#### CHAPTER 15

# Some Graphical Procedures for the Preliminary Processing of Longitudinal Data

H. Goldstein, University of London, U.K.

Longitudinal or repeated-measurement studies of the physical growth of children are typically designed so that each child is measured at a limited number of 'target' ages or occasions—often every year on his or her birthday. For a sample of children this procedure is designed to yield a set of measurements at each of these occasions. For each measurement, a common approach to summarizing the values thus obtained is to fit a low-order polynomial, typically referred to as a growth curve. An extensive literature on such growth-curve fitting procedures now exists (see Goldstein, 1979), but it usually assumes that each sample member has a measurement at exactly each target occasion. In practice, data are often missing for some occasions (a problem which is relatively straightforward to deal with), and more seriously, many children attend for measurement at times which are close to, but differ somewhat from, the target age.

This chapter will describe one method of 'adjusting' such measurements so that the adjusted values can be treated as if they originated from the target ages. The adequacy of the method will be studied using graphical techniques which will illustrate how those measurements which are inadequately adjusted can be detected, and also how certain kinds of outliers can be detected.

### 15.1 THE ADJUSTMENT PROCEDURE

Figure 15.1 shows data for a hypothetical child measured at three ages  $(x_1, x_2, x_3)$  with corresponding growth measurements  $(y_1, y_2, y_3)$ . The target age for  $x_2$ , i.e.  $x_2'$ , is also shown, and the open circle represents the adjusted or target age measurement at  $x_2'$ . This is found simply by fitting a suitable curve through the observations and interpolating the value  $y_2'$  at  $x_2'$ . Second-degree polynomials are convenient to use, being easily calculated and flexible. Other functional forms are possible but seem to possess no distinct advantages and will not be considered further.

Several questions are immediately apparent about the above adjustment

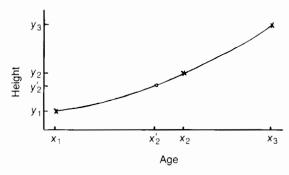


Figure 15.1 An example of adjustment of measured height  $(y_2)$  at age  $x_2$  to target age  $x_2'$  and height  $y_2'$ . Observed measurements denoted by  $\times$ , adjusted measurement by  $\bigcirc$ 

procedure. First, it is clear that when the distance  $x_2 - x_2'$  is small, the more 'acceptable' is the adjusted value since any biases are relatively small. Secondly, it is also clear that the adjustment procedure is in general more acceptable the smaller the distance  $x_3 - x_1$ , since the polynomial adjustment curve should then be a better representation of the true growth curve. Thirdly, if we wish to adjust  $x_1$  to  $x_1'$  where there are no observations at occasions below this, what curve should we use? Finally, are there any advantages to be gained from including further points and higher-degree polynomials? The first two of these points form the basis of the graphical analyses in the remainder of this paper. In response to the last two questions we argue as follows.

Adjacent adjusted measurements will, by definition, lie on polynomials of degree p with p points in common. If p is reasonably large this would effectively induce long-term regularities into the data and allow distant occasions to influence adjusted values. Since it seems desirable for the adjusted values to retain as much as possible of the true measurement variation, p should be as small as possible. A quadratic curve centred on the measurement to be adjusted provides a symmetric procedure, which a linear adjustment would not, and so seems to be the appropriate one to use in general. For an endpoint adjustment, however, a straight line seems more appropriate than a quadratic since the latter uses relatively distant information. Consequently, however, the end-point adjustments may not be so satisfactory.

## 15.2 DATA ANALYSIS

The data to be analysed are measurements of height, weight and triceps skinfold made on a sample of 62 children at five target ages (5.0, 6.0, 7.0, 8.0,

9.0 years) (Tanner et al., 1976). For each measurement of each child, adjusted values were calculated as described in the previous section. There were no missing data, so that for the first or last target ages (denoted by  $x_5$  and  $x_9$ ) linear adjustment was used, and for  $x_6$ ,  $x_7$ ,  $x_8$  quadratic adjustment was used. The data are presented in full in Table 15.1, for the single growth measure, height; age, height, target age and adjusted height are given for each of the 62 children, with five sets of measurements on each.

Figure 15.2 shows a scatterplot at age 6.0 years of the difference between the observed and adjusted values of height against the difference between the actual and target ages. This reflects the expected linear relationship for these relatively small adjustments, with an increasing variance of  $y_6' - y_6$  as  $x_6 - x_6'$  increases. (The variance is zero at  $x_6 = x_6'$ .) At first sight one or both of the points ringed seem like outliers, reflecting the inadequacy of too large an adjustment. Because of the increasing variance, however, we need to standardize the variance before being able to study outliers. If this can be done satisfactorily, then it may be possible to use the outliers in order to investigate the first two points about the adequacy of the adjustment procedure.

#### 15.3 STANDARDIZATION FOR VARIANCE

In order to allow for the changing variance we need to specify a functional relationship with  $x_i - x_i'$ . In the region of the target occasion, we assume that growth is linear. We have

$$y_i = \alpha + \beta x_i + \varepsilon_i$$
.

Then

$$var (y_i - y_i' | x_i - x_i') = 2\sigma^2 (1 - \rho_{ii}),$$

where  $y_i'$  is the value of the measurement at the target age  $x_i'$ ,  $\sigma^2$ , the variance of  $\varepsilon_i$ , is constant, and  $\rho_{ii}$  is the correlation between  $y_i, y_i'$ . Now  $\rho_{ii} = 1$  if  $x_i = x_i'$  and decreases as  $x_i - x_i'$  increases.

One reasonable choice for relating  $\rho_{ii}$  to  $x_i - x_i'$  is

$$\rho_{ii} = \exp(-A \mid x_i - x_i' \mid K)$$

which gives to a first approximation

$$var(y_i - y_i' | x_i - x_i') \propto |x_i - x_i'|^K.$$

We now can study the residuals from the weighted regression of  $y_i - y_i'$  on  $x_i - x_i'$ , standardizing the residuals by their estimated standard errors. These are given by

S.E. (residual at 
$$x_i - x_i'$$
) =  $\{\sigma^2 [1 - w_i(x_i - x_i')^2/S_i^2]/w_i\}^{1/2}$ ,

