Public Sector Wage Policy and Labor Market Equilibrium: A Structural Model

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October 2011

PRELIMINARY AND INCOMPLETE

Abstract

We develop and estimate a structural model that incorporates a sizable public sector in a labor market with search frictions. The wage distribution and the employment rate in the public sector are taken as exogenous policy parameters. Overall wage distribution and employment rate are determined within the model, taking into account the private sector’s endogenous response to public sector employment policies. Job turnover is sector specific and transitions between sectors depend on the worker’s decision to accept alternative employment in the same or different sector by comparing the value of employment in the current and prospective jobs. The model is estimated on British data by a method of moments. We use the model to simulate the impact of various counterfactual public sector employment policies.

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1 Introduction

We formulate a search-theoretic model that incorporates interactions between the public and private sectors. The wage offer distribution of the public sector is treated as an exogenous policy parameter, and conditional on this and exogenous transitional parameters, the offer and observed private sector wage distributions are then derived endogenously. Exploiting data from the British Household Panel Survey (BHPS) the model is then estimated by minimum-distance matching of some key moments from the data. These estimates allow us to make counterfactual policy analysis of different public sector wage policies.

There has been very little done in modeling the public sector explicitly within an equilibrium model of the labor market and nothing to our knowledge that estimates such a model. This is a considerably large oversight when one thinks that in our data 24% of employed individuals were employed by the public sector.\footnote{Algan et al. (2002) report that, based on a slightly narrower definition of public sector employment, the OECD finds an average public sector share of total employment of 18.8\% in 2000 over a sample of 17 major OECD countries.} It is, of course, naive to believe that with an employment share this large the public sector will not influence wage determination and by extension overall employment.

Instead of modeling the behavior of private sector firms explicitly, the literature thus far has been dominated by reduced form comparisons of the two sectors.\footnote{For a survey of the literature, see Bender (1998).} The general consensus of stylized facts emerging from the empirical literature is that the public sector wage distribution is more compressed than the private sector and workers receive a small public sector wage premium which is more prevalent in low skilled workers.

With these stylized facts being known for some time, it is fairly surprising that so little has been done in explicitly modeling the interaction between the two sectors. The existing literature that does this has largely focused on assessing the impact of the public sector on the level or volatility of aggregate wages and employment. Papers in that vein include Algan et al. (2002), Quadrini and Trigari (2008), Hörner et al. (2007), and Gomes (2011). All model search as directed to a particular sector and and none has direct job-to-job reallocation (within or between sectors). All model wages as determined by bargaining over the surplus from a match. Algan et al. (2002) find that the creation
of public sector jobs has a massive crowding out effect on private-sector job creation, such that the marginal public sector job may destroy as many as 1.5 private sector jobs in some OECD countries. This crowding out effect is especially strong when public sector wages are high and/or when public and private-sector output are close substitutes. Focusing on the cyclicality of employment, Quadrini and Trigari (2008) examine a public sector wage policy that is acyclical (a single wage) and procyclical (government wage is an increasing function of private sector wages). Calibrating the model for the US economy, they predict that the volatility of employment increased by two and four times, with the existence of the public sector over the periods 1970-2003 and 1945-1970, respectively. They postulate that this reduction over time is because of a more pro-cyclical policy adopted by the state.

Hörner et al. (2007) model two economies: one where a benevolent social planner aims to maximize individual’s welfare with public sector wages and employment (amongst other matters); the other in the absence of a public sector. The equilibrium of the model allows the authors to draw two conclusions. Firstly, that the public sector has an ambiguous effect on overall employment and secondly, that in more turbulent times there will be higher unemployment in the economy with the public sector. The latter result comes from the individuals being risk averse and therefore crowding into the safer sector (the public sector) in more uncertain times. Finally, Gomes (2011) builds a dynamic stochastic general equilibrium model with search and matching frictions calibrated to U.S. data and shows that high public sector wages induce too many unemployed to apply for public sector jobs and raise unemployment. He further argues that the cyclicality of public sector wage policy has a strong impact on unemployment volatility.

To our knowledge, other than this paper, the extent of the literature that explicitly models private sector firm behavior for a given public wage setting policy is Albrecht et al. (2011) and Burdett (2011). Albrecht et al. (2011) extend the canonical Diamond-Mortensen-Pissarides model (Pissarides, 2000) to incorporate the public sector posting an exogenous number of job vacancies. They also introduce match-specific productivity and heterogeneity in worker’s human capital, which generate wage dispersion in the private sector. Burdett (2011) is closer to ours in the sense that firms post wages rather than bargain over the surplus and, crucially, that workers are allowed to search
on the job. On-the-job search — and the possibility for workers to switch jobs between sectors — is an essential ingredient of the model as it captures rich and complex aspects of the competition between sectors over the labor services of workers. Moreover, as is understood since at least Burdett and Mortensen (1998), on-the-job search begets endogenous wage dispersion. However, in Burdett’s model the public sector sets a single wage, leading to the counterfactual prediction that the private sector’s response to competition from the public sector is to post a wage distribution with a hole in its support. In our model the public sector’s policy is to post wages from a distribution of wages. This allows us to have wage differences in the public sector and a continuous private sector wage distribution with connected support. Unlike the two models discussed we allow for differences in cross-sector job destruction and job offer arrival rates. Crucially, this paper is unique in the literature insofar as the parameters of the model are estimated.

Methodologically, a similar paper to ours is Meghir et al. (2010) who develop an equilibrium wage posting model to determine the interaction between a formal and informal sector in a developing country. Here the two sectors vary in the degree of regulatory tightness, the formal sector firms incurring additional costs to wages in the form of corporation tax, income tax, social security contributions, severance pay and unemployment insurance. While firms in the informal sector are not exposed to these labor market regulations they do face the chance (with given probability) of incurring a non-compliance cost. Unlike this model, private sector firms endogenously select into either the formal or informal sector and the equilibrium wage offer distributions of both are determined endogenously, similarly to this paper, Meghir et al. estimate their model using indirect inference.

