\ x)\left(xq_i(x) - \frac{q_i(x)}{2} - u_i(x)\right)dx = 0 \qquad \text{(E1)}$$

In any equilibrium where the marginal type is not the lowest type with whom firms trade, we have

$$M_i(u_i(k), k) = \begin{cases} \frac{s}{z} \text{ for } k \in \left[1 - z, k_i^M\right[\\ \frac{1+s}{z} \text{ for } k = \ k_i^M \\ \frac{1}{z} \text{ for } k > \ k_i^M \end{cases}$$

Moreover, from the proof of proposition 1 we know that for all $k < k_i^M$:

$$\int_k^1 M_i' \left(u_i\left(x\right),\ x\right)\left(xq_i\left(x\right) - \frac{q_i\left(x\right)}{2} - u_i\left(x\right)\right)dx =$$

$$= \frac{1-s}{2z}\left(k_i^M q_i\left(k_i^M\right) - \frac{q_i\left(k_i^M\right)^2}{2} - u_i\left(k_i^M\right)\right)$$

After substitution, condition (E1) becomes

$$-\int_{1-z}^{k_i^M} \frac{s}{z}dx - \int_{k_i^M}^{1} \frac{1}{z}dx + \phi = 0 \tag{19}$$

i.e.

$$\phi = \left(1-k_i^M\right)\frac{1-s}{z} + s \tag{20}$$

where $\phi \equiv \frac{1-s}{2z}\left(k_i^M q_i\left(k_i^M\right) - \frac{q_i\left(k_i^M\right)^2}{2} - u_i\left(k_i^M\right)\right)$.

Now consider the optimal quality allocation for $k < k_i^M$; from (E2), this satisfies

$$\frac{s}{z}\left(k - q_i\left(k\right)\right) - \frac{1-k_i^M}{z} - \frac{s\left(k_i^M - k\right)}{z} + \phi(k) = 0 \tag{21}$$

Substituting for $\phi = \left(1-k_i^M\right)\frac{1-s}{z} + s$ this yields

$$q_i\left(k\right) = 2k - 1 + z \rightarrow \ \geq k = q_i^{FB}\left(k\right) \text{ for all } k \geq 1-z, \text{ with strict inequality for } k > 1-z.$$

Second step.

Suppose that $z < \frac{1}{0.19s-0.19}\left(-\frac{1}{2}\sqrt{-0.37\,s - 0.19\,s^2 + 1.56} + 0.5\right)$, implying that at equilibrium firms trade also with types below the marginal type. If $u_i(1-z) = 0$ is optimal, it must be consistent with

$$\phi \leq \left(1-k_i^M\right)\frac{1-s}{z} + s \tag{22}$$

where

$$\phi \equiv \frac{1-s}{2z}\left(k_i^M q_i\left(k_i^M\right) - \frac{q_i\left(k_i^M\right)^2}{2} - u_i\left(k_i^M\right)\right) \tag{23}$$

From incentive compatibility we have

$$u_i\left(k_i^M\right) = u_i(1-z) + \int_{1-z}^{k_i^M} q_i(x)dx \rightarrow \ \leq u_i(1-z) + (k_i^M - 1 + z)q_i(k_i^M) \text{ from constraint IC.2}$$

If $u_i(1-z)=0$, this can be rewritten as

$$u_i\left(k_i^M\right) \leq (k_i^M - 1 + z) q_i(k_i^M) \tag{24}$$

Hence,

$$k_i^M q_i\left(k_i^M\right) - \frac{q_i\left(k_i^M\right)^2}{2} - u_i\left(k_i^M\right) \geq q_i\left(k_i^M\right)(1-z) - \frac{q_i\left(k_i^M\right)^2}{2} \tag{25}$$

Implying that

$$\phi \geq \frac{1-s}{2z} q_i\left(k_i^M\right)\left((1-z) - \frac{q_i\left(k_i^M\right)}{2}\right)$$