**Table 15.1** Ages, heights, target ages and adjusted heights for 62 children (basic data from Tanner *et al.*, 1976)

Number   Age   Value   age   Value   number   Age   Number   number   Age   Value   number   Age   Number   number   number   Age   Number   number   number   Age   Number   numbe	data from Tanner et al., 1976)									
1   6.164   117.3   6.0   116.22   2   6.091   120.4   6.0   119.80     1   7.071   123.3   7.0   122.89   2   7.008   126.2   7.0   126.1     1   9.082   132.6   9.0   132.22   2   9.134   140.2   9.0   139.44     3   5.030   113.6   5.0   113.40   4   5.041   109.6   5.0   109.37     3   6.085   120.7   6.0   120.12   4   6.071   115.3   6.0   114.88     3   7.008   127.1   7.0   127.05   4   7.052   121.4   7.0   121.06     3   7.986   133.0   8.0   133.09   4   7.997   127.8   8.0   127.82     3   9.087   140.0   9.0   139.45   4   9.030   134.4   9.0   134.21     5   5.161   104.3   5.0   103.13   6   5.000   108.6   5.0   108.60     5   5.161   104.3 </td <td></td> <td>Age</td> <td></td> <td></td> <td></td> <td></td> <td>Age</td> <td>Height value</td> <td></td> <td>Adjusted value</td>		Age					Age	Height value		Adjusted value
1   6.164   117.3   6.0   116.22   2   6.091   120.4   6.0   119.80     1   7.071   123.3   7.0   122.89   2   7.008   126.2   7.0   126.1     1   9.082   132.6   9.0   132.22   2   9.134   140.2   9.0   139.44     3   5.030   113.6   5.0   113.40   4   5.041   109.6   5.0   109.37     3   6.085   120.7   6.0   120.12   4   6.071   115.3   6.0   114.88     3   7.008   127.1   7.0   127.05   4   7.052   121.4   7.0   121.06     3   7.986   133.0   8.0   133.09   4   7.997   127.8   8.0   127.82     3   9.087   140.0   9.0   139.45   4   9.030   134.4   9.0   134.21     5   5.161   104.3   5.0   103.13   6   5.000   108.6   5.0   108.60     5   5.161   104.3 </td <td>1</td> <td>5.055</td> <td>110.0</td> <td>5.0</td> <td>109.64</td> <td>2</td> <td>5.022</td> <td>113.1</td> <td>5.0</td> <td>112.95</td>	1	5.055	110.0	5.0	109.64	2	5.022	113.1	5.0	112.95
1   8.030   127.7   8.0   127.56   2   7.992   133.7   8.0   133.75     1   9.082   132.6   9.0   132.22   2   9.134   140.2   9.0   139,44     3   5.030   113.6   5.0   113.40   4   5.041   109.6   5.0   109,43     3   6.085   120.7   6.0   120.12   4   6.071   115.3   6.0   114.88     3   7.008   127.1   7.0   127.05   4   7.052   121.4   7.0   121.06     3   7.986   133.0   8.0   133.09   4   7.997   127.8   8.0   127.82     3   9.087   140.0   9.0   139,45   4   9.030   134.4   9.0   134.21     5   5.161   104.3   5.0   103.13   6   5.000   108.6   5.0   108.60     5   5.064   116.9   7.0   117.13   6   7.008   122.4   7.0   122.35     5   7.961   122.7<	1	6.164	117.3	6.0	116.22	2	6.091	120.4	6.0	119.80
1   8.030   127.7   8.0   127.56   2   7.992   133.7   8.0   133.75     1   9.082   132.6   9.0   132.22   2   9.134   140.2   9.0   139,44     3   5.030   113.6   5.0   113.40   4   5.041   109.6   5.0   109,43     3   6.085   120.7   6.0   120.12   4   6.071   115.3   6.0   114.88     3   7.008   127.1   7.0   127.05   4   7.052   121.4   7.0   121.06     3   7.986   133.0   8.0   133.09   4   7.997   127.8   8.0   127.82     3   9.087   140.0   9.0   139,45   4   9.030   134.4   9.0   134.21     5   5.161   104.3   5.0   103.13   6   5.000   108.6   5.0   108.60     5   5.064   116.9   7.0   117.13   6   7.008   122.4   7.0   122.35     5   7.961   122.7<	1		123.3	7.0	122.89	2	7.008	126.2	7.0	126.14
1   9.082   132.6   9.0   132.22   2   9.134   140.2   9.0   139.44     3   5.030   113.6   5.0   113.40   4   5.041   109.6   5.0   109.37     3   6.085   120.7   6.0   120.12   4   6.071   115.3   6.0   114.88     3   7.086   133.0   8.0   133.09   4   7.997   127.8   8.0   127.82     3   9.087   140.0   9.0   139.45   4   9.030   134.4   9.0   134.21     5   5.161   104.3   5.0   103.13   6   5.000   108.6   5.0   108.60     5   6.964   116.9   7.0   117.13   6   7.008   122.4   7.0   122.35     5   7.961   122.7   8.0   122.96   6   7.972   127.7   8.0   122.86     5   9.013   130.5   9.0   130.40   6   9.024   133.7   9.0   133.56     7   5.06   101.5 </td <td></td> <td>8.030</td> <td>127.7</td> <td></td> <td>127.56</td> <td>2</td> <td>7.992</td> <td>133.7</td> <td>8.0</td> <td>133.75</td>		8.030	127.7		127.56	2	7.992	133.7	8.0	133.75
3     6.085     120.7     6.0     120.12     4     6.071     115.3     6.0     114.88       3     7.086     127.1     7.0     127.05     4     7.052     121.4     7.0     121.06       3     7.986     133.0     8.0     133.09     4     7.997     127.8     8.0     127.82       3     9.087     140.0     9.0     139.45     4     9.030     134.4     9.0     134.21       5     5.161     104.3     5.0     103.13     6     5.000     108.6     5.0     108.60       5     6.084     111.0     6.0     110.41     6     6.038     116.2     6.0     115.94       5     6.964     116.9     7.0     117.13     6     7.008     122.4     7.0     122.35       5     7.961     122.7     8.0     122.96     6     7.972     127.7     8.0     127.86       5     9.013     130.5     9.0     130.40     6	1		132.6		132.22	2	9.134	140.2	9.0	139.44
5     5.161     104.3     5.0     103.13     6     5.000     108.6     5.0     108.60       5     6.084     111.0     6.0     110.41     6     6.038     116.2     6.0     115.94       5     6.964     116.9     7.0     117.13     6     7.008     122.4     7.0     122.35       5     7.961     122.7     8.0     122.96     6     7.972     127.7     8.0     127.86       5     9.013     130.5     9.0     130.40     6     9.024     133.7     9.0     133.56       7     5.006     101.5     5.0     101.47     8     5.022     95.5     5.0     95.37       7     6.025     107.4     6.0     107.24     8     6.016     101.2     6.0     101.12       7     6.981     114.0     7.0     114.12     8     6.975     105.6     7.0     105.72       7     7.959     119.5     8.0     119.74     8 </td <td>3</td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td>109.37</td>	3									109.37
