Prospect Theory and Tax Evasion: A Reconsideration of the Yitzhaki Puzzle

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May 2014
Yitzhaki puzzle: The expected utility model of tax evasion predicts a negative relationship between tax rates and evasion when preferences satisfy DARA. Most empirical evidence finds the opposite.

Recent years have seen several attempts to employ the insights of prospect theory to the tax evasion decision:


This literature is reviewed by Hashimzade, Myles and Tran-Nam (2013).
Dhami and al-Nowaihi (2007: 171) claim to “…show that prospect theory provides a much more satisfactory account of tax evasion including an explanation of the Yitzhaki puzzle.”

Hashimzade et al. conclude (on the basis of several examples) that “Prospect theory does not necessarily reverse the direction of the tax effect: our examples show that certain choices of the reference level can affect the direction of the tax effect in some situations, but none of the examples is compelling.”

We investigate this dichotomy
Our Contribution

- We revisit the tax evasion model under expected utility theory and various reference dependent models.
- We allow $R$ to be a (general) decreasing function of the marginal tax rate (and the taxpayer’s declaration).
- We analyse the model both with the probability of audit:
  - fixed exogenously
  - as a function of the taxpayer’s declaration
- We find that:
  - Prospect theory does not reverse the Yitzhaki puzzle for existing psychologically plausible specifications of reference income.
  - There are clear-cut versions of prospect theory that reverse the Yitzhaki puzzle, but these have not been advocated as psychologically plausible.
Prospect theory bundles four key elements

- **reference dependence**: outcomes judged relative to a reference level of income $R$
- **diminishing sensitivity**: marginal utility is diminishing in distance from the reference income
- **loss aversion**: the disutility of a loss exceeds the utility of a gain of equal magnitude
- **probability weighting**: objective probabilities transformed into decision weights

Previous literature has not spelled out which of these concepts are needed for particular results

We analyse the effects of these elements separately and in combination
Expected Utility Model

- Income when the taxpayer is caught (audited) and when not caught are
  \[ Y^n = Y - tX; \quad Y^c = Y^n - tf [Y - X] \]
  \( f > 1 \) is the fine rate on all undeclared tax

- Objective function under expected utility given by
  \[ V = p\nu (Y^c) + [1 - p] \nu (Y^n), \]

\( \nu \) is taxpayer utility (\( \nu' > 0, \nu'' < 0 \))
\( p \in (0, 1) \) is the probability of audit
Objective function under reference-dependence given by

\[ V = p v(Y^c - R) + [1 - p] v(Y^n - R) \]

To encompass existing literature, we allow \( R(t) \) and \( R(t, X) \)
RD Framework with Diminishing Sensitivity

- Can only be introduced in combination with reference-dependence
- Replace \( v(x) \) with \( v_0(x) \) for \( x < 0 \), where \( v'' > 0 \) such that \( A(x) < 0 \)
- We focus on the only interesting case: \( Y^n > R > Y^c \)
- Objective function is

\[
V_{DS} = p v_0(Y^c - R) + [1 - p] v(Y^n - R)
\]

- No guarantee that an interior maximum exists or is unique
Loss aversion:

- May be introduced on its own, or in combination with the other elements of prospect theory
- With respect to a utility function $v$ requires that $-v(-x) > v(x)$ for $x > 0$
- This condition necessarily holds if $v$ is strictly concave
- Combined with diminishing sensitivity, loss aversion requires that

$$-v(-x) > v(x) \text{ for } x > 0$$

Probability weighting: replace $p$ with $w(p)$, where $w(0) = 0$, $w(1) = 1$ and $w' > 0$
We study interior maximum: evidence for characterising tax evasion as an all-or-nothing activity is weak.

The derivative $\frac{\partial X}{\partial t}$ at an interior maximum is

$$\frac{\partial X}{\partial t} = t \left[ (1 - p) X v''(Y^n) - p [f - 1] [X + [Y - X] f] v''(Y^c) \right]$$

where $D$ is the second order condition.

Adding and subtracting $t^{-1} D (Y - X)$ in the numerator, and applying the first order condition, (1) rewrites as

$$\frac{\partial X}{\partial t} = \frac{1}{t} \left[ [Y - X] - \frac{Y [A(Y^n) - A(Y^c)]}{[f - 1] A(Y^c) + A(Y^n)} \right]$$

$A(x) = -v''[x] / v'[x]$ is the Arrow-Pratt coefficient of absolute risk aversion.
Fixed $p$ and Expected Utility

Proposition

(Yitzhaki, 1974) Under DARA, at an interior maximum, $\frac{\partial X}{\partial t} > 0$.