For (22) to hold, it is therefore necessary that

$$\frac{1-s}{2z} q_i\left(k_i^M\right)\left((1-z) - \frac{q_i\left(k_i^M\right)}{2}\right) \leq \left(1-k_i^M\right)\frac{1-s}{z} + s \tag{26}$$

i.e.

$$\frac{1-s}{2z} q_i\left(k_i^M\right)\left((1-z) - \frac{q_i\left(k_i^M\right)}{2}\right) - \left(1-k_i^M\right)\frac{1-s}{z} - s \leq 0 \tag{27}$$

The lefthandside of (27) is concave in $q_i\left(k_i^M\right)$, reaching a maximum at $q_i\left(k_i^M\right) = 1-z \rightarrow\, > k_i^M = q_i^{FB}\left(k_i^M\right)$ for all admissible k_i^M, i.e. for all $k_i^M \geq 1-0.5z$. Now suppose that the overall optimal mechanism – that is, the optimal mechanism derived when taking both conditions IC.1 *and* IC.2 into consideration – satisfies $q_i(k) \leq q_i^{FB}(k)$ for all $k \leq k_i^M$. Notice that the overall optimal $q_i\left(k_i^M\right)$ cannot be below $3k_i^M - 2$, the optimal quality allocation when taking *only* condition IC.1 into account[19]. Hence, k_i^M and $3k_i^M - 2$ are the lower and upper bound on $q_i\left(k_i^M\right)$. Moreover, $1-z < 3k_i^M - 2$ for all $k_i^M \geq 1-0.5z$. To establish our claim, it is therefore sufficient to prove that, when evaluated at these two values, the lefthandside of (27) is strictly positive for any $k_i^M \geq 1-0.5z$.

Evaluated at $q_i\left(k_i^M\right) = 3k_i^M - 2$, the lefthandside of (27) is

$$\frac{1}{4z}\left(22k_i^M + 12s + 4z - 22k_i^M s - 6k_i^M z - 8sz + 6k_i^M sz - 9k_i^{M^2} + 9k_i^{M^2} s - 12\right) \tag{28}$$

[19] This is because if the overall optimal mechanism contains pooling, this must occur to prevent the quality schedule from being decreasing in type over some interval. We know that for $k \in [1-0.5z, 1]$, the quality allocation calculated when taking only condition IC.1 into consideration does not violate IC.2. Hence, if pooling occurs over some interval that includes type $1-0.5z$ this must be because a downward jump in the the unconstrained quality schedule occurs at some type $\leq 1-0.5z$. The pooling quality level must therefore be $\geq 1-1.5z$, the optimal quality allocation offered to $1-0.5z$ when taking *only* condition IC.1 into account.

Expression (28) is strictly increasing in k_i^M[20]. Hence, if expression (28) is strictly positive when $k_i^M = 1 - 0.5z$, it is also strictly positive for all $k_i^M > 1 - 0.5z$. Evaluated at $k_i^M = 1 - 0.5z$ expression (28) becomes

$$\frac{1-s}{z}(1-1.5z)\left((1-z)-\frac{1-1.5z}{2}\right)-1-s \tag{29}$$

Expression (29) is strictly positive when

$$z < \frac{1}{3s-3}\left(\frac{1}{2}\sqrt{96s-48s^2+208}-8.\right)$$

Now consider $q_i\left(k_i^M\right) = k_i^M$; when this is the case, the lefthandside of (27) is

$$\frac{1}{4z}\left(6k_i^M + 4s - 6k_i^M s - 2k_i^M z - 4sz + 2k_i^M sz - k_i^{M^2} + k_i^{M^2} s - 4\right) \tag{30}$$

