Gaming and Strategic Ambiguity in Incentive Provision

Florian Ederer
MIT

Richard Holden
MIT and NBER

Margaret Meyer
Oxford and CEPR

June 2008

Motivation: Very precise incentive schemes/contracts can induce "gaming". In response, some have suggested introducing more randomness or ambiguity:

Public Administration: 2001: Performance ratings for English NHS trusts. 2003: Switch from publication of "priorities" to "lists of targets" (Bevan + Hood, *BMJ*, 2004). Evidence that hospitals

- diverted resources from unmeasured services, such as computers and building maintenance (Kmietowicz, *BMJ*, 2003), and from follow-up outpatient appointments (waiting time not targeted) to new outpatient appointments (targeted) (Bevan + Hood, *BMJ*, 2006)

- inefficiently distorted behavior in other ways, e.g. kept patients waiting outside A+E to delay the "clock starting"; relocated ambulance depots from rural to urban areas to reduce response times (Bevan + Hood, 2006)

Bevan and Hood's recommendations: i) to improve the auditing of performance data and ii) *"to introduce more uncertainty in the way that performance will be assessed and thus make some kinds of managerial gaming more difficult"* (2006)

Motivation (2)

Contract Law: Legal scholars debate the relative advantages and disadvantages of **precise** and **vague** contract terms ("rules" and "standards", respectively).

- Scott and Triantis (*Yale Law J.*, 2006) argue: "Compare the incentives of an agent faced with a specific proxy for effort (in the form of a contract rule) and another agent whose behavior is governed by a broad standard of effort. The first agent has the incentive to direct her attention to satisfying the proxy alone and to ignore all other dimensions of the desired performance. When faced with a standard, however, the agent has many proxies that might bear **probabilistically** on litigation outcomes. Her optimal strategy may therefore be to focus on effort rather than on any single proxy, thereby improving her position vis-à-vis all proxies." (emphasis added)

Our objective:

To develop a model of gaming of incentive schemes, where gaming takes the form of socially inefficient focusing of efforts on a subset of tasks, and to use it to ask:

- Under what circumstances do randomized schemes reduce gaming?
- Are randomized schemes more effective in reducing gaming than deterministic schemes?
- What drawbacks do randomized schemes have?
- Can randomized schemes be superior overall to deterministic schemes?

Overview of the rest of the talk

- Related literature
- The model
- Deterministic contracts
- Random contracts: ex ante and ex post randomization
- When are deterministic contracts optimal?
- When are random contracts optimal?
- Robustness and extensions
- Conclusions

Related economics literature

- Precise contracts based on verifiable measures can be distortionary:
  - Multi-task principal-agent models (Holmström+Milgrom, 1991; Baker, 1992): "effort-substitution problem" (across tasks or states) leads to optimal incentives being low-powered.
  - Relational contracts (MacLeod+Malcomson, 1989; Baker et al, 1994) can help reduce distortions by allowing use of observable but non-verifiable information.
  - Nonlinear contracts can help reduce incentives to focus on preferred tasks (MacDonald and Marx, 2001)
- General single-task P-A models show random incentive schemes can be beneficial when A's risk tolerance varies with effort (Gjesdal, 1982), but additive or multiplicative separability rules this out (Grossman+Hart, 1983).
- Some similarities with models of policing and testing (Lazear, 2006; Eeckhout et al, 2008): How much to reveal about use of *fixed* monitoring resources?

The Model (1)

- One period; one principal (P); two tasks; one agent (A), privately informed about his cost function

- Output on task j is $x_j = e_j + \epsilon_j$, where e_j is (privately- observed) effort on task j and ϵ_j is a random shock.

- $\epsilon_1, \epsilon_2 \sim N(0, \sigma^2)$ with correlation $\rho \geq 0$

- Let $\lambda \geq 1$. With prob. $\frac{1}{2}$, A's cost function is $\frac{1}{2}(e_1 + \lambda e_2)^2$ ("type $A1$") and with prob. $\frac{1}{2}$, it is $\frac{1}{2}(e_2 + \lambda e_1)^2$ ("type $A2$").

- NB: For each type of agent, efforts are perfect substitues, but type Ai is biased toward task i, with λ measuring intensity of bias.

- Both types of agent have CARA utility, $U = -e^{-r(w - c_i(e_1, e_2))}$, and the same reservation utility (0 in CE terms).

