Should Incentive Schemes be High-powered or Low-powered in the Presence of Motivated Agents?

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(preliminary version)

Abstract

Pay-for-performance schemes that reward higher output or measurable dimensions of quality are increasingly advocated in the public sector. We investigate the optimal power of incentive schemes to finance public-service providers (e.g., in the health sector, education and child care) and identify conditions under which the presence of motivated agents implies that the power of optimal incentive schemes is high or low. The study highlights the interplay between the degree of motivation and the reservation utility. In the presence of symmetric information, we show that if the reservation utility of the agent is sufficiently low (high), the break-even (participation) constraint is binding and the optimal incentive scheme has low (high) power. We also show that there is an intermediate range of reservation utilities where both the participation and the break-even constraint are binding. We further expand the analysis to account for unobserved heterogeneity on both agents’ degree of motivation and productivity. The three equilibria derived under symmetric information also hold under asymmetric information when the principal is also concerned about rent extraction. However, the relation between the power of the incentive scheme and agent’s reservation utility can be reversed. The optimal price can now be lower when the reservation utility is high.

Keywords: motivated agents, altruism, incentive schemes. JEL: D82, I11, I18, L51.

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1 Introduction

A key feature of public sector employees (including health care, education and child care) is that providers are altruistic or intrinsically motivated. In the healthcare sector doctors may care about the health of the patients. In the education sector teachers may care about the education of pupils. In ministries bureaucrats may subscribe to the mission set by the government (bureaucrats may be driven by ‘Public Service Motivation’; Francois, 2000).

Market reforms are increasingly advocated to improve the functioning of public services. In particular, performance indicators are increasingly used to regulate providers and encourage higher production and productivity. Hospitals are financially rewarded for treating larger volumes of patients and performing better on clinical measures of quality (such as lower readmission rates and mortality rates). Schools and universities are paid on the basis on number of students and the students’ test scores. Students’ satisfaction surveys are increasingly available and in the public domain (eg in the United Kingdom).

Should policymakers use high-powered incentive schemes in the presence of motivated agents? On one hand, we may think that higher motivation implies that the agent provides more output, for a given price, and therefore the payer needs to incentivise less the agent through the incentive scheme. The incentive scheme has low power. On the other hand, motivation effectively implies a lower marginal cost of provision. This implies a higher optimal output for the principal which can be implemented with an incentive scheme with high power.

Which of these two plausible mechanisms is the correct one and under what circumstances? This study highlights that the answer depends on the interplay between the degree of motivation and the reservation utility of the agent. When the principal designs the incentive scheme it has to take into account two constraints: the participation constraint and a break-even constraint (budget must cover at least the monetary production costs).

We show that if the reservation utility of the agent is sufficiently low, then the optimal incentive scheme has low power. In this case, it is the break-even constraint that it is
binding. The optimal output for the purchaser is independent of the degree of motivation. But higher motivation implies a higher output, for a given price, and therefore the price required to implement the optimal output is reduced. The power of the incentive scheme is low.

If the reservation utility of the agent is high, then the optimal incentive scheme has high power. It is the participation constraint that it is now binding. Higher motivation reduces the marginal cost of provision and implies a higher optimal output. The higher output is implemented with a higher price. The power of the incentive scheme is high.

We also show that there is an intermediate range of reservation utilities where both the participation and the break-even constraint are binding. In this case both the optimal output and price are in-between the prices derived under low and high reservation utility. Moreover, the solution in this intermediate range smoothly connects the one under low and high reservation utility. A higher reservation utility implies higher output and price.

The results discussed above hold under symmetric information on the agents’ type. We further expand the analysis to account for unobserved heterogeneity on both agents’ degree of motivation and productivity. As with symmetric information, we focus on linear incentive scheme. Our primary reason for doing so is that linear schemes are widely employed by policymakers. Moreover, this allows a straightforward comparison with the symmetric information solution. Under asymmetric information, the policymaker also has to take into account potential rents agents may hold. We show that the three equilibria derived under symmetric information also hold under asymmetric information. However, critically, we show that the relation between the power of the incentive scheme and agent’s reservation utility can be reversed. While under symmetric information the optimal price is always higher when the reservation utility is high, the opposite may hold under asymmetric information. This is likely to arise when the dispersion in the degree of motivation is higher. When the reservation utility is high, the participation constraint of the most motivated provider is binding. Since the output of the most motivated provider can be higher than the expected output across all types, this in turn may imply that the price is distorted upwards for rent-extraction purposes. This does not arise when agent’s reservation utility is low, since in this case it is the break-even constraint of the provider with lowest degree
of motivation (and highest inefficiency) which is binding: since the output of this type is always lower than the output of the expected type, then the price is always distorted downwards.

One key policy implication is that high-powered incentive schemes are not incompatible with high degree of motivation. In the health sector, it has been argued in the UK that in the last twenty years healthcare providers have turned from *knights* (highly motivated providers) to *knaves* (non-motivated providers; Le Grand, 2003). It is unlikely that the distribution on the degree of motivation has radically changed over time. Our analysis shows, that the policy change from low- to high-powered incentive schemes does not necessarily have to do with a change in the degree of motivation. Moreover, the UK has experienced a significant increase in healthcare financing bringing health expenditure closer to European average spending. This may have contributed to make the break-even constraint less binding and to shifting the focus on the participation constraint.

The study contributes to the literature on designing incentive schemes in the presence of motivated or altruistic agents. Within the public economics literature the assumption of motivated agents is shared by Besley and Ghatak (2005, 2006), Dixit (2005), Murdock (2002), Lakdawalla and Philipson, (2006), Delfgaauw and Dur (2007, 2008), Glazer (2004), Makris (2009) and Makris and Siciliani (2013). Within the health economics literature the analytically-similar assumption of altruistic agents was introduced by Ellis and McGuire (1986), and then extended by Chalkley and Malcomson (1998), Eggleston (2005), Jack (2005), Siciliani (2009), Choné and Ma (2011), Brekke, Siciliani and Straume (2011, 2012), Kaarboe and Siciliani (2011) and Siciliani, Straume and Cellini (2013). Differently from existing studies, we allow for both motivation and positive reservation utilities (the latter being usually normalised to zero). As discussed above, this generates some new insights about the power of incentive schemes in the presence of motivated agents.

There is also an analogy between our study and the principal-agent literature with state dependent utility functions where the agent is profit-maximiser (Jullien, 2000, and Laffont and Martimort, 2003, Ch.3). In that literature, with two types and adverse selection of efficiency, a range of equilibria arise based on the rate of change of the agent’s reservation utility with her efficiency-type, which affects the relative strength of the ‘countervailing
incentives’. Our key result is derived under symmetric information (ie without adverse selection) and critically relies on the presence of motivated agents.

The paper is organised as follows. Section 2 introduces the main assumptions of the model. Section 3 provides a benchmark with no motivation. Section 3 provides the main analysis with motivated agents under symmetric information. Section 4 extends the analysis with private information on motivation and productivity. Section 5 presents conclusions and policy implications.