5     5.161     104.3     5.0     103.13     6     5.000     108.6     5.0     108.60       5     6.084     111.0     6.0     110.41     6     6.038     116.2     6.0     115.94       5     6.964     116.9     7.0     117.13     6     7.008     122.4     7.0     122.35       5     7.961     122.7     8.0     122.96     6     7.972     127.7     8.0     127.86       5     9.013     130.5     9.0     130.40     6     9.024     133.7     9.0     133.56       7     5.006     101.5     5.0     101.47     8     5.022     95.5     5.0     95.37       7     6.025     107.4     6.0     107.24     8     6.016     101.2     6.0     101.12       7     6.981     114.0     7.0     114.12     8     6.975     105.6     7.0     105.72       7     7.959     119.5     8.0     119.74     8 </td <td>3</td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td>	3									
5     5.161     104.3     5.0     103.13     6     5.000     108.6     5.0     108.60       5     6.084     111.0     6.0     110.41     6     6.038     116.2     6.0     115.94       5     6.964     116.9     7.0     117.13     6     7.008     122.4     7.0     122.35       5     7.961     122.7     8.0     122.96     6     7.972     127.7     8.0     127.86       5     9.013     130.5     9.0     130.40     6     9.024     133.7     9.0     133.56       7     5.006     101.5     5.0     101.47     8     5.022     95.5     5.0     95.37       7     6.025     107.4     6.0     107.24     8     6.016     101.2     6.0     101.12       7     6.981     114.0     7.0     114.12     8     6.975     105.6     7.0     105.72       7     7.959     119.5     8.0     119.74     8 </td <td>3</td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td>	3									
5     5.161     104.3     5.0     103.13     6     5.000     108.6     5.0     108.60       5     6.084     111.0     6.0     110.41     6     6.038     116.2     6.0     115.94       5     6.964     116.9     7.0     117.13     6     7.008     122.4     7.0     122.35       5     7.961     122.7     8.0     122.96     6     7.972     127.7     8.0     127.86       5     9.013     130.5     9.0     130.40     6     9.024     133.7     9.0     133.56       7     5.006     101.5     5.0     101.47     8     5.022     95.5     5.0     95.37       7     6.025     107.4     6.0     107.24     8     6.016     101.2     6.0     101.12       7     6.981     114.0     7.0     114.12     8     6.975     105.6     7.0     105.72       7     7.959     119.5     8.0     119.74     8 </td <td>3</td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td>	3									
5     6.084     111.0     6.0     110.41     6     6.038     116.2     6.0     115.94       5     6.964     116.9     7.0     117.13     6     7.008     122.4     7.0     122.35       5     7.961     122.7     8.0     122.96     6     7.972     127.7     8.0     127.86       5     9.013     130.5     9.0     130.40     6     9.024     133.7     9.0     133.56       7     5.006     101.5     5.0     101.47     8     5.022     95.5     5.0     95.37       7     6.025     107.4     6.0     107.24     8     6.016     101.2     6.0     105.72       7     7.959     119.5     8.0     119.74     8     7.953     110.6     8.0     110.84       7     9.006     125.9     9.0     125.86     8     9.211     117.0     9.0     115.93       9     5.011     102.3     5.0     102.27     10<	3	9.087	140.0	9.0	139.45	4	9.030	134.4	9.0	134.21
7     5.006     101.5     5.0     101.47     8     5.022     95.5     5.0     95.37       7     6.025     107.4     6.0     107.24     8     6.016     101.2     6.0     101.12       7     6.981     114.0     7.0     114.12     8     6.975     105.6     7.0     105.72       7     7.959     119.5     8.0     119.74     8     7.953     110.6     8.0     110.84       7     9.006     125.9     9.0     125.86     8     9.211     117.0     9.0     115.93       9     5.011     102.3     5.0     102.27     10     5.025     112.8     5.0     112.60       9     6.394     106.5     6.0     104.86     10     6.178     121.8     6.0     120.49       9     7.383     112.2     7.0     109.60     10     7.107     128.2     7.0     127.46       9     7.967     117.1     8.0     117.33	5							108.6	5.0	108.60
7     5.006     101.5     5.0     101.47     8     5.022     95.5     5.0     95.37       7     6.025     107.4     6.0     107.24     8     6.016     101.2     6.0     101.12       7     6.981     114.0     7.0     114.12     8     6.975     105.6     7.0     105.72       7     7.959     119.5     8.0     119.74     8     7.953     110.6     8.0     110.84       7     9.006     125.9     9.0     125.86     8     9.211     117.0     9.0     115.93       9     5.011     102.3     5.0     102.27     10     5.025     112.8     5.0     112.60       9     6.394     106.5     6.0     104.86     10     6.178     121.8     6.0     120.49       9     7.383     112.2     7.0     109.60     10     7.107     128.2     7.0     127.46       9     7.967     117.1     8.0     117.33	5						6.038			
7     5.006     101.5     5.0     101.47     8     5.022     95.5     5.0     95.37       7     6.025     107.4     6.0     107.24     8     6.016     101.2     6.0     101.12       7     6.981     114.0     7.0     114.12     8     6.975     105.6     7.0     105.72       7     7.959     119.5     8.0     119.74     8     7.953     110.6     8.0     110.84       7     9.006     125.9     9.0     125.86     8     9.211     117.0     9.0     115.93       9     5.011     102.3     5.0     102.27     10     5.025     112.8     5.0     112.60       9     6.394     106.5     6.0     104.86     10     6.178     121.8     6.0     120.49       9     7.383     112.2     7.0     109.60     10     7.107     128.2     7.0     127.46       9     7.967     117.1     8.0     117.33	5						7.008		7.0	
7     5.006     101.5     5.0     101.47     8     5.022     95.5     5.0     95.37       7     6.025     107.4     6.0     107.24     8     6.016     101.2     6.0     101.12       7     6.981     114.0     7.0     114.12     8     6.975     105.6     7.0     105.72       7     7.959     119.5     8.0     119.74     8     7.953     110.6     8.0     110.84       7     9.006     125.9     9.0     125.86     8     9.211     117.0     9.0     115.93       9     5.011     102.3     5.0     102.27     10     5.025     112.8     5.0     112.60       9     6.394     106.5     6.0     104.86     10     6.178     121.8     6.0     120.49       9     7.383     112.2     7.0     109.60     10     7.107     128.2     7.0     127.46       9     7.967     117.1     8.0     117.33	5									