The paper is organized as follows. In the next section we derive the equilibrium structural model. Section 3 gives an overview of the properties of the data used for estimation. Sections 4 and 5 outline the estimation protocol and present the estimation results. In Section 6 we use the results obtained to run counterfactual policy analysis and in Section 7 we conclude.
2 The Model

2.1 Basic Environment

We consider a model of wage-posting akin to Burdett and Mortensen (1998). Time is continuous and the economy is in steady state. A $[0, N]$ continuum of infinitely lived, risk neutral, ex-ante homogeneous workers face a fixed continuum of employers in a frictional labor market. A key aspect of our approach is that the set of employers comprises a continuum of infinitesimally small heterogeneous, profit-maximizing firms which we interpret as Private Sector employers, that coexist with a single, non-infinitesimal, non-profit maximizing employer which we interpret as the Public Sector. Private-sector firms behave in the same way as employers in the standard Burdett and Mortensen (1998) model, while public sector wages and level of labor demand are taken as exogenous. Because of the public sector’s non-infinitesimal size as an employer, changes in public sector employment policies will have a non-trivial impact on labor market equilibrium, both directly and through the private sector’s response to said changes in policy. The main objective of this paper is to quantify that impact for various policy changes.

2.2 Workers and Jobs

A worker can be in one of three states, either unemployed or employed in the public or private sector. Throughout the paper we indicate a worker’s labor market state using a subscript $s \in \{u, p, g\}$ for unemployment, employment in the private sector and employment in the public sector, respectively. The steady-state numbers of workers in each employment state are denoted as $N_u$, $N_g$ and $N_p$.

A job is fully characterized by a constant wage $w$ and the sector it is attached to. Workers receive job offers at a Poisson rate that depends on the worker’s state. Jobs are not indefinite and also face a Poisson destruction shock. The notation used to describe all those shocks is largely consistent with the previous literature, $\delta$ being used to denote job destruction shocks and $\lambda$ for job offer arrival rates. To explain the two states between which the particular worker transits a two letter index is used. The first letter designates the sector of origin and the second the sector of destination. So for example, $\lambda_{pg}$ is the arrival rate of public-sector offers to private sector employees, $\lambda_{ug}$ is arrival
rate of public-sector offers to unemployed workers, and so on. As job destruction always results in
the worker becoming unemployed, a single index is used to specify the job destruction shock, \( \delta_p \) or \( \delta_g \).

To summarize, a worker employed in sector \( s \in \{ p, g \} \) faces three random shocks: a job de-
struction shock \( \delta_s \), after which the worker becomes unemployed and gets flow utility \( b \), a within
sector job offer \( \lambda_{ss} \) and a cross-sector job offer \( \lambda_{ss'} \). While all the arrival rates are exogenous,\(^3\) the
associated acceptance decisions are determined endogenously.

As in Burdett and Mortensen (1998), a job offer from the private sector consists of a draw
from a wage offer distribution \( F_p(\cdot) \) which results from uncoordinated wage posting by the set of
infinitesimally small private employers, each maximizing its profit taking as given the strategies of
other firms and that of the public sector. \( F_p(\cdot) \) will be determined endogenously in equilibrium.
By contrast, a job offer from the public sector consists of a draw from a (continuous) wage offer
distribution \( F_g(\cdot) \) which is taken as an exogenous policy tool. We thus assume from the outset
that the public sector offers jobs at different wage levels to observationally similar workers. This
assumption is both realistic — residual wage dispersion among similar public-sector employees is
observed in the data — and needed to avoid the counterfactual prediction of Burdett (2011) that
equilibrium features a wage distribution with disconnected support.

We finally recognize that public and private sector jobs may differ along other dimensions than
just the wage and the transition parameters. There may be, for example, systematic differences
in working conditions. Also workers may enjoy a utility surplus (‘public service glow’) or suffer a
utility loss (‘public service stigma’) from working in the public sector. In order to capture those
unobserved differences in a parsimonious way, we assume that the flow utility that workers derive
from working in sector \( s \in \{ p, g \} \) for a wage of \( w \) is equal to \( w + a_s \), where \( a_s \) is a sector-specific
‘amenity’. Finally, and without further loss of generality, we normalize \( a_p \) to zero, so that \( a_g \)

\(^{3}\)For now… We will endogenize them eventually.
reflects the relative utility surplus (or loss) from working in the public sector. This utility surplus is assumed to be the same for all workers.

### 2.2.1 Worker Values and Reservation Wages

An individual’s utility is given by the present discounted future stream of wages. For a given worker, the transitional parameters will be unchanged if he moves job within a sector. The acceptance decision for an offer within the worker’s current sector is therefore entirely determined by the worker’s current wage and the new wage being offered. If the new offer, \( x \), is higher than the worker’s current wage, \( w \), he will accept and otherwise reject. However, since a change in sector is not only associated with a different wage but also with a change in transitional parameters, the acceptance decision is not so trivial when the job offer is from another sector. Thus depending on the two sets of transition parameters, an individual may accept a job offer from a different sector with a wage cut, or conversely, require a higher wage in order to accept. These acceptance decisions can be characterized by a set of reservation wages, accepting job offers greater or equal to these wages and rejecting those below. With the three states we have defined, there will be four corresponding reservation wages, which we define using the same double-index system as for transition parameters: \( R_{up}, R_{ug}, R_{pg}(w) \) and \( R_{gp}(w) \).

When employed, a worker’s reservation wage will be a function of their current wage. The reservation wage applying to private (public) sector offers made to a public (private) sector worker earning \( w \) makes said worker indifferent between his current present value and the present value of private (public) sector employment at his reservation wage. Formally, that is \( W_p(R_{gp}(w)) = W_g(w) \) and \( W_g(R_{pg}(w)) = W_p(w) \), where \( W_p(w) \) and \( W_g(w) \) are the values of working in the private and public sectors at wage \( w \). It follows from those definitions that the two reservation wages described are reciprocal of each other:

\[
R_{pg}(R_{gp}(w)) = w. \tag{1}
\]

The reservation wage of an unemployed worker receiving an offer from the public (private) sector is the wage at which they are indifferent between unemployment and the public (private) sector. Formally, the two reservation wages solve the equality, \( U = W_p(R_{up}) = W_g(R_{ug}) \), where
$U$ is the present value of a worker in unemployment. Hence applying (1) to this equality one can derive a second property of the reservation wages:

$$R_{pg}(R_{up}) = R_{ug}. \quad (2)$$

Note that the analogous property for $R_{ap}(\cdot)$ also holds.