2 The model

We will focus on an environment where in general there is asymmetric information in both the productivity and the motivation of the agent. In such an environment, the design of the optimal incentive scheme is very complicated as previous studies have demonstrated.\(^1\) The high costs involved with such complexity could be one reason why linear incentive schemes are predominant in reality. Here we take on this view and restrict attention to the design of the optimal linear incentive schemes.

2.1 The agent

Define \(\theta\) as the inefficiency parameter, \(\alpha\) as the motivation parameter and \(q\) as the output produced by the agent. Efficiency and motivation are private information of the provider. We assume that \(\theta\) and \(\alpha\) are jointly distributed over the support \(\theta \in [\underline{\theta}, \overline{\theta}], \alpha \in [\underline{\alpha}, \overline{\alpha}],\) according to the distribution \(F(\theta, \alpha)\), with the density \(f(\theta, \alpha)\). Assume that the latter exists and is continuous, and that \(F(\theta, \alpha)\) is common knowledge.

The contract offered to the agent by the principal specifies a lump-sum transfer \(T\) and a price \(p\) for every unit of output provided. By assumption \(T\) and \(p\) do not vary with the type of provider. Therefore the revenues of the agent are \(T + pq\). The profit function is \(\pi = T + pq - C(q, \theta)\), where \(C(q, \theta)\) is the monetary cost of production. We assume \(C_\theta(q, \theta) > 0, C_{\theta\theta}(q, \theta) > 0\) and \(C_{qq} \geq 0\). Inefficient types have higher costs and higher marginal costs. The cost function is weakly convex in output. The principal’s benefit from

\(^{1}\)See Rochet and Chone (1998), Armstrong and Rochet (1999), Rochet and Stole (2003).
quantity \( q \) is \( B(q) \), with \( B_q(q) > 0 \) and \( B_{qq}(q) \leq 0 \). Given that the agent is motivated, she has the following utility function: \( U = \alpha B(q) + T + pq - C(q, \theta) \). Thus, \( \alpha \) measures the degree of alignment of the agent’s and the principal’s preferences. We assume that the agent has a non-negative reservation utility equal to \( \bar{U} \).

Output is chosen by the agent. Assume that the Second Order Condition (SOC) is satisfied: \( \alpha B_{qq}(q) - C_{qq}(q, \theta) < 0 \). Therefore, for a given price \( p \), an agent with inefficiency \( \theta \) and motivation \( \alpha \) chooses at an interior solution the optimal output \( q(\alpha, \theta, p) \) that satisfies the First Order Condition (FOC):\(^2\)

\[
\alpha B_q(q) + p - C_q(q, \theta) = 0 \tag{1}
\]

The output is chosen such that the marginal non-monetary benefit (i.e. that arises from intrinsic motivation) and the monetary marginal benefit (i.e. the price) is equal to the marginal monetary cost of production.

Straightforward comparative statics gives:

\[
\begin{align*}
\frac{\partial q}{\partial p} &= \frac{1}{C_{qq}(q, \theta) - \alpha B_{qq}(q)} > 0, \\
\frac{\partial q}{\partial \alpha} &= \frac{B_q(q)}{C_{qq}(q, \theta) - \alpha B_{qq}(q)} > 0, \\
\frac{\partial q}{\partial \theta} &= \frac{-C_{q\theta}(q, \theta)}{C_{qq}(q, \theta) - \alpha B_{qq}(q)} < 0.
\end{align*}
\]

As expected a higher price or higher motivation leads to higher output. Higher inefficiency implies lower output.

The indirect utility function of a provider with inefficiency \( \theta \) and motivation \( \alpha \) is:

\[
U(\alpha, \theta, p, T) \equiv \alpha B(q(\alpha, \theta, p)) + T + pq(\alpha, \theta, p) - C(q(\alpha, \theta, p), \theta). \tag{3}
\]

\(^{2}\)We also assume Inada conditions are satisfied on \( C_q(q) - \alpha B_q(q) \), i.e. \( \lim_{q \to 0} [C_q(q) - \alpha B_q(q)] = \infty \) and \( \lim_{q \to \infty} [C_q(q) - \alpha B_q(q)] = 0 \).
Using the envelope theorem, we obtain:

\[ \frac{\partial U}{\partial \alpha} = B(q) > 0, \]
\[ \frac{\partial U}{\partial \theta} = -C_{q}(q, \theta) < 0. \]

Higher motivation increases utility, and higher inefficiency reduces utility. It will prove useful when we derive the optimal contract to note here that the indirect utility of the agent takes its lowest value (for given contract, \((T, p)\)) when \(\theta = \overline{\theta}\) and \(\alpha = \underline{\alpha}\).

The profit enjoyed by the agent is

\[ \pi(\alpha, \theta, p, T) \equiv T + pq(\alpha, \theta, p) - C(q(\alpha, \theta, p), \theta). \] (4)

Turning to the monotonicity properties of the profit function with respect to \(p\) and \(\alpha\), for any given \(T\), we have (by using the agent’s FOC) that

\[ \frac{\partial \pi}{\partial \alpha} = (p - C_{q}(q, \theta)) \frac{\partial q}{\partial \alpha} = -\alpha B_{q}(q) \frac{\partial q}{\partial \alpha} < 0, \] (5)
\[ \frac{\partial \pi}{\partial \theta} = -C_{\theta} + (p - C_{q}(q, \theta)) \frac{\partial q}{\partial \theta} = -C_{\theta} + \alpha B_{q}(q) \frac{C_{q\theta}(q, \theta)}{C_{qq}(q, \theta) - \alpha B_{qq}(q)}. \] (6)

Higher motivation reduces profit. The reason is that a motivated agent is willing to work at a negative profit margin. Thus, an increase in motivation leads to higher output and lower profits.

The effect of a marginal change in the efficiency parameter on profit cannot be signed without further restrictions on fundamentals. Higher inefficiency increases costs and therefore reduces profits (first term). This is the standard effect; it is the only effect when the agent is not motivated. At the same time, higher inefficiency implies a lower output, and since the motivated agent is working at a negative profit margin, this in turn implies an increase in profit (second term). Note that, for a given quantity, sufficiently low motivation implies that the first effect dominates. Instead, sufficiently high motivation implies that profits increase with inefficiency. This seemingly counter-intuitive result emerges (as already mentioned) because a motivated agent operates at a negative profit margin. The
relative strength of the latter effect depends also on the degree of concavity of the benefit function and the degree of convexity of the cost function. A more concave benefit function and/or a more convex cost function implies, for a given motivation, that quantity is less responsive to the inefficiency parameter and hence the second term in Equ.(6) is relatively smaller.

It will prove useful when we derive the optimal contract to note here that the agent’s profit takes its lowest value (for given contract, \((T, p)\) and inefficient type) when \(\alpha = \overline{\alpha}\). Denote with \(\tilde{\theta}(p, \alpha)\) a minimum of the profit function \(\pi(\alpha, \theta, p, T)\) with respect to \(\theta\), where we have used the fact that this minimum is independent of \(T\), as the latter enters additively in the profit function. Note again (after using the agent’s FOC (1)) that if \(\alpha = 0\), the profit is minimised, for given contract, for the most inefficient agent, ie when \(\tilde{\theta}(p, 0) = \overline{\theta}\).