7   6.025   107.4   6.0   107.24   8   6.016   101.2   6.0   101.12     7   6.981   114.0   7.0   114.12   8   6.975   105.6   7.0   105.72     7   7.959   119.5   8.0   119.74   8   7.953   110.6   8.0   110.84     7   9.006   125.9   9.0   125.86   8   9.211   117.0   9.0   115.93     9   5.011   102.3   5.0   102.27   10   5.025   112.8   5.0   112.60     9   6.394   106.5   6.0   104.86   10   6.178   121.8   6.0   120.49     9   7.383   112.2   7.0   109.60   10   7.107   128.2   7.0   127.46     9   7.967   117.1   8.0   117.33   10   8.008   134.5   8.0   134.45     9   8.981   122.2   9.0   122.30   10   9.197   142.1   9.0   140.84     11   5.005		9.013	130.5	9.0	130.40	6	9.024	133.7	9.0	133.56
7   6.981   114.0   7.0   114.12   8   6.975   105.6   7.0   105.72     7   7.959   119.5   8.0   119.74   8   7.953   110.6   8.0   110.84     7   9.006   125.9   9.0   125.86   8   9.211   117.0   9.0   115.93     9   5.011   102.3   5.0   102.27   10   5.025   112.8   5.0   112.60     9   6.394   106.5   6.0   104.86   10   6.178   121.8   6.0   120.49     9   7.383   112.2   7.0   109.60   10   7.107   128.2   7.0   127.46     9   7.967   117.1   8.0   117.33   10   8.008   134.5   8.0   134.45     9   8.981   122.2   9.0   122.30   10   9.197   142.1   9.0   140.84     11   5.005   104.6   5.0   104.56   12   5.008   112.5   5.0   112.44     11   6.962 <t< td=""><td>7</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td>95.37</td></t<>	7									95.37
7   7.959   119.5   8.0   119.74   8   7.953   110.6   8.0   110.84     7   9.006   125.9   9.0   125.86   8   9.211   117.0   9.0   115.93     9   5.011   102.3   5.0   102.27   10   5.025   112.8   5.0   112.60     9   6.394   106.5   6.0   104.86   10   6.178   121.8   6.0   120.49     9   7.383   112.2   7.0   109.60   10   7.107   128.2   7.0   127.46     9   7.967   117.1   8.0   117.33   10   8.008   134.5   8.0   134.45     9   8.981   122.2   9.0   122.30   10   9.197   142.1   9.0   140.84     11   5.005   104.6   5.0   104.56   12   5.008   112.5   5.0   112.44     11   6.003   112.7   6.0   112.68   12   6.038   119.8   6.0   119.54     11   8.011	7									
7   9.006   125.9   9.0   125.86   8   9.211   117.0   9.0   115.93     9   5.011   102.3   5.0   102.27   10   5.025   112.8   5.0   112.60     9   6.394   106.5   6.0   104.86   10   6.178   121.8   6.0   120.49     9   7.383   112.2   7.0   109.60   10   7.107   128.2   7.0   127.46     9   7.967   117.1   8.0   117.33   10   8.008   134.5   8.0   134.45     9   8.981   122.2   9.0   122.30   10   9.197   142.1   9.0   140.84     11   5.005   104.6   5.0   104.56   12   5.008   112.5   5.0   112.44     11   6.003   112.7   6.0   112.68   12   6.038   119.8   6.0   119.54     11   8.011   122.5   8.0   122.45   12   8.036   132.8   8.0   132.57     11   8.948	7									
9   5.011   102.3   5.0   102.27   10   5.025   112.8   5.0   112.60     9   6.394   106.5   6.0   104.86   10   6.178   121.8   6.0   120.49     9   7.383   112.2   7.0   109.60   10   7.107   128.2   7.0   127.46     9   7.967   117.1   8.0   117.33   10   8.008   134.5   8.0   134.45     9   8.981   122.2   9.0   122.30   10   9.197   142.1   9.0   140.84     11   5.005   104.6   5.0   104.56   12   5.008   112.5   5.0   112.44     11   6.003   112.7   6.0   112.68   12   6.038   119.8   6.0   119.54     11   6.962   118.0   7.0   118.19   12   7.096   126.5   7.0   125.87     11   8.011   122.5   8.0   122.45   12   8.036   132.8   8.0   132.57     11   8.948										
9   6.394   106.5   6.0   104.86   10   6.178   121.8   6.0   120.49     9   7.383   112.2   7.0   109.60   10   7.107   128.2   7.0   127.46     9   7.967   117.1   8.0   117.33   10   8.008   134.5   8.0   134.45     9   8.981   122.2   9.0   122.30   10   9.197   142.1   9.0   140.84     11   5.005   104.6   5.0   104.56   12   5.008   112.5   5.0   112.44     11   6.003   112.7   6.0   112.68   12   6.038   119.8   6.0   119.54     11   6.962   118.0   7.0   118.19   12   7.096   126.5   7.0   125.87     11   8.011   122.5   8.0   122.45   12   8.036   132.8   8.0   132.57     11   8.948   126.9   9.0   127.14   12   9.011   138.8   9.0   138.73     13   5.022	7	9.006	125.9	9.0	125.86	8	9.211	117.0	9.0	115.93
9   7.383   112.2   7.0   109.60   10   7.107   128.2   7.0   127.46     9   7.967   117.1   8.0   117.33   10   8.008   134.5   8.0   134.45     9   8.981   122.2   9.0   122.30   10   9.197   142.1   9.0   140.84     11   5.005   104.6   5.0   104.56   12   5.008   112.5   5.0   112.44     11   6.003   112.7   6.0   112.68   12   6.038   119.8   6.0   119.54     11   6.962   118.0   7.0   118.19   12   7.096   126.5   7.0   125.87     11   8.011   122.5   8.0   122.45   12   8.036   132.8   8.0   132.57     11   8.948   126.9   9.0   127.14   12   9.011   138.8   9.0   138.73     13   5.022   106.4   5.0   106.26   14   5.039   98.1   5.0   97.87     13   7.049						10				112.60
9   7.967   117.1   8.0   117.33   10   8.008   134.5   8.0   134.45     9   8.981   122.2   9.0   122.30   10   9.197   142.1   9.0   140.84     11   5.005   104.6   5.0   104.56   12   5.008   112.5   5.0   112.44     11   6.003   112.7   6.0   112.68   12   6.038   119.8   6.0   119.54     11   6.962   118.0   7.0   118.19   12   7.096   126.5   7.0   125.87     11   8.011   122.5   8.0   122.45   12   8.036   132.8   8.0   132.57     11   8.948   126.9   9.0   127.14   12   9.011   138.8   9.0   138.73     13   5.022   106.4   5.0   106.26   14   5.039   98.1   5.0   97.87     13   6.013   112.5   6.0   112.43   14   6.011   103.8   6.0   103.75     13   7.049	9									
9   8.981   122.2   9.0   122.30   10   9.197   142.1   9.0   140.84     11   5.005   104.6   5.0   104.56   12   5.008   112.5   5.0   112.44     11   6.003   112.7   6.0   112.68   12   6.038   119.8   6.0   119.54     11   6.962   118.0   7.0   118.19   12   7.096   126.5   7.0   125.87     11   8.011   122.5   8.0   122.45   12   8.036   132.8   8.0   132.57     11   8.948   126.9   9.0   127.14   12   9.011   138.8   9.0   138.73     13   5.022   106.4   5.0   106.26   14   5.039   98.1   5.0   97.87     13   6.013   112.5   6.0   112.43   14   6.011   103.8   6.0   103.75     13   7.049   116.5   7.0   116.25   14   7.011   106.7   7.0   106.64     13   8.033	9									