The Model (2)

- Principal's profit is $\Pi = B(e_1, e_2) - w$, where

$$B(e_1, e_2) = \min\{e_1, e_2\} + \frac{1}{\delta}\max\{e_1, e_2\}$$

- $\delta \geq 1$ measures degree of complementarity of A's efforts for P.

- If $\delta > \lambda$, efficient (surplus-maximizing) for both types of agent to exert perfectly balanced efforts on the two tasks; if $\delta < \lambda$, efficient for each type to focus exclusively on his preferred task.

- We focus on contracts that are linear ex post: $w = \alpha + \beta_1 x_1 + \beta_2 x_2$

- Deterministic (random) contract: at time signed, A is certain (uncertain) about values of α, β_1, β_2 that will be used ex post.

Deterministic Contracts (1): The Four Candidate Optima

1. **Symmetric deterministic (SD):** $\beta_1 = \beta_2 = \beta$

   - discontinuous at $\lambda = 1$: if $\lambda = 1$, can induce $e_1 = e_2 = \frac{\beta}{2}$, whereas if $\lambda > 1$, each type chooses effort on preferred task $\equiv \overline{e} = \beta$ and effort on less-preferred task $\equiv \underline{e} = 0$; for $\lambda > 1$, performance is independent of λ.

2. **One-task scheme (OT):** $\beta_1 = \beta$ and $\beta_2 = 0$

   - continuous at $\lambda = 1$; for $A1$, $e_1 = \beta$ and $e_2 = 0$; for $A2$, $e_1 = \frac{\beta}{\lambda}$ and $e_2 = 0$; performance $\downarrow$ as $\lambda \uparrow$: less effort from $A2$ and higher rent to $A1$.

3. **One-agent scheme (OA):** $\beta_1 = \beta$ and $\beta_2 = 0$, but attract only type $A1$

   - discontinuous at $\lambda = 1$, where coincides with OT; $A1$ chooses $e_1 = \beta$ and $e_2 = 0$; for $\lambda > 1$, performance independent of λ.

SD, OT, and OA all induce focused efforts from any agent who signs the contract.

Deterministic Contracts (2): The Four Candidate Optima

4. Asymmetric deterministic scheme (AD): $\beta_1 = \lambda\beta_2$

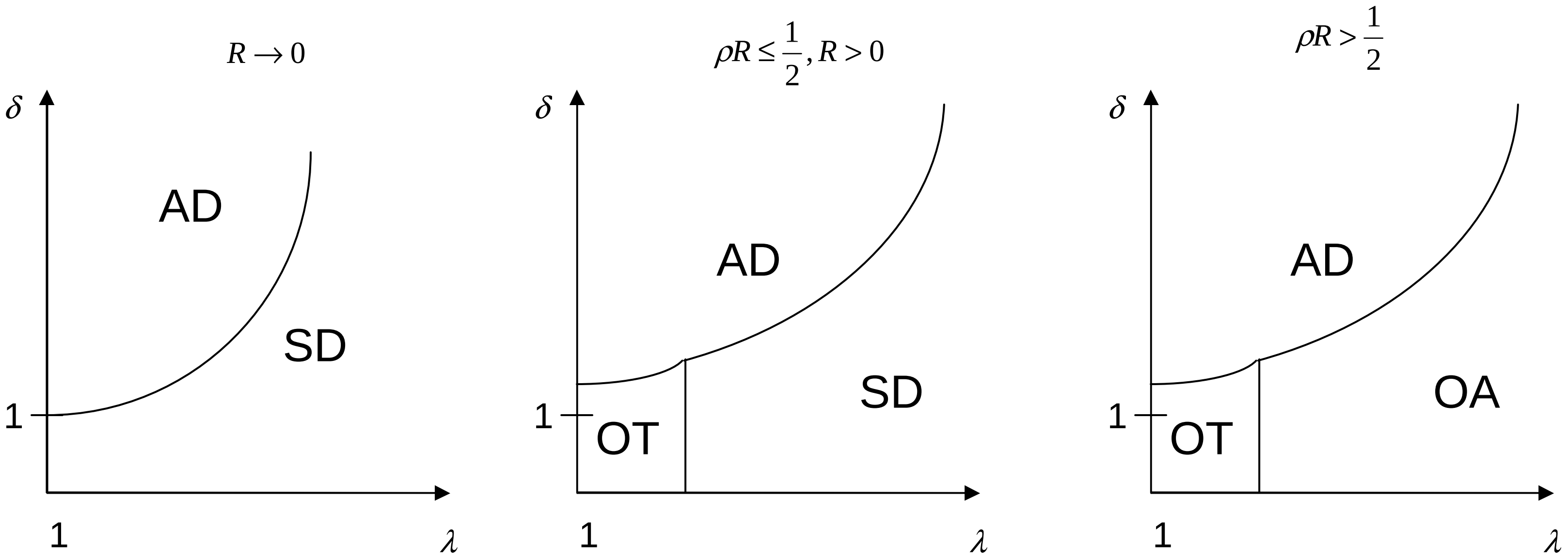
- discontinuous at $\lambda = 1$, where coincides with SD; for $\lambda > 1$, $A1$ chooses $e_1 = \lambda\beta_2$ and $e_2 = 0$; $A2$ willing to choose perfectly balanced efforts $e_1 = e_2 = \frac{\beta_2}{\lambda+1}$; $A1$ earns rent.