### 2.2 The principal

The problem of the principal is to choose \(T\) and \(p\) that maximise the difference between consumers’ benefit and the transfer to the agent.\(^3\) The principal must ensure the participation of the agent, and is also restricted by a break-even constraint that the revenues of the agent must be at least as high as monetary production costs.

Thus, the principal’s problem is:

\[
\max_{T, p} \int_{\alpha}^{\tilde{\pi}} \int_{\theta}^{\overline{\theta}} \left[ B(q(\alpha, \theta, p)) - T - pq(\alpha, \theta, p) \right] dF(\theta, \alpha) \tag{7}
\]

subject to the participation constraints:

\[
U(\alpha, \theta, p, T) \geq \overline{U}, \text{ for any } \alpha, \theta \tag{8}
\]

and the break-even constraints:

\[
\pi(\alpha, \theta, p, T) \geq 0, \text{ for any } \alpha, \theta.
\]

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\(^3\)The analysis holds under a more general utilitarian welfare function where the regulator maximises the sum of principal’s and agents’ utility but there is a distortionary effect from raising taxes, ie the opportunity cost of public funds is strictly positive. See Laффont and Tirole (1993).
Recalling our discussion about the monotonicity properties of the indirect utility and the maximised profit, we can restrict attention only to the \((\alpha, \overline{\theta})\)-type’s participation constraint:

\[ U(\alpha, \overline{\theta}, p, T) \geq \overline{U} \]  

(9)

and the \((\overline{\pi}, \tilde{\theta}(p, \overline{\pi}))\)-type’s break-even constraint:

\[ \pi(\overline{\pi}, \tilde{\theta}(p, \overline{\pi}), p, T) \geq 0. \]  

(10)

The reason is that if the optimal contract satisfies these two constraints, then it will necessarily satisfy the remaining ones. To lighten up exposition we will be referring hereafter to the \((\alpha, \overline{\theta})\)-type’s participation constraint and the \((\overline{\pi}, \tilde{\theta}(p, \overline{\pi}))\)-type’s break-even constraint as the participation and break-even constraints.

3 Benchmark Case: no motivation

We investigate the case when there is no motivation, i.e. when \(\alpha = \pi = 0\).

3.1 Symmetric Information

Let us begin with the case of symmetric information. In this case, the contract is type-dependent, and given efficiency type \(\theta\), the principal offers the contract that solves

\[ \max_{T, q \geq 0} B(q(0, \theta, p)) − T − pq(0, \theta, p) \]  

(11)

subject to the participation constraint:

\[ U(0, \theta, p, T) \geq \overline{U} \]  

(12)

and the break-even constraint:

\[ \pi(0, \theta, p, T) \geq 0. \]

Clearly, the participation constraint is more stringent than the break-even constraint.
because of the reservation utility being non-negative. Thus, in deriving the optimal contract, we can ignore the break-even constraint. The participation constraint is therefore binding, and hence $T$ is such that $U(0, \theta, p, T) = \overline{U}$. Accordingly, the optimal price is the solution of

$$\max_{p \geq 0} \ B(q(0, \theta, p)) - C(q(0, \theta, p)\theta) - \overline{U}. \tag{13}$$

Denote this optimal solution with superscripts $f$ and $n$, where $f$ refers to full information and $n$ refers to no motivation. Therefore, at an interior solution, the optimal price under symmetric information in the absence of motivation is given by\(^4\)

$$B_q(q(0, \theta, p^{f,n})) = C_q(q(0, \theta, p^{f,n}), \theta). \tag{14}$$

Thus, at optimum there is allocative efficiency. Obviously, the price is independent of the reservation utility. Combining (14) with (1) for $\alpha = 0$, we see that to induce the optimal output $q(0, \theta, p^{f,n})$ the principal only needs to set the price equal to the corresponding marginal benefit, ie $p^{f,n} = B_q(q(0, \theta, p^{f,n}))$.

### 3.2 Asymmetric Information

When the efficiency type of the agent is private information then the contract cannot be conditioned on the efficiency type. Recall that $\tilde{\theta}(p, 0) = \overline{\theta}$, and hence, the participation constraint is again more stringent than the break-even constraint. Thus, in deriving the optimal contract, we can ignore the break-even constraint. The participation constraint is binding, ie $T$ is such that $U(0, \overline{\theta}, p, T) = \overline{U}$, and the optimal price is therefore the solution of

$$\max_{p \geq 0} \int_{\tilde{\theta}} \left[ B(q(0, \theta, p)) - p q(0, \theta, p) - C(q(0, \overline{\theta}, p), \overline{\theta}) + p q(0, \overline{\theta}, p) \right] dF(\theta, 0) - \overline{U} \tag{15}$$

\(^4\)Note that the second order conditions are satisfied at the following solution by the assumed concavity of $B(q) - C(q)$ and that $\frac{\partial (B(q) - C(q))}{\partial p} > 0$.  

10
Denote the optimal solution with superscript $n$. Assume that the SOC is satisfied. Therefore, at an interior solution, the optimal price in the absence of motivation is given, after using the agent’s FOC (1), by

$$
\int_{\theta} q(0, \theta, p^n) dF(\theta, 0) = \int_{\theta} q(0, \theta, p^n) dF(\theta, 0) - q(0, \theta, p^n),
$$

(16)

This reflects the standard trade-off between rent extraction and allocative distortions. A lower price reduces information rents for any type $\theta < \theta$ but also reduces output of all types. At optimum the principal balances these two effects. The left-hand side (LHS) represents the average output distortion and the right-hand side (RHS) the gain from average rent extraction. To see the latter note that profits, at optimum, of the agent of inefficiency $\theta$ are given by $\mathcal{U} - pq(0, \theta, p) + C(q(0, \theta, p)\theta) + pq(0, \theta, p) - C(q(0, \theta, p), \theta)$. So decreasing marginally the price results in lower average rents of $\int_{\theta} q(0, \theta, p) dF(\theta, 0) - q(0, \theta, p)$. Importantly, the optimal price under asymmetric information also does not depend on the reservation utility when the agent is not motivated.

We now turn to the main focus of our study, namely the derivation of the optimal contract offered to a motivated agent: ie when $\alpha > 0$.