### 2.2.2 Bellman Equations

The value function for an unemployed worker is defined by the following Bellman equation, where $r$ is the rate of time preference, constant across workers:

$$rU = b + \lambda_{up} \int_{R_{up}}^{+\infty} [W_p(x) - U] \, dF_p(x) + \lambda_{ug} \int_{R_{ug}}^{+\infty} [W_g(x) - U] \, dF_g(x), \quad (3)$$

The first term, $b$ is the flow utility an individual gets from being in unemployment. Offers arrive from the public (private) sector at a rate of $\lambda_{up}$ ($\lambda_{ap}$). Wage offers, $x$ are drawn from the private sector from an endogenous distribution, $F_p(w)$, which will be derived from the firm side later. An unemployed worker will accept the job offer if the wage is higher than the worker’s reservation wage for that sector, the lower bound of the integral. Inside the integral is the gain the worker makes from switching from unemployment to private sector employment at wage $w$. The final term is the public sector analogue to the second. The theoretical difference between the two is that the distribution from which public-sector job offers are drawn is an exogenous policy parameter of the model.

Similar value functions define a worker employed in the private and public sectors. Below is the example for a private sector employee:

$$rW_p(w) = w + \delta_p \{ U - W_p(w) \} + \lambda_{pp} \int_w^{+\infty} [W_p(x) - W_p(w)] \, dF_p(x) + \lambda_{pg} \int_{R_{pg}(w)}^{+\infty} [W_g(x) - W_p(w)] \, dF_g(x) \quad (4)$$

A worker employed in the private sector and earning a wage $w$ has a discounted value from employment given by the right hand side of (4). The first term $w$ is the instantaneous wage paid in the current private sector firm. The next term, is the loss of value an individual would get if he
were to transit into unemployment $[U - Wp(w)]$ multiplied by the flow probability of such an event occurring, the private sector job destruction rate, $\delta_p$. At rate $\lambda_{pp}$ the worker receives an offer from another private sector firm, where the offer is drawn from the distribution $F_p(x)$. If this offer is greater than his current wage $w$ he will accept, the lower bound of the integral. Given the offer is received and it meets his acceptance criteria, the individual will make an unambiguous gain in value given by $[W_p(x) - W_p(w)]$. The next term represents the equivalent, except for offers from the public sector. Thus the wage is drawn from a different distribution and the acceptance criteria, the lower bound of the integral is instead $R_{pg}(w)$. An analogous Bellman equation defines the value function for a worker in the public sector:

$$rW_g(w) = w + a_g + \delta_g \{U - W_g(w)\}$$

$$+ \lambda_{gp} \int_{R_{gp}(w)}^{+\infty} [W_p(x) - W_g(w)] dF_p(x) + \lambda_{gg} \int_{w}^{+\infty} [W_g(x) - W_g(w)] dF_g(x). \quad (5)$$

Note the presence of the additional flow utility term $a_g$, the ‘public-sector amenity’ discussed above.

The value functions given by (3), (4) and (5) allow us to obtain the reservation wage required to leave the private for the public sector and vice-versa as a function of the transition parameters. This is done using the identity $W^p(R_{pg}(w)) = W_g(w)$ and $W_g(R_{gp}(w)) = W_p(w)$ and assuming differentiability of the value functions. This manipulation is performed in Appendix A and the solution for a private sector worker’s reservation wage from the public sector solves the following non-linear ODE:

$$R_{pg}'(w) = \frac{r + \delta_g + \lambda_{gp}\overline{F}_p(w) + \lambda_{gg}\overline{F}_g(R_{pg}(w))}{r + \delta_p + \lambda_{pp}\overline{F}_p(w) + \lambda_{pg}\overline{F}_g(R_{pg}(w))}, \quad (6)$$

with initial condition $R_{pg}(R_{up}) = R_{ug}$. It should be noted that $R_{up}$ and $R_{ug}$ themselves depend on the functions $R_{pg}(\cdot)$ and $R_{gp}(\cdot)$ as they are obtained by solving $W_s(R_{us}) = U$ for $s = p$ or $g$. However, they also depend on the $b$s, which are free parameters, so those reservation wages can themselves be estimated as free parameters.

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$^4$Here and throughout the rest of the paper, a bar over a c.d.f. denotes the survivor function, so for example $\overline{F}_p(\cdot) := 1 - F_p(\cdot)$. 

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2.2.3 Flow-Balance Equations and Worker Stocks

The economy being in steady-state, the flows in and out of any given sector, for each class of workers, are equal. Applying this to unemployment, one obtains:

\[(\lambda_{up} + \lambda_{ug}) N_u = \delta_p N_p + \delta_g N_g \tag{7} \]

The left hand side of (7) is the rate at which workers leave unemployment toward the two sectors of employment. This occurs when a worker receives a job offer from a given employment sector and the associated wage offer is higher than his appropriate reservation wage. Assuming homogeneous workers, there is no reason why a firm would offer wages below a worker’s reservation wage (and if it did, it would employ no worker and therefore become irrelevant to market equilibrium). Therefore we assume without (further) loss of generality that \(F_p (R_{up})\) and \(F_g (R_{ug})\) are equal to zero. The right hand side is at the unemployment inflow, which consists of workers being hit by job destruction shocks in their sector of employment. A worker can only be in one of three states, \(u, p\) or \(g\) so:

\[N_u + N_p + N_g = N,\] where \(N\) is the total population of workers, a given number.