No other type of deterministic contract can be optimal:

- A contract with $\beta_1 > \lambda\beta_2 > 0$ is dominated by OT.

- A contract with $\lambda\beta_2 > \beta_1 > \beta_2 > 0$ is dominated by SD.

Optimal deterministic contracts are summarized below:



$R \to 0$
δ
AD
SD
1
1
λ
$\rho R \leq \frac{1}{2}, R > 0$
δ
AD
SD
OT
1
1
λ
$\rho R > \frac{1}{2}$
δ
AD
OA
OT
1
1
λ

Ex Ante Randomization (EAR)

Contract specifies that with prob. p, $\beta_1 = \beta$ and $\beta_2 = 0$, and with prob. $1-p$, $\beta_1 = 0$ and $\beta_2 = \beta$. P commits to employ a randomization device to determine on which output pay will be based.

Proposition 1 *For parameter regions such that EAR induces interior solutions for efforts:* **i)** *It is optimal for P to commit to* $p = \frac{1}{2}$. **ii)** *For both types of agent, EAR with* $p = \frac{1}{2}$ *induces "aggregate effort"* $\overline{e}^{EAR} + \lambda \underline{e}^{EAR}$ *satisfying*

$$\beta = \partial c/\partial \overline{e} + \partial c/\partial \underline{e} = (1+\lambda)(\overline{e} + \lambda \underline{e}),$$

and a gap in efforts, $\overline{e}^{EAR} - \underline{e}^{EAR}$, *satisfying*

$$\lambda = \frac{\partial c/\partial \underline{e}}{\partial c/\partial \overline{e}} = \frac{E\left[U'(\cdot) I_{\{\underline{x} \text{ is rewarded}\}}\right]}{E\left[U'(\cdot) I_{\{\overline{x} \text{ is rewarded}\}}\right]} = \exp\left[r\beta(\overline{e} - \underline{e})\right].$$

- Optimal to commit to $p = \frac{1}{2}$: induces most balanced profile of efforts and avoids leaving rent to either agent type.

Pros and Cons of Ex Ante Randomization

- "Interim randomization", under which P chooses p at the same time as A chooses efforts, would yield the same outcome as EAR as the unique Bayesian Nash eqm. So attractive properties of EAR are not crucially dependent on P's commitment power w.r.t. randomizing probability.

- EAR pushes A toward balanced efforts as a means of (partially) insuring himself against the risk generated by the random choice of which task to reward. Extent of optimal self-insurance is greater, so $\overline{e}^{EAR} - \underline{e}^{EAR}$ is smaller, the larger is r, the higher is β, and the smaller is λ.

- Under EAR, both A's effort choices and P's profit are continuous at $\lambda = 1$, in contrast to the SD scheme $\Longrightarrow$ EAR is more robust to introduction of private information on A's part. Also, EAR is more robust to uncertainty about the magnitude of λ than is an AD scheme.

- Because optimal self-insurance is only partial (for $\lambda > 1$), EAR with coefficient β imposes **greater** risk costs on A than a deterministic OT scheme which rewards a single task at rate β.