4 Motivated agent under symmetric information

Suppose that the agent is motivated, ie $\alpha > 0$, and the principal knows the agent’s type $(\alpha, \theta)$. In this case, the contract is type-dependent, and given a type $(\alpha, \theta)$, the principal offers the contract that solves

$$
\max_{T, p \geq 0} B(q(\alpha, \theta, p)) - T - pq(\alpha, \theta, p)
$$

subject to the participation constraint:

$$
U(\alpha, \theta, p, T) \geq \mathcal{U}
$$

\footnote{A sufficient condition for this is that $C_{qqq} = 0$ (which is satisfied with quadratic cost function). To see this note that $C_{qqq} = 0$ implies that $\frac{\partial^2 q}{\partial p^2} = \frac{\partial^2 q}{\partial p^2}$, and hence that the objective is a concave function of $p$ (this follows directly after taking the objective function’s second derivative with respect to $p$).}
and the break-even constraint:

$$\pi(\alpha, \theta, p, T) \geq 0.$$  \hfill (19)

Denote with $\mu$ and $\lambda$ the Kuhn-Tucker multipliers of the participation and break-even constraints, respectively. Assume that the SOCs hold.\footnote{A sufficient condition for this is that $B_{qqq} = C_{qqq} = 0$. To see this note that $C_{qqq} = B_{qqq} = 0$ implies that $\frac{\partial^3 \pi}{\partial q^3}$, and hence that the Lagrangian of the problem is a concave function of $p$ and $T$ (this follows directly from the concavity of $B(q) - C(q)$, the linearity of the Lagrangian with respect to $T$, and after taking the Lagrangian’s second partial derivative with respect to $p$ and using the first of the following FOCs).} The FOCs with respect to $T$ and $p$ are, respectively (after some straightforward manipulation):

$$\lambda + \mu = 1,$$

$$B_q(q(\alpha, \theta, p)) - C_q((q(\alpha, \theta, p)), \theta) = -\mu \alpha B_q(q(\alpha, \theta, p)).$$

The solution to this system alongside the appropriate complementary-slackness conditions (which are not stated for clarity of exposition) depends on the level of the reservation utility.

For low enough reservation utility, the break-even constraint is binding while the participation constraint is slack (ie $\mu = 0$ and $\lambda = 1$). In this case, the optimal price, denoted by $p_{BE}^f$ is given implicitly by

$$B_q(q(\alpha, \theta, p_{BE}^f)) = C_q((q(\alpha, \theta, p_{BE}^f)), \theta).$$  \hfill (20)

This requires that $\bar{U} \leq \alpha B(q(\alpha, \theta, p_{BE}^f)) \equiv \bar{U}^{BE}$, so that the participation constraint is satisfied. Since the break-even constraint does not depend directly on the agent’s motivation, the optimal output is identical to the one obtained when the agent is not motivated. This output does not depend on the degree of motivation. However, the price implementing such optimal output still does take into account the agent’s motivation. Given that the optimal output is independent of the degree of motivation, higher motivation implies that the price required to implement such optimal quantity is reduced because the agent internalises to a greater extent the benefit function of the principal. Specifically, combining (20) with the agent’s FOC (1) we have $p_{BE}^f = (1 - \alpha)B_q(q(\alpha, \theta, p_{BE}^f))$ and $\partial p_{BE}^f / \partial \alpha = -B_q < 0.$
The optimal lump-sum transfer is equal to $T^f_{BE} = C(q(\alpha, \theta, p^f_{BE}), \theta) - p^f_{BE}q(\alpha, \theta, p^f_{BE})$ with $\partial T^f_{BE}/\partial \alpha = -q(\alpha, \theta, p^f_{BE})\partial p^f_{BE}/\partial \alpha > 0$. The lump-sum transfer increases with motivation. Higher motivation has no effect on output but reduces the optimal price which needs to be compensated with a higher lump-sum transfer to make sure that the agent breaks even. Finally, outputs, prices and lump-sum transfers do not depend on the level of the reservation utility as long as $U \leq U^{BE}$ because the participation constraint is not binding.

For high enough reservation utility, the participation constraint is binding and the break-even constraint is slack (ie $\mu = 1$ and $\lambda = 0$). In this case, the optimal price, denoted by $p^f_{PC}$ is given implicitly by

$$
(1 + \alpha)B_q(q(\alpha, \theta, p^f_{PC})) = C_q((q(\alpha, \theta, p^f_{PC})), \theta).
$$

This requires that $U \geq \alpha B(q(\alpha, \theta, p^f_{PC})) \equiv U^{PC}$, so that the break-even constraint is indeed satisfied.\footnote{Since the participation constraint is binding ($\pi + \alpha B(q) = U$), then the break-even constraint $\pi \geq 0$ can be re-written as $U - \alpha B(q) \geq 0$.} The intuition behind the above rule for the optimal price is that a marginal increase in output enhances the utility of the motivated agent, therefore relaxing the participation constraint. Accordingly (when the participation constraint is binding) the required lump-sum transfer $T$ decreases by $\alpha B_q$, and hence the marginal cost from the marginally higher output decreases by the same amount. At optimum, the marginal benefit must equal the "augmented" marginal cost, giving rise to (21). The optimal quantity is therefore increasing in the degree of motivation. Combining (21) with the agent’s FOC (1) we see that to induce the optimal output $q(\alpha, \theta, p^f_{PC})$ the principal needs to set the price equal to the corresponding marginal benefit, ie $p^f_{PC} = B_q(q(\alpha, \theta, p^f_{PC}))$. Higher motivation implies a lower price if the marginal benefit is decreasing because the higher optimal quantity triggered by higher motivation reduces the marginal benefit ($\partial p^f_{PC}/\partial \alpha = B_{qq}\partial q(\alpha, \theta, p^f_{PC})/\partial \alpha \leq 0$). The price set by the principal does not depend on motivation if the marginal benefit is constant: on one hand, higher motivation implies, for a given level of output, that the agent needs to be incentivised less and therefore the price tends to be lower; on the other hand, higher motivation increases the optimal output which tends to
increase the optimal price. If the marginal benefit is constant, the two effects cancel each other out and the price is independent from the degree of motivation. If the marginal benefit is instead decreasing, higher motivation still increases the optimal output but at lower rate, so that the second effect dominates. The optimal price now decreases with the degree of motivation in equilibrium.

Again, we emphasise that the mechanisms through which higher motivation leads to lower price when the participation constraint or the break-event constraint are binding are different. When the break-even constraint is binding, the optimal output does not depend on motivation. Since more motivated agents provide a higher output, for a given price, the price needed to implement the optimal output is lower. When the participation constraint is binding, the optimal output increases with motivation because motivation relaxes the participation constraint. If the marginal benefit is constant, both agents with high and low motivation need to be incentivised by the same price. If the marginal benefit is decreasing, it is optimal to set a lower price for more motivated agents since the marginal benefit of output is lower at the optimum.

The optimal lump-sum transfer is now:

\[ T_{PC}^f(\alpha, \theta, p_{PC}) = C(q(\alpha, \theta, p_{PC}), \theta) - p_{PC}^f q(\alpha, \theta, p_{PC}) + B(q(\alpha, \theta, p_{PC})) \]

with \( \frac{\partial T_{PC}^f}{\partial \alpha} = -B(q(\alpha, \theta, p_{PC})) - q(\alpha, \theta, p_{PC}) \frac{\partial p_{PC}^f}{\partial \alpha} \). By the envelop theorem, we can ignore the effects through which motivation affects output. Higher motivation relaxes the participation constraints and, if the marginal benefit is decreasing, reduces price. The first effect tends to reduce the lump-sum transfer and the second effect tends to increase it. If the marginal benefit is constant (or the benefit/cost function not too concave/convex), then the first effect dominates and the transfer decreases with altruism. This is in sharp contrast to the solution when only the break-even constraint is binding, when the transfer increases with motivation. Finally, although outputs and prices do not depend on the reservation utility, the lump-sum transfer increases with it since a higher reservation utility makes the participation constraint more binding.