- Result is a pure income effect: $t$ increases $\rightarrow$ taxpayers feel poorer $\rightarrow$ taxpayers become more risk averse
Fixed $p$ and $R = R(t)$

- Assume $R_t \leq 0$, $R_X = 0$
- Most popular specification is $R = Y[1 - t]$, so $R_t = -Y$
- The derivative $\frac{\partial X}{\partial t}$ now becomes

$$
\frac{\partial X}{\partial t} = \frac{1}{t} \left[ [Y - X] - \frac{[Y + R_t] \left[A(Y^n - R) - A(Y^c - R)\right]}{[f - 1] A(Y^c - R) + A(Y^n - R)} \right]
$$

- Additionally assuming diminishing sensitivity:

$$
\frac{\partial X}{\partial t} = \frac{1}{t} \left[ [Y - X] - \frac{[Y + R_t] \left[A(Y^n - R) - A(Y^c - R)\right]}{[f - 1] A(Y^c - R) + A(Y^n - R)} \right]
$$
Proposition

Assume $R_t \leq 0$ and $R_X = 0$. Then:

(i) assuming DARA, there exists a threshold level $\tilde{R}_t < -Y$ such that, at an interior maximum, $\partial X / \partial t < 0$ for $R_t < \tilde{R}_t$ and $\partial X / \partial t \geq 0$ for $R_t \geq \tilde{R}_t$.

(ii) assuming diminishing sensitivity, there exists a threshold level $\tilde{R}_{t,DS} \in (-Y, 0)$ such that, at an interior maximum, $\partial X / \partial t < 0$ for $R_t > \tilde{R}_{t,DS}$ and $\partial X / \partial t \geq 0$ for $R_t \leq \tilde{R}_{t,DS}$.

(iii) parts (i) and (ii) hold if loss aversion and/or probability weighting are additionally assumed.
Intuition

- Without diminishing sensitivity
  - $\partial X / \partial t < 0$ if reference income sufficiently *sensitive* to $t$.
  - $t$ increases $\rightarrow$ expected income increases ($R$ falls faster than expected value of the tax gamble) $\rightarrow$ taxpayers feel richer (relative to the reference income) $\rightarrow$ taxpayers become *less* risk averse

- With diminishing sensitivity
  - $\partial X / \partial t < 0$ if reference income sufficiently *insensitive* to $t$.
  - $t$ increases $\rightarrow$ expected income falls ($R$ falls slower than the expected value of the tax gamble) $\rightarrow$ taxpayers feel poorer (relative to the reference income) $\rightarrow$ taxpayers become *less* risk averse
Fixed $p$ and $R = R(t)$

\[
\begin{align*}
\frac{\partial X}{\partial t} &< 0 \text{ under RD} \\
\frac{\partial X}{\partial t} &> 0 \text{ under RD, DS} \\
\frac{\partial X}{\partial t} &\geq 0 \\
\frac{\partial X}{\partial t} &> 0 \text{ under RD} \\
\frac{\partial X}{\partial t} &< 0 \text{ under RD, DS}
\end{align*}
\]
(i) Assume $R_X = 0$, and $R_t \in \left( \tilde{R}_t, \tilde{R}_{t, DS} \right)$. Then, at an interior maximum, $\partial X / \partial t > 0$ whether or not diminishing sensitivity is assumed.

(ii) Assume $R = Y \left[ 1 - t \right]$, which implies $R_t = -Y$. Then $\partial X / \partial t = t^{-1}[Y - X] > 0$ whether or not diminishing sensitivity is assumed.

In this case, ability of prospect theory to resolve the Puzzle is strictly weaker than expected utility theory.
Implications

- Trotin (2012) – false claim
- Bernasconi and Zanardi (2004)
  - Do not specify $R$, which implies $R_t = 0$. Then $R_t = 0 > \tilde{R}_{t,DS}$ so $\frac{\partial X}{\partial t} > 0$
  - Not specifying reference income important to result

- Yaniv (1999)
  - Reference income: $R = Y - D$
  - $D$ is the amount of an advanced tax payment
  - The Yitzhaki puzzle is reversed in Yaniv’s model when the tax authority *under-estimates* taxpayer incomes
  - But *over-estimation* advocated by the literature
Fixed $p$ and $R = R(t,X)$