Equation (8) is the flow-balance equation for private sector workers, equating the flow into the private sector below a wage \(w\) to the flow out, thus imposing that not only is the share of private sector workers static in our model so is the distribution of wages amongst them. The left hand side is the flow out of private employment. \(N_p G_p(w)\) is the number of private sector workers earning less than a wage \(w\). They can exit to unemployment through job destruction shocks \(\delta_p\). The second and third terms are the exit rates into the public sector and higher paid private sector jobs, respectively, upon receiving a job offer \((\lambda_{pg} \text{ and } \lambda_{pp})\). The right hand side is the flow into private sector employment, the first term being the flow from unemployment and the second, from the public sector.

\[
N_p \delta_p G_p (w) + N_p \lambda_{pg} \int_{R_{up}}^{w} F_g (R_{pg} (x)) dG_p (x) + N_p \lambda_{pp} F_p (w) G_p (w) \\
= N_u \lambda_{up} F_p (w) + N_g \lambda_{gp} \int_{R_{ug}}^{R_{pg} (w)} [F_p (w) - F_p (R_{pg} (x))] dG_g (x) \tag{8}
\]
Rearranging equation (8) and differentiating with respect to the wage rate, $w$, one obtains:

$$\frac{d}{dw} \left\{ \left[ \delta_p + \lambda_{pp} F_p (w) \right] N_p G_p (w) \right\} + N_p g_p (w) \lambda_{pg} F_g (R_{pg} (w))$$

$$- N_g \lambda_{gp} G_g (R_{pg} (w)) f_p (w) = N_u \lambda_{up} f_p (w). \quad (9)$$

This would be a fairly straightforward ODE in $g_p (w)$ (the probability distribution of observed wages in the private sector), if it was not for the term featuring $G_g (R_{pg} (w))$. This term can be derived by manipulation of the flow balance equation for public sector workers earning less than $R_{pg} (w)$ (instead of $w$). This manipulation is performed in Appendix A. Plugging this solution into (9), we obtain an ODE that defines $G_p (w)$.

An additional hurdle at this point is the determination of $N_p$ and $N_g$ (with $N_u = N - N_p - N_g$). Those numbers are needed to solve for $G_p (\cdot)$ in the ODE resulting from the combination of (9) and the isolation of $N_g G_p (R_{pg} (w))$, given in the appendix. Now, $N_p$ and $N_g$ are jointly defined by the balance of flows in and out of employment (7), and the flow balance in and out of, say, the private sector, which is given by evaluating the flow-balance equation of private sector workers, equation (8) at $w \rightarrow +\infty$:

$$N_p \delta_p + N_p \lambda_{pg} \int_{R_{up}}^{+\infty} F_g (R_{pg} (x)) dG_p (x)$$

$$= N_g \lambda_{gp} \int_{R_{ug}}^{+\infty} F_p (R_{gp} (x)) dG_g (x) + N_u \lambda_{up} F_p (R_{up}) \quad (10)$$

The distribution, $G_g (\cdot)$, can be derived using the identity $R_{pg} (R_{gp} (w)) = w$ applied to the derivation of $G_p (R_{pg} (w))$ in the appendix. The latter equation involves $G_p (\cdot)$, which in turns depends on $N_p$ and $N_g$, so that those three objects have to be solved for simultaneously. This will be done using an iterative procedure.

### 2.3 Private Sector Firms

There exists a $[0,1]$ continuum of private sector firms who are profit maximizers and heterogeneous in their level of productivity, $y$, where $y \sim \Gamma (\cdot)$ over the support $[y_{\min}, y_{\max}]$ in the population of firms. From our analysis of workers we can deduce the mean size of a private sector firm offering a
wage of $w$ being the ratio of the density of accepted private-sector offers at wage $w$ over extended such offers:

$$\ell_p(w) = N_p \frac{g_p(w)}{f_p(w)}$$

Private sector firms are assumed to maximize their steady-state profit flow. By setting a wage $w$, a firm will be of size $\ell_p(w)$ and thus have a total profit flow given by $\pi(w; y) = (y - w) \ell_p(w)$. Although the profit per worker is decreasing in wages, the size of firm increases in wages. Application of the Envelope Theorem to the firm’s profit maximization problem establishes that firms’ optimal wage policies, $w^*_p(y)$, are defined as solutions to the following functional equation:

$$\left(y - w^*_p(y)\right) \ell_p(w^*_p(y)) = \left(y_{\min} - w^*_p(y_{\min})\right) \ell_p(w^*_p(y_{\min})) + \int_{y_{\min}}^{y} \ell_p(w^*_p(x)) \, dx. \quad (11)$$

As is well known, it follows from the monotone comparative statics properties of a firm’s profit function that any optimal wage policy $w^*_p(y)$ is non-decreasing in $y$. Therefore the least productive private firm in the economy is also the worst-paying one, so that $w^*_p(y_{\min})$ is the lower support of the wage distribution. Moreover, we infer that $F_p\left(w^*_p(y)\right) = \Gamma\left(y\left(w\right)\right)$ for any optimal wage policy.

Further assuming differentiability of the firm-size function $\ell_p(w)$ (which can be established under some regularity restrictions on $F_g(\cdot)$ outside of a finite set of wage values including $R_{up}$), solutions to the profit maximization problem can alternatively be characterized using the problem’s FOC:

$$w + \frac{\ell_p(w)}{\ell'_p(w)} = y. \quad (12)$$

### 3 Data and Estimation

We now outline our estimation protocol, which is based on minimum-distance matching of certain descriptive moments of the data.\(^5\) We set the discount rate $r$ ex-ante at 0.004 (where one unit of time is a month), implying an annual rate of approximately 5%. $\theta$, given below is the exogenous parameter vector which we intend to estimate.

$$\theta = (b, a_g, \delta_p, \delta_g, \lambda_{up}, \lambda_{ag}, \lambda_{pp}, \lambda_{pg}, \lambda_{ag}, \lambda_{gg}, F_p, F_g, \Gamma)'$$

\(^5\)For a comprehensive overview of related simulation-based methods, see Gouriéroux et al. (1993).
Note that the two offer distributions and the distribution of firm types (Γ) feature in the list of parameters. As we will argue below, those distributions are non-parametrically identified. However, for numerical tractability, we will make parametric assumptions on $F_p$ and $F_g$ as outlined later.