We now investigate the case of intermediate values of the reservation utility \( U^{BE} < \)
Note that in the presence of motivation this set is never empty since the optimal quantity when only the participation constraint is binding is strictly higher than when only the break-even constraint is binding so that $\alpha B(q(\alpha, \theta, p_{PC}^f)) > \alpha B(q(\alpha, \theta, p_{BE}^f))$. In this scenario both constraints are binding (i.e. $\mu > 0$, $\lambda > 0$) and hence the optimal price, denoted by $p_{U}^f$, is given by the solution of

$$\alpha B(q(\alpha, \theta, p_{U}^f)) = \bar{U}$$

(22)

This solution requires $0 < \mu < 1$ and hence we have from the principal’s FOCs that

$$(1 + \mu \alpha)B_q(q(\alpha, \theta, p_{U}^f)) = C_q((q(\alpha, \theta, p_{U}^f)), \theta).$$

(23)

Therefore, it also follows that $p_{BE}^f < p_{U}^f < p_{PC}^f$. When both the participation- and the break-even constraint are binding, the optimal price is intermediate between the one obtained when the break-even constraint only is binding and when the participation-constraint only is binding. If the principal would set the contract which specifies the price at the level when only the break-even constraint is binding then the participation constraint would be violated since the utility would be strictly less than the reservation one. If instead the principal would choose the contract which specifies the price at the level when the participation-constraint only is binding (ie at a higher level) then the break-even constraint would be violated since profits would be negative.

Differently from the other two scenarios, the level of reservation utility is crucial in the type of contract offered to a motivated agent. Recall that $p_{PC}^f$ and $p_{BE}^f$ are independent of the reservation utility. Totally differentiating (22) and (23) and applying Cramer’s rule, we obtain: $\partial p_{U}^f/\partial \bar{U} = 1/|\alpha B_q \partial q(\alpha, \theta, p_{U}^f)/\partial p_{U}^f| > 0$ and $\partial \mu/\partial \bar{U} = - (\mu \alpha B_{qq} - C_{qq})/\alpha^2 B_q^2 > 0$. The optimal price $p_{U}^f$ is now increasing in the reservation utility. A higher reservation utility makes the participation constraint more binding and increases the scope to increase output to relax such constraint. The higher optimal output is then implemented with a higher optimal price $(\partial q(\alpha, \theta, p_{U}^f)/\partial \bar{U} = (\partial q(\alpha, \theta, p_{U}^f)/\partial p_{U}^f)\partial p_{U}^f/\partial \bar{U} > 0)$.

Analogously, higher motivation overall relaxes the participation constraint and reduces
the optimal price: \( \frac{\partial p_{fU}}{\partial \alpha} = - \left( B + \alpha B_\theta \frac{\partial q(\alpha, \theta, p_{fU})}{\partial \alpha} \right) / \left( \alpha B_\theta \frac{\partial q(\alpha, \theta, p_{fU})}{\partial p_{fU}} \right) < 0 \). This result is qualitatively in line with those obtained under high reservation utility. The effect of motivation on equilibrium quantity is given by: 

\[
\frac{dq(\alpha, \theta, p_{fU})}{dq(\alpha, \theta, p_{BE})} = \frac{\partial q(\alpha, \theta, p_{fU})/\partial \alpha + (\partial q(\alpha, \theta, p_{fU})/\partial p_{fU})\partial p_{fU}/\partial \alpha = -B/\alpha B_\theta < 0.}
\]

In contrast the results obtained under high reservation utility, output reduces in equilibrium with higher motivation. On one hand, for a given price, motivation increases output but on the other hand it reduces prices which tends to reduce output. The second effect dominates over the first so that output reduces in equilibrium.\(^8\)

The optimal lump-sum transfer is now 

\[
T_{fU}(\alpha, \theta, p_{fU}) = C(q(\alpha, \theta, p_{fU}), \theta) - p_{fU}q(\alpha, \theta, p_{fU}).
\]

The effect of reservation utility on the optimal transfer is indeterminate.\(^9\) A higher reservation utility increases the optimal price which tends to reduce the transfer but also increases the optimal quantity which increases the net marginal cost (recall \( C_q - p_{fU} = \alpha B_\theta \)) which tends to increase the transfer. The effect of motivation on the optimal lump-sum transfer is also indeterminate.

The degree of motivation also influences the range of reservation utility levels over which the three types of equilibria arise. Higher motivation increases both \( U^{BE} \) and \( U^{PC} \) but has a stronger effect on \( U^{PC} \) than \( U^{BE} \). Therefore, higher motivation expands the range of reservation utilities over which only the break-even constraint is binding, as well as the one over which both the break-even and the participation constraint are binding. It instead reduces the range of reservation utilities over which only the participation constraint is binding. To see this note that 

\[
\frac{\partial U^{BE}}{\partial \alpha} = B(q(\alpha, \theta, p_{fBE})) \quad \text{and} \quad \frac{\partial U^{PC}}{\partial \alpha} = B(q(\alpha, \theta, p_{fPC})) + \alpha B_\theta dq(\alpha, \theta, p_{fPC})/d\alpha, \quad \text{where recall} \quad q(\alpha, \theta, p_{fPC}) > q(\alpha, \theta, p_{fBE}).
\]

This arises because motivation helps to relax the participation constraint and even more so when only the participation constraint is binding since this equilibrium is associated with higher levels of output.

\(^8\)The inequality \( p_{fPC} > p_{fBE} \) follows directly from the fact that now the principal needs to pay the agent less to ensure participation, as the agent derives utility from production due to her motivation. Therefore, the effective marginal cost under motivation is lower when the participation constraint is binding (it is equal to \( C_q(q, \theta) - \alpha B_\theta(q) \) instead of \( C_q(q, \theta) \)), and hence output is higher. Of course, to induce the higher output, the agent needs to be offered a higher price, and hence the result that \( p_{fPC} > p_{fBE} \). The monotonicity of \( p_{fU} \) in the reservation utility follows directly from its definition, \( U = \alpha B(q(\alpha, \theta, p_{fU})) \), and the fact that \( \alpha B_\theta > 0. \)

\(^9\)\( \frac{\partial T_{fU}}{\partial U} = \left[ (C_q - p_{fU})\frac{\partial q(\alpha, \theta, p_{fU})}{\partial p_{fU}} - q(\alpha, \theta, p_{fU}) \right] \frac{\partial p_{fU}}{\partial U}. \)
The following two propositions summarise the key results and emphasise the role of the reservation utility and motivation.

**Proposition 1.** Under motivation and symmetric information the optimal price and output is non-constant and (weakly) increasing in the reservation utility. More precisely, prices and output increase with reservation utility for intermediate levels of reservation utility but are constant for low and high levels of reservation utility.

**Proposition 2.** Higher motivation (weakly) decreases the optimal price. Higher motivation has no effect on the optimal price if the reservation utility is high and the marginal benefit from output is constant.