11 5.005 104.6 5.0 104.56 12 5.008 112.5 5.0 112.44   11 6.003 112.7 6.0 112.68 12 6.038 119.8 6.0 119.54   11 6.962 118.0 7.0 118.19 12 7.096 126.5 7.0 125.87   11 8.011 122.5 8.0 122.45 12 8.036 132.8 8.0 132.57   11 8.948 126.9 9.0 127.14 12 9.011 138.8 9.0 138.73   13 5.022 106.4 5.0 106.26 14 5.039 98.1 5.0 97.87   13 6.013 112.5 6.0 112.43 14 6.011 103.8 6.0 103.75   13 7.049 116.5 7.0 116.25 14 7.011 106.7 7.0 106.64   13 8.033 122.6 8.0 122.42 14 8.094 114.7 8.0 114.10   13 9.046 127.4 9.0 127.18 14 9.050 119.8 9.0 119.53   15 5.055 115.2 5.0 <td>9</td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td>	9									
11   6.003   112.7   6.0   112.68   12   6.038   119.8   6.0   119.54     11   6.962   118.0   7.0   118.19   12   7.096   126.5   7.0   125.87     11   8.011   122.5   8.0   122.45   12   8.036   132.8   8.0   132.57     11   8.948   126.9   9.0   127.14   12   9.011   138.8   9.0   138.73     13   5.022   106.4   5.0   106.26   14   5.039   98.1   5.0   97.87     13   6.013   112.5   6.0   112.43   14   6.011   103.8   6.0   103.75     13   7.049   116.5   7.0   116.25   14   7.011   106.7   7.0   106.64     13   8.033   122.6   8.0   122.42   14   8.094   114.7   8.0   114.10     13   9.046   127.4   9.0   127.18   14   9.050   119.8   9.0   119.53     15   5.055	9	8.981	122.2	9.0	122.30	10	9.197	142.1	9.0	140.84
11 6.962 118.0 7.0 118.19 12 7.096 126.5 7.0 125.87   11 8.011 122.5 8.0 122.45 12 8.036 132.8 8.0 132.57   11 8.948 126.9 9.0 127.14 12 9.011 138.8 9.0 138.73   13 5.022 106.4 5.0 106.26 14 5.039 98.1 5.0 97.87   13 6.013 112.5 6.0 112.43 14 6.011 103.8 6.0 103.75   13 7.049 116.5 7.0 116.25 14 7.011 106.7 7.0 106.64   13 8.033 122.6 8.0 122.42 14 8.094 114.7 8.0 114.10   13 9.046 127.4 9.0 127.18 14 9.050 119.8 9.0 119.53   15 5.055 115.2 5.0 114.87 16 5.027 114.1 5.0 113.94   15 6.009 121.0 6.0 120.94 16 6.000 120.0 6.0 120.00   15 7.014 128.1 7.0 <td>11</td> <td></td> <td></td> <td></td> <td></td> <td>12</td> <td>5.008</td> <td></td> <td>5.0</td> <td>112.44</td>	11					12	5.008		5.0	112.44
11 8.011 122.5 8.0 122.45 12 8.036 132.8 8.0 132.57   11 8.948 126.9 9.0 127.14 12 9.011 138.8 9.0 138.73   13 5.022 106.4 5.0 106.26 14 5.039 98.1 5.0 97.87   13 6.013 112.5 6.0 112.43 14 6.011 103.8 6.0 103.75   13 7.049 116.5 7.0 116.25 14 7.011 106.7 7.0 106.64   13 8.033 122.6 8.0 122.42 14 8.094 114.7 8.0 114.10   13 9.046 127.4 9.0 127.18 14 9.050 119.8 9.0 119.53   15 5.055 115.2 5.0 114.87 16 5.027 114.1 5.0 113.94   15 6.009 121.0 6.0 120.94 16 6.000 120.0 6.0 120.00   15 7.014 128.1 7.0 128.02 16 7.038 126.2 7.0 125.96   15 8.044 133.2 8.0 <td></td> <td></td> <td></td> <td></td> <td></td> <td>12</td> <td></td> <td></td> <td></td> <td></td>						12				
11 8.948 126.9 9.0 127.14 12 9.011 138.8 9.0 138.73   13 5.022 106.4 5.0 106.26 14 5.039 98.1 5.0 97.87   13 6.013 112.5 6.0 112.43 14 6.011 103.8 6.0 103.75   13 7.049 116.5 7.0 116.25 14 7.011 106.7 7.0 106.64   13 8.033 122.6 8.0 122.42 14 8.094 114.7 8.0 114.10   13 9.046 127.4 9.0 127.18 14 9.050 119.8 9.0 119.53   15 5.055 115.2 5.0 114.87 16 5.027 114.1 5.0 113.94   15 6.009 121.0 6.0 120.94 16 6.000 120.0 6.0 120.00   15 7.014 128.1 7.0 128.02 16 7.038 126.2 7.0 125.96   15 8.044 133.2 8.0 132.95 16 8.041 132.8 8.0 132.53										
13 5.022 106.4 5.0 106.26 14 5.039 98.1 5.0 97.87   13 6.013 112.5 6.0 112.43 14 6.011 103.8 6.0 103.75   13 7.049 116.5 7.0 116.25 14 7.011 106.7 7.0 106.64   13 8.033 122.6 8.0 122.42 14 8.094 114.7 8.0 114.10   13 9.046 127.4 9.0 127.18 14 9.050 119.8 9.0 119.53   15 5.055 115.2 5.0 114.87 16 5.027 114.1 5.0 113.94   15 6.009 121.0 6.0 120.94 16 6.000 120.0 6.0 120.00   15 7.014 128.1 7.0 128.02 16 7.038 126.2 7.0 125.96   15 8.044 133.2 8.0 132.95 16 8.041 132.8 8.0 132.53					122.45	12				
13 6.013 112.5 6.0 112.43 14 6.011 103.8 6.0 103.75   13 7.049 116.5 7.0 116.25 14 7.011 106.7 7.0 106.64   13 8.033 122.6 8.0 122.42 14 8.094 114.7 8.0 114.10   13 9.046 127.4 9.0 127.18 14 9.050 119.8 9.0 119.53   15 5.055 115.2 5.0 114.87 16 5.027 114.1 5.0 113.94   15 6.009 121.0 6.0 120.94 16 6.000 120.0 6.0 120.00   15 7.014 128.1 7.0 128.02 16 7.038 126.2 7.0 125.96   15 8.044 133.2 8.0 132.95 16 8.041 132.8 8.0 132.53	11	8.948	126.9	9.0	127.14	12	9.011	138.8	9.0	138.73
13 7.049 116.5 7.0 116.25 14 7.011 106.7 7.0 106.64   13 8.033 122.6 8.0 122.42 14 8.094 114.7 8.0 114.10   13 9.046 127.4 9.0 127.18 14 9.050 119.8 9.0 119.53   15 5.055 115.2 5.0 114.87 16 5.027 114.1 5.0 113.94   15 6.009 121.0 6.0 120.94 16 6.000 120.0 6.0 120.00   15 7.014 128.1 7.0 128.02 16 7.038 126.2 7.0 125.96   15 8.044 133.2 8.0 132.95 16 8.041 132.8 8.0 132.53										97.87
13 8.033 122.6 8.0 122.42 14 8.094 114.7 8.0 114.10   13 9.046 127.4 9.0 127.18 14 9.050 119.8 9.0 119.53   15 5.055 115.2 5.0 114.87 16 5.027 114.1 5.0 113.94   15 6.009 121.0 6.0 120.94 16 6.000 120.0 6.0 120.00   15 7.014 128.1 7.0 128.02 16 7.038 126.2 7.0 125.96   15 8.044 133.2 8.0 132.95 16 8.041 132.8 8.0 132.53										103.75
13 9.046 127.4 9.0 127.18 14 9.050 119.8 9.0 119.53   15 5.055 115.2 5.0 114.87 16 5.027 114.1 5.0 113.94   15 6.009 121.0 6.0 120.94 16 6.000 120.0 6.0 120.00   15 7.014 128.1 7.0 128.02 16 7.038 126.2 7.0 125.96   15 8.044 133.2 8.0 132.95 16 8.041 132.8 8.0 132.53										106.64
15 5.055 115.2 5.0 114.87 16 5.027 114.1 5.0 113.94   15 6.009 121.0 6.0 120.94 16 6.000 120.0 6.0 120.00   15 7.014 128.1 7.0 128.02 16 7.038 126.2 7.0 125.96   15 8.044 133.2 8.0 132.95 16 8.041 132.8 8.0 132.53										114.10
15 6.009 121.0 6.0 120.94 16 6.000 120.0 6.0 120.00 15 7.014 128.1 7.0 128.02 16 7.038 126.2 7.0 125.96 15 8.044 133.2 8.0 132.95 16 8.041 132.8 8.0 132.53	13	9.046	127.4	9.0	127.18	14	9.050	119.8	9.0	119.53
15 7.014 128.1 7.0 128.02 16 7.038 126.2 7.0 125.96 15 8.044 133.2 8.0 132.95 16 8.041 132.8 8.0 132.53										113.94
15 8.044 133.2 8.0 132.95 16 8.041 132.8 8.0 132.53										
										125.96
	15								8.0	
15 9.006 139.5 9.0 139.46 16 8.991 138.9 9.0 138.96	15	9.006	139.5	9.0	139.46	16	8.991	138.9	9.0	138.96