The rest of this section focuses on obtaining estimates for the vector $\theta$. We begin by describing the moments we match and how we obtain them from the data, we then describe in detail the estimation procedure. Results are presented in the next section.

3.1 The Sample

The data used in the analysis are taken from the BHPS, a longitudinal data set of British households. Data were first collected in 1991 and the households selected were determined by an equal probability sampling mechanism. Since then, there have been 18 further waves, collected annually. The model outlined is derived under a steady state assumption. Therefore it is necessary that the time period used is short and has approximately constant shares in each of the three states across time. We choose data from 2004 to 2008 to satisfy this assumption, allowing long enough time after the Conservatives’ drive toward privatization in the 80s and 90s but before the great recession of 2008.

Using retrospective accounts of employment history we construct a panel dataset of respondents at a monthly frequency. We include in our data those who across our panel reach at least 21 years of age and don’t exceed 60. Income is adjusted for inflation according to the consumer price index and we trim the income distributions in each sector, treating data as missing if it is below the 2nd or above the 98th percentile in the distribution of wage in either employment sector. We also exclude individuals with holes in their employment history and once someone becomes inactive they are from then on excluded. Thus, consistent with our model, an agent can be in one of three states, unemployment or employment in the private or public sectors. We define private sector employment as anyone who declares themselves as employed in a private sector firm, non-profit organization or in self-employment and public sector employment as in the civil service, central

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6From wave 9 the BHPS was extended to include Scotland and Wales and from wave 11, Northern Ireland. All three regions are over represented in the sample and therefore we weight the data accordingly.
Table 1: Descriptive Statistics

<table>
<thead>
<tr>
<th></th>
<th>Private Sector</th>
<th>Public Sector</th>
<th>Unemployment</th>
</tr>
</thead>
<tbody>
<tr>
<td>size of each sector</td>
<td>79.65%</td>
<td>14.73%</td>
<td>5.62%</td>
</tr>
<tr>
<td>part-time workers in each sector</td>
<td>4.23%</td>
<td>5.54%</td>
<td>n/a</td>
</tr>
<tr>
<td>average hourly wages</td>
<td>12.73</td>
<td>14.54</td>
<td>n/a</td>
</tr>
<tr>
<td>standard deviation of hourly wages</td>
<td>10.91</td>
<td>9.57</td>
<td>n/a</td>
</tr>
<tr>
<td>% who change employment in a given year</td>
<td>22.25%</td>
<td>19.81%</td>
<td>50.36%</td>
</tr>
<tr>
<td>% with a degree</td>
<td>19.43%</td>
<td>34.57%</td>
<td>10.87%</td>
</tr>
<tr>
<td>% with an A-level (or higher qual)</td>
<td>52.95%</td>
<td>71.17%</td>
<td>54.04%</td>
</tr>
<tr>
<td>% with a GCSE (or higher qual)</td>
<td>80.09%</td>
<td>90.56%</td>
<td>79.04%</td>
</tr>
</tbody>
</table>

or local government, the NHS, higher education, a nationalized industry, the armed forces or a government training scheme. Finally, we restrict our sample to men, to avoid issues of labor supply that are more prevalent amongst women, particularly part-time work and inactivity.

We are left with a sample of 3,299 individuals across a maximum of 5 years; there is some attrition, which we assume to be exogenous. Table 1 shows some basic descriptive statistics. Consistent with the large literature on the public-private wage gap, we observe that public sector workers appear to be paid more than their private sector counterparts and there also appears to be more equality within the public sector. Also, the proportion of public sector workers may appear low, this is because the sample comprises of only men, and women make up a sizable share of public sector employment.

Table 2 conveys information about the extent of job mobility, both within and between sectors. Our data is an unbalanced monthly panel of workers, starting from the first interview date and ending with the last. Counting in each month the number of people making each type of transition and the number in each state, we construct monthly cross-sector transition matrices. Averaging these across our time period, we obtain the transition matrix shown in Table 2.

Private sector workers are, on the whole, more mobile than public sector workers, but not massively so. A closer look reveals that private sector workers experience much more frequent within-sector job changes than their public-sector counterparts. Mobility between employment sectors, however, is dominated by public sector employees moving to the private sector, cross-
Table 2: Job mobility within and between sectors

<table>
<thead>
<tr>
<th></th>
<th>Private Sector</th>
<th>Public Sector</th>
<th>Unemployment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Private Sector</td>
<td>0.0159</td>
<td>0.0020</td>
<td>0.0045</td>
</tr>
<tr>
<td>Public Sector</td>
<td>0.0076</td>
<td>0.0075</td>
<td>0.0041</td>
</tr>
<tr>
<td>Unemployment</td>
<td>0.0844</td>
<td>0.0165</td>
<td>—</td>
</tr>
</tbody>
</table>

**Note:** Transition rates are monthly. Rows do not add up to one. The two entries on the main diagonal are the fractions of workers changing jobs within the private and the public sector, respectively.

sector mobility in the other direction being a comparatively rare event. It is also striking to see that the separation rate into unemployment is only marginally smaller in the public sector than in the private sector. Finally, perhaps the most important conclusion to be drawn from Table 2 is that direct, job-to-job reallocation *between employment sectors* is substantial: given the transition rates in Table 2 and the various sectors’ relative sizes given in Table 1, one can infer that about 20 percent of the employment inflow into the private sector comes from the public sector, and that about 30 percent of the private sector employment outflow goes into the public sector.

### 3.2 Estimation

Identification of the model’s parameters comes from two data sources: observed transitions between labor market states and observed wage distributions.

Observed sector-specific wage distributions are direct empirical counterparts to $G_p(\cdot)$ and $G_p(\cdot)$ in the model. While neither has a closed-form solution in the model, both can be simulated given parameter value. In order to map the wage distribution well, we take as moments to be matched 50 quantiles of the distributions, giving 100 moments in total.