We conclude this section by providing a simple illustrative example. We assume that the marginal benefit is constant and the marginal cost is increasing: \( B(q) = bq \) and \( C(q) = F + \theta q^2 / 2 \). Optimal prices, quantities and lump-sum transfers are respectively equal to (i) for low reservation utility: \( p^{BE} = (1-\alpha)b, q^{BE} = b/\theta, T^{BE} = F + b^2(\alpha - 1/2)/\theta \); (ii) for high reservation utility: \( p^{PC} = b, q^{PC} = b(1+\alpha)/\theta, T^{PC} = F - 0.5(1 + \alpha)^2 b^2/\theta + \overline{U} \); (iii) for intermediate reservation utility: \( p^{U} = \overline{U}/\alpha b - \alpha b, q^{U} = \overline{U}/\alpha b, T^{U} = F + \overline{U} - (\theta/2)(\overline{U}/\alpha b)^2 \). Moreover, \( \overline{U}^{BE} = ab^2/\theta < \alpha(1 + \alpha)b^2/\theta = \overline{U}^{PC} \). Figure 1 illustrates the solution when \( b = \theta = 1, \alpha \in \{0.5; 0.6\} \) and \( F = 1 \).

[Figure 1 here]

The example illustrates some general features of the optimal solutions and emphasise the implications from having higher levels of motivation (dotted lines). The relation of price and output with the reservation utility is similar. Price and output is higher under "high" reservation utility than under "low" reservation utility but do not vary with reservation utility if the reservation utility is either in the "high" or "low" range. Price and output instead increase with reservation utility for intermediate values. The lump-sum transfer does not vary with reservation utility when reservation utility is "low" but increases with it when it is "high".

Higher levels of motivation weakly reduce price across the three scenarios. When the reservation utility is "high", motivation has no effect of price since by assumption the marginal benefit from output is constant. Motivation has no effect on output when
reservation utility is "low" since the optimal output for the purchaser does not depend on motivation. Motivation instead increases the optimal output when the reservation utility is "high" since higher motivation increases the scope for relaxing the participation constraint. Interestingly, motivation mostly reduces the output, for a given level of reservation utility, when reservation utility is "intermediate". Although motivation increases output, for a given price, the reduction in the optimal price is so large that output falls.

The analysis in this section has not investigated the role of inefficiency on the optimal contract. This is briefly discussed here. The results are in line with extensive previous literature on optimal regulation (eg Baron and Myerson, 1982). Under the three scenarios, optimal prices (weakly) increase with inefficiency. Price does not vary with inefficiency if the marginal benefit is constant but only when the reservation utility is "low" and "high". For "intermediate" levels of reservation utility price reduces with inefficiency even when the marginal benefit is constant. The optimal output reduces when inefficiency is higher when the reservation utility is "low" and "high". Perhaps interestingly, it does not reduce with inefficiency when the reservation utility is "intermediate" ($\partial q^L / \partial \theta = 0$): the lower output induced by higher inefficiency is exactly compensated by the higher price. Figure 2 illustrates the solution when $b = 1$, $\alpha = 0.5$, $\theta \in \{1; 1.2\}$ and $F = 1$.

We now return to the main model where the agent is motivated and privately informed about the type.

5 Motivated agent under asymmetric information

Recall that the principal’s problem is:

$$\max_{T, p \geq 0} \int_0^\alpha \int_{\theta} \left[ B(q(\alpha, \theta, p)) - T - pq(\alpha, \theta, p) \right] dF(\theta, \alpha)$$

$$10 \partial p_{BE}^L / \partial \theta = -B_{qq} C_{q\theta} / (C_{qq} -(1 + \alpha) B_{qq}) \geq 0; \partial p_{PC}^L / \partial \theta = (1 - \alpha) B_{qq} C_{q\theta} / (C_{qq} - \alpha B_{qq}) \geq 0; \partial p_{L}^L / \partial \theta = C_{q\theta} > 0.$$
subject to the participation constraint:

\[ U(\alpha, \bar{\theta}, p, T) \geq \mathcal{U} \]  

(25)

and the break-even constraint:

\[ \pi(\bar{\alpha}, \bar{\theta}(p, \bar{\alpha}), p, T) \geq 0. \]  

(26)

Suppose that \( \bar{\theta}(p, \bar{\alpha}) \) is almost everywhere differentiable.\(^1\) To describe the solution denote, with some abuse of notation, \( \mu \) and \( \lambda \) as the multipliers of the above participation and break-even constraints, respectively. The Lagrangian is

\[
\int_{\alpha}^{\pi} \int_{\vartheta}^{\bar{\theta}} \left[ B(q(\alpha, \theta, p)) - T - pq(\alpha, \theta, p) \right] dF(\theta, \alpha) \\
+ \mu U(\alpha, \bar{\theta}, p, T) - \mathcal{U} \]  

\[ + \lambda \pi(\bar{\alpha}, \bar{\theta}(p, \bar{\alpha}), p, T) \]

Assume that the SOC\\(s\) hold.\(^2\) The FOC\\(s\) of this problem with respect to \( T \) and \( p \) are, respectively (after some straightforward manipulation that make use of the envelope theorem):

\[
\lambda + \mu = 1,
\]

\[
\int_{\alpha}^{\pi} \int_{\vartheta}^{\bar{\theta}} \left\{ \left[ (1 + \alpha)B_q(q(\alpha, \theta, p)) - C_q((q(\alpha, \theta, p)), \theta) \right] \frac{\partial q(\alpha, \theta, p)}{\partial p} - q(\alpha, \theta, p) \right\} dF(\theta, \alpha)
\]

\[ = -\lambda \left[ q(\alpha, \bar{\theta}(p), p) - \bar{\alpha}B_q(q(\alpha, \bar{\theta}(p), p)) \frac{\partial q(\alpha, \bar{\theta}(p), p)}{\partial p} \right] - \mu q(\alpha, \bar{\theta}, p). \]

\(^1\) Similar arguments to the ones below could be developed when \( \bar{\theta}(p, \alpha) \) is not differentiable, but at the cost of more cumbersome exposition, without adding much further insights, and so we abstain from discussion this case.

\(^2\) A sufficient condition for this is that \( C_{qqq} = B_{qq} = 0 \). To see this note that \( C_{qqq} = B_{qq} = 0 \) implies that \( \frac{\partial^2 C}{\partial p^2 \partial T} = \frac{\partial^2 B}{\partial p^2 \partial T} \), and hence that the Lagrangian of the problem is a concave function of \( p \) and \( T \) (this follows directly from the concavity of \( B(q) - C(q) \), the linearity of the Lagrangian with respect to \( T \), and after taking the Lagrangian’s second partial derivative with respect to \( p \), and using the agent’s FOC and the first of the following FOC\\(s\)).
To describe the solution it is convenient to define the following function:

\[ W(p) \equiv U(\alpha, \bar{\theta}, p, T) - \pi(\bar{\alpha}, \bar{\theta}(p, \bar{\alpha}), p, T) \]

\[ = \alpha B(q(\alpha, \bar{\theta}, p)) + p[q(\alpha, \bar{\theta}, p) - q(\bar{\alpha}, \bar{\theta}(p), p)] + \left[ C(q(\bar{\alpha}, \bar{\theta}(p), p, \bar{\theta}(p)) - C(q(\alpha, \bar{\theta}, p, \bar{\theta})) \right]. \]

In the absence of heterogeneity on the agent’s type on motivation and productivity, then
\( W(p) \) collapses to \( \alpha B(q(\alpha, \theta, p)) \) and we obtain as a special case the analysis provided in the section with symmetric information. It is precisely because the participation and break-even constraints bind for different agents’ types that the solution under asymmetric information differs from symmetric information. The solution to the above system, alongside the appropriate complementary-slackness conditions (which are not stated for clarity of exposition), depends on the level of the reservation utility in the following way.

