Introduction

- Cone-beam computed tomography
- Radiotherapy
- Statistical model
Computed tomography

Angle of projection

Attenuation

Distance along detector
Cone-beam CT
‘Conventional’ reconstruction

- Filtered back-projection
  - assumes linearity in physics
- Felkamp-Davis-Kress algorithm
  - approximate filtered back-projection for CBCT
Radiotherapy
Radiotherapy

- Prostate cancer most common cancer in UK men
- Bladder and rectum both radiosensitive
- Soft tissues in pelvis very mobile
Radiotherapy
Radiotherapy

- Hard to distinguish between tissues
  - gold markers implanted
  - markers easy to see on projections
  - but cause artefacts on reconstructions
Prostate fiducial markers
Statistical question

- Where is the patient’s prostate?
  - where are the markers?

- Sub-question:
  - what does the patient look like?
  - artefact removal
‘Obvious’ model

\[ \mathbf{X} = \text{parameterisation of marker in 3D} \]

\[ \mathbf{Y} = \text{projection data} \]

\[ \pi(\mathbf{x}|\mathbf{y}) \propto \pi(\mathbf{y}|\mathbf{x})\pi(\mathbf{x}) \]

\[ \mathbf{Y} \text{ is } 512 \times 512 \times 620 \sim 10^8 \]
More tractable model

\[ X = \text{marker in 2D projection} \]

\[ Y = \text{data for this projection} \]
Parameterisation of marker
Model - likelihood

\[ y_{(i,j)}|x \sim N(\mu_*, \sigma^2) \]

\[ \mu_* = \begin{cases} 
\mu_b \\
\mu_m = \mu_b + 1000 \\
\mu_h = \mu_b + 500 
\end{cases} \]

‘background’
‘marker’
‘half’
Model - prior

\[ \pi(\mathbf{x}) \propto \exp \left\{ \alpha A + \beta B + \gamma C \right\} \]

\[ A \quad = \quad \text{shortness} \]
\[ B \quad = \quad \text{wiggliness} \]
\[ C \quad = \quad \text{curviness} \]
Sampling from posterior

- $\pi(x|y) \propto \pi(y|x)\pi(x)$
- Summarising $\pi(x|y)$ still hard!
- Find candidate regions of interest
  - in a subset of projections
  - using morphological analysis
Sampling from posterior
Sampling from posterior

- Metropolis-Hastings MCMC
  - within each region of interest
- After burn-in, record:
  1. positions of marker ends after each iteration
  2. marker and posterior density
     (whether or not we move there)
Sampling from posterior

\[ \pi(\text{any marker in this ROI}|Y) \]

\[ = \sum_{\text{all markers}} \pi(\text{this marker in this ROI}|Y) \]

\[ \approx \sum_{\text{markers seen}} \pi(\text{this marker in this ROI}|Y) \]

- Pick ‘top five’ regions of interest
Finding the markers

- Recall $X$ is related to a marker at $(i, j, \theta)$
- Let $f(x|y)$ be the un-normalised density
- Parameterise markers in 3D by four ends
  - end $s$ at $(u_s, v_s, w_s) \in \mathbb{R}^3$
  - $x' = (u_1, \ldots, u_4, v_1, \ldots, v_4, w_1, \ldots, w_4)$
Finding the markers

- Define objective function:

\[
    r(x') = \sum_{i,j,\theta} \left[ f(x|y) \times (1 - \mathbb{I}_{[\text{end of marker predicted at } (i,j,\theta)]}) \right]
\]
Statistical question

- Where is the patient’s prostate?
  - where are the markers?

- Sub-question:
  - what does the patient look like?
  - artefact removal
What does the patient look like?

- Remove the markers from projections
- Reconstruct 3D volume
Further work

- Pragmatic approach
- Revisit the ‘obvious’ model
  - restricted to a small volume
  - model patient as 3D MRF
  - superimpose markers
Thanks!

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