Expression (30) is strictly increasing in k_i^M[21]. Hence, if expression (30) is strictly positive when $k_i^M = 1 - 0.5z$, it is also strictly positive for all $k_i^M > 1 - 0.5z$. Evaluated at $k_i^M = 1 - 0.5z$ expression (30) becomes

$$\frac{1-s}{z}(1-0.5z)\left((1-z)-\frac{1-0.5z}{2}\right)-1-s \tag{31}$$

Expression (31) is strictly positive when

$$z < \frac{1}{3s-3}\left(\frac{1}{2}\sqrt{96s-48s^2+208}-8\right)$$

This is exactly the same condition as that which guarantees that expression (29) is strictly positive. We conclude that when $z < \frac{1}{3s-3}\left(\frac{1}{2}\sqrt{96s-48s^2+208}-8\right)$, a situation where $u_i(1-z) = u_{-i}(1-z) = 0$ *and* $q_i(k) = q_{-i}(k) \leq q^{FB}(k)$ for all k cannot be an equilibrium. ■

References

[1] **Armstrong, M. and Vickers, J**. (1993): "Price Discrimination, Competition and Regulation", *The Journal of Indstrial Economics*, **41**, 335-359.

[20] The derivative of expression (28) with respect to k_i^M is $\frac{1}{2z}(-9k_i^M + 3sz + 11)(1-s)$, which is strictly positive for all $k_i^M \leq 1$.

[21] The derivative of expression (30) with respect to k_i^M is $\frac{1}{4z}(-2k_i^M + 2sz + 6)(1-s)$, which is strictly positive for all $k_i^M \leq 1$.

[2] **Armstrong, M. and Vickers, J**. (2001): "Competitive Price Discrimination", Rand Journal of Economics **32**, 579-605.

[3] **Berry, S., Lewinsohn, J. and Pakes, A**. (1995): "Automobile Prices in Market Equilibrium", *Econometrica* **63**, 841-890.

[4] **d'Aspremont,C.,GabszewicsJ. and Thisse, J.-F.** (1979): "On Hotelling's Stability in Competition", *Econometrica* **17**, 1145-1151.

[5] **Feenstra, R.C. and Lewinsohn J.A. (**1995): "Estimating Markups and Market Conduct with Multidimensional Product Attributes", *Review of Economic Studies*, **62**, 19-52.

[6] **Feldstein, M.S.** (1971): "Hospital Price Inflation: a Study of Nonproft Price Dynamics", *American Economic Review*, **61**, 853-872.

[7] **Goldberg, P.K. (**1995): "Product Differentiation and Oligopoly in International Markets: the Case of the US Automobile Industry", *Econometrica* **63**, 891-951.

[8] **Held, P.J. and Pauly, M. V.** (1983): "Competition and the Efficiency of the End Stage Renal Disease Program", *Journal of Health Economics*, **2**, 95-118.

[9] **Hotelling, H.** (1929): "Stability in Competition", *Economic Journal*, **39**, 41-57.

[10] **Mussa, M. and Rosen, S.** (1978): "Monopoly and Product Quality", *Journal of Economic Theory*, **18**, 301-317.

[11] **Inderst, R.** (2001): "Screening in a Matching Market", *Review of Economics Studies*, **68**, 849-868.

[12] **Laffont, J.-J. and Martimort D.** (2002): *The Theory of Incentives.* Princeton University Press.

[13] **Robinson, J. and Luft, H.** (1985): "The Impact of Hospital Market Structure on Patient Volume, Average Lenght of Stay and the Cost of Care", *Journal of Health Economics*, **4**, 333-356.

[14] **Rochet, J.-C. and Stole, L. (**2002): "Nonlinear Pricing with Random Participation", *Review of Economic Studies,* **69**, 277-311.

[15] **Salop, S.** (1979): "Monopolistic Competition and Outside Goods", *Bell Journal of Economics* **10**, , 141-156.

[16] **Stole, L.** (1995): "Nonlinear pricing and oligopoly", *Journal of Economics and Management Strategy* **4,** 529-562.

[17] **Verboven, F.** (1996): "Product Line Rivalry and Market Segemntation- with an Application to Automobile Optional Engine Pricing", *Journal of Industrial Economics,* **47**, 399-426.