Turning to transition moments, we match the eight transition rates reported in Table 2. Denoting these as $\pi_{ss'}$, where $s$ is the state of origin and $s'$ the state of destination, we thus add eight moments to match: $(\pi_{up}, \pi_{ug}, \pi_{pu}, \pi_{pp}, \pi_{pg}, \pi_{gu}, \pi_{gp}, \pi_{gg})$. The theoretical counterparts of those monthly transition rates are given by the probabilities of a certain type of transition occurring within a one-month period. The theoretical counterparts of $\pi_{pu}$, $\pi_{pp}$, $\pi_{pg}$, $\pi_{gu}$, $\pi_{gp}$ and $\pi_{gg}$ all have similar expressions: $\pi_{ss'}^{\text{model}}$ is constructed by taking the probability that an exit from state $s = p$ or $g$, given wage $w$, occurs before one month has elapsed, multiplying it by the conditional probability
of exiting toward $s'$, given that an exit occurs and given initial wage $w$, then finally integrating out $w$ using the relevant initial wage distribution, $dG_s(w)$. For example:

$$
\pi_{pp}^{\text{model}} = \int_{R_{up}}^{+\infty} \frac{\lambda_{pp}F_p(w)}{\delta_p + \lambda_{pp}F_p(w) + \lambda_{pg}F_g(R_{pg}(w))} dG_p(w),
$$

where the "×1" term in the exponential is there as a reminder that $\pi_{pp}$ is a monthly transition probability and that all the flow parameters ($\delta_p$, $\lambda_{pp}$, etc.) are monthly. The theoretical counterparts of the transition rates from unemployment are simpler (as there is no wage to integrate out):

$$
\pi_{up}^{\text{model}} = \frac{\lambda_{up} (1 - e^{-(\lambda_{up} + \lambda_{ug})})}{\lambda_{up} + \lambda_{ug}},
$$

and symmetrically for $\pi_{ug}^{\text{model}}$.

### 3.3 Estimation Procedure

We first estimate the first eleven components of $\theta$ by matching the 108 moments described above, leaving $\Gamma(\cdot)$ out. The latter is backed out in a final step as the underlying private firm productivity distribution that rationalizes the estimates of $F_p(\cdot)$ and $F_g(\cdot)$ obtained in previous steps. We also make parametric assumptions about $F_p(\cdot)$ and $F_g(\cdot)$. First we assume that:

$$
F_p(w) = \begin{cases} 
1 - \frac{(w/R_{up})^{\alpha_p}}{1-(\bar{w}_p/R_{up})^{\alpha_p}} & \text{if } w \in [R_{up}, \bar{w}_p] \\
0 & \text{if } w < R_{up} \\
1 & \text{if } w > \bar{w}_p,
\end{cases}
$$

where $\bar{w}_p$ is set equal to the top percentile in the observed wage distribution. It then proves convenient to parameterize $F_g(\cdot)$ as:

$$
F_g \left( R_{pg} \left[ F_p^{-1}(x) \right] \right) = x^{\alpha_g}, \quad \text{for } x \in [0,1].
$$

Note that the latter parameterization carries the implicit assumption that the lower support of $F_g(\cdot)$ is precisely $R_{ug}$. This assumption, although not implausible, has no real theoretical justification as the public sector is not assumed to be profit maximizing and as such may offer wages that are all strictly greater than the workers’ common reservation wage. Experimenting with richer specifications, allowing for the lower support of $F_g(\cdot)$ to be strictly above $R_{ug}$, led to the conclusion
that those two numbers are indeed very close and that (13) is a valid approximation.

In order to match the moments described we implement a two-step algorithm. In a first step, we use the eight flow parameters \( \delta_p, \delta_g, \lambda_{up}, \lambda_{ug}, \lambda_{pp}, \lambda_{pg}, \lambda_{gp}, \lambda_{gg} \) to fit the eight transition rates derived from our model to those observed in the data, conditional on initial guesses about the offer distributions \( F_p(\cdot) \) and \( F_g(\cdot) \). This first, just-identified step produces a perfect fit to observed transition rates. Then, in a second step, conditional on the transition rates obtained from the first step, we derive the offer distributions such that the distance between the vector of quantiles of the empirical and theoretical wage distributions \( G_p(\cdot) \) and \( G_g(\cdot) \) is minimized. The process is repeated until convergence. We find that this iterative two-step protocol produces a better overall fit than one-step procedures. [DISCUSS THE EQUIVALENCE BETWEEN WEIGHTED ONE-STEP AND TWO-STEP PROCEDURES.]

4 Results

4.1 Labor Market Transitions

Parameter estimates of the model are given in Table 3, of which the top panel contains all transition parameters. The unit of time associated with the transition/offer arrival rates is a month. Again, given our estimates of the rest of the parameter vector, those transition rate values produce a perfect fit to the observed monthly transition rates reported in Table 2.

A striking feature of our parameter estimates is the large on-the-job offer arrival rates for workers in both sectors. Comparison of our estimates of \( \lambda_{pp}, \lambda_{pg}, \lambda_{gp}, \) and \( \lambda_{gg} \) with the corresponding monthly transition rates \( \pi_{pp}, \pi_{pg}, \pi_{gp}, \) and \( \pi_{gg} \) (see Table 2) suggests that employed workers only accept between 2.5 and 3.5 percent of the offers they receive. Moreover, employed workers, regardless of sector, receive far more frequent offers from both sector than unemployed workers. Those results contrast sharply with standard findings from simple, one-sector wage-posting model. However, the same pattern arises in estimates obtained by Meghir et al. (2010) in a different two-sector wage posting model, with a formal and an informal sector estimated on Brazilian data.
### Table 3: Parameter Estimates

<table>
<thead>
<tr>
<th>$\delta_p$</th>
<th>$\lambda_{up}$</th>
<th>$\lambda_{pp}$</th>
<th>$\lambda_{pg}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0045</td>
<td>0.0844</td>
<td>0.5072</td>
<td>0.0793</td>
</tr>
<tr>
<td>$\delta_g$</td>
<td>$\lambda_{ug}$</td>
<td>$\lambda_{gp}$</td>
<td>$\lambda_{gg}$</td>
</tr>
<tr>
<td>0.0041</td>
<td>0.0165</td>
<td>0.2152</td>
<td>0.2686</td>
</tr>
<tr>
<td>$\alpha_p$</td>
<td>$\alpha_g$</td>
<td>$b$</td>
<td></td>
</tr>
<tr>
<td>3.1189</td>
<td>0.7565</td>
<td>10.8798</td>
<td></td>
</tr>
</tbody>
</table>