For low enough reservation utility, the break-even constraint is binding while the participation constraint is slack (ie \( \mu = 0 \) and \( \lambda = 1 \)). In this case, the optimal price, denoted by \( p^*_{BE} \) is given by

\[
\int_{\alpha}^{\bar{\alpha}} \int_{\theta}^{\bar{\theta}} \left\{ [(1 + \alpha)B_q(q(\alpha, \theta, p^*_{BE})) - C_q((q(\alpha, \theta, p^*_{BE}))\frac{\partial q(\alpha, \theta, p^*_{BE})}{\partial p}) \right\} dF(\alpha, \theta)
\]

\[ = \int_{\alpha}^{\bar{\alpha}} \int_{\theta}^{\bar{\theta}} q(\alpha, \theta, p^*_{BE})dF(\alpha, \theta) - q(\bar{\alpha}, \bar{\theta}(p^*_{BE}, \bar{\alpha}), p^*_{BE})
\]

\[ + \alpha B_q(q(\bar{\alpha}, \bar{\theta}(p^*_{BE}, \bar{\alpha}), p^*_{BE})) \frac{\partial q(\bar{\alpha}, \bar{\theta}(p, \bar{\alpha}), p^*_{BE})}{\partial p}. \]

This requires that \( U \leq W(p^*_{BE}) \), for the participation constraint to be satisfied. A marginal increase in price increases the revenue (for given output) of the type who is most motivated and has inefficiency \( \bar{\theta} \) (second term on RHS), which relaxes the break-even constraint but increases profits for the expected type (first term on RHS) which makes the break even constraint more binding. Moreover, since motivated agents work at a negative profit margin, the higher quantity induced by the higher price for type \((\bar{\alpha}, \bar{\theta})\) reduces profits which also makes the profit constraint more binding (third term on RHS). Note that the quantity of type \((\bar{\alpha}, \bar{\theta})\) can be higher than the quantity of the expected type (for example this is always the case if \( \bar{\theta} = \theta \)). Therefore, if it may be optimal to increase the
price for rent-extraction purposes. This is in contrast to standard intuition where it is typically optimal to distort prices downwards in the presence of asymmetric information on inefficiency to reduce informational rents (as we show in section 3.2 above).

For high enough reservation utility, the participation constraint is binding and the break-even constraint is slack (i.e., \( \mu = 1 \) and \( \lambda = 0 \)). In this case, the optimal price, denoted by \( p_{PC}^* \), is given by

\[
Z \int_{\alpha}^{\pi} \int_{\theta}^{\bar{\theta}} \left\{ [(1 + \alpha)B_q(q(\alpha, \theta, p_{PC}^*)) - C_q((q(\alpha, \theta, p_{PC}^*)), \theta)] \frac{\partial q(\alpha, \theta, p_{PC}^*)}{\partial p} \right\} dF(\alpha, \theta) = \int_{\alpha}^{\pi} \int_{\theta}^{\bar{\theta}} q(\alpha, \theta, p_{PC}^*) dF(\alpha, \theta) - q(\alpha, \bar{\theta}, p_{PC}^*). \tag{28}
\]

This requires that \( \bar{U} \geq W(p_{PC}^*) \), so that the break-even constraint is indeed satisfied. The LHS gives the benefit from a marginal increase in price in terms of allocative efficiency, i.e., the difference between the augmented (due to motivation) marginal benefit and marginal cost. The RHS gives the cost of a marginal increase in price in terms of informational rents: a higher price relaxes the participation constraint of the most inefficient and least motivated type but increases the informational rent for all the other types. Since the quantity of the most inefficient and least motivated type is the lowest, the first effect is smaller than the second. Therefore, the price is distorted downwards due to rent extraction purposes.

For intermediate values of the reservation utility, both constraints are binding (i.e., \( \mu > 0 \), \( \lambda > 0 \)) and hence the optimal price, denoted by \( p_{UP}^* \), is given by the solution of

\[
W(p_{UP}^*) = \bar{U}
\]

The following expression is critical in characterising the solution:

\[
D \equiv \pi B_q(q(\pi, \tilde{\theta}(\pi, p)), p) \frac{\partial q(\pi, \tilde{\theta}(\pi, p), p)}{\partial p} - \left[ q(\pi, \tilde{\theta}(\pi, p), p) - q(\alpha, \bar{\theta}, p) \right]. \tag{29}
\]

More precisely we want to determine whether the price when the participation constraint is binding is higher than when the break-even constraint is binding as in the analysis with
symmetric information and whether the price is still weakly increasing in the reservation utility. We will show that the results obtained under symmetric information do not necessarily carry over in the presence of asymmetric information. With asymmetric information the price can weakly decrease with the reservation utility.

Scenario (1). Suppose that \( D > 0 \). Then, comparing FOCs (27) and (28) (and given the SOCs), it follows that the price when the participation constraint only is binding is higher than when the break-even constraint only is binding: \( p^*_{PC} > p^*_{BE} \). The ranking of prices is analogous to the symmetric information case. Suppose further that either \( e(\pi, p) = 0 \) for any \( p \); or \( e(\pi, p) = 0 \) for any \( p \). Then, it also follows that \( W'(p) = D > 0 \) and \( W(p^*_{PC}) > W(p^*_{BE}) \). The three regions that define the cases established above in terms of low, high and intermediate reservation utility are well defined. Again, the solution is analogous to the symmetric information case. For low reservation utility, ie for any \( U \leq W(p^*_{BE}) \), then \( p^*_{BE} \) is the solution and \( \partial p^*_{BE} / \partial U = 0 \). For high reservation utility, ie for any \( U \geq W(p^*_{PC}) \) then \( p^*_{PC} \) is the solution and \( \partial p^*_{PC} / \partial U = 0 \). For intermediate reservation utility, ie for \( W(p^*_{BE}) \leq U \leq W(p^*_{PC}) \) then \( p^*_{U} \) is the solution and \( \partial p^*_{U} / \partial U = D > 0 \).

When is inequality \( D > 0 \) likely to be satisfied? Suppose the degree of heterogeneity in motivation is sufficiently small, i.e. \( \pi \rightarrow \alpha \). If \( \tilde{\theta}(\pi, p) = \tilde{\theta} \), the second term of (29) vanishes and the condition in question is always satisfied. Note that \( \tilde{\theta}(\pi, p) = \tilde{\theta} \) holds when \( \frac{\partial \pi(\alpha, \beta, p, T)}{\partial \beta} < 0 \), ie, after recalling (6), when the benefit function is sufficiently concave and/or the cost function is sufficiently convex. The solution is qualitatively similar to the one obtained under symmetric information.