Ex Post Randomization (EPR)

After observing the outputs x_1 and x_2, P *chooses* whether to pay $\alpha+\beta x_1$ or $\alpha+\beta x_2$. A anticipates receiving $w = \min\{\alpha+\beta x_1, \alpha+\beta x_2\}$.

Proposition 1 *If EPR induces interior solutions for efforts, then for both types of agent, "aggregate effort" $\overline{e}^{EPR} + \lambda \underline{e}^{EPR}$ satisfies:*

$$\beta = \partial c/\partial \overline{e} + \partial c/\partial \underline{e} = (1+\lambda)(\overline{e} + \lambda \underline{e}),$$

and the gap in efforts, $\overline{e}^{EPR} - \underline{e}^{EPR}$, satisfies:

$$\lambda = \frac{\partial c/\partial \underline{e}}{\partial c/\partial \overline{e}} = \frac{E\left[U'(\cdot) I_{\{\underline{x} \text{ is rewarded}\}}\right]}{E\left[U'(\cdot) I_{\{\overline{x} \text{ is rewarded}\}}\right]} = \exp\left[r\beta(\overline{e} - \underline{e})\right] \frac{\Phi\left(\frac{\overline{e}-\underline{e}+r\sigma^2\beta(1-\rho)}{\sigma\sqrt{2(1-\rho)}}\right)}{\Phi\left(\frac{-(\overline{e}-\underline{e})+r\sigma^2\beta(1-\rho)}{\sigma\sqrt{2(1-\rho)}}\right)}.$$

- Under EPR, *two* forces push towards balanced efforts: i) insurance motive and ii) P's strategic choice of which task to reward raises exp. marg. return to effort on task on which A exerts lower effort, rel. to exp. marg. return on other task. As a result:

$$\overline{e}^{EPR} - \underline{e}^{EPR} < \overline{e}^{EAR} - \underline{e}^{EAR} \qquad \forall\, \lambda > 1.$$

Ex Post Randomization (2)

Under EPR, the gap in efforts, $\overline{e}^{EPR} - \underline{e}^{EPR}$, is smaller

- the larger is r: stronger insurance motive is the dominant effect
- the smaller is λ
- the smaller is $\sigma^2(1-\rho)$: less uncertainty about which task will be rewarded, for any given $\overline{e} - \underline{e}$

Ex Post vs. Ex Ante Randomization

- EPR and EAR induce same aggregate effort ($\overline{e} + \lambda \underline{e}$), hence same effort costs.

- P's benefit is higher (lower) under EPR than under EAR if $\delta > \lambda$ ($\delta < \lambda$).

- EPR with coeff. β imposes **lower** risk costs on A than a deterministic OT scheme with coeff. β (and hence lower risk costs than EAR). Reason: $Var(w^{EPR}) = Var(\alpha + \beta \min\{x_1, x_2\}) < Var(\beta x_i)$.

These three points imply:

Proposition 3 *For any $\beta > 0$, if $\delta \geq \lambda$, EPR generates higher profit for P than EAR, strictly higher for $\rho < 1$.*

For a given β, aggregate effort ($\overline{e} + \lambda \underline{e}$) under EPR is only $\frac{\beta}{1+\lambda}$, while under OT and SD it is β. We need to know: Does EPR impose greater or lower risk costs than OT or SD *per unit of aggregate effort induced*?

When Are Deterministic Contracts Optimal? (1)

Proposition 4 *When $\lambda = 1$, so A has no private information about the environment, both EAR and EPR yield lower profit for P, for any $\beta > 0$, than a suitably designed SD scheme, strictly lower for $\rho < 1$.*

- When $\lambda = 1$, EAR and EPR with coeff. β both induce $\overline{e} = \underline{e} = \frac{\beta}{4}$. SD with coeff. $\frac{\beta}{2}$ also induces $\overline{e} = \underline{e} = \frac{\beta}{4}$. Hence all 3 schemes generate the same benefit for P and the same cost of effort for A, so difference in profit represents cost of compensating A for risk. Risk costs are strictly higher under both EPR and EAR than under SD if $\rho < 1$, equal under all 3 schemes if $\rho = 1$. SD displays a "diversification benefit" from averaging shocks on 2 tasks.

- Similar arguments show, for arbitrary $\lambda > 1$, that SD can induce any given level of aggregate effort $\overline{e} + \lambda\underline{e}$ at strictly lower risk cost than EPR or EAR.