Table 15.1 Continued

Serial number	Age	Height value	Target age	Adjusted value	Serial number	Age	Height value	Target age	Adjusted value	
17 17 17 17 17	5.024 5.991 6.983 8.038 9.038	110.1 116.6 121.3 126.7 132.1	5.0 6.0 7.0 8.0 9.0	109.94 116.56 121.38 126.50 131.89	18 18 18 18	5.107 6.045 7.005 8.055 8.986	114.6 120.3 125.9 131.1 137.1	5.0 6.0 7.0 8.0 9.0	113.95 120.03 125.87 130.79 137.19	
19 19 19 19	5.008 5.981 6.997 8.038 8.951	109.8 115.8 121.2 128.4 134.5	5.0 6.0 7.0 8.0 9.0	109.75 115.91 121.22 128.14 134.83	20 20 20 20 20 20	5.013 5.967 6.980 7.980 8.980	109.1 114.4 119.9 126.5 131.7	5.0 6.0 7.0 8.0 9.0	109.03 114.58 120.02 126.62 131.80	
21 21 21 21 21	5.011 5.969 6.986 8.082 9.117	106.4 111.6 117.5 123.3 128.4	5.0 6.0 7.0 8.0 9.0	106.34 111.77 117.58 122.88 127.82	22 22 22 22 22 22	5.096 6.047 7.003 8.003 8.957	103.5 108.9 116.2 122.2 127.0	5.0 6.0 7.0 8.0 9.0	102.95 108.59 116.18 122.18 127.22	
23 23 23 23 23	5.077 6.170 6.956 8.112 8.970	110.0 116.1 121.9 127.5 131.8	5.0 6.0 7.0 8.0 9.0	109.57 115.00 122.18 126.95 131.95	24 24 24 24 24	5.077 6.170 6.956 8.112 8.970	108:9 116.4 121.6 127.5 132.0	5.0 6.0 7.0 8.0 9.0	108.37 115.25 121.86 126.92 132.16	
25 25 25 25 25	5.045 6.003 7.049 8.192 8.907	117.5 123.6 131.3 137.8 143.4	5.0 6.0 7.0 8.0 9.0	117.21 123.58 130.98 136.50 144.13	26 26 26 26 26	5.055 5.951 6.951 7.940 8.942	110.5 115.5 121.2 125.9 131.4	5.0 6.0 7.0 8.0 9.0	110.19 115.78 121.45 126.21 131.72	
27 27 27 27 27	5.049 6.194 7.011 8.109 9.107	111.1 117.5 122.8 128.8 134.1	5.0 6.0 7.0 8.0 9.0	110.83 116.33 122.73 128.21 133.53	28 28 28 28 28	5.055 5.970 7.126 8.101 8.962	110.4 115.4 122.8 127.6 132.9	5.0 6.0 7.0 8.0 9.0	110.10 115.58 122.08 127.04 133.13	
29 29 29 29 29	5.047 5.967 6.943 8.088 8.935	105.1 109.9 114.8 120.9 124.6	5.0 6.0 7.0 8.0 9.0	104.85 110.07 115.09 120.48 124.88	30 30 30 30 30	5.063 5.964 7.074 7.899 9.137	103.2 109.1 114.3 118.0 125.1	5.0 6.0 7.0 8.0 9.0	102.79 109.30 113.96 118.51 124.31	
31 31 31 31 31	5.052 5.961 6.937 8.046 8.931	105.1 109.1 115.0 120.5 125.3	5.0 6.0 7.0 8.0 9.0	104.87 109.30 115.35 120.26 125.67	32 32 32 32 32 32	5.066 6.082 7.027 8.082 9.063	107.5 115.0 122.1 129.0 135.1	5.0 6.0 7.0 8.0 9.0	107.01 114.39 121.91 128.48 134.71	