#### 4.2 Wages and Worker Values

Table 3 also reports estimates of the wage offer distribution parameters, $\alpha_p$ and $\alpha_g$, and an estimate of the flow value of unemployment, $b$, at about £11/hr. The latter implies values of the unemployed workers' reservation wages of $R_{up} = £2.9/hr$ and $R_{ug} = £6.3/hr$, respectively. The fact that $R_{up}$ and $R_{ug}$ are both lower than $b$, which is unusual in empirical wage posting models, is a consequence of the relative values of offer arrival rates on- and off-the job: unemployed workers are prepared to give up a substantial amount in terms of income flow to benefit from the more efficient on-the-job search technology.

We now turn to an analysis of wage distributions. Panel (a) in Figure 1 shows the model’s fit to observed cross-section (log-) wage distributions in both sectors of employment. The fit is reasonably good in both sectors, although the model has some difficulty fitting the high quantiles of the public-sector distribution. In passing, we note that, as is well documented elsewhere in the literature, the public-sector wage distribution dominates the private-sector one except in the top two deciles. There is also markedly less wage dispersion in the public sector.

Panel (b) in Figure 1 next shows estimated log-wage offer distributions in both sectors, $F_p(\cdot)$ and $F_g(\cdot)$, together with the distributions of accepted log-wage offers, $G_p(\cdot)$ and $G_g(\cdot)$. Both offer distributions are fairly concentrated, much more so than the corresponding accepted offer distributions. Indeed the large estimated offer arrival rates imply a large extent of stochastic dominance of $G_s(\cdot)$ over $F_s(\cdot)$ for $s = p$ or $g$. We also see that the public-sector offer distribution strongly dominates its private-sector equivalent. However part of that dominance is “un-done” by
Figure 1: Wage distributions and reservation wages
later reallocation within and between sectors: the dominance of \( G_g(\cdot) \) over \( G_p(\cdot) \) is less marked than the one of \( F_g(\cdot) \) over \( F_p(\cdot) \). The main driver here is the rate \( \lambda_{gp} \) at which private sector workers receive offers of alternative jobs in the private sector, which is very high compared to its public sector counterpart \( \lambda_{gg} \), implying much quicker upward wage mobility in the private than in the public sector. As a consequence, the distribution of private sector wages \( G_p(\cdot) \) dominates the private sector offer distribution \( F_p(\cdot) \) by much more than \( G_g(\cdot) \) dominates \( F_g(\cdot) \).

Panel (c) in Figure 1 then plots \( R_{pg}(w) \), the reservation wage of private-sector employees presented with offers from the public sector. The dashed line on that graph is the main diagonal and the vertical lines materialize the deciles of the private-sector wage distribution, \( G_p(\cdot) \). It appears on this plot that, over most of the support of \( G_p(\cdot) \) (up to about the 95th percentile), \( R_{pg}(w) > w \), i.e. private-sector employees will only accept to go into the public sector with a wage increase. The likely reason is again that upward wage mobility is quicker in the private sector, mainly because of the high value of the private-sector offer arrival rate \( \lambda_{pp} \).

Finally, our model allows us to examine the public-private sector pay gap. While said pay gap is conventionally assessed in terms of wages, in our model workers care not only about their wages but also about future wages, which depend on transition rates and expected future wage progression patterns that differ between sectors. Following Postel-Vinay and Turon (2007), we thus assess the public-private pay gap in terms of lifetime values of employment, as well as raw wages. Specifically, panel (d) in Figure 1 takes up a plot of the private- and public-sector wage distributions \( G_p(\cdot) \) and \( G_g(\cdot) \), together with corresponding plots of the distributions of worker “permanent income” in both sectors, where we define permanent income as the annuitized worker value \( rW_p(w) \) and \( rW_g(w) \) as defined by equation (4). Again, the x-axis on panel (d) is on a logarithmic scale. We draw two conclusions from this graph. First, assessing the public pay gap based on wages only may lead one to overstate the public sector premium: the public- and private-sector distributions of worker values are much closer together than the corresponding wage distributions. In other words, differences in patterns of wage mobility (due to differences in offer arrival rates) between sectors rub off a large part of the public-sector wage premium. Second, even when comparing lifetime values,
a public premium persists in the lower quantiles of the value distribution in the sense that the
distribution of public-sector values dominates its private-sector equivalent in the bottom quantiles,
consistently with the documented fact that low-wage workers tend to queue for public sector jobs
much more than high wage workers. Both of those conclusions are consistent with the findings of
Postel-Vinay and Turon (2007), who estimate a partial equilibrium, descriptive model of wages on
job mobility, also on BHPS data, but with a much richer representation of worker heterogeneity
and wage dynamics.

5 Counterfactual Policy Analysis

Using the estimated parameters of the structural model we simulate the effects of various changes
in public sector wage and employment policy. It seems sensible to use our model to assess the
austerity measures being implemented across Europe. As a benchmark we will look at the effect
of the UK government’s attempt to cut their deficit through increasing public sector layoffs. The
goal of which was to reduce public sector employment by almost half a million, approximately 8%.
To determine whether this is sensible policy making we will compare this with policies undertaken
by other European governments. Spain are attempting to reduce public spending through reducing
public sector pay and Italy by reducing new public sector hires \(^7\). We attempt to fix the policies to
have approximately the same overall public sector saving and use the knock on effects on the three
sectors as metrics to assess the relative performances of the policies.

Table 4 shows the effect of increasing the rate of public sector job destruction by 2.6 times,
decreasing the public sector offer distribution by 18% across the whole distribution and reducing
all public sector offer arrival rates by 57.5%. These seem like radical policies but reducing the size
of the public sector by 8% is a fairly radical objective.