Proposition 5 *When $\lambda > 1$ and $\delta \leq \lambda$, both EPR and EAR yield strictly lower profit for P, for any $\beta > 0$, than a suitably designed SD scheme.*

- For $\delta < \lambda$, *efficient* effort allocations are *focused* and are induced by SD.

When Are Deterministic Contracts Optimal? (2)

Proposition 6 *Whenever, for a given β, EAR or EPR induce A to exert effort only on his preferred task, the same effort allocation can be induced more profitably by a SD scheme.*

- When randomized schemes induce focused efforts, they provide no benefit w.r.t. balance but impose greater risk costs than SD scheme.

Corollary 1 *Consider the limiting case where $\sigma^2 \to \infty$ and $r \to 0$ in such a way that $r\sigma^2 \to R \in [0, \infty)$. For any $\beta > 0$, both EPR and EAR induce A to exert effort only on his preferred task, hence are dominated by a SD scheme.*

- Profit from deterministic schemes depends only on $r\sigma^2$, whereas r and σ^2 individually affect efforts and profit from randomized schemes. Small r makes insurance motive very weak under EAR and EPR, and large σ^2 means $\overline{e} - \underline{e}$ has little effect on which task is rewarded under EPR. Hence incentives for balanced efforts disappear under both EPR and EAR.

Corollary 2 *If $[1 + r\sigma^2 (\lambda + 1)^2]\sqrt{\frac{(\lambda+1)\ln\lambda}{r}} > 2$, then the* **optimal** *β under EAR induces focused efforts, so EAR is dominated by a SD scheme.*

When Are Random Contracts Optimal? (1)

We identify three environments where EPR, or both EAR and EPR, dominate the best deterministic contract. In each setting, EPR, or both EAR and EPR, induce perfectly balanced efforts. In each case, there is a critical level of δ (critical degree of complementarity of tasks for P) above which the randomized scheme(s) that induce(s) perfectly balanced efforts dominate(s) the best deterministic scheme.

The limiting case as $\lambda \to 1^+$: agent has private information, but magnitude of privately-known preference across tasks is arbitrarily small

Proposition 7 *Consider the limiting case as $\lambda \to 1^+$.*

i) *There exists $\widehat{\delta}(r\sigma^2, \rho)$ above which EAR and EPR both dominate the best deterministic contract.*

ii) *$\widehat{\delta}(r\sigma^2, \rho)$ is $\uparrow$ in $r\sigma^2$ and $\downarrow$ in ρ, approaching 3.82 as $r\sigma^2 \to \infty$ and $\rho \to 0$.*