Table 15.1 Continued

Table 15	.1 0	nunuea							
Serial number	Age	Height value	Target age	Adjusted value	Serial number	Age	Height value	Target age	Adjusted value
33	5.052	109.5	5.0	109.20	34	5.093	112.3	5.0	111.72
33	6.140	115.8	6.0	114.87	34	6.071	118.4	6.0	117.97
33	6.948	121.8	7.0	122.15	34	7.063	124.2	7.0	123.84
33	7.907	127.6	8.0	128.11	34	8.071	129.7	8.0	129.35
33	9.077	133.3	9.0	132.92	34	9.145	134.2	9.0	133.59
35	5.030	109.0	5.0	108.80	36	5.038	108.8	5.0	108.55
35	6.046	115.9	6.0	115.59	36	6.035	115.3	6.0	115.10
35	7.222	123.8	7.0	122.44	36	7.033	120.0	7.0	119.83
35	8.024	128.2	8.0	128.06	36	8.027	125.4	8.0	125.25
35	9.211	135.2	9.0	133.96	36	9.044	131.5	9.0	131.24
37	5.011	113.5	5.0	113.43	38	5.003	115.5	5.0	115.48
37	6.027	120.1	6.0	119.93	38	6.005	122.9	6.0	122.87
37	6.994	125.8	7.0	125.83	38	6.997	129.0	7.0	129.02
37	8.011	131.4	8.0	131.34	38	8.014	136.0	8.0	135.91
37	9.063	136.8	9.0	136.48	38	8.995	142.2	9.0	142.23
39	5.090	106.9	5.0	106.42	40	5.099	105.8	5.0	105.42
39	6.005	111.8	6.0	111.77	40	6.003	109.3	6.0	109.29
39	6.984	116.9	7.0	116.99	40	6.984	114.2	7.0	114.27
39	7.981	123.2	8.0	123.30	40	8.011	117.8	8.0	117.76
39	9.047	127.7	9.0	127.50	40	9.036	122.6	9.0	122.43
41 41 41 41	5.211 6.003 7.003 8.039 8.995	101.9 108.8 116.0 122.1 127.8	5.0 6.0 7.0 8.0 9.0	100.06 108.78 115.98 121.87 127.83	42 42 42 42 42	5.038 5.964 6.994 8.013 8.970	109.3 114.9 120.7 125.9 130.7	5.0 6.0 7.0 8.0 9.0	109.07 115.11 120.73 125.83 130.85
43 43 43 43	5.030 6.016 6.997 8.016 9.104	101.2 108.0 113.4 118.8 125.1	5.0 6.0 7.0 8.0 9.0	100.99 107.90 113.42 118.71 124.50	44 44 44 44	5.013 6.005 7.002 8.022 8.975	113.2 118.0 123.0 127.4 133.4	5.0 6.0 7.0 8.0 9.0	113.14 117.98 122.99 127.28 133.56
45	5.032	105.1	5.0	104.92	46	5.014	112.8	5.0	112.71
45	5.978	110.3	6.0	110.41	46	5.970	118.9	6.0	119.09
45	7.002	114.7	7.0	114.69	46	6.959	125.3	7.0	125.53
45	8.035	121.7	8.0	121.51	46	7.998	130.3	8.0	130.31
45	9.076	125.6	9.0	125.32	46	8.970	135.9	9.0	136.07
47	5.115	107.1	5.0	106.41	48	5.265	102.9	5.0	101.17
47	5.929	112.0	6.0	112.42	48	6.367	110.1	6.0	108.10
47	6.984	118.2	7.0	118.29	48	7.096	112.9	7.0	112.47
47	8.030	123.3	8.0	123.15	48	8.145	118.9	8.0	118.15
47	9.074	128.5	9.0	128.13	48	9.126	123.3	9.0	122.73

Table 15.1 Continued

Tuble 15:1 Continued									
Serial number	Age	Height value	Target age	Adjusted value	Serial number	Age	Height value	Target age	Adjusted value
49 49	5.011 6.030	112.2 120.0	5.0 6.0	112.12 119.77	50 50	5.110 6.019	110.7 115.6	5.0 6.0	110.11 115.50
49	7.074	127.8	7.0	127.30	50	7.082	121.5	7.0	121.06
49	8.003	133.5	8.0	133.48	50	7.992	126.3	8.0	126.34
49	9.071	140.0	9.0	139.57	50	9.063	132.2	9.0	131.85
51	5.014	110.5	5.0	110.41	52	5.038	110.8	5.0	110.50
51	6.052	117.0	6.0	116.69	52	6.055	118.8	6.0	118.40
51	7.145	123.0	7.0	122.17	52	7.044	125.3	7.0	125.02
51	8.063	128.5	8.0	128.15	52	8.066	131.5	8.0	131.08
51	9.066	133.6	9.0	133.26	52	9.063	138.1	9.0	137.68
53	5.008	111.4	5.0	111.34	54	5.024	99.0	5.0	98.89
53	6.099	120.0	6.0	119.29	54	6.057	103.8	6.0	103.54
53	7.077	126.3	7.0	125.80	54	7.060	108.1	7.0	107.85
53	8.074	132.8	8.0	132.31	54	8.052	112.1	8.0	111.87
53	9.110	139.8	9.0	139.06	54	9.065	117.1	9.0	116.78
55	5.020	99.4	5.0	99.29	56	5.005	112.7	5.0	112.66
55	6.014	104.8	6.0	104.73	56	6.044	120.0	6.0	119.70
55	7.105	109.9	7.0	109.36	56	7.033	126.4	7.0	126.18
55	8.042	115.1	8.0	114.90	56	8.074	133.7	8.0	133.22
55	9.179	119.5	9.0	118.81	56	9.033	139.4	9.0	139.20
57	5.024	110.7	5.0	110.56	58	5.017	106.9	5.0	106.79
57	6.041	116.7	6.0	116.46	58	6.088	113.9	6.0	113.38
57	7.027	122.4	7.0	122.23	58	6.984	118.7	7.0	118.79
57	8.027	128.9	8.0	128.74	58	8.000	124.6	8.0	124.60
57	9.033	133.9	9.0	133.74	58	8.981	129.4	9.0	129.49
59	5.014	116.3	5.0	116.21	60	5.039	106.2	5.0	105.96
59	6.030	122.7	6.0	122.50	60	6.052	112.5	6.0	112.20
59	7.006	129.3	7.0	129.26	60	7.008	117.7	7.0	117.66
59	8.003	135.4	8.0	135.38	60	8.003	123.4	8.0	123.38
59	8.978	141.7	9.0	141.84	60	8.984	128.2	9.0	128.28
61	5.003	104.8	5.0	104.78	62	5.005	114.2	5.0	114.17
61	6.019	111.9	6.0	111.78	62	5.991	120.4	6.0	120.46
61	6.975	117.3	7.0	117.45	62	7.044	127.1	7.0	126.82
61	7.997	123.7	8.0	123.72	62	8.060	133.5	8.0	133.14
61	8.995	128.8	9.0	128.83	62	9.057	139.1	9.0	138.78
				_	-				

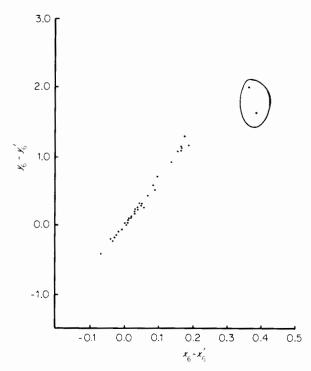


Figure 15.2 Height of 62 children at age 6. Scatter-plot of difference between observed and adjusted measurement  $(y_6 - y'_6)$  against difference between actual and target age  $(x_6 - x'_6)$ 

where

 $\sigma^2$  = residual mean square,  $w_i$  = regression weight =  $|x_i - x_i'|^K$ ,  $S_x^2 = \sum w_i(x_i - x_i')^2$ .