In all three of the policy experiments the private sector picks up some of the slack from a
reduction in government wage expenditure. The UK compensates for a public sector reduction,
\(^7\)Italy are also actively reducing public sector pay, but for convenience we restrict it to three broad classes of policy
Table 4: Counterfactual Policy Effects

<table>
<thead>
<tr>
<th></th>
<th>Original</th>
<th>UK Policy</th>
<th>Spanish Policy</th>
<th>Italian Policy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean of Private Sector log-wages</td>
<td>2.587</td>
<td>2.578</td>
<td>2.589</td>
<td>2.577</td>
</tr>
<tr>
<td>s.d. of Private Sector log-wages</td>
<td>0.513</td>
<td>0.438</td>
<td>0.494</td>
<td>0.491</td>
</tr>
<tr>
<td>Mean of Public Sector log-wages</td>
<td>2.698</td>
<td>2.457</td>
<td>2.426</td>
<td>2.637</td>
</tr>
<tr>
<td>s.d. of Public Sector log-wages</td>
<td>0.390</td>
<td>0.295</td>
<td>0.376</td>
<td>0.379</td>
</tr>
<tr>
<td>Unemployment Share</td>
<td>4.23%</td>
<td>5.31%</td>
<td>4.24%</td>
<td>4.67%</td>
</tr>
<tr>
<td>Private Sector Share</td>
<td>76.85%</td>
<td>77.33%</td>
<td>78.36%</td>
<td>81.24%</td>
</tr>
<tr>
<td>Public Sector Share</td>
<td>18.92%</td>
<td>17.36%</td>
<td>17.41%</td>
<td>14.09%</td>
</tr>
<tr>
<td>Public Sector Savings</td>
<td>30.07%</td>
<td>30.25%</td>
<td>30.80%</td>
<td>30.80%</td>
</tr>
<tr>
<td>Change in Public Sector Employment</td>
<td>8.27%</td>
<td>8.00%</td>
<td>25.54%</td>
<td></td>
</tr>
</tbody>
</table>

largely with an increase in unemployment, where as in Italy, there is large private sector expansion. In the UK, the new equilibrium appears more equitable with the second moment of the private and public sector wage distribution dropping; while Spain and Italy see increases in the first moment of the private and public sector wage distributions, respectively.

6 Concluding Remarks

COMING SOON...

References


APPENDIX

A Theory: Intermediate Derivations

A.1 Derivation of the Reservation Wage, Equation (6)

The value function for a private sector worker earning a wage \( w \), is given in equation (4). Assuming differentiability:

\[
W_p'(w) = \left[ r + \delta_p + \lambda_{pp}F_p'(w) + \lambda_{pg}F_g(R_{pg}(w)) \right]^{-1}
\]

(14)

This also gives \( W_p'(w) \) by analogy. Integrating by parts in (4) yields:

\[
(r + \delta_p)W_p(w) = w + \delta_pU + \lambda_{pp}\int_{w}^{\infty} W_p'(x) F_p(x) \, dx + \lambda_{pg}\int_{R_{pg}(w)}^{\infty} W_g'(x) F_g(x) \, dx
\]

(15)

Plugging the various value functions into the definition of \( R_{pg}(w) \) given in the paper, one obtains the following, fairly complicated expression:

\[
R_{pg}(w) = -a_g + \frac{r + \delta_g}{r + \delta_p}w + \left\{ \frac{r + \delta_g}{r + \delta_p} \delta_p - \delta_g \right\} U + \left\{ \frac{r + \delta_g}{r + \delta_p} \lambda_{pp} - \lambda_{gp} \right\} \int_{w}^{\infty} W_p'(x) F_p(x) \, dx
\]

\[
+ \left\{ \frac{r + \delta_g}{r + \delta_p} \lambda_{pg} - \lambda_{gg} \right\} \int_{R_{pg}(w)}^{\infty} W_g'(x) F_g(x) \, dx
\]

Differentiating yields (6).

A.2 Derivation of the Private-Sector Wage Distribution, Equation (9)

Equation (9) would be a simple ODE if it was not for the term featuring \( G_g(R_{pg}(w)) \). We now show how to express that term as a function of \( w \) and \( G_p(w) \). Writing the flow-balance equation for the public sector yields:

\[
\{ \delta_g + \lambda_{gg}F_g(w) \} N_gG_g(w) + N_g\lambda_{gp}\int_{R_{ug}}^{w} F_p(R_{gp}(x)) \, dG_g(x)
\]

\[
- N_p\lambda_{pg} \int_{R_{ug}}^{R_{pg}(w)} \left[ F_g(w) - F_g(R_{pg}(x)) \right] dG_p(x) = N_u\lambda_{ug} \left[ F_g(w) - F_g(R_{ug}) \right].
\]

Now applying the latter equation at \( R_{pg}(w) \) (instead of \( w \)), we get:

\[
\{ \delta_g + \lambda_{gg}F_g(w) \} N_gG_g(R_{pg}(w)) + N_g\lambda_{gp}\int_{R_{ug}}^{R_{pg}(w)} F_p(R_{gp}(x)) \, dG_g(x)
\]

\[
- N_p\lambda_{pg} \int_{R_{ug}}^{w} \left[ F_g(R_{pg}(w)) - F_g(R_{pg}(x)) \right] dG_p(x) = N_u\lambda_{ug} \left[ F_g(R_{pg}(w)) - F_g(R_{ug}) \right].
\]

(16)
Adding (16) to (8):

\[
N_p G_p (w) \{ \delta_p + \lambda_{pp} F_p (w) + \lambda_{pg} F_g (R_{pg} (w)) \} \\
+ N_g G_g (R_{pg} (w)) \{ \delta_g + \lambda_{gp} F_p (w) + \lambda_{gg} F_g (R_{pg} (w)) \} \\
= N_u \lambda_{ug} [F_g (R_{pg} (w)) - F_g (R_{ug})] + N_u \lambda_{up} [F_p (w) - F_p (R_{up})],
\]

(17)

which can be solved for \( N_g G_g (R_{pg} (w)) \). Plugging the solution into (9), we obtain an ODE defining \( G_p (w) \). Note that by considering \( w \to +\infty \) in the latter equation, one obtains (7).