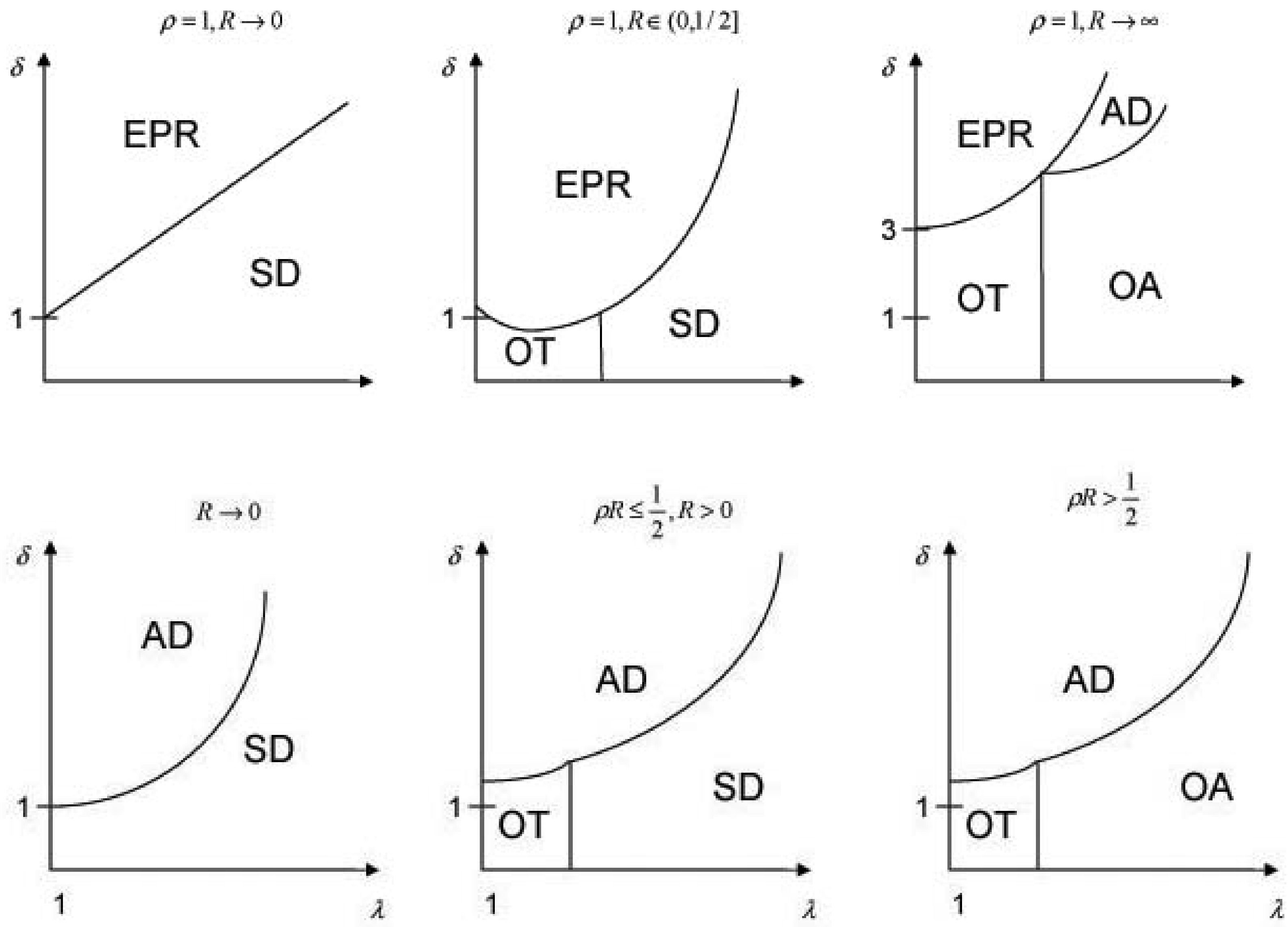
iii) *As $r\sigma^2 \to 0$ or $\rho \to 1$, both EAR and EPR dominate the best deterministic contract for all $\delta > 1$.*

When are Random Contracts Optimal? (2)

The limiting case of prefect correlation of the shocks: $\rho \to 1$

- Under EPR, A's effort choices and P's profit are continuous at $\rho = 1$.
- At $\rho = 1$, A is *certain* that wage will be based on task on which effort is lower. Hence optimal efforts are perfectly balanced: $\overline{e}^{EPR} = \underline{e}^{EPR} = \frac{\beta}{(1+\lambda)^2}$.

Optimal contracts for $\rho \to 1$ are summarized below:


$\rho = 1, R \to 0$
δ
EPR
SD
1
$\rho = 1, R \in (0, 1/2]$
δ
EPR
1
OT
SD
$\rho = 1, R \to \infty$
δ
AD
EPR
3
OT
OA
1
$R \to 0$
δ
AD
SD
1
1
λ
$\rho R \leq \frac{1}{2}, R > 0$
δ
AD
SD
1
OT
1
λ
$\rho R > \frac{1}{2}$
δ
AD
OA
1
OT
1
λ

When are Random Contracts Optimal? (3)

Proposition 8 *Consider the limiting case where $r \to \infty$ and $\sigma^2 \to 0$ in such a way that $r\sigma^2 \to R \in [0, \infty)$.*

i) *Both EAR and EPR induce both types of A to choose equal efforts on the two tasks, for any finite value of λ.*

ii) *EAR and EPR are equally profitable.*

iii) *There is a critical value of δ above which both EAR and EPR are more profitable than any deterministic contract.*

For $r\sigma^2 \to R$, contrast

- $r \to \infty$, $\sigma^2 \to 0$: perfectly balanced efforts; random contracts dominate if δ sufficiently large;
- $r \to 0$, $\sigma^2 \to \infty$: fully focused efforts; deterministic contracts optimal for all δ.