Note that the regression line is forced to go through the origin.

Figure 15.3 shows the standardized residuals plotted against the fitted values from a linear regression fitted to the data of Figure 15.2 with K = 1. It is clear that the variance of these increases with increasing  $|x_i - x_i'|$  with a single large negative residual at (2.3, -5.3)

Figure 15.4 is similar to Figure 15.3 but taking K = 2 and this seems to make a satisfactory allowance for the variance, no pattern in residuals being apparent and the outlier in Figure 15.3 no longer an extreme value. Similar residual plots have been studied for other ages and variables and the value K = 2 seems to be generally satisfactory and is used in the remainder of this chapter. Note that there are no obvious outliers in Figure 15.4.

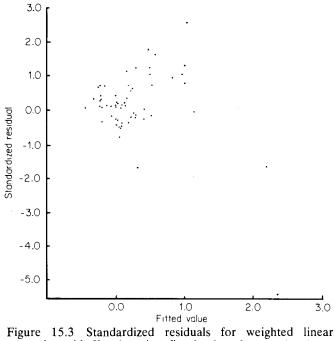


Figure 15.3 Standardized residuals for weighted linear regression with K=1 against fitted values for data in Figure 15.2. Equation of fitted line is y=6.037x

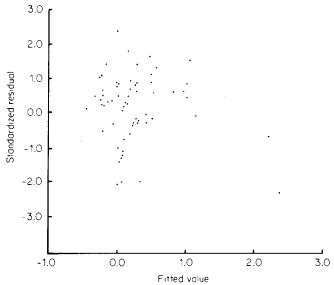


Figure 15.4 Standardized residuals from weighted linear regression with K = 2 plotted against fitted values for data in Figure 15.2. Equation of fitted line is y = 6.074x

### 15.4 OUTLIER DETECTION

In the remainder of the chapter we consider some of the outliers in the data and offer an interpretation of their significance. Figure 15.5 shows the residual plot for height measurements at age 9, with the most extreme residual being for child number 25. Figure 15.6 shows the growth measurement for this child from which it is clear that the adjusted measurement is in

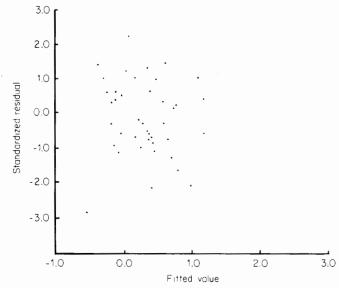


Figure 15.5 Residual plot (K = 2) at age 9 for height measurements

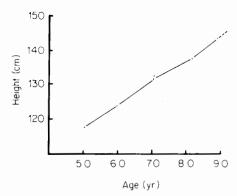


Figure 15.6 Plot of height measurements for child 25

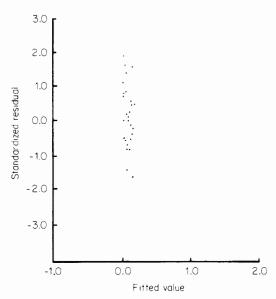


Figure 15.7 Residual plot (K = 2) at age 5 for height measurements. Subset of residuals only

fact an extrapolation outside the range of observed values and is also the 'earliest' measurement at this age. Nevertheless, the measurement itself is not very extreme and in practice we probably would not wish to exclude it from the data.

Figure 15.7 shows a plot of a subset of the standardized residuals at age 5 for height, with child number 9 as an outlier. In fact this child's growth rate was only 2.6 cm/yr, between 5 and 6 years, which is below the first percentile of the appropriate velocity standards (Tanner *et al.*, 1966), so that the adjustment for this child becomes too small in comparison to the other children. In this case, therefore, the procedure detects a measurement outlier rather than an inappropriate adjustment.

Figure 15.8, likewise, is detecting an outlier (child 25) resulting from extreme measurements of weight. This child has a growth velocity from 6–7 years below the first percentile and from 7–8 years above the 99th percentile, leading to an adjustment which is too large and suggesting that the 7-year measurement may be too low.

Finally, Figure 15.9 shows a subset of standardized residuals from skinfold measurement at age 7 with child 25 again having a large positive outlier. Figure 15.10 shows the actual skinfold measurements for child 25, illustrating as before that a small growth velocity followed by a large one is not well approximated by a quadratic curve. In this case, since the pattern is the same as with weight, we may be inclined to accept the measurements as accurate,

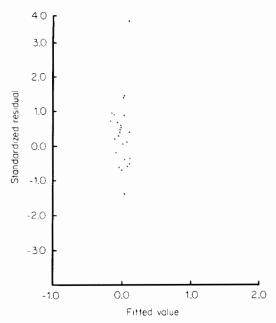


Figure 15.8 Residual plot (K = 2) at age 7 for weight measurements. Subset of residuals only

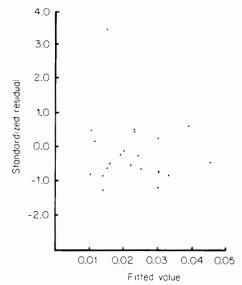


Figure 15.9 Residual plot (K = 2) at age 7 for triceps skinfold measurements. Subset of residuals only

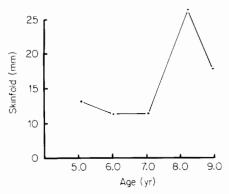


Figure 15.10 Plot of skinfold measurements for child 25

although the possibility of, for example, an incorrectly recorded age would need to be investigated. If the measurements are accepted, then we might prefer to use a linear adjustment using ages 6 and 7 (which here would cover the target age) rather than a quadratic.

#### 15.5 CONCLUSIONS

We have shown how residual plots of adjusted measurements can detect outliers resulting from a number of causes. The examples have been used to illustrate typical findings. As a rule, outliers for large values of  $x_i - x_i'$  indicate an inappropriate adjustment, either through too large a distance or with measurements too far apart for the adjusting curve to be used satisfactorily. Those outliers occurring with small values of  $x_i - x_i'$  tend to reflect data errors, but may also indicate that a different order polynomial should be used for adjustment.

The procedure described can be fully automated to detect outliers and produce relevant data plots so that editing decisions can be made. It could usefully be adopted routinely for the preliminary processing of longitudinal growth data.

#### REFERENCES

Goldstein, H. (1979) The Design and Analysis of Longitudinal Studies. London: Academic Press.

Tanner, J. M., Whitehouse, R. H., Marubini, E., and Resele, L. F. (1976) The adolescent growth spurt for boys and girls of the Harpenden growth study. *Ann. Hum. Biol.*, 3, 109–26.

Tanner, J. M., Whitehouse, R. H., and Takaishi, M. (1966) Standards from birth to maturity for height, weight, height velocity and weight velocity. British Children 1965, II. Arch. Dis. Child, 41, 613-35.