- Does the descriptive utility of the RD framework improve if we allow a dependence upon $X$ as well as $t$?
- Assume $R_t < 0$, $R_X < 0$, $R_X$ homogeneous of degree one in $X$.
- Two specifications of reference income that satisfy these assumptions:
  1. The expected value of the tax gamble, (as suggested by Kőszegi and Rabin, 2006)
     \[ R = Y - tX - pft \{ Y - X \} \]
  2. $R = (1 - t)X$ (as examined in Hashimzade et al.)
Fixed \( p \) and \( R = R(t,X) \)

- The derivative \( \partial X / \partial t \) now becomes

\[
\frac{1}{t} \left[ [Y - X] - \frac{\phi \left[ A(Y^n - R) - A(Y^c - R) \right]}{t[f - 1] - RX} \right] A(Y^c - R) + [t + RX] A(Y^n - R)
\]

\[
\phi = t [Y + R_t] + RX [Y - X]
\]

- Additionally assuming diminishing sensitivity:

\[
\frac{1}{t} \left[ [Y - X] - \frac{\phi \left[ A(Y^n - R) - A(Y^c - R) \right]}{t[f - 1] - RX} \right] A(Y^c - R) + [t + RX] A(Y^n - R)
\]
Fixed $p$ and $R = R(t,X)$

**Proposition**

Assume $R_t < 0$, $R_X < 0$, $R_{XX} = 0$ and $R_X$ homogeneous of degree one in $t$. Then parts (i)-(iii) of previous Proposition hold unchanged, and so does its Corollary.
Endogenous $p$

- Does allowing $p$ to depend upon $X$ improve the descriptive utility of the RD framework?
- Let $p = p(X)$
- Optimal auditing literature suggests $p'(X) < 0$
- Few, if any, general results hold. Instead we focus on the setting employed in Dhami and al-Nowaihi (2007)
- These authors employ a power function for $v$ (implying homogeneity) and $R = Y(1 - t)$
- They claim to reverse Yitzhaki’s puzzle in a prospect theory model of this type
Endogenous $p$

- Under the assumptions of Dhami and al-Nowaihi the objective function under reference-dependence is

$$V_{p(X)} = v[t] v[Y - X] \{ p[X] v[-(f - 1)] + 1 - p[X] \}$$

- Additionally assuming diminishing sensitivity:

$$V_{p(X)} = v[t] v[Y - X] \{ p[X] v[-(f - 1)] + 1 - p[X] \}$$

- We obtain bang-bang dynamics if $p'(X) = 0$
Endogenous $p$

If an interior solution exists, then the following Proposition holds:

**Proposition**

Assume endogenous reference dependence, $v$ homogeneous, $p'(X) \leq 0$ and $R = Y(1 - t)$. Then, at an interior maximum, $\partial X / \partial t = 0$.

Hence, allowing for $p'(X) < 0$ does not resolve the Yitzhaki puzzle.

What does explain Dhami and al-Nowaihi’s finding?
Dhami and al-Nowaihi introduce a stigma parameter that such that wealth when caught becomes

\[ Y^c = Y - tX - [s + ft] [Y - X]. \]

**Proposition**

\((\text{Dhami and al-Nowaihi, 2007})\) Assume endogenous reference dependence, stigma, \(v\) homogenous of degree \(\beta > 0\), \(p' \leq 0\), and \(R = Y (1 - t)\). Then, at an interior maximum, \(\partial X / \partial t < 0\).

So prospect theory combined with stigma reverses the Yitzhaki puzzle.
Is stigma also able to reverse the Yitzhaki puzzle when combined with expected utility theory?

Proposition

Assume expected utility theory, stigma, $p' < 0$, and risk neutrality. Then, at an interior maximum, $\partial X / \partial t < 0$.

So stigma reverses the Yitzhaki puzzle in both the expected utility and reference-dependent models.

Unclear that prospect theory is adding descriptive value over expected utility theory.
Conclusions

- There are “few well-known and broadly accepted applications of prospect theory in economics.” Barberis (2013)
- The reason, Barberis argues, is that prospect theory is not straightforward to apply
  - In particular, the most appropriate choice of the reference level is often unclear
- This general problem shines through in applications of prospect theory to tax evasion
  - Prospect theory can resolve the Yitzhaki puzzle on some well-defined sets, but not on others
  - Curiously, the sets where prospect theory does reverse the Puzzle are not those advocated by the literature