Robustness and Extensions (1)

Imperfect substitutability of efforts for the agent: As before, let agent type Ai prefer task i and the two types of agent be equally likely. Let each type's cost function be

$$c(\overline{e}, \underline{e}) = \frac{1}{2}\left(\overline{e}^2 + 2s\lambda\overline{e}\underline{e} + \lambda^2\underline{e}^2\right),$$

where $s \in [0, 1]$ measures the degree of substitutability.

Symmetric deterministic scheme induces strictly positive efforts on both tasks if and only if $s\lambda < 1$.

Even for $s < 1$, we can still identify settings where the optimal deterministic scheme is dominated by a contract involving randomization.

Robustness and Extensions (2)

Even outside the exponential-normal model, randomized schemes i) induce more balanced efforts than deterministic ones and ii) are more robust to uncertainty about A's preferences.

Consider $U(w-c(\overline{e},\underline{e}))$, where $c(\overline{e},\underline{e})=\frac{1}{2}\left(\overline{e}^2+2s\lambda\overline{e}\underline{e}+\lambda^2\underline{e}^2\right)$ and $U''<0$. Let (ϵ_1,ϵ_2) have an arbitrary symmetric joint density. For both EAR and EPR, it is still true that $\beta=\frac{\partial c}{\partial\overline{e}}+\frac{\partial c}{\partial\underline{e}}$ and $\frac{\partial c/\partial\underline{e}}{\partial c/\partial\overline{e}}=\frac{E\left[U'(\cdot)I_{\{\underline{x}\text{ is rewarded}\}}\right]}{E\left[U'(\cdot)I_{\{\overline{x}\text{ is rewarded}\}}\right]}$.

Proposition 9 i) *When SD induces interior optimal efforts ($s\lambda<1$), EAR and EPR do so as well, and both induce more balanced efforts than SD:*

$$1<\frac{\overline{e}^{EAR}}{\underline{e}^{EAR}}<\frac{\overline{e}^{SD}}{\underline{e}^{SD}} \qquad \text{and} \qquad 1<\frac{\overline{e}^{EPR}}{\underline{e}^{EPR}}<\frac{\overline{e}^{SD}}{\underline{e}^{SD}}.$$

ii) *With efforts perfect substitutes ($s=1$), as $\lambda\uparrow$ from 1, $\frac{\overline{e}^{EAR}}{\underline{e}^{EAR}}$ and $\frac{\overline{e}^{EPR}}{\underline{e}^{EPR}}$ both increase continuously from 1, whereas $\frac{\overline{e}^{SD}}{\underline{e}^{SD}}$ jumps from 1 to ∞.*

Robustness and Extensions (3): Another Random Contract

Suppose P can commit to reward A according to the "Max" contract:

$$w = \max\{\alpha + \beta x_1, \alpha + \beta x_2\}$$

- Max contract always induces focused efforts: insurance motive is outweighed by the positive influence of $(\overline{e} - \underline{e})$ on marginal return to $\overline{e}$ relative to $\underline{e}$.

- Hence the Max contract can be attractive only if balance is not too important to P.

- As $\sigma^2(1-\rho) \to 0$, $\overline{e}^{Max} \to \beta$: A becomes certain that his preferred task will be rewarded.

Proposition 10 *Let the correlation, ρ, of the shocks to outputs approach 1.*

i) *Among the contracts which induce focused efforts (SD, OT, OA, Max), the Max contract is the most profitable for P, for all $\lambda > 1$, $\delta \geq 1$, $r\sigma^2 \geq 0$.*

ii) *Every deterministic contract is dominated by either EPR or the Max contract, for all $\lambda > 1$, $\delta \geq 1$, $r\sigma^2 \geq 0$.*

Conclusions

We've developed a model of gaming of incentive schemes, where gaming takes the form of socially inefficient focusing of efforts.

- Ex ante randomization reduces gaming by inducing A to self-insure. Ex post randomization does, too, and also reduces gaming because greater focus on preferred task reduces the relative marginal return to that task.

- EAR and EPR are more robust than deterministic schemes to uncertainty about A's preferences.

- But EAR and EPR impose higher risk cost on A than deterministic schemes, per unit of aggregate effort induced.

- If A is no better informed than P about the environment, a deterministic contract is optimal.

- If A is better informed and if tasks are sufficiently complementary for P, randomized schemes are superior overall in settings where they generate very strong incentives for balanced efforts.