Scenario (2). Suppose \( D < 0 \). Then \( p^*_{PC} < p^*_{BE} \). The price ranking is now reversed compared to the symmetric-information case. Suppose further that either \( \tilde{\theta}(\pi, p) = \tilde{\theta} \) for any \( p \); or \( \tilde{\theta}(\pi, p) = \tilde{\theta} \) for any \( p \). Using these, it then follows that \( W'(p) = D < 0 \). In this case \( W(p^*_{PC}) > W(p^*_{BE}) \) and the three regions that define the cases established above are again well defined. Nevertheless, the relation between prices and reservation utility is reversed. Specifically for any \( \bar{U} \geq W(p^*_{PC}) \) then \( p^*_{PC} \) is the solution. For \( \bar{U} \leq W(p^*_{BE}) \) then \( p^*_{BE} \) is the solution, and for \( W(p^*_{BE}) \leq \bar{U} \leq W(p^*_{PC}) \) then \( p^*_{U} \) is the solution, and \( p^*_{PC} < p^*_{U} < p^*_{BE} \) with \( \partial p^*_{U} / \partial U = D < 0 \). Therefore, the price is decreasing in reservation
utility in the reservation utility lies in the "intermediate" range. This is in sharp contrast to the solution with symmetric information.

When is inequality \( D < 0 \) satisfied? If \( \tilde{\theta}(\pi, p) = \theta \) and \( \alpha = 0 \) then the bracketed term of (29) is positive and takes the highest possible value for a given \( \pi \). Moreover, if \( \tilde{\theta} \) is sufficiently high then this term is larger than the first term of (29) (which does not depend on \( \tilde{\theta} \)). \( \tilde{\theta}(\pi, p) = \theta \) holds when \( \frac{\partial \pi(\alpha, \theta, p; T)}{\partial \theta} > 0 \), ie, after recalling (6), when the benefit function is not very concave and/or the cost function is not very convex.

To summarise:

**Proposition 2:** Under motivation and asymmetric information the optimal price in non-constant. The price is (weakly) increasing in the reservation utility if i) the degree of heterogeneity in motivation is sufficiently small, and ii) the benefit function is sufficiently concave and/or cost function is sufficiently convex. Instead, the price is (weakly) decreasing in the reservation utility if: i) the degree of heterogeneity in efficiency is sufficiently large, ii) the benefit function is not very concave and/or cost function is not very convex.

Figure 3 illustrates the two possible solutions.

![Figure 3 here](image-url)

We conclude this section by providing two examples. First, we provide an example when the counter-intuitive scenario with \( D < 0 \) arises. Suppose that \( B(q) = \log(q) \) and \( C(q) = \theta \frac{q^2}{2} \). Then, \( q(\alpha, \theta, p) = \frac{p + \sqrt{p^2 + 4\alpha \theta}}{2\theta} \) and

\[
\pi(\alpha, \theta, p) = \frac{1}{8\theta} \left( p + \sqrt{p^2 + 4\alpha \theta} \right) \left[ 2p + \left( p - \sqrt{p^2 + 4\alpha \theta} \right) \right]
\]

with

\[
\frac{\partial \pi(\alpha, \theta, p)}{\partial \theta} = -\frac{1}{\theta} \pi(\alpha, \theta, p) + \left[ p - \sqrt{p^2 + 4\alpha \theta} \right] \frac{1}{2} \left( p^2 + 4\alpha \theta \right) 4\alpha
\]

which is negative for non-negative profits (which must hold by design at an individually rational contract) due to, when \( \alpha > 0, p < \sqrt{p^2 + 4\alpha \theta} \). We thus have that \( \tilde{\theta}(\alpha, p) = \theta \) for any \( p \) and \( \alpha \). Therefore,

\[
D = \frac{\alpha}{\sqrt{p^2 + 4\alpha \theta}} - \frac{\sqrt{p^2 + 4\alpha \theta}}{2\theta} + \frac{\sqrt{p^2 + 4\alpha \theta}}{2\theta}
\]

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with
\[
\frac{\partial D}{\partial \alpha} = -\frac{2\bar{\alpha} \vartheta}{(p^2 + 4\pi \vartheta)^2} < 0.
\]
Therefore, for sufficiently high \(\bar{\alpha}\), we have \(D < 0\).

6 Conclusions

Pay-for-performance schemes are increasingly advocated in the public sector. They reward higher output or measurable dimensions of quality in several sectors, such as health, education and child care. Public sector employees are typically motivated or feature altruistic preferences. This in turn may intuitively imply that they do not need to be incentivised with monetary incentive schemes. We show that whether this logic is correct depends on the interplay between the degree of motivation and the reservation utility.

The study shows that high-powered incentive schemes can be compatible with high degree of motivation. This arises when the reservation utility of the agent is high and the participation constraint is binding. In such case, the principal can exploit the effectively lower marginal cost of the provider and implement higher levels of output with a high-powered incentive scheme. The opposite holds when the reservation utility is low and the break even constraint is binding. Higher motivation does not imply a lower marginal cost of provision but implies a higher output, for a given price, which in turn implies that power of the incentive scheme is reduced. We also show that for an intermediate range of reservation utility, both the participation and the break-even constraints are binding. This solution smoothly connects the other two, so that higher reservation utility within the intermediate range implies higher output and prices. These results are generally robust when the environment allows for heterogeneity of agents’ types on the degree of motivation and efficiency, as long as the degree of dispersion in the degree of motivation is not too high. With high dispersion on motivation, the principal may find optimal to distort output upwards for rent-extraction purposes and this can make the power of the incentive scheme higher for low reservation utility.

High-powered incentive schemes therefore do not necessarily imply a mis-trust of policymakers (the principal) on how motivated public-service employees are, but a desire to
exploit their degree of motivation by inducing the agent to work harder. It is indeed not uncommon to hear for example in the health sector that employees (nurses, doctors) are ‘over-worked’ and working under significant pressure.

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Figure 1. Optimal price, output and lump-sum transfer

Price $p$

Output $q$

Lump-sum Transfer $T$

Reservation utility

Motivation $\alpha = 0.5$

Motivation $\alpha = 0.6$

Motivation $\alpha = 0.5$

Motivation $\alpha = 0.6$

$U > \bar{U}$

$U = \bar{U}$

$U < \bar{U}$

$\pi = 0$

$\pi = 0$

$\pi > 0$

$\pi = 0$
Figure 2. Optimal price, output and lump-sum transfer

Price $p$

Output $q$

Lump-sum Transfer $T$

Reservation utility

Reservation utility

Reservation utility

Inefficiency $\theta = 1$

Inefficiency $\theta = 1.2$

Low $U > U$

Intermediate $U = U$

High $U = U$

$\pi = 0$

$\pi = 0$

$\pi > 0$
Figure 3. Optimal price

Figure 3.a

Figure 3.b

Reservation utility, $W(p)$